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E.Patrikeeva@gismps.ru,  
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Igor B. Shubinsky

RELIABLE FAIL-SAFE  
INFORMATION SYSTEMS  
Methods of synthesis

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## I.B.Shubinsky Reliable Fail-safe Information Systems 2016

The book describes conceptual provisions to ensure structural and functional reliability of information systems at all stages of a life-cycle. It represents different types of redundancy taking into account limited efficiency of the failure detection system. Under these conditions a broad-based assessment of their efficiency is performed, with determination of capabilities of structural redundancy with an endless number of standby facilities. Ways to ensure functional reliability of software are represented, including the recommendations for the development of software programs requirement specification, with the description of the process of a reliable program architecture development and well proven rules and recommendations used for design and implementation of software, as well as for integration with system hardware.

The book also presents theoretical and practical provisions of adaptive fault tolerance (active protection) of information systems, including the methods and disciplines of active protection, as well as the ways of implementation. A method of synthesis of active protection and the results of research of information system reliability with various disciplines of active protection are offered. There are also certain assessments of the efficiency of active protection in relation to the traditional methods of structural redundancy.

You can find the description of the principles to ensure functional safety of information systems, with a substantiation of the possibility to restart independent channels in two-channel safe systems. The rules of determination of the allowed time for a guaranteed detection of single and double hazardous failures are developed, including the method of synthesis of a combined two-level information system developed with higher functional safety requirements.

To prove the conformance of reliability with functional safety the method of accelerated field testing of the information system has been developed. The book contains the description of this method, including the example of its practical implementation. You will also find the information about the procedures of certification tests based on the requirements of information safety and software certification conformance.

A checklist of the most complex and significant subjects is provided at the end of each chapter. The book is primarily intended for experts who are engaged in practical development, manufacture, operation and updating of information. It is intended for researchers in the field of structural reliability of different discrete systems, academic staff, post-graduate students and students specializing in the field of information systems and as well as those working in the field of automated control systems.

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Igor B. Shubinsky  
**FUNCTIONAL DEPENDABILITY  
OF INFORMATION SYSTEMS  
2012**

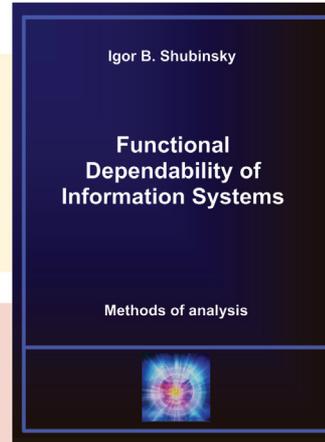
For the first time, this book presents the theory of functional dependability of information systems as a component of the general dependability theory. The book comprises basic concepts and definitions, major threats for the functional dependability of information systems, system parameters, methods for estimating the functional dependability of digital devices, and methods and models of estimating software functional dependability. A separated chapter considers the functional reliability of critical information systems, including the concept of a critical system, features of faults, estimation of functional reliability of operators, estimation of hazardous failures and risks, the requirements of functional dependability and the software architecture of critical information systems. A checklist of the most complex and significant subjects is provided at the end of each chapter.

The book is primarily intended for experts who are engaged in practical development, manufacture, operation and updating of information technologies and information systems. It is intended for researchers in the field of software-hardware of information systems, academic staff, post-graduate students and students specializing in the field of information technologies as well as those working in the field of automated control systems.

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STRUCTURAL DEPENDABILITY OF INFORMATION SYSTEMS  
Methods of analysis

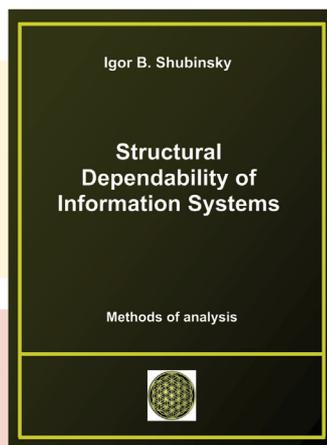
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**STRUCTURAL DEPENDABILITY  
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The book presents the basic concepts and parameters of the structural dependability of information systems. It discusses general and specific differences in dependability indices used in domestic and international standards, along with recent developments in approaches to dependability modeling. Markov reliability models together with graph semi-Markov methods for calculating reliability are described in detail and illustrated by numerous examples. Considerable attention is paid to the engineering methods of calculation and the approximate prediction of structural dependability and error estimation of information systems as well as to the statistical assessment of dependability parameters. At the end of each chapter there are checklists of the most complex and significant subjects of the chapter.

The book is intended primarily for professionals involved in practical work on the development, production, operation and modification of information systems. It is designed for scientists in the field of structural dependability of various discrete systems, academic staff and graduates (students) specializing in information systems as well as in the field of automated control systems.

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## Description of approach to estimating survivability of complex structures under repeated impacts of high accuracy

**Gennady N. Cherkesov**, Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia,  
e-mail: gennady.cherkesov@gmail.com

**Alexey O. Nedosekin**, National Mineral Resources University "Gorny", St. Petersburg, Russia,  
e-mail: apostolfoma@gmail.com



Gennady N.  
Cherkesov



Alexey O.  
Nedosekin

**Abstract. Aim.** The paper describes main concepts and definitions, survivability indices, methods used to estimate survivability in different external and internal conditions of application of technical systems, including the studies in the field of structural survivability obtained 30 years ago within the frames of the Soviet school of sciences. An attempt is made to overcome different technical understanding of survivability, which has been developed in the number of industries up to date – in ship industry, aviation, communication networks, energy, in defense industry. The question of succession between the properties of technical survivability and global system resilience is considered. Technical survivability is understood in two basic notions: a) as the system property to withstand negative external impacts (NI); b) as the system property to recover its operability after a failure or accident caused by external reasons. This paper considers the relation between the structural survivability when the system operability logic is binary, and is described by a logical function of operability, and the functional survivability when the operation of the system is described by the criterion of functional efficiency. Then the system failure is a decline in its efficiency below a preset value. **Methods.** The technical system is considered as a controlled cybernetic system, which has specialized aids to ensure survivability (SAs). Logical and probabilistic methods and results of combinatorial theory of random placements are used in the analysis. It is supposed that: a) negative impacts (NI) are occasional and single-shot (one impact affects one element); b) each element of the system has binary logic (operability – failure) and zero resistance, i.e. it is for sure affected by one impact. Henceforth this assumption is generalized for the  $r$ -time NI and  $L$ -resistant elements. The paper also describes different variants of non-point models when the system part or the system as a whole are exposed to a group affection of the specialized type. The article also considers the variants of combination of reliability and survivability when failures due to internal and external reasons are analyzed simultaneously. **Results.** Different variants of affection and functions of survivability of technical systems are reproduced. It has been educed that these distributions are based on simple and generalized Morgan numbers, as well as Stirling numbers of the second kind that can be reestablished on the basis of simplest recurrence relations. If the assumptions of a mathematical model are generalized in case of  $n$  the  $r$ -time NI and  $L$ -resistant elements, the generalized Morgan numbers used in the estimation of affection law are defined based on the theory of random placements, in the course of  $n$ -time differentiation of a generator polynomial. In this case it is not possible to set the recurrent relation between the generalized Morgan numbers. It is shown that under uniform assumptions in relation to a survivability model (equally resistant system elements, equally probable NI) in the core of relations for the function of survivability of the system, regardless of the affection law, there is a vector of structure redundancy  $F(u)$ , where  $u$  is a number of affected elements, and  $F(u)$  is a number of operable states of the technical system with  $u$  failures. **Conclusions:** point survivability models are a perfect tool to perform an express-analysis of structural complex systems and to obtain approximate estimates of survivability functions. Simplest assumptions of structural survivability can be generalized for the case when the logic of system operability is not binary, but is specified by the level of the system efficiency. In this case we should speak about functional survivability. PNP computational difficulty of the task of survivability estimation does not allow solving this task by means of a simple enumerating of states of the technical system and variants of NI. It is necessary to find the ways to avoid the complete search, as well by the conversion of the system operability function and its decomposition. survivability property should be designed and implemented into a technical system with consideration of how this property is ensured in biological and social systems.

**Keywords:** survivability, vitality, resilience, risk, negative impact, survivability margin, law of vulnerability, function of survivability.

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## 1. Introduction

A term “survivability” in relation to technical systems and in particular to a ship was for the first time introduced for consideration by the Russian admiral and scientist Stepan Osipovich Makarov. The beginning of development of the ship survivability theory should be his article “Armor-plated boat “Mermaid” published in 1870 in “Sea collected book” (No.No. 3, 5, 6), that described a number of measures taken for a ship floodability [1]. In 1875 in the article “Floodability of water crafts” (“Sea collected book”, No.6, 1875) he formulated the notion of “floodability” as the “ability to remain on the float having underwater hull breaches”. In 1876 S.O. Makarov published the articles “Antiflooding means” (“Sea collected book” No. 1, 1876) and “About maintenance of water-tight bulkheads and pumping appliances” (“Sea collected book” No. 7, 1876). In 1894 he published the work “Review of elements of vessel fighting forces” where he clarified the notion of floodability as the “ability of a ship to remain on the float and not to lose its fighting qualities due to underwater hull breaches”.

In 1897 S.O. Makarov published his articles “Maritime essays” (“Sea collected book”, No.No. 1, 2, 3, 4, 7) where he finally formulated the “survivability” as the “ability of a ship to keep a fight having damages in different fighting structures” with a proviso that a deficiency of resistance to external destructive effects is compensated by attribution of a ship with the property of survivability [14].

The academician A.N. Krylov gave the shortest and a rather pointed definition of a general sense of “survivability” and defined it as “fatigue resistance to damages”. All definitions have a positive feature – that survivability is considered as a property of a ship as a whole, which is achieved by structural organization and goal-directed behavior of its functional sets of technical facilities.

Today the notion of survivability is widely used in several sectors of engineering including transport systems (aviation, railway, automobile transport), ship industry, energy, construction, in computing systems and communication networks, in industries of defense [9, 13, 14]. A renewed system and scientific interest to technical survivability in the 1980s in the USSR was determined by a large scope of works on secret subjects related to national defense capability. In times of “perestroika” all these works were scaled down, and now we observe a certain renaissance due to a marked aggravation of international atmosphere. It turns out to be important not only to raise the works on technical survivability to a former level, but also to take a fresh look at “survivability” as a complex property of a system, come to realization that the system in terms of survivability takes from the creatures that are traditionally considered living beings. An attempt of such comparison is undertaken in paper [15] where survivability is generally called vitality, and vitality projection on social, economic and technical systems is called civilization availability, mobilization resilience and survivability, respectively.

Survivability issues are considered in foreign literature as well. Approximately up to the year 1997 the surveys used the category “survivability” more often. But then the focus of attention shifted towards an area of more common properties than survivability, and a question was more about resilience. In 1997 a presidential commission delivered a report on the protection of the most mission-critical infrastructure systems [16]. And then the USA showed special demand in resilience due to a huge damage caused by 9/11 and Katrina hurricane. It became clear that resilience must be invested significantly. As an alternative – which is not to invest in resilience – usually comes at a higher cost. Therefore investments in resilience turns out to be a business of enormous earning capacity (hundreds of annual interest).

