THE JOURNAL IS INCLUDED IN THE LIST OF THE LEADING JOURNALS AND EDITIONS OF THE HIGHER ATTESTATION COMMISSION (VAK)

ISSN 1729-2646
e-ISSN 2500-3909

THE JOURNAL PROMOTER:
"Journal "Reliability" Ltd

It is registered in the Russian Ministry of Press, Broadcasting and Mass Communications. Registration certificate ПИ 77-9782, September, 11, 2001.

Official organ of the Russian Academy of Reliability

Publisher of the journal
LLC Journal "Dependability"
Director
Dubrovskaya A.Z.
The address: 109029, Moscow, Str. Nizhegorodskaya, 27, Building 1, office 209
Ltd Journal "Dependability"
www.dependability.ru
Printed by JSC "Regional printing house, Printing place" 432049, Ulyanovsk, Pushkarev str., 27. Circulation: 500 copies.
Printing order
Papers are reviewed. Signed print ,
Volume , Format 60x90/8, Paper gloss

Papers are reviewed.
Papers are published in author's edition. The opinion of members of the editorial board may not coincide with the point of view of authors' publications. The reprint of materials is granted only with the written permission of the editorial board. Manuscripts are not returned.

THE JOURNAL IS PUBLISHED WITH THE PARTICIPATION AND SUPPORT OF THE JOINT-STOCK COMPANY «RESEARCH AND DESIGN INSTITUTE OF INFORMATISATION, AUTOMATION AND COMMUNICATION ON RAILWAY TRANSPORT» (JSC «NIIAS») AND LLC PUBLISHING HOUSE «TECHNOLOGY»

CONTENTS

Structural dependability. Theory and practice

Ankudinov AV, Antonov AV, Chepurko VA. Renewal equation for Kijima-Sumita processes ... 3

Romanov IA, Streltsov AS. Functional mathematical definition of dependability indicators and establishment of the dependency between the integrated indicator and the unique indicators at the stage of manufacture and recovery of components that define the reliability of a machine ... 10

Functional dependability. Theory and practice

Cherkesov GN, Nedosekin AO, Vinogradov VV. Functional survivability analysis of structurally complex technical systems ... 17

Plotnikov NI. Theoretical modeling of dependability resources of flight crews in commercial aviation ... 25

Safety. Theory and practice

Kibzun AI, Ignatov AN. On the task of allocating investment to facilities preventing unauthorized movement of road vehicles across level crossings for various statistical criteria ... 31

Accounts

Belousova MV, Bulatov VV. Method of conversion of MTBF from cycles to kilometers travelled ... 38

Dayev ZhA, Nurushev ET. Application of statistical criteria for improving the efficiency of risk assessment methods ... 42

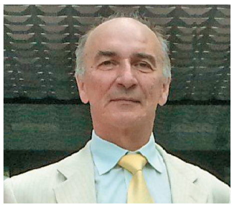
Voynov KN, Hildayati A, Ghellab Y. Ensuring reliable comprehensive protection of habitats from carcinogenic gases ... 46

Gnedenko Forum ... 50

Renewal equation for Kijima-Sumita processes

Alexander V. Ankudinov, *Obninsk Institute for Nuclear Power Engineering (IATE MEPhI), Obninsk, Russia*
Alexander V. Antonov, *JSC Rusatom Automated Control Systems, Moscow, Russia*
Valery A. Chepurko, *JSC Rusatom Automated Control Systems, Moscow, Russia*

Alexander V. Ankudinov



Alexander V. Antonov

Valery A. Chepurko

Abstract. **Aim.** *The article analyses the properties of Kijima-Sumita incomplete renewal models. These processes generalize standard renewal processes and heterogeneous Poisson processes. Being able to sufficiently simply model incomplete renewal, these models allow calculating near-real dependability indicators of technical systems. Complete, minimal renewal, "worse-than-before-the-failure" situation are modeled with the choice of the single parameter q that essentially characterizes the incompleteness of renewal. This paper is the continuation of [1], it conducts a research based on the assumption that the time to first failure has the Weibull distribution that is widely used in the dependability theory. The Kijima-Sumita incomplete renewal models appeared relatively recently ant their properties remain largely understudied. In [1], in particular, a numerical solution was obtained of the leading flow function (of renewal function) of the first Kijima process model represented as a series of functions. The aim of this paper is to derive an integral renewal equation that would associate the failure flow parameter (or renewal function) with the first time to failure distribution. Additionally, some analytical solutions are given for specific cases, a numerical solution of the resulting renewal equations is suggested. The paper analyzes the effect of the incomplete renewal coefficient on the characteristics of the Kijima model's failure flow. An interesting property of Kijima processes with a decreasing rate function of first operation time is discovered. Despite the expectations, the growth of the incompleteness of renewal in this case causes the reduction of the failure rate.* **Methods.** *The calculations were performed in the R language and various numerical methods of finding integrals and solving integral equations, including the non-uniform mesh trapeze method and grand total method (GTM).* **Conclusions.** *The paper deduces the renewal equation for the Kijima incomplete renewal processes. It also identifies some analytical solutions that demonstrate that the traditional renewal process and heterogeneous Poisson process are particular cases of the Kijima process. The results of numerical solutions for the Weibull distribution of first operation time are provided.*

Keywords: *renewal process, failure flow parameter, rate function, virtual age, renewal incompleteness coefficient, complete renewal, minimal renewal.*

For citation: *Ankudinov AV, Antonov AV, Chepurko VA. Renewal equation for Kijima-Sumita processes. Dependability 2018;2: 3-9. DOI: 10.21683/1729-2646-2018-18-2-3-9*

Introduction

Let us introduce some essential names and definitions. Let the renewable technical system start operating at the moment of time $t = 0$. $\Delta_1, \Delta_2, \ldots$ is the the series of times between failures (repairs) of such system (Figure 1). Accordingly, $\tau_k = \sum_{i=1}^{k} \Delta_i$ are the moments of failure that form a certain renewal process $\tau_1, \tau_2, \ldots$. Additionally, we will assume that a failed system is repaired immediately. If we talk about a real technical system, it ages naturally, which is usually expressed in a gradual increase of the failure rate or reduction of the intervals between consecutive repairs in the stochastic sense. Beside the chronological age of the technical system t, the "virtual" age could be taken into consideration [1-4] that describes the state of such system at the moment of time t. A more appropriate term would be "real" age, as in this case the focus is on the actual state of the technical system. Clearly, this age must depend on the prior operation times $\Delta_1, \Delta_2, \ldots$ and the quality of the prior repair.

In the classic renewal theory it is assumed that after repairs the failed technical system returns to the original state, "as new". However, even after an overhaul, replacement of a number of elements with new ones, a system as a whole can hardly be considered totally new. It is just an idealized assumption that helps simplify mathematical calculations.

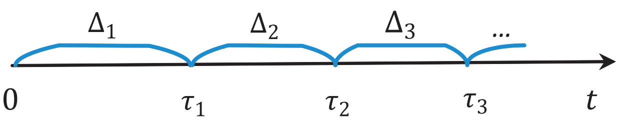


Figure 1. Renewal process

Let us consider the concept of virtual age. Let the first Kijima-Sumita model, it is directly proportional to the chronological age with the proportionality factor q. Per the second model, the correlation is somewhat more complex, but in both cases, if the prior repair is "perfect" [2-3], in other words the renewal is complete and $q = 0$, the virtual age will be equal to zero. If the renewal is minimal, i.e. the repair eliminated the cause of failure, but the system returned in the state condition-wise equivalent to "as before the failure", the virtual age will match the chronological age.

The term "virtual age" was first proposed in [3]. The authors proposed a mathematical model that allows, by choosing the non-negative parameter q, obtaining both homogeneous and heterogeneous renewal process, in which this parameter defines the noncompleteness of renewal (Figure 2). Thus, if $q = 0$ the renewal will be complete, i.e. after the repair the system will be as new. If $q = 1$, the system will return in the state preceding the failure, which corresponds to minimal renewal. In this case a heterogeneous Poisson process is obtained [4]. The intermediate values of q within the interval between 0 and 1 characterize incomplete and non-minimal system renewal, better than before the failure, but worse that at the initial moment. If $q > 1$, the system renews, but its state is worse than the state preceding the repair. A situation like that is possible in case of poor quality or unqualified repair.

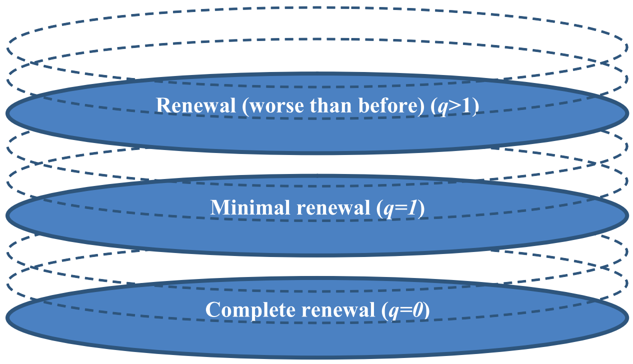


Figure 2. Capabilities of the Kijima model

Let us give a strict definition of heterogeneous renewal flow in accordance with the Kijima-Sumita models.

Definition. *A heterogeneous flow of Kijima-Sumita model is a flow formed by operation times* Δ_n *with the following conditional distribution functions:*

$$F_{\Delta_n}\left(x \mid V_{n-1} = y\right) = \frac{F(x+y) - F(y)}{1 - F(y)}; \qquad (1)$$

where V_n is the system's virtual age. The distribution function of the first operation time $F_{\Delta_1}(x) = F(x)$.

As stated above, the authors of [3] considered two models of the general renewal process. For the first model we will use the designation GRP-1. Mathematically, this model establishes the direct proportion between the virtual age at the n-th moment of renewal of V_n and the chronological age $\tau_n = \sum_{i=1}^{n} \Delta_i$:

$$V_n = V_{n-1} + q\Delta_n = q\sum_{1}^{n} \Delta_i, \;\; V_0 = 0 - \text{(General Renewal Process: GRP-1)}.$$

The second model involves direct proportion between the virtual age at the n-th moment of renewal of V_n and virtual age at the n-1-th moment of renewal of V_{n-1} and the last operation time Δ_n. Let us denote the process related to this model as GRP-2.

$$V_n = q\left(V_{n-1} + \Delta_n\right) = \sum_{1}^{n} q^{n+1-i} \Delta_i, \;\; V_0 = 0 - \text{(GRP-2)}.$$

The GRP-1 model is better suited for researching the failure flow parameter (FFP) and its properties. Let us quote some results that primarily cover the GRP-1 model and in some cases generalize to GRP-2.

For the GRP-1 processes renewal, equations can be deduced that in general may not have a solution. Let us find the recurrence relations that associate the distributions of two consecutive times to failure. The distribution density of the n-th moment of the flow is defined by formula

$$f_{\tau_n}(x)=\frac{d}{dx}\int_0^x\int_0^{x-x_1} f_{\tau_{n-1}}(x_1)f_{\Delta_n}(x_2\,|\,\tau_{n-1}=x_1)dx_2dx_1 =$$

$$\frac{d}{dx}\int_0^x\int_0^{x-x_1} f_{\tau_{n-1}}(x_1)f_{\Delta_n}(x_2\,|\,V_{n-1}=qx_1)dx_2dx_1.$$

By taking a derivative we deduce:

$$f_{\tau_n}(x)=\int_0^x f_{\tau_{n-1}}(u)K_f(x-u,qu)du, \tag{2}$$

where $K_f(a,b)=\dfrac{f(a+b)}{1-F(b)}$ is the kernel of the integral operator.

Now, let us examine the FFP: $\omega(x)=\sum_1^\infty f_{\tau_n}(x).$ (3)

As when deducing the ordinary renewal equation, let us separate the first summand from the remaining sum and apply recurrent equation (2) to each summand of that sum. We will obtain the following assertion.

Theorem. *In case of convergence of functional series (3), the FFP for model GRP-1 will satisfy the following integral (renewal) equation*

$$\omega(x)=f(x)+\int_0^x\omega(u)K_f(x-u,qu)du, \tag{4}$$

where $K_f(a,b)$ is defined in (2).

Let us examine the FFP. Let q=0. This situation ensures complete renewal of the failed system. In this case we obtain

$f_{\tau_n}(x)=\int_0^x f_{\tau_{n-1}}(u)f(x-u)du$ – an ordinary convolution integral.

As $K_f(x-u,0)=f(x-u)$, then equation (4) becomes an ordinary renewal equation.

Now let us consider the minimal renewal. Let q=1. $K_f(x-u,u)=\dfrac{f(x)}{1-F(u)}$. Let us analyze the FFP functional series. The recurrence equation (2) will be as follows:

$$f_{\tau_n}(x)=f(x)\int_0^x\frac{f_{\tau_{n-1}}(u)}{1-F(u)}du.$$

If n=2, we deduce

$$f_{\tau_2}(x)=f(x)\int_0^x\frac{f(u)}{1-F(u)}du=-f(x)\ln(1-F(x)).$$

If n=3,

$$f_{\tau_3}(x)=f(x)\int_0^x\frac{f_{\tau_2}(u)}{1-F(u)}du=f(x)\frac{\ln^2(1-F(x))}{2}.$$

By induction, if q=1 the distribution density of the k-th moment of the flow will be defined by the formula: $f_{\tau_k}(x)=f(x)\dfrac{(-1)^{k-1}\ln^{k-1}(1-F(x))}{\Gamma(k)}$. By substituting this into series (3) we obtain:

$$\omega(x)=f(x)\sum_{n=1}^\infty\frac{(-1)^{n-1}\ln^{n-1}(1-F(x))}{(n-1)!}=\frac{f(x)}{1-F(x)}. \tag{5}$$

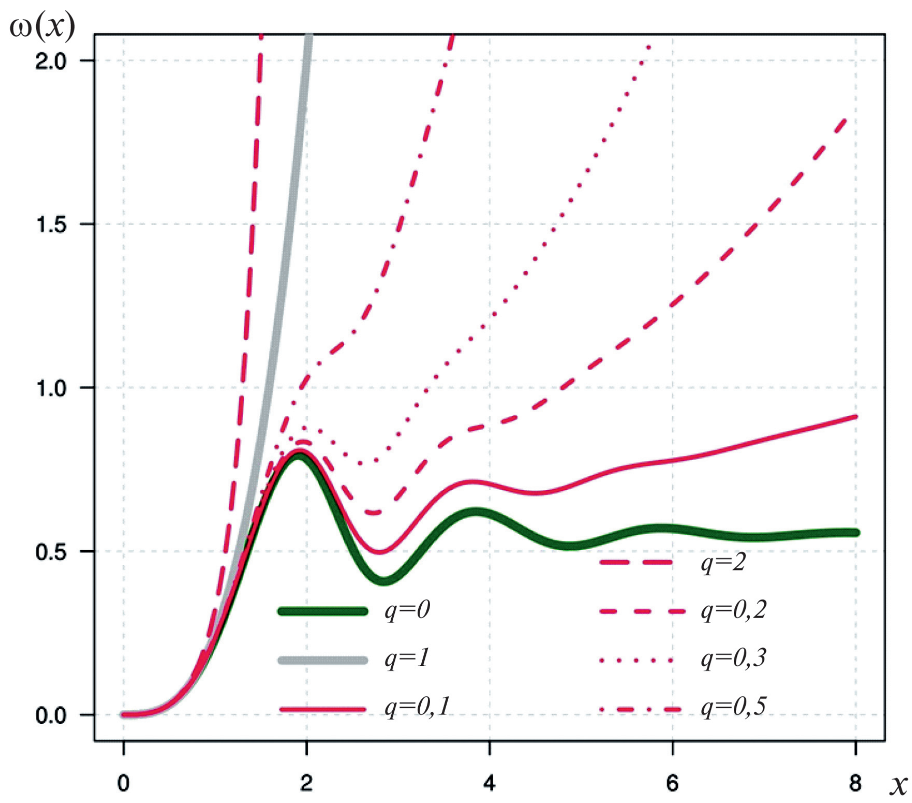


Figure 3. FFP for renewal processes of model GRP-1. Increasing rate function.

If q=1, the virtual age defined by both Kijima-Sumita models is identical and equal to the real one. $V_n = V_{n-1} + \Delta_n = \sum_1^n \Delta_i$. Therefore, the conclusions obtained under q=1 for GRP-1 are true for GRP-2 as well. We obtain an interesting property first deduced in [1, 2].

Property. *If q=1, in Kijima-Sumita models the FFP matches the rate function.*

Equation (4) under a specified value of parameter q is a Volterra integral equation of the second kind,that are easily solved by numerical techniques, e.g. grand total method (GTM) [7]. The calculations were performed in the R free software development environment [8-10]. Let us consider the graphs of the resulting numerical solutions. The calculations were performed on the assumption of Weibull distribution of operation time, the density and rate function of which are respectively $f(x) = \frac{a}{b}\left(\frac{x}{b}\right)^{a-1} e^{-\left(\frac{x}{b}\right)^a}$, $\lambda(x) = \left(\frac{x}{b}\right)^a$.

As it is known, parameter a defines the formula for the rate function. If $a > 1$, the rate $\lambda(x)$ increases, if $a = 1$, it is constant, if $a \in (0,1)$, it decreases.

In Figure 3, we can note a logical result: if parameter q increases, FFP (failure flow rate) gradually increases, i.e. the higher is parameter q (the worse is the technical system renewal), the higher is the failure flow rate. Figure 4 shows the opposite trend: as the indicator q of the incompleteness of renewal increases, the FFP goes down. This strange result will be explained below. The important point is that asymptotically FFP in case q>0 is approximated by nonlinear function.

For the first Kijima-Sumita model GRP-1 it is obvious that the trajectories of the FFP if $q \in (0,1)$ will be between the respective trajectories if q=0 and q=1 (see Figures 3 and 4). For the FFP shown in Figure 3 the parameter values were a = 4, b = 2, for the FFP in Figure 4 the parameter values were a = 0.8, b = 0.5. For a = 1, calculations have not been conducted, as in this case the FFP will not depend on q and all graphs match. That is due to the fact that if a = 1 the distribution of the first operation time is exponential, i.e. $f(x) = \frac{1}{b} e^{-x/b}$. If that is the case, the kernel of integral operator $K_f(x-u, qu) = f(x-u)$ and we obtain an ordinary renewal equation that yields constant $\omega(x) = \frac{1}{b}$ as the solution for the FFP.

Let us dwell upon the numerical GTM solution of the renewal equation in the case when the shape variable $a \in (0,1)$. In an explicit form, numerical solution (4) yields an incorrect result. That is due to the fact that around point 0 the constant term of the equation goes to infinity.

In order to obtain an adequate numerical solutions, FFP takes the form

$$\omega(x) = f(x) + \omega^*(x), \tag{5}$$

where $\omega^*(x) = \sum_2^\infty f_{\tau_n}(x)$ is the remainder term of series (3).

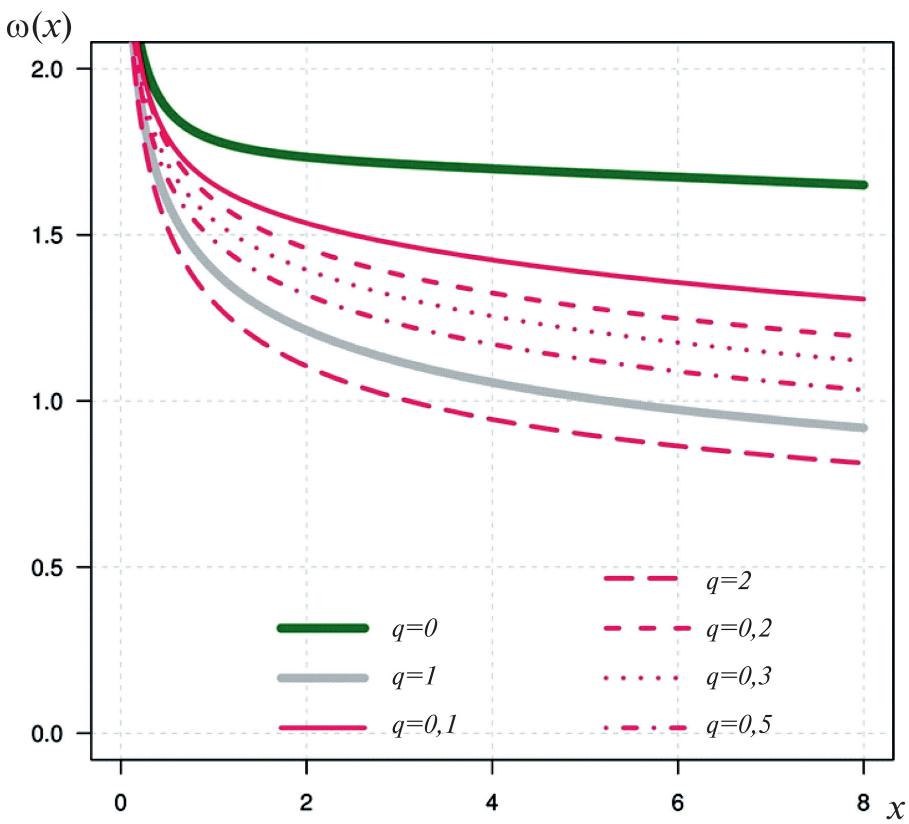


Figure 4. FFP for renewal processes of model GRP-1. Decreasing rate function

The remainder term, obviously, must satisfy the following integral renewal equation

$$\omega^*(x) = f_{\tau_2}(x) + \int_0^x \omega^*(u) K_f(x-u, qu)du, \qquad (6)$$

where $f_{\tau_2}(x)$ is the distribution density of the second failure moment of the Kijima process. In order to identify it let us use (2)

$$f_{\tau_2}(x) = \int_0^x f(u) K_f(x-u, qu)du = \int_0^x K_f(x-u, qu)\, dF(u) =$$

$$\int_0^{F(x)} K_f\left(x - F^{-1}(v), qF^{-1}(v)\right)dv, \qquad (7)$$

where $F^{-1}(x) = b\left(-\ln(1-x)\right)^{1/a}$ is the function opposite to the Weibull distribution function. Integral (7) was calculated using the trapezoidal method. Then, by solving through GTM equation (6) and its substitution into (5) the final solution was obtained.

The renewal equation can be deduced for the renewal function (RF). For the distribution function, simple integration can produce a recurrence equation equivalent to (2)

$$F_{\tau_n}(x) = \int_0^x \int_0^t f_{\tau_{n-1}}(u) \frac{f\left(t-(1-q)u\right)}{1-F(qu)} dudt =$$

$$= \int_0^x \frac{F\left(x-(1-q)u\right) - F(qu)}{1-F(qu)} dF_{\tau_{n-1}}(u). \qquad (8)$$

Integration by parts produces the property

$$F_{\tau_n}(x) = \int_0^x F_{\tau_{n-1}}(u) K_F(x-u, qu, q)du, \qquad (9)$$

where the kernel of the integral operator is a function of three variables

$$K_F(a,b,q) = \frac{(1-q) f(a+b)}{1-F(b)} + \frac{qf(b)\left(1-F(a+b)\right)}{\left(1-F(b)\right)^2}. \qquad (10)$$

As with the deduction of renewal equations for ordinary renewal processes we obtain the following assertion.

Theorem. *In case of convergence of functional series* $\Lambda(x) = \sum_1^\infty F_{\tau_n}(x)$ *the RF of* $\Lambda(x)$ *for model GRP-1 will satisfy the following integral equation.*

$$\Lambda(x) = F(x) + \int_0^x \Lambda(u) K_F(x-u, qu, q)du. \qquad (11)$$

Let q=0. This situation corresponds to complete renewal of the failed system.

$F_{\tau_n}(x) = \int_0^x F_{\tau_{n-1}}(u) f(x-u)du$ – a convolution integral.

As $K_F(x-u,0,0) = f(x-u)$, equation (4) becomes an ordinary renewal equation.