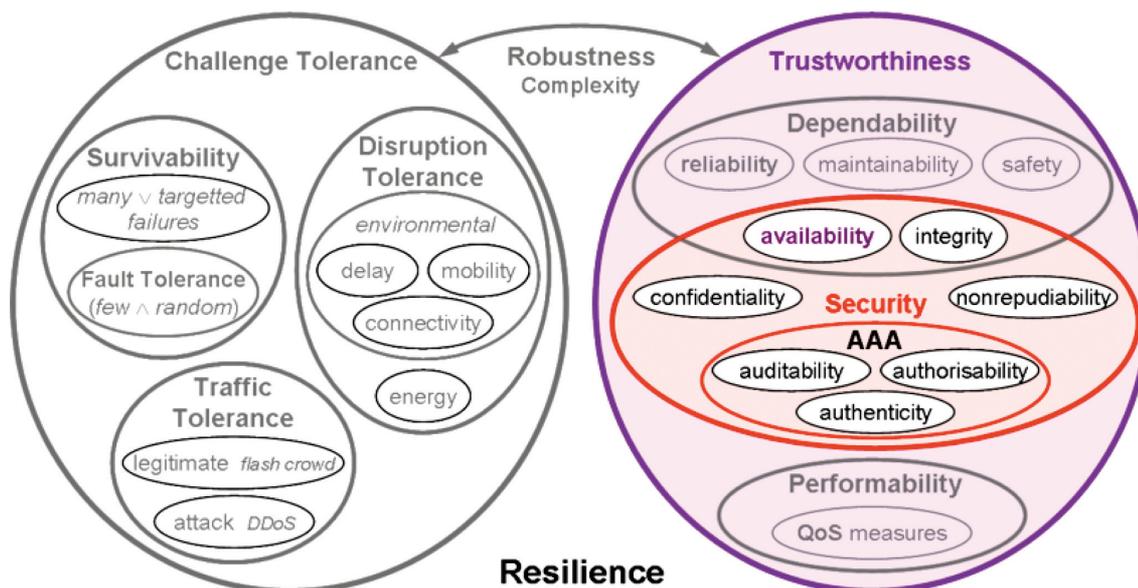


Fig. 1. Classification of resilience in relation to computing networks. Reference: [18]

That is why, for example, in [17] we read: “*Resilience attracts attention as a denominator to move beyond survivability and even to succeed in aggressive conditions ... Resilience is an emergent property related to the ability of an organization to proceed with its mission despite damages, through awareness, dexterity provided by resources, flexible infrastructures and restorability.... Therefore resilience is a combination of technical structural features such as reliability and dependability and organizational features such as awareness, training and decentralized decision making*”.

And [18] provides the following sector classification of resilience (Figure 1).

We can see from the given classification that resilience is considered as a global property that takes up the properties of dependability (in all senses), survivability, safety, operational perfection. Such view has not been conceived by the Russian sector science yet, and it is unlikely to be conceived without a critical analysis and natural antagonism. We are not going to get involved in this polemic, but we would like to attract a reader’s attention to the following paradox. Many companies and certain experts study technical survivability and consider it if their projects. However, in practice there is no consistent system of notions, survivability indices, conditions of functioning, with availability of which survivability and survivability requirements should emerge, i.e. exactly that makes a core of the respective theory. There is also no unity in understanding of the most efficient means of survivability assurance for different classes of systems and definite scenarios of external impacts on the structure and algorithms of functioning. To make the picture of survivability developments complete we should mention about a total absence of national standards reflecting the issues of terminology, survivability indices, classification, methods and recommendations on the order of system design by survivability criteria.

We also should keep in mind that the issues of technical survivability shall not be considered locally, but in the context of a more common demand in a mobilization resilience of a state and a country in general [15, 19, 21, 22]. In limited investment opportunities of a state, defense budgets shall be sequestered, and projects shall be loaded only with the specific properties that shall be economically efficient in a broad sense. In terms of technical survivability (as well as of reliability) – a right to live will go only to those design implementations that proved an optimal proportion between the strength of property and the expenses spent on its realization, and besides, an optimum has been found for return of capital employed. I.e. with time it is necessary to learn not only to estimate survivability, but also to introduce economic and financial measures to this estimate. Thirty years ago there was no need to think about it (defense money was not watched); but now we live in another epoch.

This article represents topical questions of system design by the criteria of technical survivability that may

be considered as possible directions for the development of survivability theory as a general technical discipline. Basic study in this sphere was made 30 years ago, and it is not reproduced here. However, there is a number of new circumstances that may influence a new revealing of survivability, the character of development of the respective branch of science, and we shall speak about it in this work as well.

## 2. Main concepts and definitions

There are several industrial definitions and a common technical definition of survivability. GOST 19176–80 [1] defines survivability of the system of ship facilities control as a constituent part of complex property of the control system functioning which emerges in case of part damages of equipment and communication lines. Survivability **involves** maintaining of operability of a ship which was not affected by emergency environmental impacts, as well as fail-safety of set of technical facilities under violations of control system. The work [3, p.194] defines survivability of a ship as the ability to withstand wind strength and wave force, fires, enemy’s weapons, and if damages occurred to keep and recover sea capabilities and combat qualities either totally or partially. Survivability of a ship is provided by structural design and equipment efficiency, as well as by allocation of tight junctions, hatches, handholes, doors, glass parts, signaling systems, automatic protective devices. Let us note that this definition indicates the **conditions** when survivability emerges (spontaneous forces of wind and wave, fires, weapons), **stages** of the process development and the **degree of severity** of negative impacts (to withstand damages, if a damage occur to keep sea capabilities and combat qualities, and in case of their loss to recover them either totally or partially). And the methods to provide survivability are listed (limitation of adverse consequences, structural design efficiency, tight junctions, signaling and control: signaling systems, protective devices). Such a large structure of definition could be repeated for other sectors of engineering

In electric power industry [4] survivability is understood as the property of an object to withstand perturbing actions, avoiding their successive development with mass supply interruptions. Here we should pay our attention to the requirement to the system that it must withstand deactivation of its components due to technologically related failures caused by violations of external conditions of functioning. The paper [6] gives such example of violation of external conditions under a system failure. When removing of one of two 220 kV power lines out of service for repair the unit of a condensing plant was disconnected due to a boiler damage. The other power line was overloaded and caused a blow out of a wire in a faulty contact joint. After this line was also disconnected under relay protection, asynchronous operation was phased out which disconnected 110 kV power lines. Then frequency

reduction caused the activation of frequency relief devices machinery of thermal power station, etc. As the result it interrupted a normal supply mode of the whole district for 15 hours. A “domino” effect in the system is caused by successive violation of functioning that leads to a supply disconnection.

In computing systems [2] survivability is connected with loss free conditions of any task (function) under a loss of certain resource caused by negative external impacts.

An attempt to give a common technical definition to survivability was taken in the work [9]. Here survivability is defined as the property of a system to keep and recover the ability to perform basic functions in a prescribed scope during a specified operating hours to failure under a change of the system structure and (or) algorithms and terms of its functioning due to external negative impacts (NI) that were not specified by the rules of normal operation. Basic functions and specified operating hours can be determined not only for one, but also for several NI different in severity. This definition admits the **consideration of various NI consequences**, affecting the task execution, including:

- loss of operability of the elements and their links due to their physical destruction or integrity damage;
- change (deterioration) of their technical characteristics (speed, productivity, capacity, etc.);
- distortion of algorithms of functioning;
- reduction of structure redundancy, level of production stock;
- deterioration of failure-free operation of the elements, system controllability;
- change of external terms of functioning (sharp reduction of increase of loading, loading redistribution, change of dynamic characteristics of loading).

More severe consequences of NI are also probable: inherent loss of operability, accident with possible partial or total system breakdown.

### 3. Evolution of the system states after negative impacts

NI are followed by primary consequences that are expressed in the deterioration of performance of the elements or functional connections, distortion of algorithms of functioning of functioning [9].

The system that has the property of survivability develops it in the property of gradual **degradation** that occurs due to introduction of passive and active survivability aids (SAs). Information about primary consequences goes to SAs that include operability control facilities, tools of emergency protection, means of reconfiguration and control. SAs influence the development of primary consequences. Depending on the intensity of processes, certain external conditions, SAs efficiency, the system finally passes to one of possible resilient states. This process is stochastic by its nature.

We know from the example described in section 2 that after certain intermediate states the system passed to a resilient state under which the units of a condensing plant were disconnected and machinery of thermal power station were cut off. After transition to a new state the estimation of primary consequences is performed as the result of which the system state is referred to one of three classes: operable, inoperable (or non-emergency), emergency. Based on the results of this classification the estimation for survivability by the system state is carried out. Under an operable state the system turns to the task execution immediately. If the state is inoperable the system may turn to the task execution after certain recovery procedures. The transition of the system to a new resilient state does not complete the struggle for survivability, as under further functioning secondary consequences of NI may occur before the task execution.

Secondary consequences are farther but not less dangerous than the primary ones. They are related to uncontrolled or ill controllable thermal, electric and other proc-

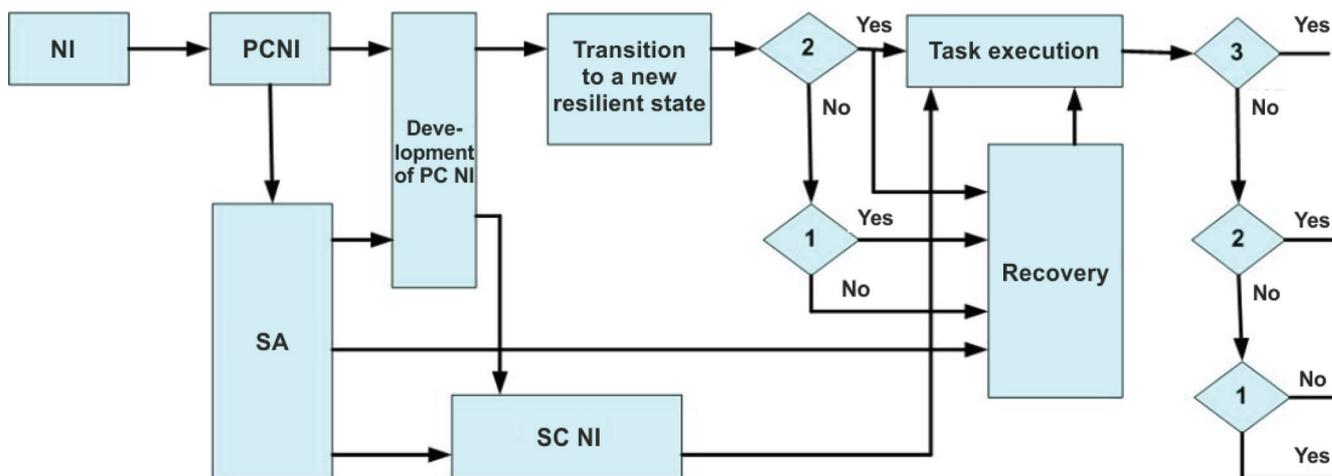


Fig. 2. Evolution of the system states after a NI  
(PC – primary consequences, SC – secondary consequences, SAs– survivability aids,  
1 – accident; 2 – operable; 3 – task execution)

esses (fire spread, cooling of space in heat supply system, etc.). The rate of development of secondary consequences and the end result depend significantly on SAs operation at struggle for survivability. After a certain period of time, the estimation of the task execution results is performed with four possible outcomes. Therefore, at struggle for survivability we can allocate three stages. The first stage includes the efforts for accident prevention, the second stage struggles for the system operability, and the third stage includes the struggle for a successful execution of a task despite primary and secondary consequences of NI. Accordingly, there are two tasks for estimation and assurance of survivability.

Trajectories of evolution of the system states with the consequences different in intensity and severity fit in the general scheme (Figure 2), but only in the cases when an impact is single. The scheme becomes much more complicated when the impacts are multiple and the processes of consequences of different NI overlap. In addition to that, in all possible schemes a “racing effect” plays a great role: the processes of consequences development and the processes of struggling for survivability proceed through time. That is why the severity of NI consequences, system state and, finally, its destiny are mainly determined by SAs capabilities, their operational efficiency and effectiveness. A certain productivity margin of SAs creates favorable conditions for a timely decision that makes it possible to limit secondary NI consequences and keep the system operability at least with a bit worse technical characteristics. That is why it is important to underline the following: struggle for survivability often takes place under severe time constraints. And therefore, the survivability models should be dynamic. A racing effect could not be taken into account, we could use static models in two extreme cases when the speeds of the competing processes are essentially different.

In the first case a SA has time to complete its algorithms and perform the required disconnection, activation and change-over switching before the technologically interconnected failures occur. **In the second case** a SA does not have time to step in high-speed processes of the development of primary NI consequences, and the transition to a new resilient state is implemented without SAs. Only later survivability aids will influence the secondary NI consequences and recovering processes. In both cases a role of stochastic factors decreases, because a final state of the system can be definitely traced by the system and NI characteristics.

#### 4. Factors and scenarios that are considered in survivability models

All factors that specify the system survivability can be divided into three groups by a functional feature: 1) factors of negative impacts; 2) **factors** that specify the system and its elements in terms of survivability; 3) factors that specify **external aids** of survivability.

**The first group** includes the scope of NI (a point, a closed figure on a plane, in space), number of **affecting** factors and their characteristics, NI duration (**impulse** and with **finite duration**), degree of NI, strategy of multiple NI, internal and external sources of NI that require the creation of survivability aids.

**The second group** is formed by:

1) factors that specify the system and its elements in terms of survivability (resistance of the elements, topology of the system and its elements, resilience to the development of NI consequences of a certain type, speed of processes caused by NI; fail-safety of the elements);

2) factors that specify **internal SAs** (due notice of a NI danger; emergency protection; redundancy; factors of localization and elimination of secondary NI consequences; factors of recovery of technical characteristics of survivability: fire resistance, strength, etc.).

**The third group** includes the factors that specify **external survivability aids** and perform the functions of rescue services and mobile centralized redundancy used for recovering.

Based on the combination of assumptions about current factors the scenarios of impacts on the system, as well as the scenarios of struggling for survivability occur. For example, we may take a scenario of multiple negative impulse impact with one affecting factor of high intensity (affection is guaranteed) and a high accuracy with availability of the structure of a certain class with no SAs. Alternative scenarios may consider zero resistance of the elements; non-ordinary flows of NI affecting several element at once; development of the impact with time, making it possible to take a countermeasure and analyze a racing effect; availability of the reserved time sufficient for probable recovery of operability with external SAs and further execution of the prescribed works, etc.

Scenarios are getting much more complicated if multi-serial NI are considered not only with a strategy of the system protection from NI, but also with a strategy of influence on the number and degree of impacts in the mode of antagonistic, management game and efficient counter efforts against a game partner.

A concept of survivability model is more restricted and specific than the concept of survivability struggle scenario. And each scenario could be generally correlated with a variety of models. A survivability model is not only used for a quantitative estimation, it also has a calculation base and further comparison with the requirements as one of major targets of development at the designing and at the operation of mission-critical systems. For comparison and analysis the survivability indices are required.

#### 5. Survivability indices

Proposals for survivability indices in technical literature first occurred in the 1970-1980s [2-7], in work [9] in more

detail. For the ranking purposes they should be classified by two features. **According to the first feature** indices are divided into two groups: indices used to estimate survivability by the system state and by the results of task execution. Indices of the first group estimate the system property to keep operability after NI. Indices of the second group estimate the ability not only to withstand NI, but also to execute the prescribed task successful in future despite NI. **According to the second feature** indices are divided into additive and mini-max indices. They differ in the way of leading of a vector index to a scalar one. Additive indices also include probabilistic indices based on the formula of total probability.

### 5.1. Indices of survivability by the system state

Let us use  $A_n$  to indicate the event of  $n$ -tuple occurrence of NI, and  $F$  to indicate a logic function of the system operability that takes a value 1 if the system is operable, and 0 if its is inoperable. Then a conditional law of vulnerability

$$Q(n) = P\{F = 0 | A_n\} \quad (1)$$

is a probability of loss of operability in case of a  $n$ -tuple NI.

Survival rate of the system under  $n$ -tuple NI

$$R(n) = P\{F = 1 | A_n\}. \quad (2)$$

Margin of survivability ( $d$ -survivability)

$$d = C - 1 \quad (3)$$

is a critical number of defects  $C$  decreased by one. Defect is a unit of measurement of damage of the system caused by a negative impact. It could be one element removed from the system as the result of NI, certain nominal capacity in energy system, lost for consumers as the result of NI, etc. A word "critical" is used to call a minimum number of defects occurrence of which leads to the loss of operability.

Maximum margin of survivability ( $m$ - survivability)

$$m = \max_{(i)}(m_i) \quad (4)$$

is a maximum number of defects that could be suffered by the system without loss of operability.

Average number of negative impacts that cause loss of operability

$$\bar{\omega} = \sum_{n=0}^{\infty} R(n) \quad (5)$$

is an expected value of the number of NI that is set by distribution (1).

Average margin of survivability

$$\bar{d} = \bar{\omega} - 1. \quad (6)$$

This is not a negative value because  $\bar{\omega} \geq 1$ . It follows from (5), as  $R(0) = 1$ . Indices (1), (2), (5) and (6) are probabilistic, (3) and (4) are deterministic.

Deterministic indices also include index  $K_S^A$  which is a minimum number of affected elements with total damage for the system not less than  $A$ , offered in [6]. Let a certain system consist of  $n_s$  objects,  $S$  is a number of the system variant. Single negative impact on the  $i$ -th element causes the damage  $C_i^S$ . The elements are ranged in order of damage decrease  $C_1^S > C_2^S > \dots > C_{n_s}^S$ . Let us set a threshold acceptable value of damage  $A$  and assume that in case of multiple negative impact different elements are affected, and first of all the elements with the most damage. And damage for the system as a whole is obtained by addition of damages of separate elements. Then  $K_S^A$  is defined by formula

$$K_S^A = \min_{(C_S > A)} K_S, C_S = \sum_{i=0}^{K_S} C_i^S. \quad (7)$$

Here  $K_S$  is the number of faulty elements or elements lost as the result of NI in  $S$  structure.

Besides we can amplify a model perception of survivability by introducing the following additional characteristics to a NI model and its consequences:

$r$  – **frequency** of NI is a number of simultaneously affected elements or subsystems by one NI. In this case by the result of one NI we can observe  $r$  defects in the system. Such approach is used for dispersed systems in which a single NI causes multiple consequences (for example, act of nature or a military strike);

$L$  – **resistance** of the element to an affecting impact. It is an integer number of NI endured by an element without loss of operability. In a more general case a deterministic  $L$ -criterion of resistance should be substituted by the function of resistance that may have a stochastic or a fuzzy set nature. We speak about resistance when the survivability of the system element is provided by external SAs, for example, be means of defense measures (air defense, underground fortifications, etc.). For the models that are considered here  $L = 0$ .

### 5.2. Indices of survivability based on the results of task execution

**Let now** the system with a basic structure  $S_0$  execute a certain task during a period of time  $t$ . As the result of NI a new structure  $S_i$  may occur in the system, one of the variety of operable structures  $S^O = \{S_i, i=1, \dots, N_p\}$  or inoperable structures  $S^{IO} = \{S_i, i = N_p+1, \dots, N\}$ . After a  $n$ -tuple NI the system with a new structure should start the execution of a prescribed task and complete it within the time period  $t$ . The estimation of survivability based

on the results of task execution is carried out using the following indices.

Conditional function of survivability

$$G_i(t) = G(t | S_i) = P(t | S_i) / P(t | S_0) \quad (8)$$

is a relation of probabilities of task execution by the system for two cases: for a basic structure  $S_0$  and for a new one  $S_i$ . And it may be possible that for a new structure  $S_i$  the task will be formulated differently than for  $S_0$  structure. However, in this case  $G_i(t) < 1$  have to be fulfilled. If recovery is available inoperable structures ( $i > N_p$ ) also could be considered, because for them  $P(t/S_i) > 0$  also may hold true. If recovery is unavailable  $P(t/S_i) = 0$  with  $i > N_p$ .

The function of the system survival in case of a  $n$ -tuple impact (an event  $A_n$ ):

$$G(t, n) = G(t | A_n) = \sum_{k=1}^N P_n(k) G_k(t) \quad (9)$$

is the survivability function averaged by all possible structures;  $P_n(k)$  is a probability of occurrence of structure  $S_k$  after a  $n$ -tuple NI.

Absolute function of survivability

$$G(t) = \sum_{n=1}^{\infty} P(A_n) G(t | A_n) = \sum_{k=1}^N P(S_k) G_k(t) \quad (10)$$

is the function of survival averaged by all possible events  $A_n$ . Probability  $P(S_k)$  is defined by formula

$$P(S_k) = \sum_{n=1}^{\infty} P(A_n) P_n(k). \quad (11)$$

Indices (9) and (10) refer to the additive class and they ensure a turning of the vector index  $\{G_k(t), k = 1, \dots, N\}$  into a scalar one. If there is no consistent information about probabilities  $P_n(k)$  and  $P(S_k)$  they can be substituted with weight coefficients  $\alpha_k$  and  $\beta_k$ , that are assigned expertly. If it is also difficult, then it is necessary to proceed with mini-max indices.

Sequence  $G(t, n)$  is a decreasing function  $n$  and it changes from 1 with  $n = 0$  to 0 with  $n \rightarrow \infty$ . That is why an average number of NI that causes non-execution of a task is defined by formula

$$\bar{\omega}(t) = \sum_{n=1}^{\infty} n(G_i(t, n-1) - G(t, n)) = \sum_{n=0}^{\infty} G(t, n). \quad (12)$$

with  $t = 0$  or  $\lambda_i = 0$  (elements are absolutely reliable) formulas (9) and (12) change into (2) and (5) respectively. Indeed, with  $t = 0$  function  $G_k(0) = 1$  for  $k \leq N_p$  and  $G_k(0) = 0$  for  $k > N_p$ . Based on (9) we have the function of survival with a zero duration of a task:

$$G(0 | A_n) = \sum_{k=1}^{N_p} P_n(k) = R(n), \quad (13)$$

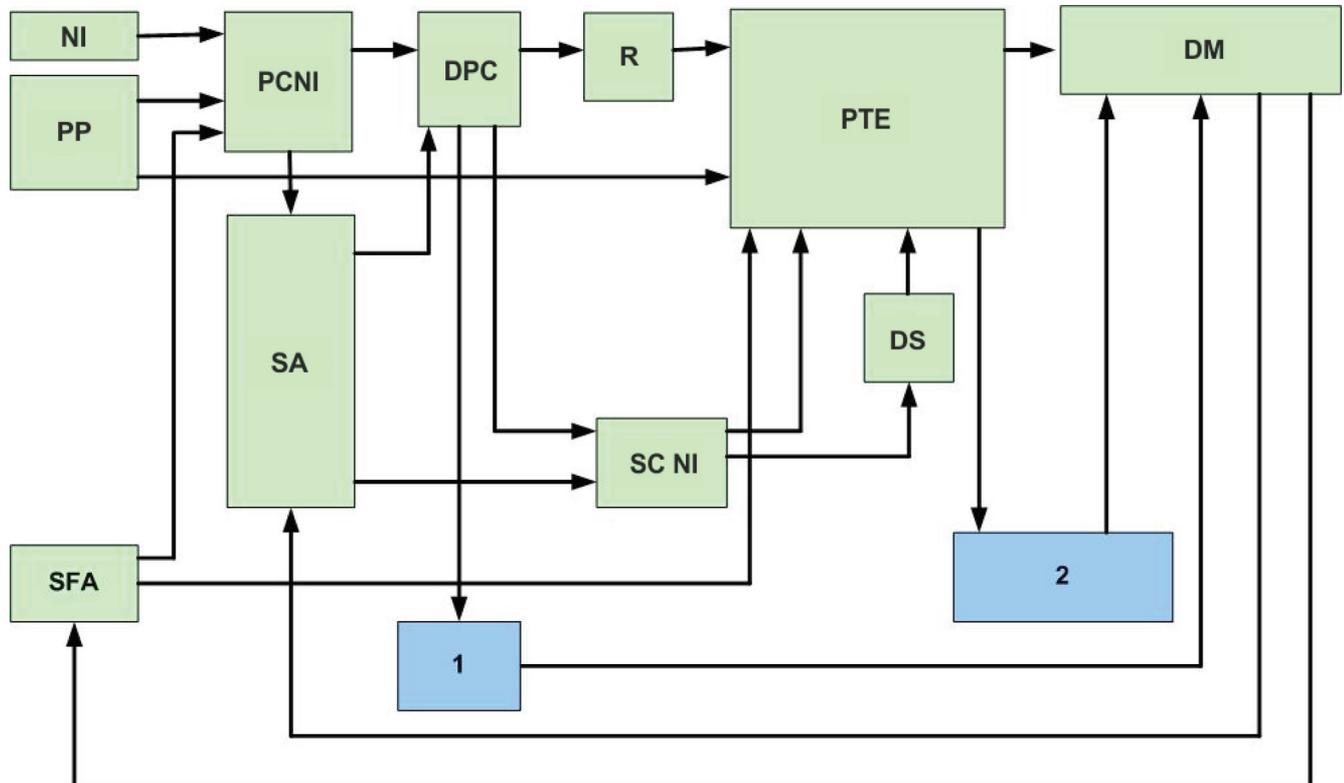


Fig. 3. Structure scheme of the survivability model

(1 – estimation of survivability by the system state, 2 – estimation of survivability by the results of task execution); PP – physical processes, DPC – development of primary consequences, R – reliability, PTE – process of task execution, DM – decision making, SC NI – secondary consequences of NI, DS – development of secondary consequences, SFA – structure, function, algorithm)

and based on (10) we obtain an absolute function of survivability with a zero duration of a task

$$G(0) = R = \sum_{n=1}^{\infty} P(A_n)R(n).$$

Indices (8) – (13) could be generalized also for the case of branching and multipoint structures. For this purpose probability of task execution in (8) should be substituted with a certain quality factor  $E(S)$ . So for a system with branching structure, a functioning within a time interval  $t$  can be expressed by a composed function

$$E(t, S) = \varphi (P(t/S)), \tag{14}$$

where  $P(t/S) = \{P_m(t/S), m=0, \dots, M\}$  is the distribution of the number of inoperable branches at the moment of time  $t$  provided that initially the system had  $S$  structure. Then a conditional function of survivability is defined by formula

$$G_i(t) = G(t/S_i) = E(t, S_i) / E(t, S_0). \tag{15}$$

With  $M = 1$  we shall get  $E(t/S) = P(t/S)$ , and formula (15) changes into (8). Other indices can be found by formulas (9) – (12).

### 6. Survivability models

Model of survivability of a complex system is actually a set of a large number of particular models of different application that use both deterministic and probabilistic methods to describe processes (Figure 3).

**NI model.** By the scope of application we can distinguish point models and spatial models. In point models NI is assumed to affect one or several elements. In the latter case the scope of NI is a group of points in which the system elements are located. That is why the number

of elements in the system is always more than the number of points in the scope of NI. For each element or a group of elements a probability of occurrence in the scope of NI is set. If the scope is single-point the following distribution is set  $\{\alpha_i, i = 1, \dots, N\}$ , where  $N$  is the number of the system elements,  $\alpha_i$  is a probability that the  $i$ -th element occurs in the scope of NI. One of possible distributions is a n equal distribution  $\alpha_i = 1/N$ . For a multipoint scope the following distribution is set  $\{\beta_i = P(X = i), i = 1, \dots, N\}$  where  $\beta_i$  is a probability that  $i$  elements occur in the scope. In models we can use, for example, truncated binomial distribution

$$\beta_i = C_N^i p^i (1 - p)^{N-i} / (1 - p^N), i = 1 \dots N \tag{16}$$

or a truncated Poisson distribution

$$\beta_i = \frac{a^i}{i!} / \sum_{k=1}^N \frac{a^k}{k!}, i = 1 \dots N; a = -\ln(Np).$$

In spatial models it is necessary to set two-dimensional distribution of orthogonal coordinates of the NI epicenter  $p_2(x_0, y_0)$  and distribution of the radius of the circle  $p_0(r_0)$ , where NI is observed.

Based on the type of distribution of NI intensity we can distinguish NI with an infinite intensity, with a constant intensity  $I$  on the whole scope, and with an intensity decreasing from the epicenter by a certain law  $I(r, \varphi)$ , in particular, in accordance with Rayleigh rating:

$$I(r, \varphi) = I_0 \exp(-r^2 / ar_0^2), \tag{17}$$

where  $I_0$  is a maximum intensity in the epicenter,  $r_0$  is a radius of a circle of the NI scope,  $a$  is a constant parameter,  $r$  and  $\varphi$  are polar coordinates of a point with the origin in the epicenter.

By duration we can distinguish impulse NI (zero duration), with a constant  $\tau$  and random duration  $T$ , set by dis-

Table 2

Factors	NI model						
	1	2	3	4	5	6	7
Scope	point	point	group of points	group of points	area	area	area
Intensity	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$I_0$	$I_0$
Duration	0	0	0	0	0	$\tau$	$\tau$
Strategy	1	2	1	2	1	1	2

Table 3

Factors	System model				
	1	2	3	4	5
Type of element	Point	Point	Point	Point	Area
Resistance	0	0	0	0	0
System topology	arbitr.	arbitr.	set	set	set

tribution  $F_r(t)=P(T < t)$ . Under a constant duration a range of disturbance  $I_0$  can be set as a time function, for example, using formulas:

$$I_0(t) = I_0^0(1 - t / \tau); I_0(t) = I_0^0 \exp(-t^2 / b\tau^2). \quad (18)$$

where  $b = 0,3 \div 0,5$  is a parameter. Similar dependences are also set under a random duration, only in this case  $\tau$  in (18) is substituted with a random  $T$ .

In case of multiple NI the simplest strategies of the choice of characteristics of a recurrent NI are the strategy of independent NI (strategy 1) and the strategy with an exclusion of affected elements from the scope of a recurrent NI (strategy 2). By the distinguished characteristics different models could be created. Some of them are given in Table 2.

**System model.** *SFA*-model [10] gives a description of technical, functional and algorithmic structure of the system including the models of functioning and characteristics of the elements, system topology, traffic of information, material and energy flows, functional and structure hierarchy, purpose tree.

Let us take a closer look at four characteristics of the model: dimensions of the elements, their reliability, resistance and system topology. In terms of dimensions elements may be point, linear, flat with a boundary of arbitrary shape, solid with a boundary of simply connected surface. In terms of reliability level of the elements, the models can have absolutely reliable elements and the elements of limited reliability. The first case is an idealization used to estimate the survivability by the system state. In terms of resistance we can distinguish the elements with zero resistance and the elements with non-zero resistance. The first case is an idealization which is used in order to consider all elements occurring in the scope of NI to be inoperable. In the second case a probability of disturbance of operability depends on the NI intensity and on the size of the part of area (or scope) of the element that occurred in the scope of NI.

By the system topology let us distinguish the models with arbitrary and specified topology. A model of the first type can be used with point elements and point NI. The second type model is used with spatial NI and flat or solid elements.

Combinations of three characteristics lead to the models of the system that are listed in Table 3.

**A model of physical processes (PP).** This model is used for analysis of transient processes in the system after a NI. It describes a trajectory of the process of functioning occurred as the result of its own movement.

**A model of primary consequences (PC)** is obtained as the result of interrelation of a PP model with a model of NI. Disturbances related to NI are imposed into a PP model, with consideration of deterministic transient processes occurred as the result of own movements and forced movements caused by disturbances, but without any controlling actions of SAs.

**A SA model** reflects the characteristics of control means, emergency protection, reconfiguration and control. Decision algorithms of struggle for survivability which are the part of this model, form certain controlling actions aimed at the change of a structure and parameters of the system, as well as at the use of internal reserve created for the operation in extreme situations. Characteristics of external SAs should also be considered in this model.

**A model of development of primary consequences (DPC)** is obtained as the result of combination of a PP model and a SA model. It makes it possible to find a trajectory of the controlled transient process taking into account SA actions. A final objective of the analysis of a DPC model is a determination of a new resilient state of the system. Since certain SA characteristics are probabilistic, the results of DPC analysis can also be represented in a probabilistic form.

**A dependability model (D)** contains the information about reliability and maintainability of the elements, the system of maintenance, a system response to certain failures of the element, as well as about the influence of different affecting factors of NI on the reliability of the elements. This model is used to estimate the survivability by the results of task execution.

**A model of secondary consequences (SC)** reflects those late consequences of NI that may occur in the system as the result of reduction of scope of functions and deterioration of technical characteristics. Secondary consequences include a longer time of function performance, a faster ageing and deterioration of the elements, an additional expansion of errors in information systems, an increased consumption of energy and materials for performance of the same functions, and other consequences leading to the reduction of available reserves in the system and to a further degradation of technical characteristics.

**A recovery model (R)** contains the description of emergency resources, rules and methods of their use in extreme situations in order to recover technical and functional algorithmic structure of the system part which executes the prescribed task. It could be interpreted as a model of the system development after a NI.

**A model of the processes of task execution (PTE)** is obtained as the result of combination of five models (*SFA*, PP, D, R, SC). The analysis of this model helps to estimate the survivability by the results of task execution.

When developing a system, it would be very useful to provide a developer with a model of decision making (DM) about how to improve survivability, if the estimates show its unsatisfactory level. A model helps to formulate recommendations for developers related to a change of the system structure and parameters, as well as to an additional development of SAs.

## 7. Calculation and analysis of survivability

When describing the elements we assume that each element may be in one of three states:  $e_0$  – the element

is operable and put into operation;  $e_1$  – the element is operable but is taken out of operation due to different reasons;  $e_2$  the element is inoperable. State transitions are determined by four groups of factors: natural failures of the elements, recovery of operability, switches by actuation of emergency protection and reconfiguration, external disturbances. Connections between elements are determined and stationary in time, so the element state could be determined at any time by the state of operability of this element and of the states of other elements. Features of the system operability are permanent in time and help to define the system state by the set of states of its elements.

To calculate the survivability indices we can take one of the following approaches: approach 1 based on a logical and probabilistic method or approach 2 based on the results of theory of random placements including **Morgan and Stirling** numbers.

### 7.1. Methods of calculation based on logical and probabilistic method

Let us consider the basic stages of the **analysis of the system survivability based on a logical and probabilistic model**.