Figures 5 and 6 show the RF graphs. In Figure 5 the parameter values were a = 4, b = 2, in Figure 6 the parameter values were a = 0.8, b = 0.5. It can be noted that in the first case when parameter q increases, the RF (average number

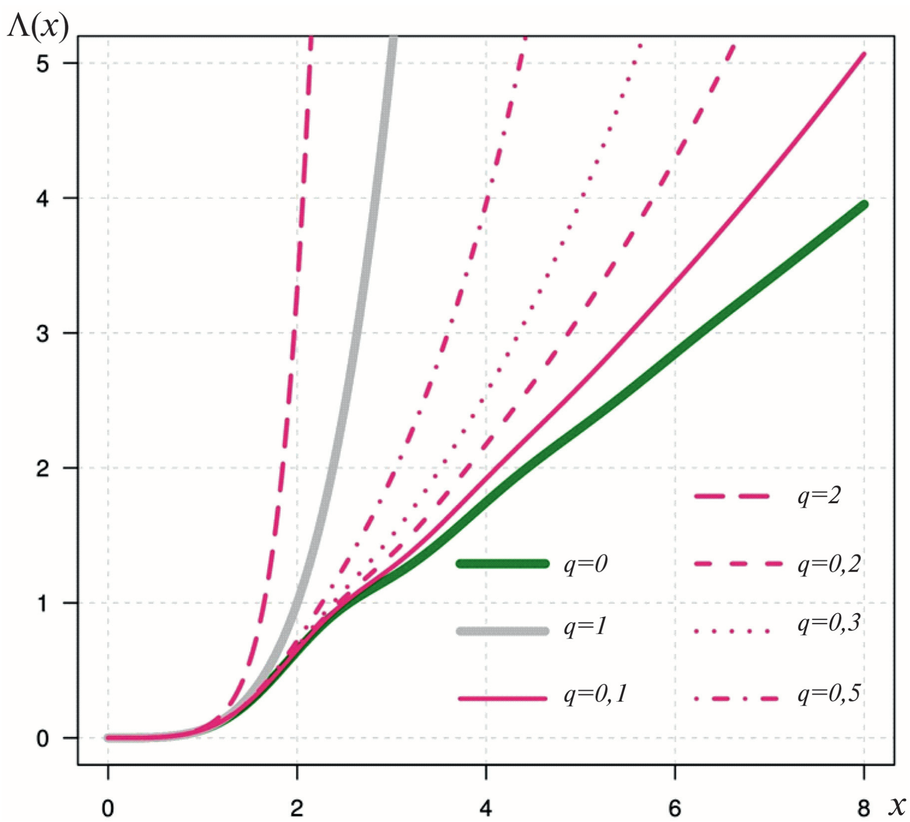


Figure 5. RF for renewal processes of model GRP-1. Increasing rate function

of failures) increases by degrees. The worse is the technical system renewal, the higher is the rate of failure flow. Figure 6 shows the opposite trend: as the indicator q increases, the incompleteness of RF renewal goes down. In order to explain this paradoxical result, let us analyze the integral operator of formula (8).

First, let us proceed to the probability of no failure, $P(x)=1-F(x)$:

$$\frac{F\left(x-(1-q)u\right)-F\left(qu\right)}{1-F(qu)}=1-\frac{P\left(x-(1-q)u\right)}{P\left(qu\right)}=$$
$$=1-\frac{P\left(\alpha+qu\right)}{P\left(qu\right)}=1-\frac{P\left(\alpha+t\right)}{P\left(t\right)},$$

where $a\in[0,x]$, $t=qu$. Let us denote $\varphi\left(t\right)=1-\dfrac{P\left(\alpha+t\right)}{P\left(t\right)}$.

Then, in case of Weibull distribution of the first operation time, the equation is true

$$\ln\left(1-\varphi\left(t\right)\right)=\ln P\left(\alpha+t\right)-\ln P\left(t\right)=\left(\frac{t}{b}\right)^{a}-\left(\frac{t+\alpha}{b}\right)^{a}. \quad (12)$$

The function on the right side of (12) increases in variable t if $a > 1$, is constant (does not depend on t) if $a = 1$, and finally decreases in variable t if $a\in(0,1)$.

Thus, if $a > 1$, function $\varphi(t)$ will decrease as variable t increases, therefore, the integral operator kernel $\dfrac{F\left(x-(1-q)u\right)-F\left(qu\right)}{1-F(qu)}$ will decrease as the incompleteness of renewal parameter q increases. Due to that the distributions of the second, third, etc. moments of the Kijima flow will shift to the left. I.e. if $a > 1$, for each $n = 2, 3,\ldots$

$$F_{\tau_n}\left(x\,|\,q_1\right)>F_{\tau_n}\left(x\,|\,q_2\right) \text{ if } q_1>q_2. \quad (13)$$

Similarly, if $a\in\left(0,1\right)$, for each n = 2, 3,…

$$F_{\tau_n}\left(x\,|\,q_1\right)<F_{\tau_n}\left(x\,|\,q_2\right) \text{ if } q_1>q_2. \quad (14)$$

I.e. if the rate function decreases, the distributions of the Kijima flow moments will shift to the right. Therefore, both RF and FFP will decrease. That explained the previously obtained paradoxical result. Under certain conditions, decreasing quality of renewal in the Kijima model causes the reduction of the average number of failures and failure flow rate. The conditions are solely associated with the distribution of the first operation time. In this particular case the distribution had a decreasing rate function.

Conclusion

The paper analyses some properties of the Kijima renewal processes. The undeniable benefit of such processes in the dependability theory consists in the capability to take into consideration incomplete renewal of technical systems. The process model enables the modeling of conventional, completely recoverable technical systems, thus generalizing them. The paper introduces an equation of renewal for the failure flow parameter and renewal function of Kijima renewal processes. Some specific analytical solutions were obtained, the results of numerical calculations are given. It

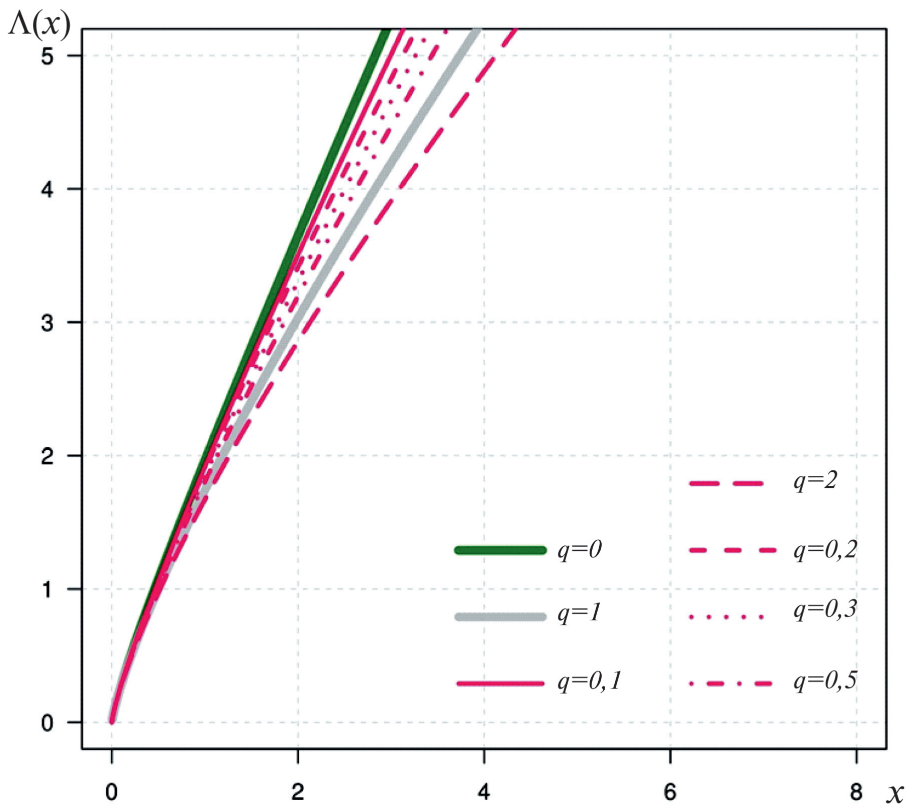


Figure 6. RF for renewal processes of model GRP-1. Decreasing rate function

also analyzes the effect of incomplete renewal on the flow rate and renewal function.

References

[1]. Chumakov IA, Antonov AV, Chepurko VA.On some properties of Kijima incomplete renewal models. Dependability 2015;3(54):10-15.

[2]. Chumakov IA. Metody rascheta resursnykh kharakteristik oborudovaniya slozhnykh sistem s uchetom rezhimov obsluzhivaniya i nepolnogo vosstanovleniya: dissertatsiya kandidata tekhnicheskikh nauk [Methods of calculation of life characteristics of complex system equipment subject to maintenance modes and incomplete renewal: Candidate of Engineering dissertation]. Obninsk, Russia; 2014 [in Russian].

[3]. Kijima M, Sumita N. A useful generalization of renewaltheory: Counting process governed by non-negative markovianincrements. Journal of Applied Probability 1986;23:71-88.

[4]. Finkelstein M. Failure rate modelling for reliability and risk. Verlag London Limited: Springer; 2008.

[5]. Antonov AV, Nikulin MS, Nikulin AM, Chepurko VA. Teoria nadiozhnosti. Statisticheskie modeli. Ouchebnoie posobie [Dependability theory. Statistical models. A study guide]. Moscow: NITS INFRA-M; 2015 [in Russian]. + Additional Materials,<http//www.znanium.com>.

[6]. Chepurko SV, Chepurko VA. Modeli neodnorodnykh potokov v teorii vosstanovlenia. Monografia [Non-uniform streams models in the renewal theory. A monograph]. Obninsk: IATE; 2012.

[7]. Kopchionova NV, Maron IA. Vychislitel'naya matematika v primerakh i zadachakh [Computational mathematics in examples and problems]. Moscow: Nauka; 1972 [in Russian].

[8]. R Core Team. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria,<http://www.R-project.org/>.

[9]. Crawley MJ. The R Book. John Wiley & Sons, Ltd. Chichester; 2007.

[10]. Kabakov RI. R v deystvii. Analiz i vizualizatsiyadannykh v programme R [R in action. Analysis and visualization of data in R]. Moscow: DMK Press; 2014 [in Russian].

About the authors

Alexander V. Ankudinov, post-graduate student, Intelligent Cybernetic Systems Division, Obninsk Institute for Nuclear Power Engineering (IATE MEPhI),Obninsk, Russia, e-mail: anck93@yandex.ru

Alexander V. Antonov, Doctor of Engineering, Professor, chief expert of the Division for Justifying Calculations of Design Solutions, JSC Rusatom Automated Control Systems, Moscow, Russia, e-mail: anck93@yandex.ru

Valery A. Chepurko, Candidate of Physics and Mathematics, Associate Professor, Chief Specialist of Division for Justifying Calculations of Design Solutions, JSC Rusatom Automated Control Systems, Moscow, Russia, e-mail:v.a.chepurko@mail.ru, VAChepurko@rasu.ru

Received on 15.11.2017

Dependability, vol. 18 no.2, 2018 Original article
DOI: 10.21683/1729-2646-2018-18-2-10-16

Functional mathematical definition of dependability indicators and establishment of the dependency between the integrated indicator and the unique indicators at the stage of manufacture and recovery of components that define the reliability of a machine

Igor O. Romanov, *NAKS-Khabarovsk Evaluation Center, Khabarovsk, Russia*
Anatoly S. Streltsov, *NAKS-Khabarovsk Evaluation Center, Khabarovsk, Russia*

Igor O. Romanov

Anatoly S. Streltsov

Abstract. *As of today, the dependability of machines in the process of their operation and repair is considered as a whole, while the norms and specifications of GOST 27.002-2015, reference and scientific/technical literature do not provide any strict mathematical definitions of dependability indicators of primary elements/components. At the same time, the parametric dependability of a machine is based on the paradigm of evaluation of dependability indicators over the whole operation period from the stage of manufacture to end of service life subject to possible repairs.* **Aim.** *Given the above, the aim of this paper is to evaluate the effect of unique dependability indicators on the integrated indicator I_D (efficiency retention coefficient) at the stage of manufacture and recovery of machine components.* **Methods.** *The paper is based on the mathematical device for the identification of linear dependency between the integrated indicator and unique indicators that involves the identification of the integrated indicator when the unique indicator under consideration changes its value from the basic (first) level to the high (forth) level, while all the other unique indicators remain at the basic level, which rules out the correspondence between the unique indicator under consideration and the other indicators in the process of integrated indicator calculation.* **Results.** *Calculations show that the optimal option for increasing I_D is to increase the unique indicators according to their priorities. Thus, coordinated increase at the stage of manufacture and recovery of only three unique indicators in some instances ensures 75 percent growth of the integrated indicator.* **Conclusions.** *It is suggested to classify machine components into three groups based on the value of reliability indicator of the initially installed machine component: ones that define the life until discarding (I_{R1}>1); ones that define machine service life (I_{R1}=1) and ones that define machine reliability (I_{R1}<1). The identification of the dependability indicators of the components in each group is based on the provisions of GOST 27.002-2015, but each group has its own unique features that must be taken into consideration in the functional mathematical definition of the dependability indicators of components in relation to the dependability of machines as a whole. For the components of the third group functional mathematical definitions were developed, dependencies and priorities between the unique indicators and increased integrated indicator were identified. Using a specific example, the economic feasibility of increasing the integrated indicator was calculated. It was established that the most promising solution would be a coordinated increase of the integrated indicator at the stages of manufacture and recovery that enables a more that a double reduction of costs, while ensuring 61 percent profitability.*

Keywords: *reliability, maintainability, storability, longevity, integrated dependability indicator, priorities, profitability.*

For citation: *Romanov IO, Streltsov AS. Functional mathematical definition of dependability indicators and establishment of the dependency between the integrated indicator and the unique indicators at the stage of manufacture and recovery of components that define the reliability of a machine. Dependability 2018;2: 10-16. DOI: 10.21683/1729-2646-2018-18-2-10-16*

Introduction

Despite the progress in engineering, due to insufficient dependability of the machines produced, consumers have to deal with significant repair and maintenance costs that over the entire period of operation many times exceed the cost of a new machine. Thus, for trucks, the cost exceeds 6 times, and for metal-working machines, the cost exceeds 8 times. At the same time, the major part of the costs is accounted for the current repair and maintenance of the failed components [1]. Formally, a machine is a system with functionally connected elements (components), the dependability of which makes and defines the system dependability as a whole. Therefore, identifying the dependability indicators of the components relative to the machine dependability and the estimation of their contribution to this dependability will enable a differentiated approach to the calculation of unique dependability indicators in the design process and their assurance at the manufacturing and recovery stages, that is a relevant task both theoretically and practically.

Unfortunately, modern reference and scientific and technical literature in the field of machine dependability [2, 3, 4, 5, 6] does not provide any specific interrelated solutions to the task.

Table 1. Unique dependability indicators determined the machine reliability

Indicators	Functional definitions	Mathematical definitions
1	2	3
Component dependability		
1.1 Component dependability of the initially installed component, I_{R1}	Part of the machine life, which is realized by the component of the first installation	$I_{R1} = \frac{\overline{L_1}}{\overline{L_i}}$,
1.2 Reliability retention coefficient of the first replacement component, γ	This coefficient indicates how much the reliability indicator of the first replacement component is changed, I_{RO1}	$\gamma = \frac{\overline{L_{O1}}}{\overline{L_1}} = \frac{I_{RO1}}{I_{R1}}$ where $\overline{L_{O1}}$ is the time to failure of the first replacement component
1.3 Reliability indicator of the i-the replacement component, I_{Ri}	Part of the machine life, which is realized by the component of the i-th replacement	$I_{Ri} = I_{R1} \cdot \gamma^{i-1}$
1.4 Number of replaced components in operation, N_O	Number of components replaced in operation for the machine life realization	$N_O = \frac{\ln(I_{R1} + \gamma - 1) - \ln(I_{R1}\gamma)}{\ln\gamma}$
1.5 Average reliability indicator of replaced components, $\overline{I_{RO}}$	Machine resource part, which is realized on the average by the replaced components	$\overline{I_{RO}} = \frac{(1 - I_{R1})}{N_O}$
Component maintainability		
2.1 Reliability recovery coefficient, I_{Rec}	This coefficient indicates how much the reliability indicator of recovered component is changed	$I_{Rec} = \frac{\overline{L_{Rec1}}}{\overline{L_{O1}}} = \frac{I_{RRec1}}{I_{RO1}}$ where $\overline{L_{Rec1}}$ is the time to failure of the first recovery component, I_{RRec1} is the reliability indicator of the first recovery component
2.2 Reliability indicator of the i-th recovery, I_{RReci}	Part of machine life, which is realized by the component of the i-th recovery	$I_{RReci} = I_{Rec} \cdot I_{R1} \cdot \gamma^{i-1}$
2.3 Component maintainability indicator, I_m	Part of machine life, which is realiazed by the component through N-fold recovery	$I_m = \frac{I_{Rec} \cdot I_{R1} \cdot \gamma \cdot (1 - \gamma^{N_{Rec}})}{(1 - I_{R1}) \cdot (1 - \gamma)}$ where N_{Rec} is the number of technologically possible recoveries
2.4 Estimated number of component recoveries, $N_{Rec.Est}$	Number of the component recoveries for the full machine life realization under given I_{Rec}	$N_{Rec,Est} = \frac{\ln\left[1 - \frac{(1 - I_{R1}) \cdot (1 - \gamma)}{I_{Rec} \cdot I_{R1} \cdot \gamma}\right]}{\ln\gamma}$
2.5 Estimated reliability recovery coefficient, $I_{Rec.Est}$	Reliability recovery coefficient for the machine life realization under given number of recoveries, N_{Rec}	$I_{Rec.Est} = \frac{(1 - I_{R1}) \cdot (1 - \gamma)}{I_{R1} \cdot \gamma \cdot (1 - \gamma^{N_{Rec}})}$

Table 1 continued

1	2	3
Component storability		
3.1 Probability of component storage in operation, $P_{P.O}$	Relative number of absence of operational failures from the total sample of components	$P_{P.O} = \frac{N_0 - N_O}{N_0}$ where N_O is the number of operational failures
3.2 Probability of the component preservation during storage and transportation, $P_{P.S.T}$	Relative number of stored components during storage and transportation from N_O	$P_{P.S.T} = \frac{N_0 - N_{S.T}}{N_0}$ where $N_{S.T}$ is the number of lost components
3.3 Probability of component preservation during recovery, $P_{P.MD}$	Relative number of components without manufacturing defect during component recovery	$P_{P.MD} = \frac{N_0 - N_{MD.R.}}{N_0}$ where $N_{MD.R}$ is the number of components with manufacturing defect
3.4 Probability of one-time component recovery, P_{Rec}	Relative number of components one-time recovered from N_O	$P_{Rec} = P_{P.O} \cdot P_{P.S.T} \cdot P_{P.MD}$
3.5 Indicator of component storability, I_{St}	Probability of component preservation with its N-fold recovery	$I_{St} = \frac{P_{Rec} \cdot (1 - P_{Rec}^{N_{Rec}})}{N_{rec} \cdot (1 - P_{Rec})}$
Component longevity		
4.1 Indicator of component longevity, I_L	Part of machine life realized by I_{R1}, I_m, I_{St}	$I_L = I_{R1} + I_m \cdot I_{St} \cdot (1 - I_{R1})$
4.2 Actual number of component recoveries, $N_{Rec.Act}$	Number of recoveries of the failed component	$N_{Rec.Act} = \frac{I_m \cdot I_{St} \cdot (1 - I_{R1})}{I_{Rec} \cdot \overline{I_{RO}}}$
4.3 Additional number of new components, $N_{N.Act}$	Number of new components replaced in operation when $(I_m \cdot I_{St}) < 1$ for the machine life realization	$N_{N.Act} = \frac{(1 - I_m \cdot I_{St}) \cdot (1 - I_{R1})}{\overline{I_{RO}}}$

Functional mathematical definition of unique dependability indicators of components

According to GOST 27.002-2015, the unique dependability indicator of a component is defined by the mean time to first failure that is eliminated by current machine repair by replacing the failed component.

Considering a component as an independent object, it is worth to take the following ratio for the reliability indicator in relation to the machine:

$$I_{R1} = \frac{\overline{L_1}}{\overline{L_i}}, \tag{1}$$

where I_{R1} is the reliability indicator of the initially installed component; $\overline{L_1}$ is the mean time to first failure of the initially installed component; $\overline{L_i}$ is the machine mean lifetime.

From the physical point of view, the reliability indicator determines which part of the machine's life is implemented by the initially installed component.

It is worth classifying machine components into three groups based on I_{R1}: ones that define the life until discarding (I_{R1}>1); ones that define machine service life (I_{R1}=1) and ones that define machine reliability (I_{R1}<1).

Undoubtedly, the identification of the dependability indicators of the components in each group is based on the provisions of GOST 27.002-2015, but each group has its own unique features that must be taken into consideration in the functional mathematical definition of the dependability indicators of components in relation to the dependability of machines as a whole.

For the components of the third group, functional mathematical definitions of the unique dependability indicators [7] were developed by the authors. The unique dependability indicators are given in Table 1.

Functional mathematical definition of integrated dependability indicator of component and its dependence on unique indicators

From the technical point of view, the longevity indicator, that includes all unique indicators, can be taken as the integrated indicator of reliability. However, the consumer is primarily concerned with in the economic aspect of dependability, i.e. the operation costs which the consumer will bear when replacing failed components during machine life realization. According to GOST 27.002-2015, such is the efficiency retention coefficient, which is defined by the ratio of the value of object use efficiency indicator over certain duration of operation to the nominal value of this indicator, calculated under condition that there are no object failures during the same period of time.

If the cost of a new part is taken as the nominal value of the efficiency indicator, while the operation costs for replacing the failed components when the machine realizes its life are taken as the value of the efficiency index, the integrated dependability indicator of the component is defined by the following ratio

$$I_D = \frac{C_D}{C_D + N_{D.L} \cdot (C_D + C_O) + N_{\mathrm{Rec}.Act} \cdot (C_{\mathrm{Rec}} + C_O)}, \quad (2)$$

where I_D is the integrated dependability indicator of the component; C_D is the cost of a new component; C_{Rec} is the cost of the recovered component; C_0 is the cost of the component replacement in operation and economic losses caused by machine downtime; N_D is the number of new components replaced in operation in addition to the recovered components; $N_{Rec.Act}$ is the actual number of component recoveries.

Let us divide the numerator and denominator of formula (2) into C_D and introduce the term of the relative cost, determined by the following ratios

$$\alpha = \frac{C_{\mathrm{Rec}}}{C_D}; \beta = \frac{C_O}{C_D}, \quad (3)$$

where α is the relative cost of the recovered component; β is the relative cost of the component replacement in operation.

Taking into account formula (3), sub-paragraph 4.2 and 4.3 of Table 1, after the corresponding transformations, formula (2) takes the following expression

$$I_D = \frac{1}{1 + \frac{(1 - I_{R1})}{\overline{I_{RO}}} \cdot (1+\beta) \left\{ (1 + I_m \cdot I_{St}) \cdot \left[\frac{(\alpha+\beta)}{I_{\mathrm{Rec}} \cdot (1+\beta)} - 1 \right] \right\}}. \quad (4)$$

From the economic point of view, the integrated dependability indicator determines what part of new component cost is from the total cost of operation to replace failed components.