**Stage 1. Description of states of elements.** For each element two logic variables are set:  $x_i$  is an indicator of operability of the  $i$ -th element ( $x_i = 1$ , if it is operable and  $x_i = 0$  if it is not),  $y_i$  is an indicator of the state of an operable element ( $y_i = 1$ , if the element is in operation,  $y_i = 0$  if it is not). To counter disturbances which affect the elements indicators are set  $z_{ij}$  ( $z_{ij} = 1$ , if a disturbance of the  $j$ -th kind affects the  $i$ -th element,  $z_{ij} = 0$  if it does not) and  $z_i$  is a logical sum by all kinds of disturbances. Then the indicators of three states of an element are set:

$$\begin{aligned} u_{i0} &= 1[e_0] = x_i \overline{y_i} \overline{z_i}; \quad u_{i1} = 1[e_1] = x_i \overline{y_i} z_i; \\ u_{i2} &= 1[e_2] = \overline{x_i} \vee x_i z_i \end{aligned} \quad (19)$$

**Stage 2. Construction of logical dependences.** Based on the preliminary analysis of dynamic models of physical processes, with consideration of operations of the means of emergency protection, reconfiguration and control, logic equations are constructed in relation to unknown states of operable elements:

$$y_i = f_{y_i}(x_k, y_j, z_k, k = 1, \dots, N; j \in M_i), i = 1, \dots, N, \quad (20)$$

where  $N$  is a number of elements in the system,  $M_i$  is a variety of elements, neighboring to the  $i$ -th element. A set of equations like (20) makes a closed system of logic equations represented in a vector form:

$$Y = f_Y(X, Y, Z) \quad (21)$$

An advantage of this equation is that when describing a state of an operable element we use only its immediate surround, and it is not necessary to check the whole system. Later these particular and rather simple dependences are used to find an explicit dependence of the state of an operable element on the operability of the rest elements and characteristics of NI.

The system operability is determined by operability of its elements and by dependences (21). For many systems the main state is the state of a relatively small group of output elements. However, due to the availability of indirect connections reflected in (21), the system operability is determined by the state of all other elements as well. For a single-functional system a logic function of operability (LFO) is written as

$$F = f(X, Y, Z) \quad (22)$$

In a multi-functional system the dependence (22) is constructed for each function separately. If simultaneous performance of all functions is required, then

$$F = \&_{(i)} f_i(x, y, z). \quad (23)$$

where  $f_i$  is a logic function – the indicator of performance of the  $i$ -th function of the system. This method of description of the system state does not require a combinatorial enumerating of all states of the elements, and functions  $f_i$  are formally found from the systems of logic equations.

**Stage 3. Solution of logic equations.** Equation system (21) is linear and it can be transformed into:

$$y_i = a_i \vee a_{i1} y_1 \vee a_{i2} y_2 \vee \dots \vee a_{iN} y_N; a_{ii} = 0, \quad (24)$$

where  $a_i$  and  $a_{ij}$  are the coefficients expressed through  $x_i$  and  $z_j$ . There are different ways to solve logic equations, including a method of determinants, substitution method, matrix method, etc. The method of determinants, as well as its application in relation to dependability is described in paper [11]. Solution (24) in form of  $Y = g_Y(X, Z)$  should be put into (22) or (23) and obtain an explicit expression

$$F = f(X, g_Y(X, Z), Z) = g(X, Z). \quad (25)$$

Solution of the system of logic equations should be carried out repeatedly: one time for a basic structure  $S_0$ , when all  $z_{ij} = 0$ , and several times more, depending on the number of different types of disturbances. Searching through all variants under a single and multiple impacts, it is possible to obtain a complete set of operable structures in the system. Function (25) allows for analysis of  $d$ - and  $m$ -survivability by means of enumerating of the vector of states of the elements.

**Stage 4. Probabilistic description of the elements and external disturbances.** Each element is represented in a

probabilistic model by probability  $p_i = P(x_i = 1)$  that at this moment or at any arbitrary moment the element of operable. When disturbance  $z_{ij} = 1$  occurs, the resistance of the  $i$ -th element in relation to the  $j$ -th disturbance can be taken into account using probabilities  $\alpha_{ij}$  that an element will keep its operability in case of disturbance. Besides, the probabilities are set that an element will occur in the scope of the  $j$ -th factor of NI.

**Stage 5. LFO conversion to a form of transition to substitution.** According to [8] we can distinguish the forms of transition to a total and partial substitution. Forms of transition to a total substitution are a full disjunctive normal form, a noniterated form in a “joint-denial” basis, a disjunction of orthogonal noniterated forms. After the reduction to one of these forms, a one step substitution of logic variables and logic operations with probabilities and arithmetic operations is made. If such conversions are difficult to realize due to their complexity, one can use a form of transition to a partial substitution. Current varieties of these forms and conversion rules are listed in [8].

**Stage 6. Writing a mixed form.** Substitution of noniterated variables in the conversed LFO is a partial substitution as the result of which certain logic variables and operations are substituted with probabilities and arithmetic operations, and the rest variables and operations transit into the indices of arithmetic expressions. The obtained form is called a mixed form because it contains logic variables and probabilities and two groups of operations: logic and arithmetic. Methods and algorithms of transition to a mixed form are described in [11].

**Stage 7. Determination of survivability indices.** Multistep substitution of logic variables in mixed forms constructed for a basic structure  $S_0$  and other operable structures  $S_i$  is used to find probabilities  $P(t/S_0)$  and  $P(t/S_i)$ , and then formula (8) is used to find a conditional function of survivability  $G_i(t)$ . Formulas (9) – (12) are used to find a function of survival, absolute survivability function, average number of NI.

For the systems of branching structure after stage 6 it is necessary to perform three more stages (stages 8, 9 and 10) and only then get back to stage 7.

**Stage 8. Constructing a generator polynomial of the distribution of probabilities of states of the  $i$ -th branch [11]:**

$$\Phi_i(z, X) = 1 + (z - 1)Q(X), \quad (26)$$

where  $Q(X) = P\{F(X) = 0\}$  is a mixed form,  $X$  is a vector of non-substituted logic variables.

**Stage 9. Constructing a generator polynomial for the system.** If the structure is isotropic a polynomial (26) is raised to a power equal to a branching coefficient at a bottom layer of a branching structure. Then logic variables of the next layer are substituted, and again raising to a power, substitution, etc. As the result of a multi-step procedure we obtain a polynomial whose coefficients express probabilities that certain amount of branches is inoperable. If the struc-

ture is not isotropic, a raising into a power is substituted by multiplying of polynomials.

**Stage 10. Determination of survivability indices.** Stage 9 includes distributions  $P(t/S_0)$  and  $P(t/S_i)$ , then scalar indices  $\varphi(P(t/S_0))$  and  $\varphi(P(t/S_i))$  are calculated, formulas (9) – (15) are used to find  $G_i(t)$ ,  $G(t, n)$ ,  $G(t)$  and other survivability indices.

## 7.2. Estimation of survivability by the system state based on the theory of random placements

Let a two-pole system contain  $k$  subsystems and  $N$  point elements with arbitrary connections and have the LFO

$$F = f(X), X = \{x_1, x_2, \dots, x_N\}$$

The system is a subject to the flow of  $n$  point independent NI with equally probable affection of each element at the occurrence of NI. We consider the elements resistance to be 0, and the intensity of NI is insufficient to ensure a transition of the element that occurred in the scope of NI, to an inoperable state. Let us estimate the survivability by indices (2) – (6).

Survival rate of the system with a  $n$ -tuple NI can be represented as follows

$$R(n) = \sum_{X \in X_1} P(X | A_n) = P\{F = 1 | A_n\}, \quad (27)$$

where  $X_1$  is a subset of vectors  $X$ , corresponding to operable states of the system. Probability  $P(X | A_n)$  is found by formula:

$$P(X | A_n) = \sum_{\bar{n} \in M_n} P(\bar{n})P(X | \bar{n}), \quad (28)$$

where  $\bar{n} = (n_1, n_2, \dots, n_k)$  is a vector of the number of NI affecting  $k$  subsystems,  $M_n$  is a set of vectors that fulfill condition  $n_1 + n_2 + \dots + n_k = n$ . Probability

$$P(\bar{n}) = \frac{n!}{n_1! n_2! \dots n_k!} \gamma_1^{n_1} \gamma_2^{n_2} \dots \gamma_k^{n_k}, \quad (29)$$

where  $\gamma_i$  is a probability that the  $i$ -th subsystem is within the scope of NI. In particular, it can be that  $k = N$ . At an equally probable affection of the elements formulas (27) – (29) could be clarified.

By representing LFO as an orthogonal disjunctive normal form

$$F = \bigvee_{i=1}^m Q_i, \quad (30)$$

we shall write (27) as follows

$$R(n) = \sum_{i=1}^m P(Q_i = 1 | A_n). \quad (31)$$

For implicants with  $l_i = 0, 1$  or 2 denials, formulas in (31) can be written in explicit form:

$$P(Q_i = 1 | A_n) = (1 - s_i / N)^n, l_i = 0, n \geq 1, \quad (32)$$

$$P(Q_i = 1 | A_n) = \sum_{j=1}^n C_n^j (1 - s_i / N)^{n-j} / N^j, l_i = 1, n \geq 1,$$

$$P(Q_i = 1 | A_n) = \sum_{k=2}^n \sum_{j=1}^{k-1} C_k^j (1 - s_i / N)^{n-k} / N^n, l_i = 2, n \geq 2,$$

where  $s_i$  is a number of letter in implicant  $Q_i$ . These formulas refer to a particular case of (29) with  $k = 2$  and different values of  $n_1$  and  $n_2$ .

### 7.3. Estimation of survivability by the system state based on a combinatorial method

A basic structure  $S_0$  is used to define all possible operable structures  $S_i, i = 1, \dots, N_p$ . then:

$$R(n) = \sum_{j=1}^{N_p} r_j(n) / N^n = r_n / N^n, \quad (33)$$

where  $r_j(n)$  is a number of cases when  $S_j$  structure occurs at a  $n$ -tuple NI. This number is defined by formula

$$r_j(n) = \sum_{(k)} L_{nk} B_{kj}, \quad (34)$$

where  $L_{nk}$  is a number of conversions from  $n$  elements of  $k$  kinds,  $B_{kj}$  is a number of different vectors  $X$  with  $k$  zeros that lead to  $S_j$  structure. Since parameters  $d$  and  $m$  from formulas (3) and (4) are usually very large, it is not difficult to find  $B_{kj}$  by simple enumeration of vectors. A maximum number of vectors under test is  $mN$ , but in practice it is much smaller.

Numbers  $L_{nk}$  are so called **Morgan numbers**. They are related to **Stirling numbers of the second kind** by formula

$$L_{nk} = k! S_{nk}, \quad (35)$$

where  $S_{nk}$  could be found using a recurrence relation

$$S_{nk} = S_{n-1,k-1} + k S_{n-1,k}; S_{nk} = 0 \text{ with } n < k; S_{nn} = 1. \quad (36)$$

But numbers  $L_{nk}$  could be calculated by formula:

$$L_{nk} = \sum_{i=1}^k C_k^i i^n (-1)^{k+i}. \quad (37)$$

We obtained the result of this paragraph already in 1987. In the course of research work we also obtained calculation formulas for the case of  $r$ -tuple NI and the systems with  $L$ -resistant elements [20].

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### About the authors

**Gennady N. Cherkosov** – Dr. Sci., professor, professor of Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia,

e-mail: [gennady.cherkosov@gmail.com](mailto:gennady.cherkosov@gmail.com)

**Alexey O. Nedosekin** – Dr. Sci., Ph.D., academician of the International Academy of Ecology, Man and Nature Protection Sciences, professor of National Mineral Resources University “Gorny”, St. Petersburg, Russia,

e-mail: [apostolfoma@gmail.com](mailto:apostolfoma@gmail.com)

## About trigonometric distributions for the description of failures of technical devices

**Vladislav A. Volodarsky**, chair of train protection systems, Krasnoyarsk Institute of Railway Transport, Krasnoyarsk, Russia, e-mail: volodarsky.vladislav@yandex.ru



Vladislav A. Volodarsky

**Abstract.** The purpose of this article is to offer and examine nonconventional trigonometric distributions in order to describe degradation failures of technical devices. Two methods for approximate description of reliability indices are proposed for an estimated value of mean time to failure. Firstly, as the parameter of a failure flow with operation time equal to mean time to failure tends to its stationary value equal to the opposite value of mean time to failure, it is offered to approximate the dependence of the parameter of a failure flow of the operation time by a piecewise linear function. Other reliability indices are defined using the Laplace transformation. For instance, the probability of reliable operation can be described by the cosine function, and the failure rate – by the tangent function. Secondly it is proposed to approximate the dependence of the density of failures distribution depending on the operation time by the sine function. Other reliability indices are defined using the Laplace transformation. For instance, the probability of reliable operation can be described by the function of squared cosine, and the failure rate – by the double tangent function. As a result of studies it has been concluded that since a failure rate of the offered distributions increases as the operation time increases, and the coefficient of variation is less than one, they can be used to describe degradation failures of technical devices. The obtained results have shown that reliability indices at these distributions are expressed by elementary functions and it can simplify the calculation of reliability indices of systems with different connections of their constituent elements.