The analysis of formula (4) shows that the product in front of the curly brackets is the relative costs in operation with no component recovery (I_m=0). The sum in the curly brackets defines the amount of change of relative costs for the component recovery, and the difference in the square brackets defines the ratio of relative costs in operation with one-time component recovery. With a positive value, the operation costs increase, with a negative value, the operation costs decrease. In this context the assessment of the feasibility of the component recovery is determined by the following inequality

$$I_{\mathrm{Rec}} > \frac{\alpha+\beta}{1+\beta}. \quad (5)$$

Identification of the linear dependency between the increasing integrated dependability indicator and the unique indicators, their prioritization (by weight) and economic efficiency assessment due to increasing integrated indicator

In order to identify the linear dependency between the integrated dependability indicator and the unique indicator, a mathematical method is used [8], that involves finding the integrated indicator using formula (4) when the unique indicator under study changes its value from the basic (first) level to the upper (fourth) level, and all the others unique indicators are at the basic levels, that eliminates the relationship between the unique indicator under study and the others when calculating the integrated indicator.

Table 2. The results of the linear dependency calculation between the integrated and unique indicators

Level	Dependability indicators of the component							
	Component manufacturing (I_m=0)							
	I_{R1}	$\overline{I_{RO}}$	I_D	γ	$\overline{I_{RO}}$	I_D	β	I_D
Basic 1	0.3	0.18	0.14	0.8	0.18	0.14	0.5	0.14
2	0.4	0.28	0.24	0.85	0.2	0.16	0.4	0.16
3	0.5	0.39	0.34	0.9	0.25	0.19	0.3	0.18
Upper 4	0.55	0.45	0.4	0.95	0.3	0.22	0.2	0.2
	Component recovery (I_{R1}=0.3. γ=0.8. β=0.5							
	I_m	I_D	I_{St}	I_D	α	I_D	I_{Rec}	I_D
Basic 1	0.5	0.16	0.8	0.16	0.5	0.16	1.0	0.16
2	0.6	0.17	0.85	0.165	0.4	0.17	1.4	0.2
3	0.8	0.19	0.9	0.17	0.3	0.18	1.8	0.24
Upper 4	1.0	0.22	0.95	0.18	0.2	0.2	2.2	0.29

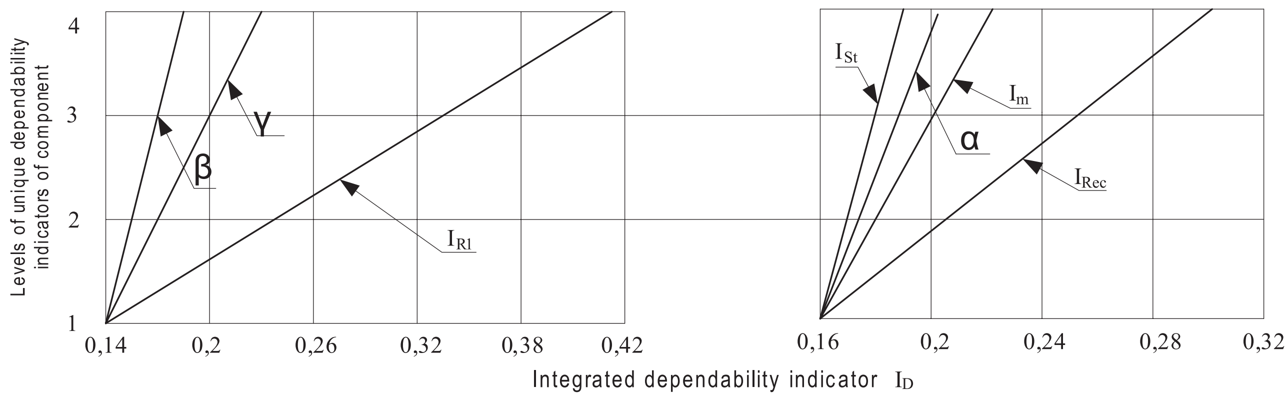


Figure 1. Graphical dependency between the integrated dependability indicator and unique indicators in the component manufacturing and recovery processes

Table 2 shows the results of the integrated dependability indicator calculation, and Figure 1 shows the graphical linear interpretation.

The weight of the unique indicator in the increasing integrated indicator is identified based on the results of the assessment of the linear dependency between the integrated indicator and the unique indicator changes and is estimated by the following ratio:

$$\eta_i = \frac{\Delta I_{Di}}{\sum_1^n \Delta I_{Di}}, \qquad (6)$$

where η_i is the weight of the i-th unique indicator in the increasing integrated indicator; ΔI_{Di} is the increase of the integrated indicator from the i-th unique indicator defined by the difference

$$\Delta I_{Di} = I_{Dimax} - I_{DiBasic}, \qquad (7)$$

where I_{Dimax} is the maximum value of the integrated indicator for the i-th unique indicator, when it is at the upper fourth level; $I_{DiBasic}$ is the value of the integrated indicator for the i-th unique indicator, when it is at the basic level.

From the physical point of view, the weight of the i-th unique indicator η_i shows what part of the increase of the integrated indicator is attributed to the i-th unique indicator out of all unique indicators.

The priority of the i-th unique indicator is determined by ranking of the weight of all unique indicators.

The weight of the unique indicators is calculated by formulas (6) and (7), and in accordance with priorities, is presented in Table 3, while Figure 2 shows the graphical interpretation.

The analysis of calculation results (Table 2 and Table 3) shows that the optimal option for increasing I_D is to change the unique indicators in accordance with their priorities. Thus, in case of simultaneous increase at the stages of manufacture and recovery, only three unique indicators ensure an increase of the integrated indicator by 75 %.

The unique dependability indicators of the component are calculated from the primary factors that are formed during design, manufacture, recovery and operation of machines, the implemented of which by means of technological, process-engineering, organizational and other methods allows such indicators achieving specified values of the priority unique indicators. This requires certain expenditures, which affects the cost of new and recovered components, as well as operation costs associated with the replacement of failed components. The economic viability of costs is determined both by the resulting economic effect and profitability that defines the payback period of these costs.

Table 3. The results of the calculation of the weight of unique dependability indicators in the increase of the integrated indicator and their prioritization

Manufacturing				Recovery				Combined			
Pr	I_{Di}	ΔI_{Di}	η_i	Pr	I_{Di}	ΔI_{Di}	η_i	Pr	I_{Di}	ΔI_{Di}	η_i
1	I_{R1}	0.26	0.65					1	I_{R1}	0.26	0.376
2	γ	0.09	0.22					2	I_{Rec}	0.15	0.217
3	β	0.05	0.13					3	γ	0.09	0.13
				1	I_{Rec}	0.15	0.52	4	I_m	0.06	0.086
				2	I_m	0.06	0.2	5	α	0.05	0.072
				3	α	0.05	0.17	6	β	0.05	0.072
				4	I_{St}	0.03	0.11	7	I_{St}	0.03	0.04
	$\sum \Delta I_{Di}$	0.4	1.0		$\sum \Delta I_{Di}$	0.29	1.0		$\sum \Delta I_{Di}$	0.69	1.0

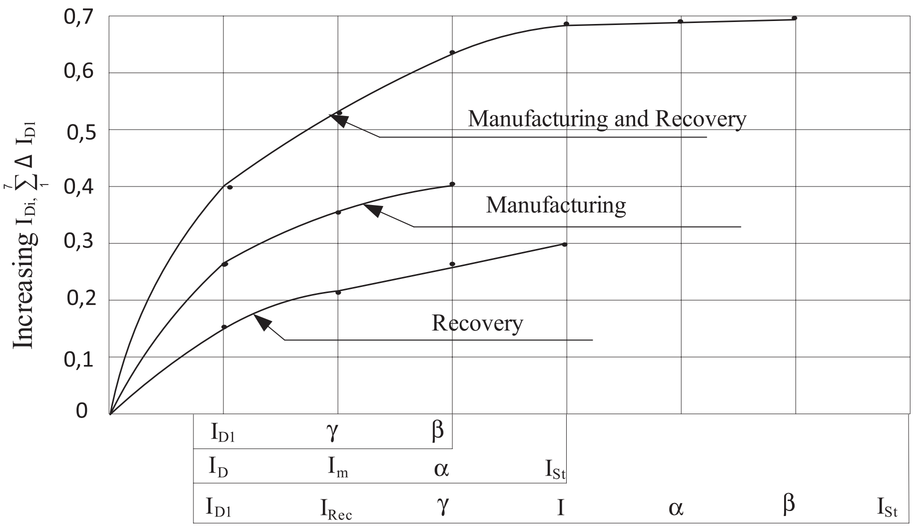


Figure 2. Graphical dependence between I_D increase and priorities of the unique dependability indicators of the component

Table 4. The results of the calculations of the dependability indicators and economic efficiency of increasing I_D from 0.17 to 0.41.

Dependability indicators of component								C_O	C_{EE}	R
α	β	γ	I_{R1}	I_m	I_{St}	I_{Rec}	I_D	Thousand rubles	Thousand rubles	%
Component manufacturing stage										
-	0.25	0.8	0.3	-	-	-	0.17	19.5	-	-
-	0.2	0.8	0.4	-	-	-	0.28	12.8	6.7	34
Component recovery stage										
0.5	0.25	0.8	0.3	0.5	0.85	1.0	0.2	16.8	2.7	14
0.65	0.25	0.8	0.3	1.0	0.9	1.5	0.26	14.2	5.3	27
Manufacturing and recovery										
0.65	0.2	0.8	0.4	1.0	0.9	1.5	0.41	8.4	11.1	61

The operation costs for replacing failed components during the realization of machine life are defined by the product

$$C_O = (\frac{1}{I_D} - 1)\cdot C_{NC}, \quad (8)$$

where C_O is the operation costs; I_D is the value of the integrated dependability indicator; $C_{DC}$is the cost of the new component (repair component).

The economic effect of increasing the integrated dependability indicator from I_{D1} to I_{D2} is defined by the difference

$$C_{EE} = C_{O1} - C_{O2}, \quad (9)$$

where C_{EE} is the economic effect of increasing the integrated dependability indicator.

The profitability of increasing the integrated dependability indicator is defined by the ratio

$$R = \frac{C_{EE}}{C_O}\cdot 100\%, \quad (10)$$

where R is the profitability of costs in %.

Let us consider, as an example, an assessment of the economic feasibility of increasing the integrated dependability indicator of a component. The initial values of the indicators are as follows: I_D = 0.17;C_{NC}=4 thousand rubles; C_{Rec} = 2 thousand rubles; C_O = 1 thousand rubles; I_{Rec} =1; I_{R1}=0.3; I_m = 0.5; I_{St}=0.85. The required values of the indicators are as follows: I_D = 0.4; C_{Rec} = 2.6 thousand rubles; C_{NC}= 5 thousand rubles; I_{R1}= 0.4; I_m= 1; I_{St}= 0.9; I_{Rec} = 1.5.

The results of the calculations are given in Table 4.

Conclusion

1. It is proposed to classify machine components into three groups based on the value of reliability indicator of the initially installed machine component: ones that define the life until discarding; ones that define machine service life; and ones that define machine reliability.

2. For the components of the third group functional mathematical definitions were developed, dependencies and priorities between the unique indicators and increased integrated indicator were identified.

3. Using a specific example, the economic feasibility of increasing the integrated indicator was calculated. It was established that the most promising solution would be a coordinated increase of the integrated indicator at the stages of manufacture and recovery that enables a more than a double reduction of costs, while ensuring 61 percent profitability.

References

[1]. Pronikov AS. Nadezhnost' mashin [Dependability of machines]. Moscow: Mashinostroenie, 1978 [in Russian].

[2]. Beliaev YuK et al. Ushakov IA, editor. Nadiozhnost tekhnicheskikh sistem: Spravochnik [Dependability of technical systems: Reference book]. Moscow: Radio i sviaz; 1985 [in Russian].

[3]. Polovko AM, Gurov SM. Osnovy nadiozhnosty [Fundamentals of dependability]. BHV-Petersburg; 2006 [in Russian].

[4]. Pronikov AS. Parametricheskaya nadezhnost mashin [Parametric dependability of machines]. Moscow: Bauman MSTU Publishing; 2002 [in Russian].

[5]. Antsupov AV, Antsupov AV (Jr.), Antsupov VP. Methodology of machine elements' reliability prediction by means of various criteria. Dependability 2013;3(46):15-23.

[6]. Berdichevsky BE, editor. Nadiozhnost mashin [Dependability of machines]. Moscow: MIR; 1970 [in Russian].

[7]. Streltsov AS, Romanov IO. Sistema otsenok pokazateley nadezhnosti mashin i diagnostiki ekonomicheskoy tselesoobraznosti vosstanovleniya i uprochneniya detaley mashiny [System for evaluation of machine dependability indicators and diagnostics of economic feasibility of machine parts recovery and strengthening]. Svarka i diagnostika 2016;5(59):30-35 [in Russian].

[8]. Johnson N, Leone F. Statistics and experimental design in engineering and physical sciences. Moscow: Mir; 1980.

About the authors

Igor O. Romanov, Candidate of Engineering, Lead Welding Engineer, NAKS-Khabarovsk Evaluation Center, Russia, Khabarovsk, e-mail: ig_romanov@mail.ru.

Anatoly S. Streltsov, Candidate of Engineering, Worker, NAKS-Khabarovsk Evaluation Center, Russia, Khabarovsk.

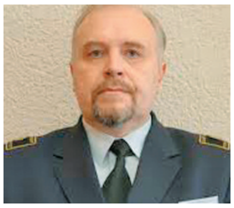
Received on 07.03.2017

Original article
DOI: 10.21683/1729-2646-2018-18-2-17-24

Functional survivability analysis of structurally complex technical systems

Gennady N. Cherkesov[1], *Peter the Great St. Petersburg Polytechnic University, Saint Petersburg, Russia*
Alexey O. Nedosekin, *Saint Petersburg Mining University, Saint Petersburg, Russia*
Valentin V. Vinogradov, *Saint Petersburg State University of Aerospace Instrumentation, Saint Petersburg, Russia*


Gennady N. Cherkesov


Alexey O. Nedosekin


Valentin V. Vinogradov

Abstract. Aim. *The paper analyzes the functional survivability of structurally complex technical systems. This approach is the evolution of the structural survivability paradigm, when the system/element failure criterion is binary. The paper shows that given a wide variety of probabilistic scenarios of adverse effects (AE) on a system, an invariant model kernel is identified that is responsible for the interpretation of functional redundancy. The aim is to identify the proportion of retained operable states within the acceptable computational time, when the fixed number u of elements is disabled as the result of AE. In this case the analysis of survival law is conducted at the confluence of functional redundancy analysis and probabilistic AE models of arbitrarily wide variety.* **Methods.** *A technical system is considered a controllable cybernetic system equipped with specialized survival facilities (SF). System survivability analysis uses logic and probabilistic methods, as well as the results of the combinatorial theory of random allocation. It is assumed that: a) AE are localized and single (one effect affects exactly one element); b) each of the system's elements has a binary logic (operability – failure) and zero resilience, i.e. destruction after one effect is guaranteed. Subsequently this assumption is generalized for the case of r-fold AE and L-resilient element.* **Results.** *The paper reconstructs a number of variants of the destruction law and survivability functions of technical systems. It is identified that those distributions are based on prime and generalized Morgan numbers, as well as Stirling numbers of the second kind that can be recovered using the simplest recurrence formulas. While the assumptions of the mathematical model are generalized for the case of nr-fold AE and L-resilient elements, the generalized Morgan numbers involved in the estimation of the destruction law are identified using the random allocation theory by means of n-fold differentiation of the generating polynomial. In this case it does not appear to be possible to establish a recursive relation between the generalized Morgan numbers. It is shown that under homogeneous assumptions regarding the survivability model (equally resilient system elements, equally probable AEs) in the correlation kernel for the system survivability function, regardless of the destruction law, is the functional redundancy vector $F(u, \varepsilon)$, where u is the number of affected elements, ε is the system's limiting efficiency criterion, below which its functional failure is diagnosed, $F(u, \varepsilon)$ is the number of system states operable in terms of ε under u failures (destructions) of its elements.* **Conclusions.** *Point models of survivability are an excellent tool of express analysis of structurally complex systems and tentative estimation of survivability functions. The most simple assumptions of structural survivability can be generalized in cases when the system's operability logic is not binary, yet is associated with the level of system operation efficiency. In this case we must speak of functional survivability. The PNP computational complexity of the survivability evaluation problem does not allow solving it by means of a simple enumeration of the system states and AE variants. Ways must be found of avoiding simple enumeration, e.g. by using conversion of the system operability function and its decomposition by means of generalized logical and probabilistic methods.*

Keywords: *functional survivability, structural survivability, adverse effect (AE), functional redundancy, structural redundancy, survivability function, generalized logical and probabilistic method (GLPM).*

For citation: *Cherkesov GN, Nedosekin AO, Vinogradov VV. Functional survivability analysis of structurally complex technical systems. Dependability 2018; 2: 17-24. DOI: 10.21683/1729-2646-2018-18-2-17-24*

[1] Doctor of Engineering, Professor, Peter the Great St. Petersburg Polytechnic University. Deceased in 2016. The paper contains unpublished materials from Professor's archive.

Introduction.

In [1-3], technical survivability is defined as the property of a structurally complex technical system to maintain its operability under a wide spectrum of adverse effects (AE). If we talk about survivability as a function of structural redundancy, it is a case of **structural survivability**. If the system's functional efficiency and capability to maintain at least a part of its functionality are evaluated, then we talk about **functional survivability**.Our understanding is that structural survivability is a special property of functional survivability that is primarily ensured by the presence of structural redundancy features in the system along with specialized survivability facilities (SF).

At this point we must specify some terms. First, let us compare the categories of "dependability" and "survivability". From our perspective, the conceptual separation of the above properties is along the line of the causes of operability deterioration and associated reduction or complete loss of functional efficiency of a technical system. In dependability those are strictly internal reasons that cause failures and faults; in survivability those are strictly internal reasons of operability deterioration (destruction)of individual elements. "Destruction" can be understood as either failures and faults or direct destruction of elements caused by AE. Large systems energy engineers do not agree with this terminological separation (in their practice survivability per [4] is an individual special property of dependability]. Computer system developers also think that survivability is a special case of dependability (e.g. see [5, p. 179]); there, it is synonymous with fault-tolerance. In this paper we ignore the above differences and understand survivability the way stated above.

We also must separate the definitions of structural and functional survivability. A similar separation is made in [6,7] in the context of information systems dependability. I.B. Shubinsky believes that structural dependability is the dependability of products (objects, elements, systems), while the functional dependability is the dependability of service provision (performance of the processes of collection, processing, transmission of information, management of subordinated objects). We do not completely agree with this dichotomy, at least as regards technical systems. Stated above is, in our opinion, the functional dependability ***in the narrow sense***. But, if we associate the property of the system's functional dependability with the property of its efficiency, the structure evidently contributes to the property of functional dependability. If dependability is not ensured at the level of system components, if the available structural redundancy is not properly managed, functional dependability is not ensured either. It turns out that functional dependability that is understood ***in the wide sense*** contains specific properties of structural dependability and functional dependability in the narrow sense. Equally, structural survivability is a separate specific property of functional survivability in the wide sense, as we noted at the very beginning of the paper.

The interpretation that we indirectly propose is substantiated as part of functional survivability standardization. Such standardization goes down two lines: the line of standard accepted efficiency and the line of maximum allowed probability of system survival. The harder is the standard requirement for the maximum allowed (from below) level of retained system efficiency after AE, the lower is the expected structural redundancy in the course of survival, the lower will be the survival probability and the harder must be the requirements for SF (that are formally only assigned to the technical system and are not its components). Naturally, the opposite is also true: the softer are the requirements for efficiency, the higher is the contribution of the structural redundancy into the system's survival.

Here the line must be drawn between the structural and functional redundancy in the narrow sense. In [6, p. 18], redundancy is a property of most existing technical objects (systems) to perform more functions than required and have more resources than required for the performance of only the required functions. In our opinion that is the definition of the functional redundancy in the wide sense that encompasses structural redundancy and functional redundancy in the narrow sense (as the capability to perform the same work using different means [6, p. 48]). The level of functional redundancy in the wide sense is defined in close connection with the standard level of efficiency. For example, if during a special period it is required to maintain 10% of output capacity of a power system after AE (level of emergency reserve), that corresponds to the maximum level of functional redundancy accumulated by the system under normal operational conditions.

Let us touch upon the subject of integration of various types of redundancy for the purpose of survivability (in [6] such integration is called multilevel redundancy). Structural redundancy and functional redundancy in the narrow sense always act together. A separate role is played by the information and algorithmic redundancy concentrated in the object's systems control supersystem. As regards the redundancy of the SF, it is localized outside the technical system. For instance, in the context of special military facilities, appropriate SFs are assigned to all technical systems within the facility together, rather than being part of one of the systems. Accordingly, we cannot assert that redundancy within a system and redundancy of the SFs are integrated for the purpose of ensuring system survivability. They operate in different ways, which can be clearly seen during simulation (we will emphasize it further).

Given the above, the indicator of functional redundancy should be the probability $R(n, \varepsilon)$ of the system retaining functional efficiency at level ε in fractions of its standard level under n AEs [2, 3]. The derived indicator of structural survivability as a separate special property is probability [2, 3].

The central methodological problem of the survivability science is the fact that AEs are not stochastic, manifest themselves as single events that cannot be interpreted in terms of the classic probability theory. Changing from statistical to axiological probabilities in the curse of AE scenario definition is a makeshift solution that is used temporarily for the purpose of identifying the property of survivability. In whole, the probabilistic concept of survivability as at the decline. In the new scientific paradigm there are two main approaches to survivability analysis:

• transition from probabilistic description of AEs and system's reaction to AEs to **fuzzy set models**. This subject requires separate consideration and it is not examined in this paper;

• designing a feasible **AE test** of system (not assuming high accuracy of real AE simulation) and associating the designed AE tests and the system's reaction to it. The purpose of such simulation experiment is to make the system manifest its survivability property and quantify the degree of this property's manifestation. In this case the system will primarily demonstrate the structural and functional types of redundancy.In other words, it will degrade due to AE not immediately, but gradually while retaining some resilience to the effects. Among other things, such gradual degradation will be ensured by efficient algorithms of system reconfiguration and exclusion of destroyed fragments (manifestation of functional redundancy in the narrow sense).

As of today, the most evident scientific results have been achieved with the proposal of the so-called **point model** of AE, when the AE is aimed at destroying an individual system element that has binary operation (operability or failure). This model can be easily generalized for the case of r-fold AEs for the case of a system with L-resilient elements [8]. In this paper we will demonstrate the application of this approach.

Thus, the aim of this paper is to establish the connection between functional survivability and redundancy in structurally complex systems by identifying this connection by means of AE tests of two types:

• **independent strategy:** AE against a system element can repeat;

• **dependent strategy:** a system element previously affected by AE cannot be targeted by AE again.

This paper examines equally probable AEs (in the axiological sense), i.e. there is no AE preference pattern. It can be compared to a system with homogeneous dependability, in which the elements have the same probability of no failure. We can generalize this result for the case of different AE probabilities in an exhaustive event, but it will in no way contribute to the aim of this paper. Additionally, we are ready to prove the redundancy that we have identified will manifest itself under a wide spectrum of AEs, and the **redundancy monotonicity of survivability** (the more redundant is the system, the more survivable it is) will be scientifically substantiated.

A brief description of the approach to survivability analysis used in this paper

There is a well-known Shannon's formula of reliability of structurally complex homogeneous non-renewable technical systems [9, p.161]:

$$P(t) = F_N(0)*p(t)^N + F_N(1)*p(t)^{N-1}(1-p(t)) + \dots + F_N(N-1)*p(t)(1-p(t))^{N-1}, \quad (1)$$

where$t$ is the reliability evaluation period, $p(t)$ is the probability of no failure of an individual system element, $F_N(u)$, $u = 0\ldots N$ is the number of operable system states under the condition that u of its elements simultaneously failed within the period of reliability evaluation t. Also, in the dependability theory $F_N(u)$ is the number of **disconnecting sets** consisting of u elements. We can also write $F_N(u) = F_N(u, \varepsilon=1)$ while making provisions for the possible extension of the given structural model to the level functional redundancy in a general sense.