**Keywords:** probability of reliable operation, failure rate, cosine distribution, the distribution of the cosine of the square.

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### Postulates

At present the reliability indices of technical devices (TD) are normally calculated presuming a constant rate of failure of their constituent elements. It corresponds with the case when the elements are affected only by sudden failures caused by external influences. Degradation failures of elements, related to internal processes of wear and ageing, are not taken into consideration. The latter does not correspond with reality. For instance, [1, 2] contain detailed descriptions of degradation processes causing wear and ageing of the elements of railway power supply, signalling and communication systems. Such processes lead to degradation failures of elements and are described in the reliability theory by a distribution class with an increasing failure rate function, i.e. IFR-distributions [3]. Let us call the elements with degradation failures the elements of an ageing type.

By means of acquisition and processing of information about failures recovered during the operation of the indicated system elements, only the estimations of constant parameter values of a failure flow  $\omega$  [1] or mean times to failure as  $\Phi = 1/\omega$  [2] are obtained. However, it does not mean that the failure rate of the ageing type elements is a constant value.

According to the definition, the parameter of a failure flow is a ratio of the number of items failed in the time interval

$n(dt)$  to the number of items under test in this interval  $dt$  provided that the failed items are replaced by the operable (new or repaired) ones, i.e.  $\omega(t) = n(dt)/Ndt$ , where  $N$  is the number of items under test which remains constant. From the reliability theory we know that the parameter of a failure flow at any distribution during operation tends to the stationary value equal to  $\omega = 1/T$ . It becomes apparent at the acquisition of statistics about failures of TD under actual operating conditions.

According to the definition, a failure rate is a ratio of the number of items failed in the time interval  $n(dt)$  to the average number of the items  $N_{av}$ , operating without fail in this time interval  $dt$ , i.e.  $\lambda(t) = n(dt)/N_{av}dt$ . Due to failures of items,  $N_{av}$  decreases with each interval, however  $\lambda(t)$  of the ageing type elements increases.

As the laws of distribution of mean time to failure of the ageing type elements are normally not known, the task of calculation of TD reliability indices has to be solved in the context of uncertainty.

**The purpose of the paper** is to offer and examine nonconventional trigonometric distributions in order to describe degradation failures of technical devices.

When only the value of mean time to failure can be estimated, for example, from the expression  $T = 1/\omega$ , then two methods can be offered for an approximate description of TD reliability indices.

## Cosine distribution

Firstly, as the parameter of a failure flow with  $t = T$  tends to its stationary value equal to  $1/T$ , it is proposed to approximate the dependence  $\omega(t)$  by a piecewise linear function of the following type [4]:

$$\text{with } t < T \omega(t) = t/T^2; \text{ with } t \geq T \omega(t) = 1/T. \quad (1)$$

Other indices are defined using the Laplace transformation. The distribution density  $f(t)$  shall be found from the equation connecting it in an operator form with the parameter of a failure flow  $f(s) = \omega(s)/(1+\omega(s))$  as

$$f(t) = (1/T)\sin(t/T). \quad (2)$$

Then, the probability of reliable operation  $P(t)$  and the failure rate  $\lambda(t)$  shall be defined from the equations:

$$P(t) = 1 - \int_0^t f(t)dt = \cos(t/T), \quad (3)$$

$$\lambda(t) = f(t)/P(t) = (1/T)\text{tg}(t/T). \quad (4)$$

The argument  $t/T$  in the formulas used to define reliability indices is measured in radians. Let us call the obtained distribution the cosine distribution whose range of definition lies within the interval  $0 < t/T < \pi/2$ .

Improper integral of distribution density within the range of distribution definition as per [5] shall be equal to one. Let us check

$$\int_0^{\pi/2} (1/T)\sin(t/T)dt = 1.$$

Coefficient of variation of distribution is defined from the equation

$$V = \mu_2^{0.5} / \mu_1, \quad (5)$$

where  $\mu_1$  is the first initial moment;  
 $\mu_2$  is the second central moment;

$$\mu_1 = \int_0^{\pi/2} tf(t)dt = \int_0^{\pi/2} (t/T)\sin(t/T)dt = T,$$

$$\mu_2 = \int_0^{\pi/2} (t - \mu_1)^2 f(t)dt =$$

$$= \int_0^{\pi/2} (t - T)^2 (t/T)\sin(t/T)dt = (\pi - 3)T^2.$$

Putting the values  $\mu_1$  and  $\mu_2$  into equation (5), we shall get  $V = 0,376$ .

As a failure rate of this distribution in accordance with (4) is a monotonously increasing function of time, and the value of the coefficient of variation is less than one, it refers to the class of IFR-distributions and can be used to describe degradation failures of technical devices. Article [4] contains

the definitions of asymmetry and kurtosis coefficients and notes that the cosine distribution can be represented in the Pearson's range by a point with the coordinates  $p^2 = 0,18$  и  $\beta = 2,23$ . It is shown that according to [3] a cosine function is also the distribution with an increasing mean failure rate, the distribution of a "new is better than the used" type and the distribution of a "new is averagely better than the used" type.

Equations (1), (2), (3) and (4) were used to define the dependences of reliability indices of the relative time of operation  $t/T$  of the distribution offered. The calculation results are brought together in Table 1.

**Table 1**

$t/T$	0	0,2	0,4	0,6	0,8	1,0	1,2	1,4	$\pi/2$
$Tf(t)$	0	0,20	0,39	0,56	0,72	0,84	0,93	0,98	1,0
$T\lambda(t)$	0	0,20	0,42	0,68	1,03	1,56	2,57	5,80	$\infty$
$T\omega(t)$	0	0,20	0,40	0,60	0,80	1,0	1,0	1,0	1,0
$P(t)$	1	0,98	0,92	0,83	0,70	0,54	0,36	0,17	0

Distribution of the cosine of the square

Secondly, it is proposed to approximate the dependence of density of TD failures distribution  $f(t)$  depending on the operation time  $t$  by the sine function of the following type

$$f(t) = (1/T)\sin(2t/T) \quad (6)$$

with a range of definition  $0 < t < \pi/2$ .

Improper integral of distribution density within the range of distribution definition as per [5] shall be equal to one. Let us check

$$\int_0^{\pi/2} (1/T)\sin(2t/T)dt = 1.$$

Considering that  $\sin(2t/T) = 2\sin(t/T)\cos(t/T)$ , let us introduce the equation (6) in the following form

$$f(t) = (2/T)\sin(t/T)\cos(t/T). \quad (6a)$$

The probability of reliable operation  $P(t)$  with consideration of (6) is defined based on the equation

$$P(t) = 1 - \int_0^t f(t)dt = (1 + \cos(2t/T))/2. \quad (7)$$

Considering that  $\cos(2t/T) = \cos^2(t/T) - \sin^2(t/T)$ , let us introduce the equation (7) in the following form

$$P(t) = \cos^2(t/T). \quad (7a)$$

Let us call the obtained distribution the distribution of the cosine of the square.

A failure rate  $\lambda(t)$  with consideration of (6a) and (7a) is defined as

$$\lambda(t) = f(t)/P(t) = (2t/T)\text{tg}(t/T). \quad (8)$$

The parameter of a failure flow  $\omega(t)$  is defined through the Laplace transformation of the following type  $\omega(s) = f(s) / (1 - f(s))$  with consideration of (6) as follows

$$\omega(s) = (\sqrt{2}/T) \sin(\sqrt{2} \cdot t/T). \quad (9)$$

The coefficient of variation of distribution is defined using the equation (5).

$$\mu_1 = \int_0^{\pi T/2} t f(t) dt = \int_0^{\pi T/2} (t/T) \sin(2t/T) dt = \pi T/4,$$

$$\mu_2 = \int_0^{\pi T/2} (t - \mu_1)^2 f(t) dt = \int_0^{\pi T/2} \left(t - \frac{\pi T}{4}\right)^2 \sin(2t/T) dt = \left(\frac{\pi^2}{16} - 0,5\right) T^2.$$

Putting the values  $\mu_1$  and  $\mu_2$  into equation (5), we shall get  $V = 0,435$ .

Equations (6), (7a), (8) and (9) were used to define the dependences of reliability indices of the relative time of operation  $t/T$  of the distribution offered. The calculation results are brought together in Table 2 and are represented in Figures 1 and 2.

Table 2

$t/T$	0	0,2	0,4	0,6	$\pi/4$	1,0	1,2	1,4	$\pi/2$
$Tf(t)$	0	0,39	0,72	0,93	1,0	0,91	0,68	0,33	0
$T\lambda(t)$	0	0,40	0,84	1,36	2,0	3,12	5,14	11,6	$\infty$
$T\omega(t)$	0	0,39	0,76	1,06	1,26	1,40	1,39	1,30	1,10
$P(t)$	1,0	0,96	0,85	0,68	0,50	0,29	0,13	0,03	0

Formula (8) and Figure 1 show that a failure rate is monotonously increasing with the time of operation and, considering that the value of the variation coefficient is less than one, the offered distribution refers to the class of IFR-distributions and can be used to describe degradation failures of technical devices' elements. Figure 1 also shows that the parameter of a failure flow tends to the value equal to  $1/T$ . It confirms a famous provision of the reliability theory that at any distribution, the parameter of a failure flow in the course of operation tends to the steady value which is the opposite to the mean time to failure value.

As it is seen from Figure 2, the probability of TD reliable operation decreases in the course of operation, and with the value  $t = \pi T/2$  it verges towards the zero value. And a curve  $P(t)$  is at first up-convex, and then down-convex.

For comparison Figures 3 – 5 show the curves of distribution density, failure rate and the parameter of a failure flow of the cosine distribution (marked with blue 1) and the distribution of the cosine of the square (marked with red 2), built with the use of data from Tables 1 and 2. As

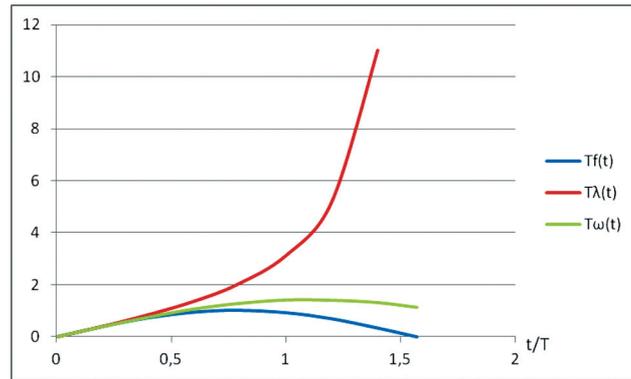


Fig. 1

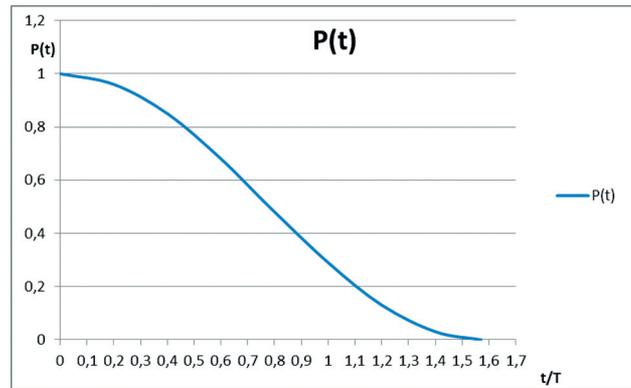


Fig. 2

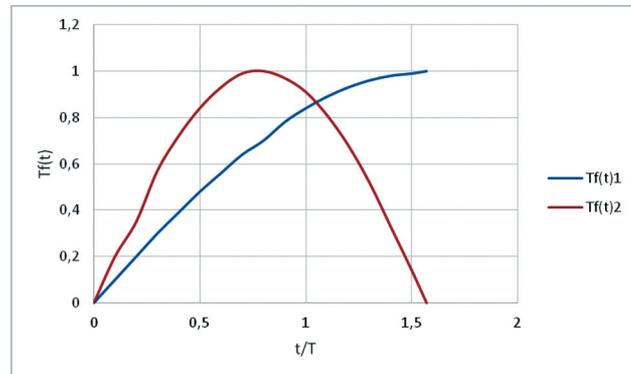


Fig. 3

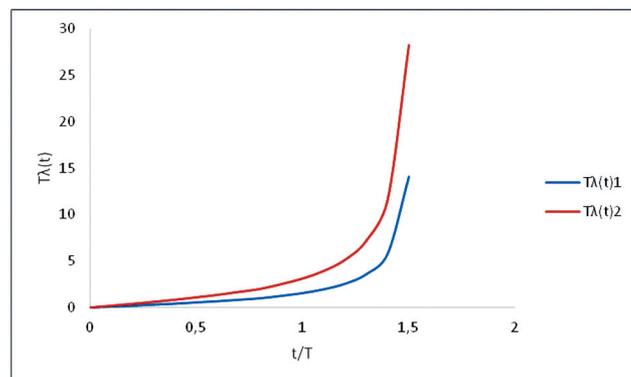


Fig. 4

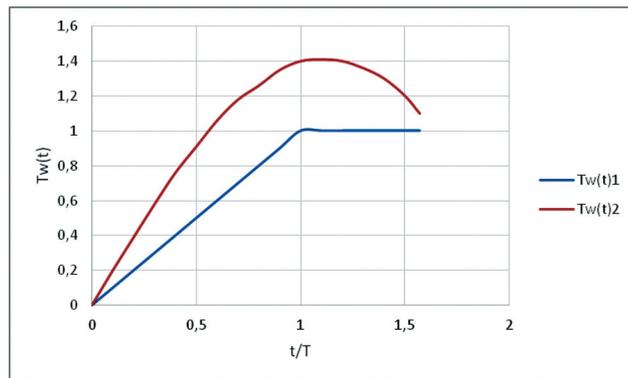


Рис. 5

we can see from formulas (4), (8) and Figure 4, a failure rate at the distribution of the cosine of the square increases in the course of operation is twice as fast as in case of the cosine distribution.

## Conclusion

In the case when only mean time to failure is defined, and we know that TD elements are exposed to wear and ageing, it is reasonable to use the offered cosine distribution and distribution of cosine of the square to describe TD degradation failures in the context of uncertainty. The obtained results show that reliability indices at these distributions are expressed by elementary functions and

it can simplify the calculations of reliability indices of systems with different connections of their constituent elements.

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## About the author

**Vladislav A. Volodarsky** – PhD, professor, senior researcher of the chair of train protection systems, Krasnoyarsk Institute of Railway Transport. Krasnoyarsk, Russia. Tel.: +7 (391) 221 60 72, e-mail: [volodarsky.vladislav@yandex.ru](mailto:volodarsky.vladislav@yandex.ru)

## Special aspects of calculation of lifting equipment reliability

**Vladimir A. Ermolenko**, Kaluga branch of the Bauman Moscow State Technical University, Kaluga, Russia, e-mail: tvermolenko@rambler.ru

**Pavel V. Vitshuk**, Kaluga branch of the Bauman Moscow State Technical University, Kaluga, Russia, e-mail: tvermolenko@rambler.ru



Vladimir A. Ermolenko



Pavel V. Vitshuk

**Abstract. Aim.** When designing lifting equipment as a whole, as well as of its elements it is desirable to perform not only deterministic strength estimations, but also a probabilistic calculation of major reliability indices. Theoretical approach to the calculation of major reliability indicators of lifting equipment is described by V.I Braude. In practice the calculation of reliability of lifting equipment is usually quite difficult, because the information about values for certain indices provided in literary sources is incomplete and discordant. It causes the necessity to use average reliability indices and to introduce different assumptions to the calculation. And the calculation results turn out to be rather approximate. At the same time an approximate calculation of reliability indices allows to decide on efficient use of one or another design layout of lifting equipment and/or its structural unit. **Methods.** To demonstrate the logical arguments that could be used at the calculation of reliability of lifting equipment, the article describes an example of calculation of probability of reliable operation for the lifting gear of an overhead crane, executed by a "detailed" scheme and consisting of nine elements: a three-phase induction electric motor with a short-circuit rotor; a parallel shaft double-stage gear box; a block brake with locking movement actuated by a coil spring and with breaking actuated by a short-stroke alternating electromagnet; flexible bolt coupling (with brake pulley); load drum; drum axle (or shaft); drum support; load cable and its mountings; hook assembly. Structurally, the elements of a lifting gear are connected in-series, i.e. in case of a failure of any element, the operable state of the gear is violated (a failure occurs). **Results.** The known experience of operation of lifting equipment shows that the most probable failures of a lifting gear's elements are the following failures: turn-to-turn short circuit of electric motor; wear out of bearings and gear teeth; turn-to-turn fault of a coil of a brake electromagnet; tearing up of a pulley of a flexible bolt coupling and break cheek wear out; fatigue breakdown of a drum and a bearing block, built into a drum; fatigue breakdown of a drum axle (or shaft); wear out of drum axle bearings, built into a drum; wear out (breakage) of wires and strands of a load cable; hook wear out and bearing freezing of a hook assembly. That is why the reference data used for calculation usually describe the probability of occurrence or a rate of these particular failures. Calculation was carried out with the following assumptions: Degradation (wear rout) failures were not taken into account, since they are anticipated during the phase of technical maintenance and repair; failures, caused by the violations of the rules of safe operation, were refer not to the crane failures, but to the failures of other systems. For descriptive reasons the elements of a lifting gear were chosen from the catalogue with a certain "margin" and without taking a load-bearing mode into account. **Conclusions.** The calculation results showed that neglecting various load-bearing factors (for instance, a gear box underload by a rotation moment) may lead to excess reliability of a crane as a whole, its machinery and structure components.

**Keywords:** probability of reliable operation, lifting equipment, lifting gear, overhead crane, reliability indices, designing, calculation.

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The calculation of reliability indices of lifting equipment as a whole, as well as of its components is quite difficult, because the values for certain indices provided in literary sources [1–3, etc.], are incomplete and discordant. To define missing values, average indices have to be used. And the calculation results turn out to be rather approximate.

The values of time, for which the probability of reliable operation is calculated, can be taken as  $t = 1$  year. The

number of hours of a gear's operation for 1 year shall be defined as follows [4]:

$$t = 8760 \cdot K_{\text{гг}} \cdot K_{\text{гд}} \cdot \text{ПИБ}, \text{ hour}, \quad (1)$$

where  $K_{\text{гг}}$  and  $K_{\text{гд}}$  are the factors of use of a calendar time of the year and of a day respectively; ПИБ is a duty rating.

For certain lifting, construction equipment and road machinery in general the values of these factors are listed

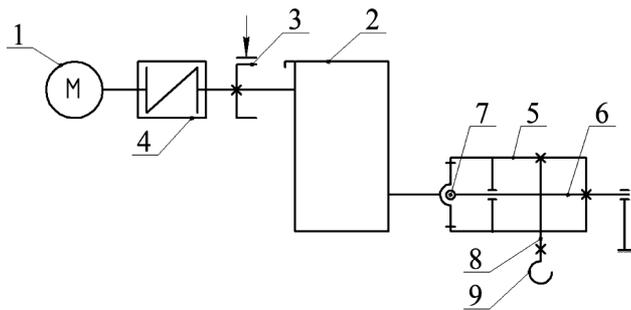


Fig. 1. Kinematic scheme of an overhead crane lifting gear: 1 – electric motor; 2 – gear box; 3 – electromagnet block brake; 4 – flexible bolt coupling (with brake pulley); 5 – drum; 6 – drum axle (or shaft); 7 drum support; 8 – load cable; 9 – hook assembly

in [4]. For crane electrical equipment the values of factors  $K_{HF}$  and  $K_{HC}$  are given in [2] and in [5].

According to VNIPTMASH [5] for the operation modes L; M; H; VH the estimated times of operation of crane electro motors are not more than 250; 1000; 3000; 4000 hrs/year respectively. The operation modes L; M; H; VH are given by obsolete rules of Gosgortekhnadzor of 30.12.1969. Correlation between operation modes of cranes and crane gears according to the Rules of Gosgortekhnadzor, GOST 25835-83, GOST 25546-82 and ISO 4301/1 is provided in [10]. Correlation between operation modes of cranes and crane gears for different foreign standards is given in Tables 1 and 2.

Theoretical approach to the calculation of major reliability indicators of lifting equipment is described by V.I Braude [6]. Let us dwell on a pragmatic angle of this issue. Standards have provisions for the probability of reliable operation as the main reliability index of an object. That is why let us consider the calculation of reliability indices of lifting equipment on the example of calculation of the probability of reliable operation of an overhead crane lifting gear (Figure 1) that consists of 9 elements [7]. Structurally, the elements of a lifting gear are connected in-series, i.e. in case of a failure of any element, the operable state of the gear is violated (a failure occurs).

1. Electric motor. From Guide [2] it is known that for 4A electric motors the probability of reliable operation is equal to 0,9 with 10000 hrs of operation time.

Based on the assumption about the exponential law of distribution:

$$P(10^4) = \exp\left(-\frac{10^4}{T_1}\right) = 0,9,$$

where  $T_1$  is a mean time to failure of an electric motor, hrs.

$$T_1 = \frac{10^4}{\ln 0,9} = \frac{10^4}{(-0,105)} = 9,5 \cdot 10^4.$$

2. Gear box. According to VNIPTMASH [5] a failure rate  $\omega_2=0,2$  per 1 thousand hrs. Then the gear box's mean time to failure is:

Table 1. Correlation of operation modes of a crane for foreign standards

ISO 4301/1	PN-79 M-06503 (Poland)	BS 466-84 (Great Britain)	SFS 4300-79 (Finland)	DIN 15018 (Germany)	B 4004-1 (Austria)
A1	1	A1	1	B1	T1
A2		A2	2		T2
A3		A3	3	T3	
A4	2	A4	4	B3	T3
A5	3	A5	5	B4	T4
A6	4	A6		T4	
A7	5	A7	6	B5	T5
A8	6	A8		B6	T6

Table 2. Correlation of operation modes of crane gears for foreign standards

ISO 4301/1	CT CЭB 2077-80	CSN 27009 (Czech Republic)	BS 466-84 (Great Britain)	SFS 4020-80 (Finland)	DIN 15018 (Germany)	FEM 9.661
M1	1		M3	ImB	IEm	IDm
M2					IDm	
M3					ICm	IBm
M4	2		M4	ImA	IAm	IAm
M5	3		M5	2m	2m	2m
M6	4		M6	3m	3m	3m
M7	5		M7	4m	4m	4m
M8	6		M8	5m	5m	5m

$$T_2 = \frac{1}{\omega_2} = \frac{1000}{0,2} = 5 \cdot 10^3 \text{ hrs.}$$

If under the calculation of the gear box a loading mode was not taken into account, there is usually an underload by an equivalent rotation moment, i.e. we will have a longer mean time to failure:

$$T_2' = T_2 / K_Q,$$

where  $K_Q$  is an equivalent loading coefficient [8].

$$K_Q = \sqrt[m]{\sum \left( \frac{Q_i}{Q_H} \right)^m \frac{t_i}{t}}, \quad (2)$$

where  $Q_i$  is a random value of the lift load's weight (defined by a load schedule for the respective operation mode);  $t_i$  is time spent on operation with load goods  $Q_i$ ;  $t$  is total time;  $m$  is a degree of durability line.