Formula (1) can be rewritten as follows:

$$P(t) = \sum_{u=0}^{N} \Pr_N\left(pc|u\right) * Pr_N(t,u), \quad (2)$$

where

$$Pr_N(t,u) = \binom{N}{u} \{p(t)\}^u (1-p(t))^{N-u} - \quad (3)$$

isthe unconditional probability law of occurrence in a system of N elements of exactly u failures within time t (naturally, here the binomial distribution law is a standard Bernoulli scheme), and

$$\Pr_N\left(pc|u\right) = F_N(u) / \binom{N}{u} - \quad (4)$$

isconditional probability that the system remains operational if u random elements are removed from it.

Formula (4) can be named the **law of degradation** (for the case of dependability) or the **law of destruction** (for the case of survivability). That is the model of how natural failures or AEs are distributed in the system and cause degradation of its structure and functionality.

Let us return to the problem of functional survivability analysis. If the AE strategy is dependent (elements are chosen in the system consecutively, one after another), the survivability function is the probability of retention by the system of its operability under n single AEs [1-8]:

$$R^*(n, \varepsilon) = f(n, \varepsilon) = F_N(n, \varepsilon) / \binom{N}{u}. \quad (5)$$

The * sign indicates that the survivability was evaluated on the assumption of dependent strategy. Naturally, in case of dependent strategy $n \le N$. We can rewrite (5) as follows:

$$R(n, \varepsilon) = \sum_{u=0}^{N} \Pr_N\left(pc|u,\right) * Pr_N(n,u), \quad (6)$$

where $\Pr_N\left(pc\,|\,u,\right)$ is defined out of (4), with extension for the case $\varepsilon < 1$, while $Pr_N(n, u)$ is the destruction law for the case when under n AEs exactly u out of N system elements are affected is determined using formula:

$$Pr_N(n, u) = \begin{cases} 1, u = n \\ 0, u \; n. \end{cases} \quad (7)$$

If the AE strategy is independent, the number n can be arbitrary and in this case the law of destruction formula is correct [1-8]:

$$Pr_N(n, u) = N^{-n} * \binom{N}{u} * M(n, u) =$$

$$= N^{-n} * \binom{N}{u} * \sum_{v=0}^{u} \binom{u}{v} (-1)^{u+v} * v^n, \quad (8)$$

where $M(n, u)$ are Morgan's combinatorial numbers. For combinatorial Morgan's numbers partition, equation [6] is true:

$$\sum_{u=0}^{N} \binom{N}{u} * M(n,u) = N^n. \quad (9)$$

Destruction law (8) can be developed for the case of r-fold AE, when the scope of a single AE simultaneously covers r elements. In this case [4]

$$Pr_N(n, u, r) = \binom{N}{r}^{-n} * \binom{N}{u} * M(n, u, r) =$$

$$= \binom{N}{r}^{-n} * \binom{N}{u} * \sum_{v=r}^{u} \binom{u}{v} (-1)^{u+v} * \binom{v}{r}^n, \quad (10)$$

where$M(n, u, r)$ are generalized Morgan's numbers for the case of r-fold AEs. As with (9), a combinatorial set can be written:

$$\sum_{u=0}^{N} \binom{N}{u} * M(n,u,r) = \binom{N}{r}^n \quad (11)$$

Distribution of type (10) could be named a Markov-Nedosekin distribution, as A.A. Markov first suggested an individual specific case of this distribution (quoted per [18]), while A.O. Nedosekin first formulated this generalization [14]. Out of (10) under $r = 1$ easily follows (8).

If we make another round of generalization and assume that elements have a determinate resilience L to adverse effects, i.e. are destroyed exactly after (L+1) strikes, then (8) and (10) rewrite as follows:

$$Pr_N(n, u, r, L) = \binom{N}{r}^{-n} * \binom{N}{u} * M(n,K,u,L) =$$

$$= \binom{N}{r}^{-n} * \binom{N}{u} \sum_{=0}^{\binom{N}{r}} Q(n,K,,L) * \sum_{v=r}^{u} \binom{u}{v} (-1)^{u+v} * \left(\binom{v}{r} \right), \quad (12)$$

where $K = \binom{N}{r}$, $M(n,K,u,L)$ are generalized Morgan's numbers for the case of r-fold AEs and L-resilient elements, and

$$Q(n, K, \omega, L) = \frac{d^n}{dt^n} \{(e^t - g(t, L))^{\omega} * (g(t, L))^{K-\omega}\} |_{t=0},$$

$$g(t, L) = \sum_{k=0}^{L} \frac{t^k}{k!}. \quad (13)$$

Result (13) was obtained by A.O. Nedosekin in [8] using the method of generating functions in noncommutative nonsimmetrical K-basis with n-samples [17, p. 222].

If $r = 1$, formula (2) after a series of combinatorial transformations becomes as follows:

$$Pr_N(n, u, r, L) = N^{-n} * \binom{N}{u} * M(n,N,u,L) =$$

$$= N^{-n} * \binom{N}{u} * Q(n,N,u,L). \quad (14)$$

Finally, by substituting $L = 0$ into (14), in the course of a series of transformations we obtain standard Morgan's operands of the form (8). In this particular case the following is true:

$$M(n, u) = \frac{d^n}{dt^n} (e^t - 1)^u |_{t=0}. \quad (15)$$

If we compare formulas (2) and (6), we will see a certain conceptual invariant. Functional redundancy in the system is demonstrated by vector $F_N(u, \varepsilon)$ or conditional probability of the form (4), which is identical. The application to such redundancy of the corresponding law of degradation or destruction of the form (3), (7), (8), (10) or (12) generates a corresponding probability response in the system. AE laws change, the system's responses to AEs change, but the kernel of the model, the **redundancy vector**, remains unchanged. Therefore, our primary aim is to establish the form of the redundancy vector for a milti-element structurally complex system. When the redundancy vector has been established, evaluating the probability of system survival for various AE scenarios though is a technical problem

It also must be noted that the property of element resilience characterized by parameter L is in fact not a property of the element itself, but rather an attribute of the survivability facilities that are intended to provide the system with the properties of resilience. For example, in terms of system survivability under seismic impacts, the vibroplatform on which elements of the technical system are installed (one, several or all) has the resilience property. Such platform must be able to withstand an impact characterized by an acceleration multiple of g (gravity factor). If the impact is divisible by (L+1), the vibroplatfrom partially loses stability and is destroyed, while the elements installed upon it are either destroyed or loose connection to the system, which is equivalent in terms of the consequences. The multidirectional manifestation of structural dependability and resilience can be indirectly observed in formulas (6) and (12), where the structural redundancy is associated with one of the probabilities, while the resilience is associated with the other.

Here, we put the emphasis on the fact that identifying the redundancy vector is not an easy task at all. It is NP-hard [3], as it is associated with complete enumeration of 2^N system states with the division of such states into two classes, i.e. functionally operable and functionally fallible. The general logical and probabilistic method (GLPM) comes to help though [11, 12]. It allows overcoming the "curse of dimensionality" by means of methods of decomposition of the initial logical operabilityfunction (LOF) with its preliminary identification based on the formalization of the system operation rules, with the identification of the full list of operability paths and minimal failure cross-sections. In today's conditions of industrial automation, this work is performed by the ARBITR software system (developed by SPIKSZMA, Saint Petersburg, Russia). The scientific component of the system was developed by the school of Prof. A.S. Mozhaev.

Thus, let us proceed to the multivariant analysis of survivability using the examples of two trial computational schemesand formulas (4) – (15). In order to simplify the demonstration, let us assume that ε = 1, i.e. we are solving the problems of structural survivability in particular by evaluating the effect of structural redundancy on the survivability. Examples for the case when ε < 1 can also be easily provided. The results will be published in the following papers.

Analysis of structural survivability for three calculation examples

Example 1. Bridge-type structure system (N = 5 elements)

Let the system have a two-pole operability model (bridge-type, Figure 1), for which the operability function is as follows [3, 9, 12]:

$$F = x_1x_3 \vee x_1'x_2x_4 \vee x_1\, x_2x_3'x_4 \vee x_2'x_3'x_1x_4x_5 \vee x_1'x_4'x_2x_3x_5. \quad (16)$$

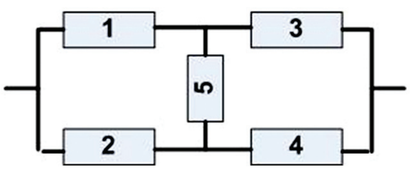


Figure 1. Bridge-type structure system

In this example 1, as the complete number of system states is $2^5 = 32$, all states can be easily enumerated manually in order to choose the operable ones (16 in total). The redundancy vector and conditions probability of the form (4) are given in Table 1.

The survival law $R^*(n)$for dependent AE strategy is the last column of Table 1 on the assumption that $n = u$. In order to perform the analysis for dependent AE strategy let us first recover the table of Morgan numbers per (8) for N = 5. The data is given in Table 2.

Table 1. Redundancy vector and conditions of probability of operability per case 1

u	$F_N(u)$	Number of combinations of N by u	$\Pr_N(pc\,|\,u)$
0	1	1	1
1	5	5	1
2	8	10	0.8
3	2	10	0.2
4	0	5	0
5	0	1	0

Table 2. Morgan numbers M_5 (n, u)

n	$M_5(n, u), u = 0…5$					
	$u = 0$	$u = 1$	$u = 2$	$u = 3$	$u = 4$	$u = 5$
1	0	1	0	0	0	0
2	0	1	2	0	0	0
3	0	1	6	6	0	0
4	0	1	14	36	24	0
5	0	1	30	150	240	120
6	0	1	62	540	1560	1800
7	0	1	126	1806	8400	16800

The data in Table 2 is used together in calculations according to formulas (6) and (8). The values of $R(n)$ in case of $n \leq 7$ are given in Table 3.

Table 3. Function $R(n)$

n	1	2	3	4	5	6	7
$R(n)$	1	0.8400	0.5200	0.3024	0.1744	0.1012	0.0592

As an integral factor that can be used as a proper convolution of the redundancy vector, the mean number of AEs that causes the loss of operability in case of dependent AE strategy the following can be used:

$$\overline{\omega} = \sum_{n=0}^{\infty} R^*(n) = \sum_{u=0}^{N} F_N(u) \Big/ \binom{N}{u}. \quad (17)$$

In the case of bridge-type structure, $\overline{\omega} = 3$. That means that the system can be intentionally disabled at an average with three strikes. In order to remove the N-dependence in choosing the optimal survivability design solution, the system survivability index (SI) can be used:

$$SI = \overline{\omega} / N. \quad (18)$$

In our case SI = 0.600. To understand whether that is much or little, many networked systems must be evaluated. Such evaluations are not within the scope of this paper. However, formula (18) is another example of a distinct connection between structural redundancy and survivability.

Let us now complicate the problem definition. Let us assume that in one AE $r = 2$ elements are simultaneously affected. In this case the use of formula (10) results in the destruction law as shown in Table 4.

Table 4. Destruction law Pr_5 (n, u, r=2)

n	Pr_5 (n, u, r=2), u = 0…5					
	$u = 0$	$u = 1$	$u = 2$	$u = 3$	$u = 4$	$u = 5$
1	0,000	0,000	1,000	0,000	0,000	0,000
2	0,000	0,000	0,100	0,600	0,300	0,000
3	0,000	0,000	0,010	0,240	0,570	0,180
4	0,000	0,000	0,001	0,078	0,489	0,432
5	0,000	0,000	0,000	0,024	0,340	0,635
6	0,000	0,000	0,000	0,007	0,219	0,774
7	0,000	0,000	0,000	0,002	0,136	0,862

The combined application of (6) and (10) results in the values of $R(n)$ shown in Table 5. Naturally, in case of square independent AEs the system degrades faster that in the case described in Table 3.

Table 5. Function $R(n)$

n	1	2	3	4	5	6	7
$R(n)$	0,8000	0,2000	0,0560	0,0164	0,0049	0,0015	0,0004

Example 2. Three-generator electric energy system (N = 10 elements)

[13] and [11] describe a three-generator electric energy system (EES, Figure 2). Its operability diagram is shown in Figure 3.

The operability function established based on the diagram in Figure 3 is as follows [11, p. 30]:

$$Yp = y_7 = x_{67} \wedge x_{26} \wedge x_{25} \wedge x_{15} \vee x_{67} \wedge x_{26} \wedge x_{12} \vee \vee x_{67} \wedge x_{36} \wedge x_{13} \vee x_{47} \wedge x_{25} \wedge x_{24} \wedge x_{15} \vee x_{47} \wedge x_{36} \wedge \wedge x_{26} \wedge x_{24} \wedge x_{13} \vee x_{47} \wedge x_{24} \wedge x_{12} \vee x_{57} \wedge x_{36} \wedge x_{26} \wedge \wedge x_{25} \wedge x_{13} \vee x_{57} \wedge x_{25} \wedge x_{12} \vee x_{57} \wedge x_{15} \quad (19)$$

The total number of operable states in the diagram is 554 of 2^{10} = 1024. By making a complete enumeration of system functions per LOF of type (21) we arrive at Table 6 that contains the redundancy vector. According to this definition of the problem, all effects are single, while the system's elements have zero resilience.

The destruction law per example 2 is shown in Table 7; the survival law for the independent AE strategy is shown in Table 8. For the case of Example 2 we also have $\overline{\omega} = 5.737$, SI = 0.574. As we can see, the "specific survivability" of EES of Example 2 turns out to be even slightly lower than the bridge-type structure's. We can speak of redundancy concentration, when the growing number of elements does not cause qualitative improvements to the system's survivability performance. Nevertheless, due to the growing hardware component, the AE-related system degradation is smoother than that of the bridge-type operability logic.

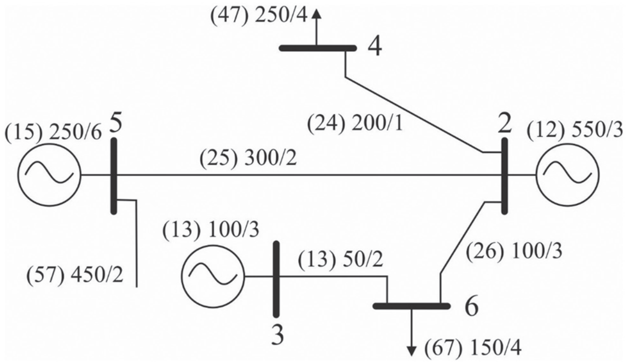


Figure 2. Three-generator EES diagram.*Source: [11]*

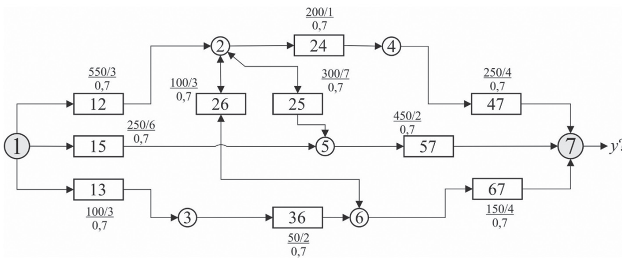


Figure 3. EES operability diagram.*Source: [11]*

Table 6. Redundancy vector and conditional probability of operability per example 2

u	$F_N(u)$	Number of combinations of N by u	$\Pr_N(pc\,|\,u)$
0	1	1	1.000
1	10	10	1.000
2	45	45	1.000
3	116	120	0.967
4	175	210	0.833
5	137	252	0.544
6	57	210	0.271
7	12	120	0.100
8	1	45	0.022
9	0	10	0.000
10	0	1	0.000

Conclusion

The structural survivability know-how developed by Soviet/Russian scientist over the last 30 years significantly help achieving a new level of modeling and analysis of survivability and resilience of complex systems (not necessarily technical ones). The primary goal is the transition

Table 7. Destruction law Pr_{10} (n, u)

n	$Pr_{10}(n, u)$, u = 0…10										
	u = 0	u = 1	u = 2	u = 3	u = 4	u = 5	u = 6	u = 7	u = 8	u = 9	u = 10
1	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.100	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.010	0.270	0.720	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.001	0.063	0.432	0.504	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.014	0.180	0.504	0.302	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.003	0.065	0.328	0.454	0.151	0.000	0.000	0.000	0.000
7	0.000	0.000	0.001	0.022	0.176	0.423	0.318	0.060	0.000	0.000	0.000
8	0.000	0.000	0.000	0.007	0.086	0.318	0.402	0.169	0.018	0.000	0.000
9	0.000	0.000	0.000	0.002	0.039	0.210	0.400	0.279	0.065	0.004	0.000
10	0.000	0.000	0.000	0.001	0.017	0.129	0.345	0.356	0.136	0.016	0.000
11	0.000	0.000	0.000	0.000	0.007	0.075	0.271	0.387	0.216	0.042	0.000
12	0.000	0.000	0.000	0.000	0.003	0.042	0.200	0.379	0.289	0.081	0.000
13	0.000	0.000	0.000	0.000	0.001	0.023	0.141	0.346	0.345	0.130	0.000
14	0.000	0.000	0.000	0.000	0.001	0.012	0.096	0.298	0.379	0.186	0.000
15	0.000	0.000	0.000	0.000	0.000	0.006	0.064	0.247	0.393	0.244	0.000

from structural to functional survivability. The first steps in this direction have already been made [14-16], however the work must continue with the aim of automatic construction of LOF for multiple systems with arbitrary performance criteria. By changing the level of retained efficiency ε, at the stage of manual search already it can be observed that as ε grows the level of available structural and functional redundancy slowly goes down. Manual search should be abandoned through automated construction and examination of a set of LOFs responsible for various levels of required efficiency ε.

Secondly, AE scenario tolerances should be formulated more strictly. That involves progressive replacement of probabilistic combinatorial models with their simplistic hypotheses of effects on models, where the effect is formulated in terms of the adverse factors themselves. In this case fuzzy logic AE modeling suggests itself, as well as elements' resilience to effects, including the efficiency of survivability facilities. That is the subject of our future activities.

Table 8. Function $R(n)$

n	$R(n)$	n	$R(n)$
0	1	8	0.2509
1	1	9	0.1490
2	1.0000	10	0.0849
3	0.9760	11	0.0469
4	0.9016	12	0.0253
5	0.7720	13	0.0134
6	0.5850	14	0.0070
7	0.3987	15	0.0036

References

[1]. Cherkesov GN. Metody i modeli otsenki zhivuchesti slozhnykh sistem [Methods and models of evaluation of complex systems survivability]. Moscow: Znanie; 1987 [in Russian]. Also at: <http://www.gcherkesov.com/articles/article02.pdf> .

[2]. Cherkesov GN, Nedosekin AO. Description of approach to estimating survivability of complex structures under repeated impacts of high accuracy. Dependability 2016;2:3-15.

[3]. CherkesovGN, Nedosekin AO. Description of approach to estimating survivability of complex structures under repeated impacts of high accuracy (Part 2). Dependability 2016;3:26-34.

[4]. Nadiozhnost sistem energetiki. Sbornik rekomenduemykh terminov [Dependability of energy systems. Recommended terminology reference book]. Moscow: Energia; 2007 [in Russian].

[5]. Khoroshevsky VG. Inzhenernyy analiz funktsionirovaniya vychislitelnykh mashin i sistem [Engineering analysis of computer systems operation]. Moscow: Radio i sviaz; 1987 [in Russian].

[6]. Shubinsky I.B. Nadiozhnie otkazoustoychivie informatsionnie sistemy. Metodysinteza [Dependable failsafe information systems. Synthesis methods]. Moscow: Dependability Journal LLC; 2016 [in Russian].

[7]. Shubinsky IB. O poniatii funktsionalnoy nadiozhnosti [On the concept of functional dependability]. Dependability 2012;4:74-84 [in Russian].

[8]. Nedosekin AO. Primenenie teorii sluchaynykh razmeshcheniy k analizu zhivuchesti tekhnicheskikh sistem [Application of the random placement theory in the

survavability analysis of technical systems]. Kibernetika AN USSR 1991; 6 [in Russian]. Also at: <http://www.ifel.ru/surv/Res_5.pdf> .

[9]. Cherkesov GN. Nadezhnost apparatno-programmnykh kompleksov [Dependability of hardware and software systems]. Saint Petersburg: Piter; 2005 [In Russian].

[10]. Riordan J. Combinatorial identities. Moscow: Nauka; 1982.

[11]. Mozhaeva IA. Metodiki strukturno-logicheskogo modelirovaniya slozhnykh sistem s setevoy strukturoy [Methods of structural logical modeling of complex networked systems]. Candidate of Engineering thesis. Saint Petersburg; 2015. Also at: <http://www.ifel.ru/surv/Res_8.pdf> .

[12]. Mazhaev AS. Obshchiy logiko-veroyatnostnyy metod [General logical probabilistic method]. Saint Petersburg: VMA; 1988 [in Russian]. Also at: <http://www.ifel.ru/surv/Res_9.pdf> .

[13]. Guk YuB, Karpov VV. Teoriya nadezhnosti v elektroenergetike: konspekt lektsiy [Dependability theory in the electric power industry: Compendium of lectures]. Saint Petersburg: SPbSTU; 1999 [in Russian].

[14]. Nedosekin AO. Analiz zhivuchesti sistem energetiki kombinatorno-veroyatnostnymi metodami [Survivability analysis of power systems through combinatorial probabilistic methods]. Proceedings of the Russian Academy of Sciences. Power engineering 1992;3:48–58 [in Russian]. Also at: <http://www.ifel.ru/surv/Res_6.pdf> .

[15]. Nedosekin AO. Strukturnyy analiz zhivuchesti EES kombinatorno-veroyatnostnymi metodami [Structural survivability analysis of electrical power systems through combinatorial probabilistic methods]. MVIN BSE. Irkutsk: SPII RAS SP; 41 [in Russian].

[16]. Nedosekin AO. Strukturnyy analiz zhivuchesti EES kombinatorno-veroyatnostnymi metodami [Structural survivability analysis of electrical power systems through combinatorial probabilistic methods]. MVIN BSE. Irkutsk: SPII RAS SP; 43; 1992 [in Russian]. Also at: <http://www.ifel.ru/surv/Res_7.pdf> .

[17]. Sachkov VN. Vvedenie v kombinatornye metody diskretnoy matematiki [Introduction to combinatorial methods of discrete mathematics]. Moscow: Nauka; 1982 [in Russian].

[18]. Kolchin VF, Sevastianov VA, Chistiakov VP. Sluchaynyerazmeshcheniya [Random palacement]. Moscow: Nauka; 1976 [in Russian].

About the authors

Gennady N. Cherkesov, Doctor of Engineering, Professor, Peter the Great St. Petersburg Polytechnic University, Saint Petersburg, Russia

Alexey O. Nedosekin, Doctor of Economics, Candidate of Engineering, member of the International Academy of Ecology, Safety of People and Nature, Professor, Saint Petersburg Mining University, Director General, SI-FINANS, Saint Petersburg, Russia, e-mail: apostolfoma@gmail.com

Valentin V. Vinogradov, postgraduate student, Saint Petersburg State University of Aerospace Instrumentation, Saint Petersburg, Russia

Received on 05.12.2017

DOI: 10.21683/1729-2646-2018-18-2-25-30

Theoretical modeling of dependability resources of flight crews in commercial aviation

Nikolay I. Plotnikov, *Aviamanager Research and Design Institute of Civil Aviation, Novosibirsk, Russia*

Nikolay I. Plotnikov

Abstract. *The paper develops theoretical models of dependability of commercial aviation (CA) flight crews based on the resource method of designing organizational social objects. The aim was to provide an objective description of flight crew activity. Formal models of crew composition were constructed. Definitions of dependability of intense profession members are presented using the example of CA crews. The competitive environment of the open global air transportation market is leveled against the standardization of airline activities and primary object of aviation, i.e. CA pilot and flight crew. Air disasters of the last few decades highlight the primary causes, i.e. professional properties deficiency in pilots and excessive workload of flight crews in CA operations. This situation is caused by not only the pressure of the business environment, but also by the critical insufficiency of scientifically grounded methods of managing flight operations in terms of the human component. The paper developed theoretical models of dependability of flight crews based on classical logic and resource method of designing organizational social objects of the transportation industry (airline). The essence of the problem. In commonly known literature there still is no theoretical framework, formal models that could be used for calculation and management of dependability of activities. Crew resources are researched in terms of dependability and efficiency. In general, crew dependability is understood as the sum of dependabilities of crew members for the completion of the assigned tasks. The dependability depends on the composition of specialized skills and individual qualifications of the crew members. The efficiency is the result of three components: communications, decisions, delegation. These interactions can be formal and informal. The scientific substantiation and definition of the parameters of the crew's assignment in terms of the estimated dependability and efficiency parameters are the solution of the problem. Problem formalization. In order to formalize the problem of objective description of flight crew activity, the crew may be considered as a class of individuals. The logic of classes (sets) uses the class-forming operator C, for "class", predicate of inclusion of individuals into class ∈, a binary predicate, predicate of inclusion of a class into a class. In order for a class to exist it suffices for it to be formed out of the range of values of term t. Class generation principles are expressed in the following axioms: Each element of a class can be chosen regardless of the class formation, the independence principle. A class of individuals exists (does not exist) if it is formed (not formed) in accordance with the definition of class formation and formation axioms. Subsequent statement of the problem must be directed in detail, specific solutions for the development of models suitable for calculation and management of flight operation. Thus, the development of the theoretical essence of the composition and size of crew is a relevant problem and can be solved based on classical logic, managerial control theory, information theory.*

Keywords: *pilot, crew, modeling, class, individual, composition, power of class.*

For citation: *Plotnikov NI. Theoretical modeling of dependability resources of flight crews in commercial aviation. Dependability 2018;2: 25-30. DOI: 10.21683/1729-2646-2018-18-2-25-30*

Introduction

Essence of the problem. The competitive environment of the open global air transportation market is leveled against the standardization of airline activities and primary object of aviation, i.e. pilot and flight crew of commercial aviation (CA) aircraft (AC). Air disasters of the last few decades highlight the primary causes, i.e. professional properties deficiency in pilots and excessive workload of flight crews in CA operations. This situation is caused by not only the pressure of the business environment, but also by the critical insufficiency of scientifically grounded methods of managing flight operations in terms of the human component.

The Air Code of the Russian Federation provides the following description of flight crew: "The crew of an aircraft consists of a flight crew (the captain, other members of the flight crew) and the cabin crew (operators and stewards). The flight of a civil aircraft shall be prohibited if the number of flight crew members is less than the established minimum" [1]. This description was developed empirically over the course of the the global aviation history. In literary sources known to the author there still is no theoretical framework, formal models that could be used for calculation and management of AC flight operation. The research can start with the examination the problems of identification of the object of activity through logical analysis. In [2], the formal foundations of the CA pilot activity are put together. Observation (measurement, evaluation) in the genus-species classification of the property of an individual's dependability allows assigning numbers and calculating states. The paper sets forth the results of calculation of states for the purpose of managing AC flight operation: resources of individual dependability (RID), resources of professional dependability (RPD), resources of operational dependability (ROD) [3].

Compared to the results for individuals, the formalization and calculation of the properties of a social group (crew) are undeveloped. Since the 1980s, the aerospace industry has been developing the concept and technologies of cockpit resource management (CRM) [4]. Practically, the technologies are used as a combination of educational programs and training sessions aimed at the development of the skills related to decision, communication and delegation grouped within the concept of efficiency. However, the scientific foundations of the theory and methods of CRM calculation have not been created.

This is the statement of the overall problem, a part of which is structured as follows: 1) establishment of the method; 2) establishment of the terms of objective description of the activity of objects in accordance with the provisions of classical logic; 3) development of terms and definitions; 4) formalization of the problem of calculation of the properties of the flight crew for the purpose of managing AC flight operation.

Definition of the terms of description of the objective scope

The logical analysis of subject field terms is motivated by the following. In engineering, the "commonly accepted" and "commonly known" concepts, definitions and terms are not really such, as they have not been submitted to humanities and logical research. According to A.A. Zinoviev "... in general, it is impossible to judge the applicability of formal constructs in the research of some subject field, if there is no prior knowledge of it, if it is not studied to some extent at the descriptive level" [7, p. 7]. For that very reason we deem it essential to determine the subject field of the social groups theory and social science terms examined below.

Let us determine the objective meaning of the terms "class", "composition", "individual". The problem may be considered within the concept theory (a division of logic) [11, 12, 13], the class (sets) logic [7]. We believe that identifying the meaning of identical terms, they must be researched simultaneously in all the above theories. In the concept theory, each concept has a content (set of diverse attributes) and size (number of identical elements). The law of reverse genus-species relations established: the richer is the content, the smaller is the size and vice versa. The concept of "individual" has a large content and is generic for such concepts as "class", "composition" that have large sizes.

In the class logic [7] a social group is regarded as a class of individuals. The class-forming operator C is used, for "class", predicate of inclusion of individuals into class $\in$, a binary predicate, predicate of inclusion of a class into a class. Class formation is defined as follows.

D 1. To form (and select) a class of individuals is to construct the term "class of individuals from range *t*, where *t* is the given term and *t* is a subject" [7, p. 176].

The definition of class formation contains notation C*t*, where C is the class-forming operator. Individuals from range *t* are elements of C*t*. In order for a class to exist it suffices for it to be formed out the range of values of term *t*. The range of values *t* (pilot) is the individual whose range of values (purpose) is defined by the ability to control an AC in a three-dimensional airspace.

A 1. The term "individual" (pilot) and term "class" (crew) are terms ["individual" · "crew"], i.e. an object denoted by each of the terms, of which the meaning is known.

Class formation principles are expressed in the following axioms:

A 2. Each element of a class can be chosen regardless of the class formation, the principle of independence of elements from class.

That means that each and every pilot can be included in any class (crew). In a particular case, an individual is identical to a class, if the crew consists of one individual.

A 3. Regarding any individual it can be established whether he/she is an element of a given class, the principle of certainty.

The certainty is defined by the existence of education, qualification, experience and pilot's permission to fly.

D 2. A class of individuals exists (does not exist) if it is formed (not formed) in accordance with the definition of class formation and formation axioms.

Let us consider the primary terms of the class theory: power, composition.

D 3. **Power of class.** "The power of a class is the number of its elements. The existing (possible) power of a class is the number of existing (possible) individuals that are its elements" [7, p. 187].

Out of this definition follows the conclusion regarding the identity of two logical concepts (power ≡ size): power is a concept of class logic, size is a term of concept theory.

D 4. **Composition of class.** "Determining the composition of a class means determining what individuals are included in it. Determining the existing (potential) composition of a class means determining what individuals that are its elements exist (are possible)" [7, p. 187].

As we can see, in the definition with the wording "... determining, what..." there are no attributes (content) of the "composition" concept. Therefore, the concept of "content" has a wider generic scope that is to be divided into specific concepts.

It must be determined, what competences, training, qualifications of AC crew members are included in the class. From the history of aviation it is known that the highest competences are defined for and concentrated in the profession of pilot. With the automation of the modern commercial aviation, the professions of navigator, flight engineer, radio operator, etc. disappeared from the crew. The diversity (intensity) of the pilot's functions causes the reduction of the power (size) of the class, i.e. the number of crew members. Here we can clearly see the effect of the law of the genus-species relations. Let us complement the definition of the "composition of class": determining the composition of a class (crew) means determining the attributes of diversity of the content (intensity) an individual (pilot) must be included in the class. Thus, in the class logic, the key terms are the composition and power of class. In the concept theory those are the content and scope of a concept. Additionally, let us also examine the dictionary definitions of the term "composition".

D 5. Composition is an object (set) that includes a set of parts (elements, components), as well as the description of the quality, quantity and other characteristics of the parts of such object (set) [8].

D 6. The set of parts, elements that make a whole [9].

The dictionary definitions also indicate that "composition" is and abstract umbrella term, i.e. it has a large scope and can therefore be used as a generic term. According to the inverse relation law of the scope and content of a term, the following structure of the terms can be defined:

$$(N)\ \text{composition}:\ \mathrm{C}\{\text{content}(\text{intensity})\}\ 1\,/$$
$$/\ V\left\{\text{power},\ \text{size}\left(\textit{number}\right)\right\}, \tag{1}$$

where (N) is the introduced notation of composition (of a class, crew), C is the diversity of attributes (intensity) of each and every out of the i-th individuals of the class, V is the size, power, i.e. the number of individuals in the class, $(\cdot)$ is read like operator *and*.

The composition of a class is defined in terms of the time {past ↔ present (now) ↔ future} of observation of the following binary antonymic relations of terms:

a. existing	a'. potential
b. permanent	b'. variable
c. limited	c'. unlimited
d. finite	d'. infinite
e. known	e'. unknown
f. definite	f'. indefinite

These relations and the number of their mutual combinations create the *multiaspect* context of the problem:

$$(N):\ \mathrm{C}\left(\overline{a,n}\right)\cdot\left\{1\,/\,V\left\{\overline{a',n'}\right\}\right\}, \tag{2}$$

where the symbols make the above conventional notations.

The existing class may be defined in such a way as to include only those individuals that are placed in time {past ↔ present}. A finite numbers class my be infinite in terms of professions. A class restricted in professions can be defined unrestricted in terms of the number of individuals. We may avoid restricting the number of individuals, but in the {present ↔ future} future new elements will not appear. Although exceptions should be kept in mind and taken into consideration, when the inverse relation law C: $1/V$ does not work in the concept theory (for discordant concepts). That is the cause of the extreme complexity of problem definition and solution using only the classical logic tools. Nevertheless, the proposed problem structure can be used in further research. The identification and formalization of relations can probably be continued using pseudophysical logic.

For the purpose of solving the problem by expert (heuristic) method we create a convolution: the set of existing professions of crew member individuals are *known* and *finite* in terms of power (content, professions) and size (number).

This statement is empirical, based on historical experience of aviation, as well as crew composition in terms of professions and number.

Development of terms and definitions of CA AC flight crew dependability

The definition will be based on the previous terminology work subject to the mentioned limitations and assumptions. The following definitions are established.

D 7. Composition N is the class defined by the assignment of power and size:

$$N:\{C\cdot 1/V\}.$$

D 8. Crew (social group), the controlling subject of vehicle $(Cr) \geq 1; \overline{1,\ n}$.

D 9. AC cockpit crew, the controlling subject that performs activities in accordance with the AC flight purpose.

D 10. The ability to control an aircraft in a three-dimensional airspace is called the crew's resource of purpose.

The essence of the category of purpose can be easily understood by comparing the movement of an aircraft in a three-dimensional (X, Y, Z) space and the movement on a plane (X, Y) by a car.

D 11. Dependability is the set of properties and states of the object within the metric of the standard activity space.

D 12. The crew dependability is defined as the set of properties and states of the member individuals for the purpose of completing the purpose (flight).

Problem of calculation of flight crew dependability

Let us form the content of the problem of calculation of CA AC flight crew dependability. Let us use the above terms, definitions, work formalizations [10] in the context of the problems considered in this paper.

D 13. Dependability calculation is defined as the observation (measurement, evaluation) of the properties and states of the flight crew, performance of standard operational procedures (SOP) within the specified parameters and indicators corresponding to safe and efficient execution of flights.

Conditions of restrictions: number of members (V) and professions (C) of crew members is known and finite;

N_0 is the existing final crew composition;

N is the target state of the crew as the result of the control task solution;

N' is the set of ways of establishing the target state, ground set of the crew $N \subseteq N'$, $N_0 \subseteq N'$;

$\Phi=(N, N_0)$ is the functional that associates the initial and final states, dependability of control;

$|N|$ is the standard crew composition;

$|N|>|N_0|$ is the extended crew composition: double, enhanced, with inspectors, trainees onboard;

$|N|<|N_0|$ is the reduced composition of crew: absence of navigator, radio operator, other specialists;

$|N|\neq|N_0|$ is the replacement of crew composition: quantitative (replacement of the aircraft captain (ACC) or copilot) and/or specialized (inclusion of navigator authorized to act as radio operator).

The problem of definition of crew composition with no initial composition $N_0=\varnothing$ has the following form:

$$\Phi(N,\varnothing)\to \max_{N\in 2^{N'}},$$

where $N \in 2^{N'}$ is the first defined crew composition: ACC, copilot.

The problem of possible modification of composition in case of fixed initial composition N_0 has the following form:

$$\Phi(N,N_0)\to \max_{N\in 2^{N'}},$$

where $N \in 2^{N'}$ is the possible crew composition; example: ACC (replacement), copilot; ACC, copilot (replacement).

The problem of extended crew composition under initial number n and m additional members has the following form:

$$\Phi(N,N_0)\to \max_{N\in 2^{N'}:N_0\ \ N,\ |N|\leq n+m},$$

where $N \in 2^{N'}$ is the defined composition, $N_0 \subseteq N$, if $|N| \leq n+m$ is the extended composition; example: addition of one trainee and one inspector.

The problem of reduced crew composition under initial number n and m reduced members is formulated by the search for the set $\Delta^- \subseteq 2^{N_0}$ that maximizes the dependability (under the condition $\Delta^+ = \varnothing$) and has the following form:

$$\Phi(N,N_0)\to \max_{N=N_0\setminus\Delta^-,\ |\Delta^-|\geq m},$$

where $N = N_0 \setminus \Delta^-,\ |\Delta^-| \geq m$ is the description of conditions; example: requirement to replace the ACC with flight instructor and exclusion of one of the specialists (radio operator, navigator, loadmaster).

The problem of replacement of crew members under initial number n and m replaced members that maximizes the dependability has the following form:

$$\Phi(N,N_0)\to \max_{N=2^{N'},|\Delta^-|=\Delta^+=m},$$

where $N = 2^{N'},|\Delta^-| = \Delta^+ = m$ is the description of the cindition; example: replacement by a more experienced crew member.

In this class of problems the variables not described above are not taken into consideration. The main limiting factor of formalization is the introduction of simplification: N_0 is the existing defined crew composition instead of: N_0 is the existing defined quantitative $\{V:1,2,\ldots,n\}$, $v_i \in V$ and specialized $(\mathrm{C}:a,\ b,\ \ldots,\ k)$, $\mathrm{c}_i \in \mathrm{C}$ crew composition consisting of n individuals of k professions, $|N_0| = n,\ k$.

Additionally, the above binary (probably, unary) relations of class composition terms are not formalized. In whole, it can be said that formal constructions can be used for calculation of crew composition and subsequent development of automated control software.

Example of calculation of pilot and flight crew dependability

Let us give an example of calculation of pilot and flight crew dependability based on two selected indicators that are associated with the states of the dependability property. The states are evaluated using a nominal scale and

Table 2. Pilot dependability evaluation metric

Indicators	States (indicator values)		
	Green	Yellow	Red
	1	2	3
Age	40 years	30 years	65 years
Flight hours	10 000 hours	3000 hours	20 000 hours
Evaluation	complete	acceptable	inadequate

an ordinal scale of three-level risk matrix: "red-yellow-green". Example: let three pilots aged 40, 30 and 65 have 10 000, 3000 and 20 000 flight hours respectively (Table 2).

Compliance with purpose is evaluated as follows: the 40-year-old pilot complies with the assignment in terms of two indicators, the 30-year-old pilot acceptably complies in terms of the same two indicators. The age indicator "65 years old" is called "critical state" (CS) that is sufficiently easily predicted and calculated. Therefore, despite the "green" level of risk per another indicator, i.e. 20 000 flight hours, the general score of the 65-year-old pilot is "non-compliant". This example of evaluation of two indicators is a simple demonstration of the resource method of calculation of object states in risk matrices. The complete structure consists of 43 indicators and is a scientifically substantiated standard activity space [3].

The problem of calculation of the dependability of a flight crew of one pilot is identical to the calculation of an individual's dependability. The calculation of the dependability of a flight crew of n individuals is based on the premise that not a single indicator of not a single crew member (except the trainees) must be outside the "acceptable" score.

Conclusion

The problem and task of identifying the object of individual and social group through the example of CA objects is considered in terms of purpose and dependability of activity. The concept of "purpose" can be considered generic with a large scope that is difficult to use "directly" in the observation of properties and states of objects. Observation is possible if the scope of the concept is divided into specific concepts, i.e. efficiency, safety, dependability that have smaller scopes, but larger content (attributes). Thus the object of activity is identified.

In terms of assignment the objects "individual" and "group" are identical. In terms of dependability they are different. In the simplest case the dependability of a group is the sum of the properties of individuals. The dependability of an AC crew members is identified based on the differences between special knowledge and skills for controlling AC functional systems. The growth of technology dependability and automation lead to the universalization of knowledge and skills within the single profession of pilot.

The proposed definitions and models of AC flight crew are the initial formal tools that allow controlling the crew composition. As its is shown, the number of combinations of time-to-space relations constitutes a long list of relevant problems that require a formal description.

This paper proposes the terminology related to the object of dependability of CA AC flight crew. We assume that the definition of the terms "dependability" and "crew composition" completely comply with logical provisions. The property (purpose) of an object can be observed (measured, evaluated) in terms of the states of a previously developed standard space of dependability.

The objective meaning of the term "dependability" is the static characteristic of the subject of activity that can be structured in order to evaluate states and calculations. The formalized mathematical description of efficiency is even more complicated compared to the above stated problem of calculation of CA AC flight crew dependability. In [1], mathematical models of efficiency – decisions, communications and delegations of powers, as well as crew member responsibilities – are set forth.

References

[1]. Air Code of the Russian Federation dd 19.03.1997 no. 60-FZ (ed. 18.07.2006 no. 114-FZ) [in Russian].

[2]. Plotnikov NI. Resursy pilota. Nadiozhnost. Monografia [Pilot's resources. Dependability. [A monograph]. Novosibirsk: Research project center AviaManager; 2013 [in Russian].

[3]. Plotnikov NI. Razdel 3. Metod otsenki riskov dlya bezopasnosti poletov aviakompanii na osnove upravleniya i prognoza resursov pilota. [Section 3. A method for evaluating the air safety risks of airlines based on pilot resources management and prediction]. In: Avtomatizirovannaya sistema prognozirovaniya i predotvrashcheniya aviatsionnykh proisshestviy pri organizatsii i proizvodstve vozdushnykh perevozok. Promezhutochny etap no. 4: Adaptatsiya razrabotannykh algoritmov i programmnykh sredstv AS [Automated system for prediction and prevention of air accidents in prganization and perfromance of air transportation. Intermediate stage 4: Adaptation of developed AS algorithms and programs]. Science and engineering report, 2010-218-02-068, official registration no. 01201150118 dd 12.01.2011, identification no. 194. Ulianovsk; 2012: 154-238, 1048-1258 [in Russian].

[4]. Wiener EL, Kanki BG, Helmreich RL. Cockpit Resource Management. New York: Academic Press; 1993.

[5]. Plotnikov NI. Resursy bezopasnosti transportnykh kompleksov. Monografia [Transportation systems safety resources. A monograph]. Novosibirsk: Research project center AviaManager; 2013 [in Russian].

[6]. Plotnikov NI.Osnovaniya teorii nadezhnosti cheloveka-operatora (pilota) [The bases of human operator (pilot) dependability theory]. Dependability 2015;2(53):94-97.

[7]. Zinoviev AA. Logika naouki [The logic of science]. Moscow: Mysl; 1971 [in Russian].

[8]. <https://ru.wikipedia.org/>

[9]. Oushakov DN, editor. Explanatory Dictionary of the Russian Language. Vol. 4. Moscow: Gosudarstvenny Institut Sovietskaya entsiklopedia; OGIZ; Gosudarstvennoye izdatelstvo inostrabnnykh i natsinalnykh slov; 1935-1940 [in Russian].

[10]. Novikov DA. Teoriya upravleniya organizatsionnymi sistemami [Theory of management systems control]. 2nd edition. Moscow: Fizmatlit; 2007 [in Russian].

[11]. Lossky NO. Logika [Logic]. Obelisk; 1923 [in Russian].

[12]. Getmanova AD. Ouchebnik po logike [Logic textbook]. 2nd edition. Moscow: Vlados; 1995 [in Russian].

[13]. Goncharov SS, Yershov YuL, Samokhvalov KF. Vvedenie v logiku i metodologiyu naouki [Introduction into the logic and methodology of science]. Moscow: Interpraks, Novosibirsk: RAS SB Institute of Mathematics; 1994 [in Russian].

[14]. Books by the author of this paper can be downloaded at: http://aviam.org/index.php/layout/library

About the author:

Nikolay I. Plotnikov, Candidate of Engineering, Senior Researcher, Director General, Aviamanager Research and Design Institute of Civil Aviation, Novosibirsk, Russia, e-mail: ama1990@mail.ru

Received on 07.08.2017

Dependability, vol. 18 no.2, 2018 Original article
DOI:10. 21683/1729-2646-2018-18-2-31-37

On the task of allocating investment to facilities preventing unauthorized movement of road vehicles across level crossings for various statistical criteria[1]

Andrey I. Kibzun, *Moscow Aviation Institute, Moscow, Russia*
Alexey N. Ignatov, *Moscow Aviation Institute, Moscow, Russia*

Andrey I. Kibzun

Alexey N. Ignatov

Aim. *Railway transportation is affected by a whole range of transportation incidents, both related to rolling stock, i. e. vehicle-to-vehicle collisions, derailments, broken cast parts of bogies, etc. , and infrastructure, i. e. broken rail, fires at railway stations and terminals, broken catenary, etc. Among the above incidents, collisions at level crossings are the most likely to cause a public response, as collisions between trains and road vehicles often cause multiple deaths that are reported in national media, which entails significant reputational damage for JSC RZD. Additionally, collisions often cause derailment of vehicles, which may cause deaths and major environmental disasters, if dangerous chemical products are transported. Beside the reputational damage, collisions at level crossings cause significant expenditure related to the repair of damaged infrastructure and rolling stock, as well as damage caused by trains idling due to maintenance machines operation at the location of incident. That brings up the issue of optimal allocation of investment to facilities preventing unauthorized movement of road vehicles across level crossings (hereinafter referred to as protection systems). This problem is of relevance, as replacing level crossings with tunnels and viaducts is not going fast and does not imply the eventual elimination of all level crossing. Hence is the requirement for rational allocation of funds to the installation of protection systems over the extensive railway network. Given the above, the aim of this paper is to develop decision-making guidelines for the reduction of the number of transportation incidents in terms of statistical criteria, i. e. quantile and probabilistic.* **Methods.** *The paper uses methods of deterministic equivalent, of equivalent transformations, of the probability theory, of optimization.* **Results.** *The problem of maximizing the probability of no incidents is reduced to integer linear programming. For the problem of minimizing the maximum number of incidents guaranteed at the given level of dependability, a suboptimal solution of the initial problem of quantile optimization is suggested that is obtained by solving the integer linear programming problem through the replacement of binomially distributed random values with Poisson values.* **Conclusions.** *The examined models not only allow developing an optimal strategy with guaranteed characteristics, but also demonstrate the sufficiency or insufficiency of the investment funds allocated to the improvement of level crossing safety. Decision-making must be ruled by the quantile criterion, as the probability of not a single incident occurring may seem to be high, yet the probability of one, two, three or more incidents occurring may be unacceptable. The quantile criterion does not have this disadvantage and allows evaluating the number of transportation incidents guaranteed at the specified level of dependability.*

Keywords: *level crossing, collision, probability, quantile, integer programming.*

For citation*: Kibzun AI, Ignatov AN. On the task of allocating investment to facilities preventing unauthorized movement of road vehicles across level crossings for various statistical criteria. Dependability 2018; 2: 31-37. DOI:10. 21683/1729-2646-2018-18-2-31-37*

[1] The work was performed with the support of the Russian Science Foundation (project no. 16-11-00062) (paper sections 2, 3, 4. 1) and the support of the Russian Foundation of Basic Research and JSC RZD as part of research project no. 17-20-03050 ofi_m_RZD (paper sections 4. 2, 5).

1. Introduction

According to [1], the value of risk is a functional that associates the probability (and frequency) of an event and the expectation of the aftermath (damage) of such event. The general method of evaluation of risks associated with the above and other adverse events was addressed in [2, 3].

Most research dedicated to disasters in level crossings deals with either regressive models of correlation between the number of incidents and various factors [4, 5], or obtaining a certain cumulative index that characterizes the level of hazard/safety of a level crossing [6, 7]. A general concept of development of a strategy for protection system deployment is set forth in [7-9]. In [7], various approaches are discussed to the evaluation of the efficiency of installation of specific protection systems at specific level crossings that was based on a set of average characteristics. In order to solve the problem related to the deployment of protection systems throughout the railway network, it was suggested to use a deterministic number that characterizes the number of transportation incidents per year at a specific level crossing. However, the number of railway incidents is a random value, while the problem of rational allocation of funds was described only verbally. In [8], the problem of rational allocation of funds to protection systems installation was defined mathematically, yet as the measure of the value of a system's installation the average profit is used. However, average characteristics cannot be used to obtain any guaranteed characteristics that play a very important role in railway processes that may cause harm to people. In [9], the unit of the utility of installation of a protection system is a certain deterministic value that is obtained from an expected number of incidents at a level crossing.

This paper examines the problem related to the allocation of funds to the installation of protection systems over a railway network. Each crossing may have a unique number of protection systems available for installation, while their number may be random. It is assumed that the set of protection systems already installed at a crossing is specified. In order to define the optimal strategy of protection systems deployment, the probability is examined of not a single transportation incident occurring over a period of time. The maximum number of transportation incidents that will occur at the given level of dependability is studied as well.

2. Primary designations and assumptions

Let us consider a railway network that consists of N level crossings, in which i-th crossing may be equipped with any available M_i different protection systems, $i=\overline{1,N}$. Note that the number of protection systems available for installation may vary depending on the crossing due, for example, to the geographical features of the crossing location. Hence, it might turn out that M_1=8, while M_2=9. A protection system is understood as set of measures aimed at preventing transportation incidents (e. g. automatic level crossing signalling with automatic barriers, automatic level crossing signalling with automatic barriers and rising barrier, etc.). Let the j-th system at the i-th level crossing be characterized by the probability $P_{i,j}$ of collision between an automotive vehicle and railway rolling stock, $i=\overline{1,N},\ j=\overline{1,M_i}$. Let us assume that the protection systems are sorted based on the safety level, i.e. $\forall i \in \{1,2,\ldots,N\}$ and $\forall j \in \{1,2,\ldots,M_i-1\}$ the following is true

$$P_{i,j+1} \le P_{i,j}. \tag{1}$$

Let us assume that over a long time period (month, year) T the i-th level crossing is crossed by n_i trains, $i=\overline{1,N}$. On a line section with 2 or more tracks 2 or more trains can simultaneously be on a level crossing. Without loss of generality, further we will omit this case that can be taken into consideration within the given model, if we understand n_i as the number of cases when a level crossing was occupied by trains.

Let the variable u_i^0 designate the number of the protection system currently installed at the i-th level crossing, while the variable $u_{i,j}^0$ characterize whether a protection system with the number j is installed at the i-th level crossing: 0 if not installed, 1 if installed. Let us introduce control variables: let the variable u_i designate the number of the protection system currently installed at the i-th level crossing, while the variable $u_{i,j}$ characterize whether a protection system with the number j is installed at the i-th level crossing: 0 if not installed, 1 if installed.

Also, let the cost of installation of the j-th system at the i-th level crossing be $c_{i,j}$ currency units $i=\overline{1,N},\ j=\overline{1,M}$, while the total investment fund of protection systems installation is C^0 currency units. As the i-th level crossing is already equipped with the protection system with the number u_i^0, it is not required to install it again, i. e. $c_{i,u_i^0}= 0$, $i=\overline{1,N}$. Further, in virtue of (1) the installation at the i-th level crossing of the protection system with the index u_i^0 is impossible, so we can assume $c_{i,j}$=0 for $1 \le j < u_i^0, i=\overline{1,N}$. It must be noted that the remaining coefficients $c_{i,j}$ also depend on u_i^0, $i=\overline{1,N},\ j=\overline{1,M_i}$. Further, we will assume that

$$\sum_{i=1}^{N} \max_{1 \le j \le M_i} c_{i,j} > C^0,$$

as otherwise the cost of a set of the most expensive protection systems does not exceed the investment fund, which makes the problem of optimization related to the resource distribution trivial.

Let us introduce the following designations:

$$u^0 \overset{\text{def}}{=} \begin{pmatrix} u_1^0, u_2^0, \ldots, u_N^0, u_{1,1}^0, u_{1,2}^0, \ldots, u_{1,M_1}^0, u_{2,1}^0, \\ u_{2,2}^0, \ldots, u_{2,M_2}^0, \ldots, u_{N,1}^0, u_{N,2}^0, \ldots, u_{N,M_N}^0 \end{pmatrix},$$

$$u \overset{\text{def}}{=} \begin{pmatrix} u_1, u_2, \ldots, u_N, u_{1,1}, u_{1,2}, \ldots, u_{1,M_1}, u_{2,1}, \\ u_{2,2}, \ldots, u_{2,M_2}, \ldots, u_{N,1}, u_{N,2}, \ldots, u_{N,M_N} \end{pmatrix}.$$

Then the set of acceptable strategies $U(u^0)$ that depends on the initial system state, i.e. already installed set of protection systems, consists of various vectors u to which restrictions are applied:

$$u_i \in \{1,2,\dots,M_i\}, i=\overline{1,N},$$

$$u_{i,j} \in \{0,1\}, i=\overline{1,N}, j=\overline{1,M_i},$$

$$\sum_{j=1}^{M_i} u_{i,j} = 1, i=\overline{1,N}, \quad (2)$$

$$\sum_{j=1}^{M_i} j u_{i,j} = u_i, i=\overline{1,N}, \quad (3)$$

$$u_i \geq u_i^0, i=\overline{1,N}, \quad (4)$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M_i} c_{i,j} u_{i,j} \leq C^0. \quad (5)$$

Restrictions (2) – (3) guarantee that each level crossing can be equipper with only one protection system. Restriction (4) in virtue of (1) guarantees that the selection and installation of new protection systems will not increase the probability of collision between trains and automotive vehicles. Restriction (5) regards the maximum amount of funds that can be directed towards the installation of new protection systems, i.e. is a budget restriction.

3. Problem definition

Under the made assumptions we conclude that when one passenger or freight train passes over a level crossing the probability of its collision with automotive vehicles is

$$P_i = \sum_{j=1}^{M_i} u_{i,j} P_{i,j}. \quad (6)$$

Therefore, the number of collisions X_i within the time period T between automotive vehicles and passenger/freight trains is described with a binomial random value with the parameters n_i and P_i, i.e. $X_i \sim \mathrm{Bi}(n_i, P_i)$.

Let us introduce a new random value X that has the meaning of the total number of collisions throughout the railway network over the time period T:

$$X(u) \overset{\text{def}}{=} X_1(u) + X_2(u) + \dots + X_N(u).$$

Let us consider the probability function

$$P_\phi(u) \overset{\text{def}}{=} P\{X(u) \leq \phi\}, \phi \in \mathbb{Z}_+,$$

and quantile function

$$\phi_\alpha(u) \overset{\text{def}}{=} \min\{\phi : P_\phi(u) \geq \alpha\}, \alpha \in (0,1).$$

Function $P_\phi(u)$ characterizes the probability that within the time period T not more than φ transportation incidents occur throughout the railway network. Function $\phi_\alpha(u)$ characterizes the maximum number of incidents at the specified level of dependability α. As the given problem concerns the improvement of system dependability, further we will be considering only the case α>1/2.

Using probability and quantile functions, let us formulate two problems

$$u_0^* = \arg\max_{u \in U(u^0)} P_0(u), \quad (7)$$

$$u_\alpha^* = \arg\min_{u \in U(u^0)} \phi_\alpha(u). \quad (8)$$

Problem (7) concerns the search for the optimal strategy that would ensure the maximum probability of not a single incident occurring over the given period of time. Note that a similar problem was researched in [11], where the problem of the probability of at least one collision between shunting consists and passenger/freight trains within a given period of time was examined. However, [11] examined the analysis problem, while this paper looks at the synthesis problem. Problem (8) concerns the search for the strategy that would allow minimizing the maximum number of incidents guaranteed at the given level of dependability.

4. Solution of the problem

4. 1 Probability function optimization problem

Let us find the value of the probability of not a single incident occurring over the given period of time T. Due to the fact that the number of transportation incidents cannot be negative, we obtain

$$P_0(u) = P\{X(u) \leq 0\} = P\{X(u) = 0\} = \\ = P\{X_1(u) + X_2(u) + \dots + X_N(u) = 0\}. \quad (9)$$

As the number of transportation incidents at each level crossing cannot be negative either, out of (9) follows

$$P_0(u) = P\{\{X_1(u)=0\} \cdot \{X_2(u)=0\} \cdot \dots \cdot \{X_N(u)=0\}\}.$$

Given that the number of transportation incidents at one level crossing does not affect the number of transportation incidents at the others, the random values $X_1(u)$, $X_2(u)$, …, $X_N(u)$ are independent in total, therefore according to the formula of multiplication of probabilities [10]

$$P_0(u) = P\{X_1(u)=0\} \cdot P\{X_2(u)=0\} \cdot \dots \cdot P\{X_N(u)=0\}. \quad (10)$$

Out of (6) and (10) follows that

$$P_0(u) = (1-P_1)^{n_1} \cdot (1-P_2)^{n_2} \cdot \ldots \cdot (1-P_N)^{n_N} =$$
$$= \left(1-\sum_{j=1}^{M_1} u_{1,j}P_{1,j}\right)^{n_1} \cdot \left(1-\sum_{j=1}^{M_2} u_{2,j}P_{2,j}\right)^{n_2} \cdot$$
$$\cdot \ldots \cdot \left(1-\sum_{j=1}^{M_N} u_{N,j}P_{N,j}\right)^{n_N}. \quad (11)$$

Through equivalent transformations let us reduce the resulting nonlinear programming problem to a linear programming problem. For this purpose, let us consider a new function

$$\hat{P}_0(u) \stackrel{\text{def}}{=} \ln(P_0(u))$$

and define the problem

$$\hat{u}_0^* = \arg \max_{u \in U(u^0)} \hat{P}_0(u). \quad (12)$$

Note, that the solutions of problems (7) and (12) will be identical, as the logarithm is a monotonic increasing function. Let us consider in detail the structure of function $\hat{P}_0(u)$:

$$\hat{P}_0(u) = \ln\left(\begin{array}{c}\left(1-\sum_{j=1}^{M_1} u_{1,j}P_{1,j}\right)^{n_1} \cdot \left(1-\sum_{j=1}^{M_2} u_{2,j}P_{2,j}\right)^{n_2} \cdot \\ \cdot \ldots \cdot \left(1-\sum_{j=1}^{M_N} u_{N,j}P_{N,j}\right)^{n_N}\end{array}\right) =$$
$$= n_1 \ln\left(1-\sum_{j=1}^{M_1} u_{1,j}P_{1,j}\right) + n_2\left(1-\sum_{j=1}^{M_2} u_{2,j}P_{2,j}\right) + \ldots +$$
$$+ n_N \ln\left(1-\sum_{j=1}^{M_N} u_{N,j}P_{N,j}\right) = \sum_{i=1}^{N} n_i \ln\left(1-\sum_{j=1}^{M_i} u_{i,j}P_{i,j}\right).$$

Function $\hat{P}_0(u)$ is nonlinear again, yet due to the fact that according to the problem's definition under a certain fixed i out of all variables $u_{i,j}$ only one takes on the value equal to one, while all the others equal to zero, by making the change of variables

$$\hat{P}_{i,j} = \ln\left(1-P_{i,j}\right),$$

we obtain a representation of function $\hat{P}_0(u)$ linear in the controllable variables:

$$\hat{P}_0(u) = \sum_{i=1}^{N} n_i \sum_{j=1}^{M_i} u_{i,j}\hat{P}_{i,j}. \quad (13)$$

Thus, the optimization of nonlinear function (11) is reduced to the problem of optimization of linear function (13) in the set of acceptable strategies $U(u^0)$, and the problem of integer linear programming is obtained that can be solved in IBM ILOG Cplex and belongs to the class of knapsack problems [12].

4. 2. Quantile function optimization problem

Let us now find the expression for quantile function $\phi_\alpha(u)$. By definition we obtain

$$P_\phi(u) = P\{X(u) \le \phi\} = P\{\{X(u) = 0\} +$$
$$+\{X(u) = 1\} + \ldots + \{X(u) = \phi\}\}.$$

As for $k_1 \neq k_2$ the events $\{X(u)=k_1\}$ and $\{X(u)=k_2\}$ are incompatible, due to the fact that within one given period of time T different numbers of incidents cannot occur, using the formula of composition of probabilities [10] we obtain

$$P_\phi(u) = P\{X(u)=0\} + P\{X(u)=1\} + \ldots + P\{X(u)=\phi\}. \quad (14)$$

As shown above, identifying the probability of not a single incident $P\{X(u)=0\}$ occurring over the given period of time T itself is not trivial, let alone identifying other probabilities in formula (14). Thus, in finding the quantile function let us use the Poisson approximation, as n_i is a large number, while due to P_i being close to zero, out of the problem definition we obtain

$$\mathbf{M}[X_i] = n_iP_i \approx n_iP_i - n_iP_i^2 = n_iP_i(1-P_i) = \mathbf{D}[X_i],$$

i.e. let us consider new random values

$$\tilde{X}_i(u) \sim \Pi(n_iP_i),\ \tilde{X}(u) \stackrel{\text{def}}{=} \tilde{X}_1(u) + \tilde{X}_2(u) + \ldots + \tilde{X}_n(u)$$

and new functions

$$\tilde{P}_\phi(u) \stackrel{\text{def}}{=} P\{\tilde{X}(u) \le \phi\},$$

$$\tilde{\phi}_\alpha(u) \stackrel{\text{def}}{=} \min\{\phi : \tilde{P}_\phi(u) \ge \alpha\}, \alpha \in (0,1).$$

Let us define a new problem

$$\tilde{u}_\alpha^* = \arg \min_{u \in U(u^0)} \tilde{\phi}_\alpha(u). \quad (15)$$

Note, that the solutions of problems (8) and (15) may not be identical, yet the solution of problem (15) will be suboptimal for problem (8).

Let us find the analytic expression of function $\tilde{P}_\phi(u)$. As random values $X_1(u), X_2(u), \ldots, X_N(u)$ are independent in total, random values $\tilde{X}_1(u), \tilde{X}_2(u), \ldots, \tilde{X}_N(u)$ are independent in total as well. Therefore,

$$\tilde{X}(u) \sim \Pi\left(\sum_{i=1}^{N} n_iP_i\right),$$

$$\tilde{P}_\phi(u)=\exp\left\{-\sum_{i=1}^{N}n_iP_i\right\}\sum_{k=0}^{\phi}\frac{\left(\sum_{i=1}^{N}n_iP_i\right)^k}{k!}=$$

$$=\exp\left\{-\sum_{i=1}^{N}n_i\sum_{j=1}^{M_i}u_{i,j}P_{i,j}\right\}\sum_{k=0}^{\phi}\frac{\left(\sum_{i=1}^{N}n_i\sum_{j=1}^{M_i}u_{i,j}P_{i,j}\right)^k}{k!}.$$

As in order to find strategy $\tilde{u}_\alpha^*$ functions $\tilde{P}_\phi(u)$ must be optimized for different φ, in order to simplify the optimization let us introduce a new function

$$L_\phi(u)=\ln(\tilde{P}_\phi(u))=-\sum_{i=1}^{N}n_i\sum_{j=1}^{M_i}u_{i,j}P_{i,j}+\ln\sum_{k=0}^{\phi}\frac{\left(\sum_{i=1}^{N}n_i\sum_{j=1}^{M_i}u_{i,j}P_{i,j}\right)^k}{k!}.$$

and define new problems

$$u_{L_\phi}^*=\arg\max_{u\in U(u_0)}L_\phi(u), \tag{16}$$

where $\phi = 0,1,2,\ldots$. Note, that (16) are problems of mixed integer nonlinear programming and can be solved using Opti Toolbox. Let

$$\phi^*=\min\{\phi\in \mathbb{Z}_+ : \exp\{L_\phi(u_{L_\phi}^*)\}\geq\alpha\},$$

then $\tilde{u}_\alpha^*=u_{L_{\phi^*}}^*, \tilde{\phi}_\alpha(\tilde{u}_\alpha^*)=\phi^*$.

5. Example

Let a railway network comprising 10 level crossings be equipped with the following systems preventing unauthorized movement of road vehicles across level crossings:

(i) signs warning of the approach to a level crossing
(ii) automatic signalling
(iii) automatic signalling with blinking lunar white aspect
(iv) automatic signalling with semi-automatic barriers
(v) automatic signalling with automatic barriers
(vi) automatic signalling with rising barriers
(vii) automatic signalling with a full barrier that creates a physical obstacle to unauthorized movement of road vehicles across the crossing when a train approaches
(viii) viaduct.

Let us define a set of protection systems installed on the railway network, the number of trains travelling across a crossing every 24 hours, as well as the cost of various protection systems and the probability of collision according to information from publicly available sources, expert evaluations and [7].

Let us comment on the choice of collision probability numbers in Table 2. According to [1], "when calculating event probabilities, it is assumed that according to expert data 5 percent of pedestrians do not evaluate the danger caused by the approaching train, 10 percent of pedestrians evaluate the danger incorrectly (believing they will be able to cross the track before the approaching train, etc.)", while according to [13, 14] the probability of signal violation by a shunting engine driver is around 10^{-4}, therefore in real life the numbers given in Table 2 below may turn out to be higher. Let us also note that this example refers to cases when all level crossings are equipped with identical protection systems with identical collision probabilities.

In Table 3, highlighted in grey are those protection systems that definitely will not be installed at level crossings due to condition (1).

Let us assume that total funds allocated for the installation of protection systems are C^0=2 mil. rubles, while the time T of observation of transportation incidents is one year. Let us find optimal strategies of maximizing the probability function, as well as the suboptimal strategy of optimizing the quantile function if $\alpha = 0{,}95$.

Before finding the solution of the optimal protection system installation problem let us note that the example under consideration cannot be fully interpreted as real-life example, as a real railway network has much more level crossings than ten, while data regarding collision probability is confidential.

As it follows from Table 4, a criterion in the form of probability identifies the most "vulnerable" spot of the railway network, that is level crossing no. 3, as it has the

Table 1. Data regarding the preinstalled protection systems at level crossings and trains going across them used in solving the problems (7) and (8)

Crossing number	1	2	3	4	5	6	7	8	9	10
Number of trains running across the level crossing (per 24 hours)	11	20	100	35	9	8	5	20	50	60
Preinstalled protection system	i	ii	i	ii	i	i	i	ii	iv	iv

Table 2. Data regarding the probability of collision at the moment of train going across various level crossings equipped with various safety solutions

Crossing number	Possible protection systems (probability of collision)							
Any	i ($5\cdot10^{-4}$)	ii (10^{-5})	iii ($8\cdot10^{-6}$)	iv ($6\cdot10^{-6}$)	v ($2\cdot10^{-6}$)	vi (10^{-6})	vii ($5\cdot10^{-7}$)	viii (0)

Table 3. Data regarding the cost of installation of various protections systems at various level crossings

Crossing number	Possible protection systems (probability of collision)							
1	i (0)	ii (0.6)	iii (0.7)	iv (0.9)	v (1)	vi (1.2)	vii (1.5)	viii (800)
2	i	ii (0)	iii (0.1)	iv (0.3)	v (0.4)	vi (0.6)	vii (0.9)	viii (800)
3	i (0)	ii (0.6)	iii (0.7)	iv (0.9)	v (1)	vi (1.2)	vii (1.5)	viii (800)
4	i	ii (0)	iii (0.1)	iv (0.3)	v (0.4)	vi (0.6)	vii (0.9)	viii (800)
5	i (0)	ii (0.6)	iii (0.7)	iv (0.9)	v (1)	vi (1.2)	vii (1.5)	viii (800)
6	i (0)	ii (0.6)	iii (0.7)	iv (0.9)	v (1)	vi (1.2)	vii (1.5)	viii (800)
7	i (0)	ii (0.6)	iii (0.7)	iv (0.9)	v (1)	vi (1.2)	vii (1.5)	viii (800)
8	i	ii (0)	iii (0.7)	iv (0.9)	v (1)	vi (1.2)	vii (1.5)	viii (800)
9	i	ii	iii	iv (0)	v (0.1)	vi (0.3)	vii (0.6)	viii (800)
10	i	ii	iii	iv (0)	v (0.1)	vi (0.3)	vii (0.6)	viii (800)

Table 4. Optimal strategies of protection systems installation

Problem \ Crossing number	1	2	3	4	5	6	7	8	9	10
Maximization of probability function	ii	ii	iii	ii	ii	i	i	ii	iv	v
Minimization of quantile function	ii	ii	ii	ii	ii	i	i	ii	v	v

highest rate of trains travelling across it and the installed protection system allows for a high probability of collision. Both criteria are characterized by the fact that the strategies they produce "suggest" maximizing the quality of the level crossings equipped with protection systems no. 1, but not maximizing the quality of the level crossings with high traffic volume (nos. 9 and 10). It should be noted that in this example when substituting the quantile-optimal strategy into function $P_0(u)$ we obtain practically the same value as $P_0(u_0^*)$. Decision-making must be ruled by the quantile criterion, as the probability of not a single incident occurring may turn out to be high, while the probability of one, two, three or more incidents occurring may be unacceptable. The quantile criterion does not have this disadvantage and allows evaluating the number of transportation incidents guaranteed at the specified level of dependability. $\tilde{\phi}_\alpha(\tilde{u}_\alpha^*)=6$, while $P_0(u_0^*)=0{,}0422$, which means that the investment fund in this example is not sufficient for satisfactory operation (from the safety point of view) of level crossings.

6. Conclusion

The paper considers the problem of allocating investment to facilities preventing unauthorized movement of road vehicles across level crossings. It also examines the feasibility of both installing protection systems at an unequipped level crossing and improving the existing protection systems. The problem of maximizing the probability of no collisions occurring is reduced to the problem of integer linear programming (this result, as well as problem definition, were obtained with the support of the Russian Science Foundation (project no. 16-11-00062)). For the problem of minimizing the maximum number of transportation incidents occurring at the specified level, a suboptimal solution was proposed that is obtained by solving integer linear programming problems (this result, as well as the results of computational modeling, were obtained with the support of RFB and JSC RZD as part of research project no. 17-20-03050 ofi_m_RZD). The obtained optimal strategies allow making a range of managerial solutions that can be later used by a decision-maker. Additionally, the value $P_0(u_0^*)$ of the probability of no collisions occurring allows judging the sufficiency of investment funds, while the value $\tilde{\phi}_\alpha(\tilde{u}_\alpha^*)$ characterizes the number of transportation incidents that will occur in the future with the predefined probability α, which allows judging the level of risk of collision at level crossings.

References

[1]. GOST 33433-2015. Functional safety. Risk management in railway transportation [in Russian].

[2]. Shubinsky IB, Zamyshliaev AM et al. Evaluation of risks related to signal violations by shunting consists or passenger trains. Dependability 2016;3:39-46.

[3]. Ignatov AN, Kibzun AI, Platonov EN. Metodologiya otsenki i minimizatsii riskov na zheleznodorozhnom transporte [Method of evaluation and minimization of risks in railway transportation]. In: Proceedings of the Third Science and Engineering Conference Intelligent Control Systems in Railway Transportation. Computer and mathematical simulation; 2014. p. 177—179 [in Russian].

[4]. Saccomanno F, Fu L, Miranda-Moreno L. Risk-based model for identifying highway-rail grade crossing blackspots. Transportation Research Record: Journal of the Transportation Research Board 2004; 1862 Traffic Control

Devices, Visibility, and Rail-Highway Grade Crossings: 127-135.

[5]. Cameron AC, Trivedi PK. Regression Analysis of Count Data. New York: Cambridge University Press; 1998.

[6]. Austin R, Carson J. An alternative accident prediction model for highway-rail interfaces. Accident Analysis and Prevention 2002;34(1):31-42.

[7]. Railroad-Highway Grade Crossing Handbook. Revised Second Edition; August 2007.

[8]. Ryan TA. Priority Programming Methodology for Rail-Highway Grade Crossings. Transportation Research Record 1991; 1327 Visibility, rail-highway grade crossings, and highway improvement evaluation: 21-26.

[9]. Konur D, Golias MM, Darks B. A mathematical modeling approach to resource allocation for railroad-highway crossing safety upgrades. Accident Analysis and Prevention 2013;51:192-201.

[10]. Kibzun AI, Goriaynova ER, Naumov AV. Teoriya veroyatnostey i matematicheskaya statistika: Bazovyy kurs s primerami i zadachami [Probability theory and mathematical statistics: Basic course with examples and problems]. Moscow: Fizmatlit; 2007 [in Russian].

[11]. Ignatov AN, Kibzun AI, Platonov EN. Otsenka veroiatnosti stolknoveniya zheleznodorozhnykh sostavov na osnove puassonovskoy modeli [Poisson model-based probability estimation of train-to-train collision]. Avtomatika i telemekhanika 2016;11:43-59 [in Russian].

[12]. Bretthauer KM, Shetty B. The nonlinear knapsack problem – algorithms and applications. European J. of Operational Research 2002;138(3):459–472.

[13]. Shubinsky IB. Funktsionalnaya nadezhnost informatsionnykh sistem. Metody analiza [Functional dependability of information systems. Analysis methods]. Ulianovsk: Oblastnaya tipografia Pechatny dvor; 2012.

[14]. Zamyshliaev AM, Shubinsky IB, Ignatov AN et al. Metodika vychisleniya veroyatnosti stolknoveniya passazhirskogo poezda s manevrovym sostavom na zheleznodorozhnoy stantsii [Method of calculation of probability of a passenger train collision with a shunting consist in a railway station]. In: Proceedings of the Fourth Science and Engineering Conference Intelligent Control Systems in Railway Transportation; 2015:124-127 [in Russian].

About the authors

Andrey I. Kibzun, Doctor of Physics and Mathematics, Professor, Moscow Aviation Institute, Head of Chair, Moscow, Russia, phone: +7 (499) 158 45 60, e-mail: kibzun@mail.ru

Alexey N. Ignatov, Candidate of Physics and Mathematics, Moscow Aviation Institute, postgraduate student, Moscow, Russia, phone: +7 (906) 059 50 00, alexei.ignatov1@gmail.com

Received on 16.07.2017

Original article
DOI: 10.21683/1729-2646-2018-18-2-38-41

Method of conversion of MTBF from cycles to kilometers travelled

Maria V. Belousova, *PF KMT, Saint Petersburg, Russia*
Vitaly V. Bulatov, *PF KMT, Saint Petersburg, Russia*

Maria V. Belousova

Vitaly V. Bulatov

Abstract. **Aim.** *At machine-building enterprises, dependability indicators are evaluated at the stage of product design and later based on operational data. At the design stage, automated software systems are widely used and employ a number of methods of dependability indicators calculation: fault trees, Markov chains, etc. The input data for such calculations are based on the analysis of a product design and properties of its units and elements.In operation, the dependability indicators are analyzed totally differently. Failure information processing involves deficiency reports filed by customers and operating organizations to the service departments of manufacturing enterprises. The total number of failures for all types of products must be evaluated by the dependability service/unit within a specified period of time. This failure data processing procedure is required for the calculation of reliability and maintainability indicators. The resulting numerical characteristics are compared with the standard values set forth in the technical documentation. Based on this comparison the conclusion is made regarding the compliance or non-compliance of a specific product with the specified dependability requirements. The values of the dependability indicators given in the technical documentation are based on the results of dependability testing of prototypes. However, due to the difference in the test conditions, results recording procedures and measurement units, the values of dependability indicators set forth in the technical documentation and collected in the course of operation are not comparable. In the wagon-building industry, the operation time of rolling stock is normally measured in kilometers travelled. However, the operation of a large number of wagon components is measured in cycles, hours, etc. In most cases these measurement units are used to express the values of dependability indicators obtained during prototype testing. In the process of reliability evaluation of plug doors installed in commuter trains, it became necessary to approximately convert the operation time expressed in opening/closing cycles into operation time expressed in kilometers travelled. In light of the emerged problem it was decided to construct a mathematical model that would best reflect the association between the two values. In most cases mathematical models are constructed and verified using the initial observations of the given indicator and the explanatory factors. In this case, the input data is one factor (opening/closing cycles) and one indicator (kilometers travelled), therefore the pair linear regression model can be used.* **Results.** *The correlation between the opening/closing cycle of plug doors and the kilometers travelled by a commuter train was analyzed. The model of pair linear regression was then generated. Verification was conducted, the outcome of which gives ground for the conclusion regarding the representativeness of the resulting data.* **Conclusions.** *The presented method of calculation of the generalizing controllable dependability indicator (mean time between failures) with the example of plug doors shows that the model of pair linear regression can be used for conversion of mean time between failures from cycles into kilometers travelled required for the evaluation of dependability indicators in operation.*

Keywords: *dependability, mean time between failures, rolling stock, plug door, pair linear regression*

For citation: *Belousova MV, Bulatov VV. Method of conversion of MTBF from cycles to kilometres travelled. Dependability 2018;2: 38-41. DOI: 10.21683/1729-2646-2018-18-2-38-41*

Introduction

In the wagon-building industry, the most important factor in the evaluation of dependability indicators is obtaining reliable information on the number and type of failures. Many units of passenger cars operate in cycles, e.g. stepboards, suspensions, doors, etc. The specifications for rolling stock components must contain the values of dependability indicators expressed in cycles, hours, kilometers travelled. The value of operation time in cycles is identified by calculation and confirmed through dependability tests. Evaluating the mean time between failures in cycles over the course of operation is complicated or in some cases impossible. For this reason calculations normally involve the time to failure expressed in kilometers travelled, as in the process of operational testing monitoring statistical data using this continuous value is most practical.

This poses the question of the need for a reliable procedure of operation time values conversion. In Russian literature this matter has not been considered in depth despite its high practical significance. Given the above, the need for the mentioned method is ever more relevant.

Input data

The aim of this research is to identify the type of dependence between the dependent variable y (kilometers travelled by train) and dependent variable x (opening/closing cycles of doors). In similar cases, in technical, socioeconomic and other research, regression analysis is used.

Let us consider the use of the pair linear regression model with the example of conversion of the values of operation time from cycles to kilometers for plug doors of commuter trains.

In accordance with the technical documentation, the controllable indicator of door dependability is the door's mean time to failure $\left(T_{\mathrm{TD}}^{\mathrm{KM}}\right)$, not less than 300 000 opening/closing cycles.

At the first stage, a sample was generated that covered 17 EMU depots (Figure 1) and 27 commuter lines.

The distance between the stations of each line of the sample was evaluated using open source information [2, 3].

As the subjects of research were chosen l_i, the number of opening/closing cycles, and S_i, the distance in kilometers travelled by the train within the number of cycles l_i in the i–th direction, $i = 1, 2, \ldots, n$ (Table 1).

Table 1

i	l_i	S_i, km	i	l_i	S_i, km	i	l_i	S_i, km
1	11	20.853	10	19	40.0389	19	27	93.6574
2	12	36.5727	11	21	53.6669	20	28	62.5396
3	12	26.9275	12	21	61.2759	21	29	66.8559
4	13	34.0387	13	22	58.8737	22	30	62.365
5	13	35.8461	14	22	46.6185	23	34	64.4684
6	13	55.4732	15	23	50.6731	24	39	106.451
7	13	34.7382	16	23	42.7334	25	42	105.483
8	14	32.3032	17	24	59.7328	26	47	129.837
9	15	28.8071	18	24	43.8535	27	47	102.54

According to the primary premises of regression analysis, the number of observations must exceed the number of regression parameters included in the model, otherwise the regression parameters become statistically insignificant [5].

Model of pair linear regression

The empirical method of identifying the functional dependence comes down to evaluating unknown parameters using the least squares method. It is assumed that the factor and indicator are associated with the $y = \lambda+\beta x+\varepsilon$ dependence. First, function $\hat{f}(x)$ is chosen, the values of the parameters of which are identified in such a way as to minimize the sum of deviation squares of actual values of the attribute y_i from the expected value $\hat{y}_i = \hat{f}(x_i)$:

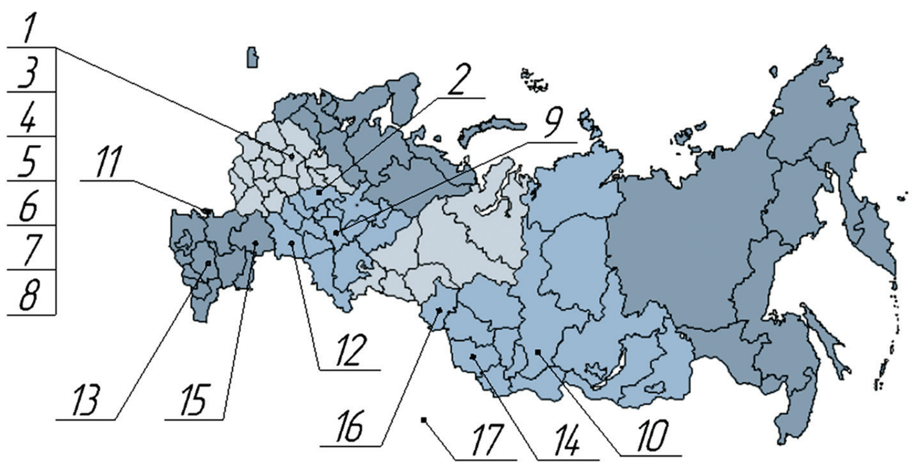


Figure 1.Multipleunitdepot: 1, Aeroxpress; 2, Gorky-Moskovsky; 3, Lobnia (TCh-14 MSK); 4, Aprelevka (TChPRIG-20); 5, Moskva 2, Yaroslavskaya; 6, Nakhabino (TCh-17 MSK); 7, Ramenskoye (TCh-7 MSK); 8, Zhelezhodorozhnaya; 9, Kazan (TChM-17); 10, Karsnoyarsk; 11, Rostov; 12, Anisovka (TChM-14); 13, MoneralnyeVody; 14, Altayskaya; 15, Volgograd; 16, Omsk; 17, Karaganda (Kazakhstan)

$$\min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2.$$

Based on the input data, the diagram was constructed of the dependence between the indicator (km) and the factor (cycles), then the a and b regression coefficients were calculated, as well as the values of $\hat{y}_i$, the regression line was plotted on the correlation field (Figure 2).

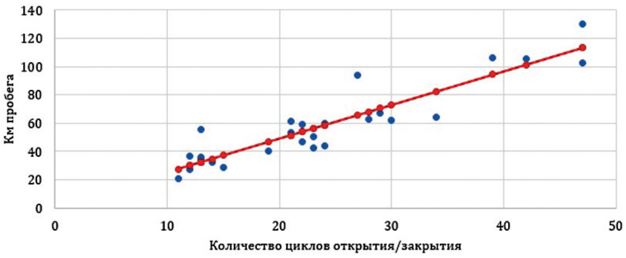


Figure 2. Diagram of dependence between indicator (km) and factor (cycles) and linear regression

Thus, the equation of pair linear regression is as follows:

$$\hat{y} = 2,38684x + 1,27483.$$

Verification of the model

In order to prove the correctness of the resulting equation of pair linear regression, let us use hypotheses of statistical significance of the obtained evaluations.

3.1 Verification of the significance of the correlation coefficient

The measure of the strength of linear connection between two random variables is the Pearson linear correlation coefficient that is evaluated with the sample correlation coefficient r_{xy}. In this case r_{xy} = 92%. In order to verify the hypothesis $H_0$of statistical significance of coefficient r_{xy}statistic $t_r = \frac{r_{xy}\sqrt{n-2}}{\sqrt{1-r_{xy}^2}}$ is calculated, which– if the alternative hypothesis H_1 is true – has the Student's distribution with the number of degrees of freedom n–2. We obtained t_r = 57.5, which is higher than t_{tabl} = 2.06, that is identified out of the Student's distribution table under n–2 degrees of freedom as the critical point that corresponds to the two-sided critical region with the level of significance of 5%. Therefore, coefficient r_{xy} can be deemed significant and, according to the Chaddock's scale, the strength of connection between the indicator and the factor is quite high.

3.2 Verification of the significance of the linear regression

Let us also verify the significance of the linear regression in general. To do that, let us calculate the determination coefficient R^2 and statistic F in formula:

$$F = \frac{R^2}{1-R^2} \cdot (n-2).$$

If the value of this statistic is higher than the critical value under the level of significance of 5%, then hypothesis H_0 on the insignificance of linear regression is discarded. By substituting input data we obtain F = 132.4, which is higher than the critical point per Fisher's distribution table F_{tabl} = 4.24 with (1, n–2) degrees of freedom, therefore the constructed regression equation is statistically significant.

3.3 Verification of homoscedasticity hypothesis

One of the primary assumptions of regression analysis is the homoscedasticity assumption that consists in the equality of dispersions of observations:

$$D(y_i) = \sigma^2, \quad i = 1, \ldots, n.$$

Non-fulfilment of this assumption deteriorates the quality of evaluation of unknown parameters. Homoscedasticity is identified using the Goldfeld-Quandt method [1]. For that purpose, m central observations are excluded from the sample and two independent regression models are constructed, for each of which residual sums of squares are calculated:

$$\tilde{S}_1^2 = \sum_{i=1}^{\frac{n-m}{2}} (y_{1_i} - \hat{y}_{1_i})^2,$$

$$\tilde{S}_2^2 = \sum_{i=\frac{n+m}{2}+1}^{n} (y_{2_i} - \hat{y}_{2_i})^2.$$

Then, statistic $\tilde{F} = \frac{\tilde{S}_2^2}{\tilde{S}_1^2}$ is calculated. If the hypothesis is correct, the F-statistic has Fisher's distribution with $\left(\frac{n-m}{2} - 2; \frac{n-m}{2} - 2\right)$ degrees of freedom. We obtained the value $\tilde{F} = 2,58$, while the critical value per Fisher's distribution table is F_{tabl} = 3.79. As $\tilde{F} < F_{\text{tabl}}$, the homoscedasticity hypothesis is accepted.

Conclusion

Let us construct a point prediction $\hat{y}_p = a + b \cdot x_p$ of indicator $y_p$for the cased when x_p = 300000 and find the confidence interval of the resulting prediction with the level of confidence of 0.95:

$$\hat{y}_p - m_y \cdot t_{tabl} < y_p < \hat{y}_p + m_y \cdot t_{tabl},$$

$$\text{where } m_y = S_{res} \cdot \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}.$$

We will obtain the following prediction value: y_p = 1.274 83 + 2.38684·x_p = 716 053, for which the confidence interval is (58788; 844 217).

Thus, we obtain the value of the controllable dependability indicator (mean time between failures) $T_{TD}^{KM} = 716053 \approx 700000$ that was found using the pair linear regression equation.

The obtained results are to be used as the controllable dependability indicator in the evaluation of the reliability level of plug doors. This approach can be recommended for the evaluation of the mean time to failure of other components of ground passenger transport vehicles that operate in cycles.

References

[1]. Bure VM, Yevseev EA. Osnovyekonomentriki [The fundamentals of econometrics]. Saint Petersburg: IzdatelstvoSPbGU; 2004 [in Russian].

[2]. <https://www.google.ru/maps> (accessed 05.05.2017).

[3]. <https://yandex.ru/maps> (accessed 05.05.2017).

[4]. GOST 27.301-95. Industrial product dependability. Dependability calculation. General provisions [in Russian].

[5]. Förster E, Rönz B. Methods of correlation and regression analysis. Guidebook for economists. Moscow: Financy i statistika; 1983 [in Russian].

About the authors

Maria V. Belousova, Dependability Engineer, PF KMT, Saint Petersburg, Russia, e-mail: 27bmw1993@mail.ru

Vitaly B. Bulatov, Candidate of Engineering, Dependability Engineer, PF KMT, Saint Petersburg, Russia, e-mail: bulatov-vitaly@yandex.ru

Received on 06.11.2017

Dependability, vol. 18 no.2, 2018 Original article
DOI: 10.21683/1729-2646-2018-18-2-42-45

Application of statistical criteria for improving the efficiency of risk assessment methods

Zhanat A. Dayev, *Baishev University Aktobe, Aktobe, Kazakhstan*
Ermek T. Nurushev, *Saint Petersburg State University of Aerospace Instrumentation, Saint Petersburg, Russia*

Zhanat A. Dayev

Ermek T. Nurushev

Abstract. Aim. *The aim of this paper is to improve the efficiency of the failure mode and effects analysis (FMEA) method through the verification of expert judgment correctness by means of statistical methods. Therefore, the paper deals with the matters of improving the quality of products and services in various enterprises through risk-oriented approaches.* **Methods.** *For the purpose of improving the efficiency of the failure mode and effects analysis (FMEA) method, it is suggested to increase the number of experts, while making the expert evaluation an independent process, i.e. by separating the experts from each other. The resulting expert judgment is proposed to be considered as a random value. The correctness of the expert judgment is suggested to be evaluated by means of statistical criteria methods, e.g. Grubbs' criterion methods. The proposed evaluation methods are not limited to the Grubbs' criterion methods. This criterion can be replaced by the Cochran's criterion or Shewhart charts. Each of the suggested methods enables more efficient estimates with lower risks in the process of service provision or product manufacture. The paper proposes statistical methods with the example of the Grubbs' criterion. All indicators of the integral estimation of the failure mode and effects analysis are submitted to statistical verification.* **Results.** *Such data verification as part of an independent expert evaluation enables a higher reliability of expert judgment and significantly reduces the number of risks at the enterprise. Such risks may include bribing or collusion of experts involved in the performance of the failure mode and effects analysis. Independent expert judgments after expert evaluation are verified by means of statistical methods. Sharp spikes in independent expert opinions will justify repeated expert evaluation, while complete agreement of evaluations will eliminate doubts regarding the quality of the performed assessment. The use of statistical methods for the evaluation of every indicator of the integral FMEA estimation will allow increasing its reliability. A combination of those approaches enables an independent estimation as part of various projects evaluation, including the evaluation of industrial products or provided service using failure mode and effects analysis, elimination of the human factor in the estimation procedure, significant reduction of risks.* **Conclusions.** *Failure mode and effects analysis (FMEA) was performed. Additionally, the method was improved by means of independent expert assessment. The consistency of the* **results** *of such evaluation is verified by means of statistical methods. The performance of such verification of independent expert opinion is demonstrated through the Grubbs' criterion. Expert opinion can also be verified by means of Cochran's criterion, Shewhart charts. The proposed approach is a combination of the failure mode and effects analysis (FMEA) method and statistical methods with the example of Grubbs' criterion.*

Keywords: *risk, Grubbs' criterion, FMEA, expert evaluation, quality.*

For citation: *Dayev ZhA, Nurushev ET. Application of statistical criteria for improving the efficiency of risk assessment methods. Dependability 2018;2: 42-45. DOI: 10.21683/1729-2646-2018-18-2-42-45*

Introduction

According to [1], the new edition of the ISO 9000 series of quality standards explicitly regulates the concept of risk and its consequences in respect to the earlier versions. Taking into account the concept of risk, this approach comes from the need to quantify the undesirable consequences that may arise in various enterprises. Therefore, the new edition of standards aims to minimize them. The global experience, described in [1], shows that not all quality management system processes have the same level of risk in terms of the organization's ability to perform its tasks, and the consequences of a process, a product, a service or a system inadequacy vary among organizations. However, there is always a need for their quantification.

There are various methods of risk assessment that are described in detail in [2, 3]. One of the most relevant and well-tested is the Failure Mode and Effect Analysis (FMEA) [4, 5]. This method has proven to be useful in many industries. Nevertheless, in this article the authors aim to improve this method by means of statistical methods for better quality control of products and services at various enterprises.

Problem definition

According to [5], FMEA is a systematic method used for identifying the types of potential failures, their causes and effects as well as the effect of failures on the functioning of the system (whole system or its components). The method is based on the assessment of the system and its components by a group of experts who evaluate any potential defect or possible failure based on three factors:

1. S, the severity of the failure;
2. O, the probability of failure occurrence;
3. D, the probability of the failure being detected before any effects occur.

Each of the factors is rated on a scale from 1 to 10. The first two factors have a direct scale, i.e. the higher is the failure severity or the probability of occurrence, the higher is the corresponding score. The third factor has an inverse scale, i.e. the higher is the probability of the failure being detected, the lower is the corresponding score. The integral estimation of the failure criticality (RPN, Risk Priority Number) is the result of multiplying these three scores. The RPN values range from 1 to 1000 and are used for evaluating the level of risk of the failure [4]. According to [5], the boundary value (RPN_b) is considered acceptable if it is lower than 125. The requirements for the competence of experts conducting FMEA assessment are very high. The recommended number of experts varies from 4 to 8. Each of the experts is a person with a lot of experience and knowledge in a certain field.

According to [2], the most complicated and the most vulnerable part of any system is the human being. The negative influence of human behavior and performance on safety is called the "human factor". In the FMEA method the expert can be this weakest link. Therefore, there might be some shortcomings in the risk and failure assessment by means of FMEA in the form of human factor. In this regard, given the importance and significance of ensuring the quality of manufactured goods and services in enterprises, we propose to improve the FMEA method in order to eliminate the human factor by means of statistical methods. The result of expert estimation is the general population. There might be situations when one or several estimations deviate from the general population. In statistics these estimations are called outliers [6, 7]. To ensure the validity, the outliers have to be excluded from the data set by means of statistical methods.

Use of Grubbs' criterion

In [4, 5], the experts collectively evaluate the severity, the probability of occurrence and the detectability of a defect or a failure during brainstorming. The expert evaluation procedure is proposed to be carried out as an independent process. The result of the evaluation is the population of estimations. Due to various factors, the resulting scores may differ with some of them being "suspicious". Therefore, it is proposed to exclude such results using the Grubbs' criterion.

The resulting expert judgment is considered a random value. Suppose that there is one outlier among the results. Let the observed sample consist of n expert opinions. The observed sample is the following series:

$$X_{(1)}, X_{(2)}, \dots X_{(n)}$$

Let us form an ordered series for this sample by ranking the results:

$$X_1, X_2, \dots X_n.$$

According to [6], the hypothesis that all results belong to the same population is tested. The alternative hypothesis is that the extreme values of the ordered series may belong to another distribution law and may be outliers from the general population.

To test whether the maximum value is an outlier, the Grubbs' statistic is calculated:

$$G_{\max} = \frac{X_n - \overline{X}}{s}, \tag{1}$$

where the expected value or the sample mean is:

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{2}$$

and the root-mean-square deviation (RMSD) is:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2}. \tag{3}$$

To test whether the minimum value is an outlier, another Grubbs' statistic is calculated:

$$G_{\min} = \frac{\overline{X} - X_1}{s}. \tag{4}$$

Using formulas (1) to (4) the statistical measures and Grubbs' statistics are calculated. The maximum or the minimum element is considered an outlier if the corresponding test value is greater than the critical value:

$$\begin{cases} G_{\max} \geq G_{n,1-\alpha} \\ G_{\min} \geq G_{1,1-\alpha} \end{cases}, \tag{5}$$

where α is the significance level. The significance level can be chosen in accordance with the adopted approaches in [6, 7].

If the inequations (5) are false, the extreme values of the evaluation results are considered outliers that need to be excluded, and the experts that made these evaluations have to be interviewed in order to identify the reasons behind their choice of scores.

The critical values are chosen according to the random values distribution law. These values for a normally distributed population can be found in [7]. If there are supposedly two outliers, the population can be tested with the two-sided Grubbs' test presented in [6, 7].

After the exclusion of the outliers, the mean value of the evaluations can be seen as the resulting score. The mean value of scores can be considered the most reliable result based on the practical certainty principle that is explained in [8].

Results and discussion with the example

According to [4], the recommended number of participants is 4 to 8. Supposing a team of eight experts conducts an independent assessment of an aircraft's elements condition by scoring every potential failure on the severity (S), the probability of occurrence (O) and the detectability (D) factors.

The scores of independent expert FMEA evaluation of an aircraft element's potential failure are distributed as in Table 1.

Table 1. Results of independent expert evaluation

Expert	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
S	5	5	6	6	5	5	8	6
O	1	1	2	2	1	2	5	1
D	9	9	9	10	10	9	2	9

The scores should be ranked and the Grubbs' statistical test should be carried out using formulas (1) – (4) to detect outliers. The results of calculations are shown in Table 2.

Let the significance level be α = 5%. Then, according to [6, 7], the critical values for Grubbs test are: $G_{8/0.95} = 2.032$ and $G_{1/0.95} = 1.653$.

When comparing the extreme values of the obtained series with the critical values, it is clear that the maximum value is an outlier.

Table 2. Results of data processing

Parameter	$\overline{X}$	s	$G_{\max}$	$G_{\min}$
S	5,750	1,035	2,173	0,724
O	1,875	1,356	2,304	0,645
D	8,375	2,615	0,621	2,437

In other words, the scores given by expert X_7 are the outlier because the values of Grubbs' statistics from Table 2 do not comply with condition (5).

Conclusion

In conclusion, the provisions for increasing the effectiveness of the FMEA method by introducing an independent expert assessment with a group of experts when assessing separate failures were considered. The Grubbs' criterion is adopted as such statistical criterion. It allows one or two outliers from expert judgments in the FMEA assessment to be detected, increasing the reliability and effectiveness of this method.

References

[1]. Golubinsky YuM, Elistratov AG, Piskunova VA, Chernova ES. Primenenie risk-orientirovannogo myshleniya v novoy versii standarta ISO 9001:2015 [Application of risk-oriented thinking in the new version of the ISO 9001:2015 standard]. Izmerenie. Monitoring. Oupravlenie. Kontrol 2016;2(16):21-27 [in Russian].

[2]. Vishniakov YaD, Nadaiev NN. Obshchaya teoria riskov [General risk theory]. Moscow: Akadenia; 2008 [in Russian].

[3]. Pacaiova H, Sinay J, Nagyova A. Development of GRAM – A risk measurement tool using risk based thinking principles. Measurement 2017;100:288-296.

[4]. GOST R 51901.12 – 2007. Risk management. Procedure for failure mode and effects analysis. Moscow: IPK Izdatelstvo standartov, 2008 [in Russian].

[5]. Rozental R. Metodika FMEA. Put povyshenia kachestva produksii [FMEA method. A way of improving product quality]. Elektronika: naouka, tekhnologia, biznes 2010;7:90-95 [in Russian].

[6]. Lemeshko BYu, Lemeshko SB. Rasshirenie oblasti primeneniya kriteriev tipa Grabbsa, ispolzuemykh pri otbrakovke anomalnykh izmereniy [Expanding the application field of Grubbs' criteria in anomalous measurements]. Izmeritelnaya tekhnika 2005;6:13–19 [in Russian].

[7]. GOST R ISO 5725-2-2002. Accuracy (trueness and precision) of measurement methods and results. Part 2. Basic method for the determination of repeatability and reproducibility of a standard measurement method. Moscow: IPK Izdatelstvo standartov, 2002 [in Russian].

[8]. Shishkin IF. Teoreticheskaya metrologia. Chast 1. Obshchaya teoria izmereniy [Theoretical metrology. Part 1. General measurement theory]. Saint Petersburg: Piter; 2010 [In Russian].

About the authors

Zhanat A. Dayev, PhD, Candidate of Engineering, Associate Professor, Head of Information and Measurement Systems Laboratory, Zerek Technology Park, Baishev University Aktobe, Aktobe, Kazakhstan, phone: +7 7771566720, e-mail: zhand@yandex.ru.

Ermek T. Nurushev, postgraduate student, Department of Innovation Theory and Integrated Quality Systems, Saint Petersburg State University of Aerospace Instrumentation (SUAI), Saint Petersburg, Russia, phone: +7 7078836452, e-mail: sanshine-13@mail.ru.

Received on: 10.04.2017

Original article
DOI:10. 21683/1729-2646-2018-18-2-46-49

Ensuring reliable comprehensive protection of habitats from carcinogenic gases

Kirill N. Voynov, *Saint Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Saint Petersburg, Russia*
Annisa Hildayati, *Saint Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Saint Petersburg, Russia*
Yahia Ghellab, *Saint Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Saint Petersburg, Russia*

Kirill N. Voynov

Annisa Hildayati

Yahia Ghellab

Abstract. *In Russia, 2017 was declared the Year of Ecology. Indeed, the rapid development of industry and transportation, including automobiles, airplanes, trains, ships, rockets, etc., heavily pollutes the environment with exhaust fumes and carcinogenic smoke of industrial enterprises, e.g. factories, boilers, power plants, specialized laboratories. Consequently, the atmosphere often contains unacceptable levels of harmful chemicals that gradually settle on the ground, including precipitation in the form of acid rain. The pollution of air, land and water resources causes extremely undesirable effects on the health of all living things. Additionally, the climate on the planet is gradually changing, increasing atmospheric temperature with the appearance of greenhouse effect, ozone holes, rapid melting of glaciers that causes rising water levels in the oceans, etc. Thus it becomes a global problem that must be solved. This paper shows a tentative reliable and radical technical solution. This situation has been basically resolved, yet given the chemical composition of the gases, it can be partially updated.*

Keywords: *dependability, habitat, atmospheric pollution, smoke-stack gases, protection.*

For citation*: Voynov KN, Hildayati A, Ghellab Y. Ensuring reliable comprehensive protection of habitats from carcinogenic gases. Dependability 2018; 2: 46-49. DOI: 10.21683/1729-2646-2018-18-2-46-49*

Introduction

Thousands of tons of gas/smoke constantly emitted into the Earth's atmosphere by industrial enterprises contain carbon dioxide, unburned fuel particles (e.g. coal), soot, various oxides, including those of heavy metals and nitrogen, end up in the air basin and are spread by the wind hundreds of kilometers around, polluting the environment and causing various medical conditions in people, including cancer. Not less harmful in this respect are exhaust fumes of cars and trucks that use various types of fuel. The attempts of many scientists and experts to reliably protect our living environment from smoke and gas have not yet yielded significant results [1-9]. In particular, the installation of filters in smoke-stacks of industrial enterprises turns out to be inefficient in combating the smoke they emit, as small particles in heated gases practically freely escape into the atmosphere and subsequently settle on the ground, polluting it. Additionally, there are no universal filters that would provide reliable protection against all chemical elements contained in smoke/gas. The screens of standard filters are unreliable, they burn out, get clogged with soot and unburned large particles, and therefore require repairs or replacement. Tall smoke-stacks also do not solve the problem, as even from the height of 150 m carcinogenic chemical elements still settle on the ground a long distance around the industrial enterprise.

A practically identical negative situation has arisen with the exhaust fumes created by a multi-million mass of automobiles. In this case, the gasses are emitted in immediate proximity of the pedestrians, automobile repair shop workers, in garages, engine run-in or repair stations, specialized laboratories. When engines are let to idle for long periods of time in places, where vehicles await the initiation of motion, or, for instance, in case of excavators waiting for the arrival of a truck to transport soil, refuse or rock, a large amount of harmful exhaust fumes are emitted into the atmosphere, which inevitably harms human health.

We propose a technical solution that ensures comprehensive and reliable protection of the environment from various gases and fumes.

The essence of the technical solution

This paper examines in depth the contents of the Russian patent [10].

The essence of the invention is shown in Figure 1 that symbolically depicts the general design layout of the proposed method of comprehensive purification of air from industrial waste emitted from smoke-stacks.

The notations for figure 1 are as follows:

1, enterprise or organization (factory, power plant, research institute, boiler plant, laboratory, workshop, etc.), that conventionally could emit harmful gases or substances into the air through smoke-stacks;

2, main discharge pipe with a suction setup/pump;

3, backup discharge pipe with a suction setup/pump;

4, dual-section stack gas analyzer;

5 and *5***'**, valves;

6 and *6'*, pressure relief valves;

7, dual-section main reservoir;

*7***'**, dual-section backup reservoir;

8, main reservoir waste removal ducts;

*8***'**, backup reservoir waste removal ducts;

9 and *9***'**, disinfectant or neutralizing solutions supply ducts.

The technical result is achieved through a multilevel cascade of airtight vessels (e.g. in the form of reservoirs, containers, collectors or basins) isolated from ambient air or water and normally featuring multi-stage automatic, semi-automatic or manual operation of the process of purification of harmful gasses or solutions (waste) without protruding smoke-stacks. Additionally, main elements, units and reservoirs can be backed-up both 1:1 or, alternatively, reduced in scale. The latter must be coordinated with the

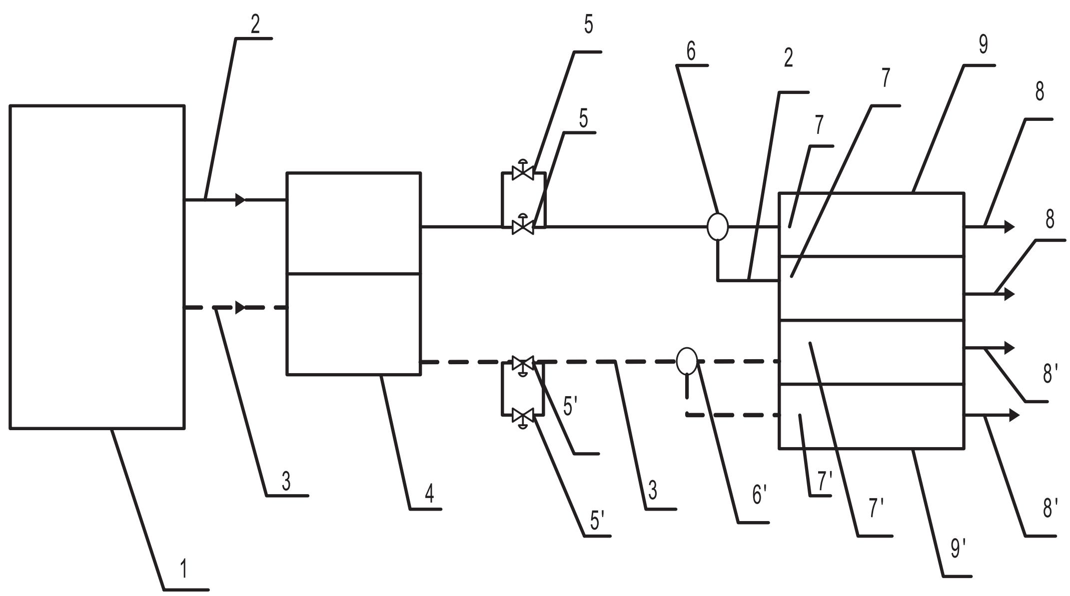


Figure 1. General design layout of removal of harmful gas emitted by smoke-stacks of any industrial enterprises and laboratories

duration of repairs, maintenance, replacement or purging of main reservoirs or equipment of industrial products being or already purified.

That means the longer is the cycle of purging, removal of waste or repair of primary equipment and reservoirs, the larger must be the backup reservoirs in order to prevent the interruption of the air purification process.

The bold line shows the main purging process, while the dash line shows the backup path that increases the overall dependability of the presented facility.

Let us assume that there is an enterprise or organization *1*, the production activities of which may cause the emission of harmful gases and elements. The industrial heater or unit is connected to a discharge pipe *2*, that does not rise up. Instead, a suction unit or extraction pump are installed. In parallel, an identical discharge pipe *3* is installed as backup. Installed next is a dual-section stack gas analyzer 4 (gas and/or chemical) (one of the sections is redundant). Sets of valves *5* and *5*' on pipes *2* and *3* are installed after the analyzer. Then, pressure relief valves *6* and *6*' are installed in parallel on those pipes. The pipes out of the valves are connected to the main reservoir *7* and the backup reservoir *7*' respectively. Ducts *8* and *8*' ensure not only the release of substances, but also the access to reservoirs *7* and *7*' for the personnel. The required neutralization or disinfection of carcinogenic emissions in the main and backup reservoirs is ensured by supplying through pipes *9* and *9*' of liquid, gasiform or powdery solutions in required quantities, particle size and chemical composition.

In the given design, through the main discharge pipe *2* the waste is directed to the gas analyzer or chemical analyzer (hereinafter referred to as the analyzer) *4*, which is ensured by the suction pump built in the pipe *2*. In parallel is an identical backup setup with discharge pipe *3* that is used only if pipe *2* and pump are damaged or undergo maintenance. The fact that the analyzer *4* has two independent sections allows using them one by one in case of failure of one of the sections. This allows regulating the composition of the content of the main reservoir 7 and the backup reservoir *7*', that operate by rotation. After analyzer *4*, via pipes *2* and *3* through open valves *5* and *5*' the drained and purged content enters the main reservoir 7 or backup reservoir *7*' if the main reservoir is undergoing repairs or purging. Both the main discharge pipe *2* and the backup discharge pipe *3* are plunged into water or another disinfecting fluid in the reservoirs *7* and *7*'. If required, the pressure relief valves *6* and *6*' can redirect outgoing carcinogenic substances into reservoir *7* (main) or *7*' (backup). The size and capacity of the backup reservoir *7*' are such as to allow not to interrupt the process of even continuous purification of the waste produced by the enterprise/organization *1*, as that is enabled by the maximum duration of main reservoir *7* purging or maintenance, i.e. the size of the backup reservoir may be smaller than those of the main one. Each of the reservoirs has an obligatory lid (whole or composite, if large), which allows vapours to turn into drops and trickles on the bottom side of the lid and fall back into the reservoir, i.e. an almost complete isolation of the purification process from the outside environment is insured. The disinfected content is finally removed through duct *8* and/or *8*'.

In order to ensure high system dependability, pumps, valves and other units controlling the process of comprehensive purification of air from industrial waste are redundant as well, which ensures continuous enterprise/organization operation under any ambient temperature and high degree of air purification.

Similarly, efficient air purification can be organized at automobile, tractor and other vehicle repair shops, where the exhaust fumes can be directed into small reservoirs or pools through hoses.

Some designs and calculations related to the dependability of equipment

It is known that probability p of fault-free operation of the system is within the range $0 \leq p \leq 1$ (the closer p is to one, the higher is the dependability). The dependability calculation formula for variant *a)* of the block diagram in Figure 2 is as follows:

$$P=[1-(1-p)^2]^n,$$

while for the variant *b)* the formula is as follows:

$$P=[1-(1-p)^n]^2,$$

As it can be seen, 1+1 element-wise redundancy ensures higher dependability that 1+1 redundancy of the total system. This is true for pumps, valves, pipes, reservoirs, instruments.

Conclusion

Given the above, the following conclusions can be made. The proposed method of comprehensive air purification can be used to eliminate the environmental effects of vapours,

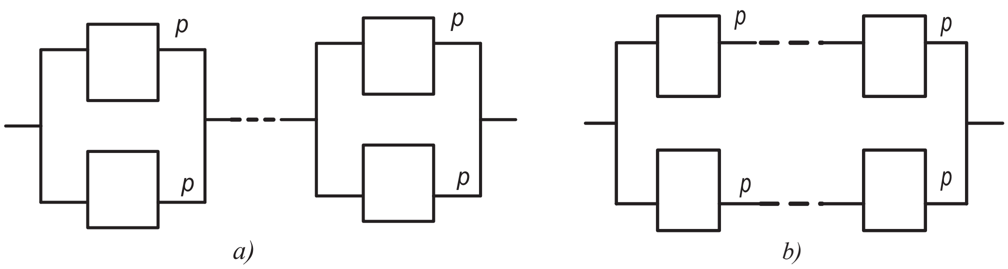


Figure 2. Logical diagrams of evaluation of redundant systems dependability:
a) element-wise and b) system in total

gases, smoke, aerosols, solid particles, soot, ash, dust, oil mist, odours, etc., or the combination of those. Additional positive effects of the implementation of the design include: the environment is practically completely protected from hazardous industrial emissions/waste, as well as exhaust fumes of cars and similar vehicles; the development of greenhouse effect is eliminated; tall smoke-stacks are not required (absence of hazard to low-flying helicopters/airplanes, especially in case of fog or low visibility); filters that get clogged soon and are not always efficient and dependable are not required.

References

[1]. Bazovsky I. Reliability: Theory and Practice. Moscow: Mir; 1965.

[2]. Barlow R, Proschan F. Mathematical theory of reliability. Moscow: Sovietskoie radio; 1969.

[3]. Gertsbakh IB, Kordonsky KhB. Modeli otkazov [Failure models]. Moscow: Sovietskoie radio; 1966 [in Russian].

[4]. Gnedenko BV, Beliaev YuK, Soloviev AD. Matematicheskie metody v teorii nadezhnosti [Mathematical methods in the dependability theory]. Moscow: Nauka; 1973 [in Russian].

[5]. Gorsky LK. Statisticheskie algoritmy issledovaniya nadezhnosti [Statistical algorithms of dependability research]. Moscow: Nauka; 1970 [in Russian].

[6]. Polovko AM. Osnovy teorii nadezhnosti [Introduction into the dependability theory]. Moscow: Nauka; 1964 [in Russian].

[7]. Hildayati A. Voynov KN, editor. Problemy, svyazannye s zagryazneniem vozdushnogo basseyna [Problems related to air pollution]. International encyclopedia. Volume XII: Osobennosti raboty tribosopryazheniy v spetsificheskikh usloviyakh [Specifics of tribocouplings operation in particular conditions]. Saint Petersburg: Nestor-Istoria; 2017 [in Russian].

[8]. Voynov KN, Yahia G. O zagryaznenii vozdukha avtomobil'nymi gazami [On air pollution with automobile exhaust fumes]. International encyclopedia. Volume XII: Osobennosti raboty tribosopryazheniy v spetsificheskikh usloviyakh [Specifics of tribocouplings operation in particular conditions]. Saint Petersburg: Nestor-Istoria; 2017 [in Russian].

[9]. Voynov KN, Vasiliev VA, Hildayati A, Ghellab Y. Novy podkhod k umensheniyu zagryazneniya vozdushnogo prostranstva [New approach to reducing air pollution]. International encyclopedia. Volume XII: Osobennosti raboty tribosopryazheniy v spetsificheskikh usloviyakh [Specifics of tribocouplings operation in particular conditions]. Saint Petersburg: Nestor-Istoria; 2017 [in Russian].

[10]. Patent no. 2604816: Sposob kompleksnoy ochistki vozdushnogo basseyna ot proizvodstvennykh otkhodov/vybrosov iz trub [A method of comprehensive treatment of an air basin for industrial waste/stack emission]. Registered in the National register of inventions of the Russian Federation on November 22, 2016. Author: Voynov KN.

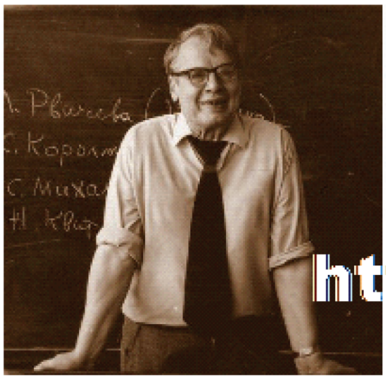
About the authors

Kirill N. Voynov, Doctor of Engineering, Professor, academy member, professor of Industrial Climate Engineering Systems and Engineering Design, Saint Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Saint Petersburg, Russia, phone: +7 (812) 376 89 72, e-mail: forstar@mail.com

Annisa Hildayati, postgraduate student, Department of Industrial Climate Engineering Systems, Saint Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Saint Petersburg, Russia, phone: +7 (931) 227 98 86, e-mail: hildayati.annisa@mail.ru

Ghellab Yahia, postgraduate student, Department of Industrial Climate Engineering Systems, Saint Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Saint Petersburg, Russia, phone: +7 (900) 656 45 12, e-mail: ghellabyahia@yahoo.com

Received on: 09.06.2017



Gnedenko
e-Forum
International Group on Reliability

http://Gnedenko-Forum.org/

Dear colleagues!

In 2005 the informal Association of Experts in Reliability, Applied Probability and Statistics (I.G.O.R.) was established with its own Internet website GNEDENKO FORUM. The site has been named after the outstanding mathematician Boris Vladimirovich Gnedenko (1912-1995). The Forum's purpose is an improvement of personal and professional contacts between experts in the mathematical statistics, probability theory and their important branches, such as reliability theory and quality control, the theory of mass service, storekeeping theory, etc.

Since January 2006, the Forum has published a quarterly international electronic magazine

"Reliability: Theory and Applications".

The magazine is registered with the Library of Congress in the USA (ISSN 1932-2321). All rights reserved for authors so that articles can be freely published in any other publications or presented at conferences.

Join Gnedenko Forum!

Welcome!

More than 500 experts from 44 countries worldwide have already joined us!

To join the Forum, send a photo and a short CV to the following address:

Alexander Bochkov, PhD

a.bochkov@gmail.com

Membership is free.

SUBSCRIBER APPLICATION FOR DEPENDABILITY JOURNAL

Please subscribe us for 20___
from No. ________ to No._______ number of copies _________

Company name	
Name, job title of company head	
Phone/fax, e-mail of company head	
Mail address (address, postcode, country)	
Legal address (address, postcode, country)	
VAT	
Account	
Bank	
Account number	
S.W.I.F.T.	
Contact person: Name, job title	
Phone/fax, e-mail	

Publisher details: Dependability Journal Ltd.
Address of the editorial office: office 209, bldg 1, 27 Nizhegorodskaya Str., Moscow 109029, Russia Phone/fax: 007 (495) 967-77-02, e-mail: evgenya.patrikeeva@yandex.ru
VAT 7709868505 Account 890-0055-006
Account No. 40702810100430000017
Account No. 30101810100000000787

Address of delivery:

To whom:____

Where:____

To subscribe for Dependability journal, please fill in the application form and send it by fax or email.
In case of any questions related to subscription, please contact us.
Cost of year subscription is 4180 rubles, including 18 per cent VAT.
The journal is published four times a year.

THE JOURNAL IS PUBLISHED WITH PARTICIPATION AND SUPPORT

OF JOINT-STOCK COMPANY RESEARCH & DESIGN INSTITUTE
FOR INFORMATION TECHNOLOGY, SIGNALLING AND TELECOMMUNICATIONS
ON RAILWAY TRANSPORT (JSC NIIAS)

JSC NIIAS is RZD's leading company in the field of development of train control and safety systems, traffic management systems, GIS support technology, railway fleet and infrastructure monitoring systems

Mission:

transportation

- ❑ efficiency,
- ❑ safety,
- ❑ reliability

Key areas of activity

- Intellectual control and management systems
- Transportation management systems and transport service technology
- Signalling and remote control systems
- Automated transportation management centers
- Railway transport information systems
- Geoinformation systems and satellite technology
- Transport safety systems
- Infrastructure management systems
- Power consumption and energy management systems
- Testing, certification and expert assessment
- Information security
- Regulatory support

www.vniias.ru