At the calculation of work surfaces of gear boxes gear teeth for back-to-back endurance  $m=3$  [9]. If we do not know a load schedule, then we can take an assumption: common cranes lift 15% of loads with nominal weight and 85% of loads with weight  $0,5 Q_H$  [10]. Then based on the formula (2) we will have:

$$K_Q = \sqrt[3]{1^3 \cdot 0,15 + 0,5^3 \cdot 0,85} \approx 0,63.$$

Now, with consideration of equivalent loading we get a higher value of the gear box's mean time to failure:

$$T_2' = T_2 / 0,63.$$

If under the calculation of the gear box a loading mode was taken into account, then the estimations by formula (2) are not made, i.e.  $T_2' = T_2$ .

At the arrangement of a lifting gear, unified and normalized assembly units are applied [11]. That is why a gear box is usually chosen from a catalogue with a capacity margin, i.e. there is an underload in capacity which causes a higher value of mean time to failure:

$$T_2'' = T_2' \left( \frac{N_K}{N_H} \right)^3, \quad (3)$$

where  $N_K$  and  $N_H$  is the gear box's capacity according to the catalogue and its specified capacity, respectively.

Let the following values be obtained at the design phase  $N_K = 10 \text{ kW}$ ;  $N_H = 5 \text{ kW}$ , then:

$$T_2'' = T_2' \cdot \left( \frac{10}{5} \right)^3 = 8T_2'.$$

Finally we have the gear box's mean time to failure:

$$T_2'' = T_2' \cdot \frac{8}{0,63} = 12,7 \cdot T_2' = 12,7 \cdot 5000 = 6,35 \cdot 10^4 \text{ hrs.}$$

At first sight such mean time to failure of the gear box seems to be incredible – about 7 years of reliable operation. But we should remember that we have almost a quadruple

unload, and besides a perfect compliance with a maintenance schedule is provided (including regular change of oil and cup seal). There are no ageing components in the gear box.

3. Brake. We know from the guide [2] that MO brake magnets admit up to 600 activations per hour. However considering their limited wearing capacity, the application of this type of brake gears should be limited by the frequency of activations of not more than 300 1/hrs – for electromagnets MO 100B and not more than 150 1/hrs – for electromagnets MO 200B. Under these operation modes and voltage fluctuations within 85...105% of rated voltage, electromagnets have the probability of reliable operation about 0,95 per a year of operation. With great probability we can assume that the value of reliable operation of an electromagnet is given for the operation mode “H” (MO 100B) and for the mode “M” (MO 200B) in view of the limitation of activation frequencies, as well as for a maximum value of a braking moment  $M_{Tmax}$ , for which we have a maximum force of tightening and current in a coil.

Time of electromagnet operation per year by formula (1) is:

$$\text{for MO 100B: } t = 2,3 \cdot 10^3 \text{ hrs;}$$

$$\text{for MO 200B: } t = 0,6 \cdot 10^3 \text{ hrs.}$$

Based on the assumption about the exponential law of distribution:

for MO 100B:

$$P(2,3 \cdot 10^3) = \exp\left(-\frac{2,3 \cdot 10^3}{T_3}\right) = 0,95 \Rightarrow T_3 = 44,8 \cdot 10^3 \text{ hrs;}$$

for MO 200B:

$$P(0,6 \cdot 10^3) = \exp\left(-\frac{0,6 \cdot 10^3}{T_3}\right) = 0,95 \Rightarrow T_3 = 11,7 \cdot 10^3.$$

The most probable failures are the punctures of turn-to-turn insulation of an electromagnet's coil. They occur as the result of insulation ageing, cracks, lacquer and fabric peeling. It is encouraged by heat and coil vibration. If the brake is adjusted for a smaller braking moment  $M_T$  a spring force will be reduced, coil current, its heat and vibration will be reduced as well, and a mean time to failure of a coil will increase in accordance with quadratic dependence [2]:

$$T_3' = T_3 \left( \frac{M_{Tmax}}{M_T} \right)^2. \quad (4)$$

Let (hypothetically)  $M_{Tmax} = 2M_T$ , then:

$$T_3' = T_3 (2M_T / M_T)^2 = 4T_3.$$

It means that we have mean values of time to failure:

$$\text{for MO 100B: } T_3' = 4 \cdot 44,8 \cdot 10^3 \approx 18 \cdot 10^4 \text{ hrs;}$$

$$\text{for MO 200B: } T_3' = 4 \cdot 11,7 \cdot 10^3 \approx 4,7 \cdot 10^4 \text{ hrs.}$$

Let us introduce the latter value to the further calculation.

4. Flexible bolt coupling with brake pulley and brake cheeks. We suppose that if maintenance schedule is met,

loose hubs and brake cheeks are changed periodically. Tearing up of a brake pulley that causes early wear of cheeks is considered as a sudden failure, whose probability is exponentially distributed with parameter  $T_4$  [6]. Let us nominally consider a coupling and a friction pair equally reliable in relation to an electromagnet's coil:

$$T_4 \approx T_3 = 4,7 \cdot 10^4 \text{ hrs.}$$

5. Drum. We consider a failure of a drum's bearing and cable mountings to be possible only when a crane is tested for a lifting with a load weight  $Q=1,25Q_H$  [11]. That is why the above listed details are considered almost failure free in operation:

$$P_5(1 \text{ year}) \approx 0,99.$$

6. Drum axle (or shaft). Estimation of an axle or shaft of a drum is carried out in the following order [12]:

6.1. Axle or shaft are calculated for fatigue, load factor  $n_t$  in weak section with the time of operation  $t=1$  year is also determined. Let  $n_t=2$ .

6.2. The value of coefficients of variation of endurance limit of detail  $V_{-lg}$  is validated. This limit is determined by dispersion of a scaling factor, stress concentration factors, melting dispersion of steel chemistry. We can take  $V_{-lg}=0,1$ .

6.3. The coefficient of variation of an equivalent cycle amplitude  $V_a$  is validated. It is determined by the difference of crane operation modes from the estimated ones. We can take  $V_a=0,3$ .

6.4. The probability of axle fatigue breakdown in a weak section is defined:

$$F(t) = F_0 \left( \frac{1 - n_t}{\sqrt{n_t^2 \cdot V_{-lg}^2 + V_a^2}} \right), \quad (5)$$

where  $F_0(x)$  is the function of normal distribution [13].

$$\begin{aligned} F_6(1 \text{ год}) &= F_0 \left( \frac{1 - 2}{\sqrt{2^2 \cdot 0,1^2 + 0,3^2}} \right) = \\ &= F_0(-2,77) \approx 0,002. \end{aligned}$$

As we have two weak sections (in support), let us double the obtained value. In other sections a load factor is higher, but a certain probability of a breakdown still exists, that is why let us double the value of a breakdown probability once more:

$$F_6(1 \text{ year}) = 2 \cdot 2 \cdot 0,002 = 0,008 \approx 0,01,$$

$$P_6(1 \text{ year}) = 1 - F_6(1 \text{ year}) = 1 - 0,01 = 0,99.$$

7. Drum axle bearing. When calculating a bearing for durability, a calendar operation time is taken into account,

and based on formula (1) the number is found, and then the number of load cycles (resource), after that considering an equivalent loading the bearing is chosen [8, 14].

Under such calculation  $\alpha\%$  of bearings shall exceed the specified life, i.e. the probability of reliable operation of the chosen bearing during  $T$  years is more than  $\alpha$ .

Let  $\alpha=0,9$ ,  $T_7=10$  years. It is necessary to define the probability of a failure of the bearing after one year of operation, but meant for 10 years of operation.

Share of the expired operation life is 0,1 of the calculated operation life:

$$\gamma = t/T_7 = 0,1.$$

If the bearing is taken with a margin of lift capability  $C_D$ , then its life increases in accordance with a cube dependence, as the exponent of the bearing's curve durability  $m=3$  [14].

Let  $C_D=1860$ , and according to the catalogue we have  $C'_D=1400$ , then:

$$T'_7 = \left( \frac{C_D}{C'_D} \right)^3 T_7 = \left( \frac{1860}{1400} \right)^3 T_7 = 1,33 T_7.$$

The relation  $T_7/T'_7$  is 1,33 smaller than  $t/T_7=0,1$ , i.e.:

$$\gamma' = \frac{0,1}{1,33} = 0,07.$$

The probability of reliable operation of the bearings is determined by Weibull distribution [15]:

$$P'_7(T_{7\gamma'}) = \exp\left(-\frac{\gamma'}{5,35}\right)^{1,34} = \exp\left(-\frac{0,07}{5,35}\right)^{1,34} = 0,997.$$

On the drum axle there is a joint 7 on the right end of the axle (Figure 1). The inner and outer rings of the joint do not have relative rotation, since the drum 5 and the drum axle 6 rotate with at the same speed. The bearing which does not rotate is considered less reliable. Let us take for it  $P''_7=0,995$ . Finally we have the probability of reliable operation of the drum axle bearing and the joint:

$$P'''_7 = P'_7 P''_7 = 0,997 \cdot 0,995 = 0,992 \approx 0,99.$$

8. Load cable with mountings. At under-control operation broken cable wires are calculated and registered in the log book [16]. As soon as a rejection number of broken wires is achieved, the cable is changed. In this case if no rules are violated [16] the cable failures are considered improbable. Let us take:

$$P_8(1 \text{ year}) \approx 0,99.$$

9. Hook assembly. Usually it has a multiple strength margin, it undergoes testing and is almost failure-free. Let us take:

$$P_9(1 \text{ year}) \approx 0,99.$$

All data obtained as the result of the above listed reasoning and calculations are brought together in table 3.

10. Calculation of probability of a lifting gear's reliable operation as a whole

Mean time to failure of the elements 1...4, connected in series:

$$\frac{1}{T} = \frac{1}{9,5 \cdot 10^4} + \frac{1}{6,35 \cdot 10^4} + \frac{1}{4,7 \cdot 10^4} = 0,47 \cdot 10^{-4} \text{ 1/hrs}$$

$$\Rightarrow T = 2,1 \cdot 10^4 \text{ hrs.}$$

Based on formula (1) let us define the time of operation of a lifting gear at the operation mode "H":

$$t = 8760 \cdot 1 \cdot 0,66 \cdot 0,4 = 2,3 \cdot 10^3 \text{ hrs.}$$

The probability of reliable operation of the elements 1...4, connected in series:

$$P_{1...4}(t) = P_{1...4}(1 \text{ год}) = P_{1...4}(2,3 \cdot 10^3 \text{ час}) =$$

$$= \exp\left(-\frac{2,3 \cdot 10^3}{2,1 \cdot 10^4}\right) = 0,853.$$

Then the probability of reliable operation of a lifting gear is:

$$P_{\text{МГ}} = 0,853 \cdot 0,99^5 \approx 0,85.$$

To calculate the reliability level of a crane as a whole let us consider that a crane usually has 3 gears (lifting gear, gear of trolley movement and gear of crane movement), steel construction and control equipment, i.e. 5 systems connected in-series. Supposing that (in order to simplify) they are equally reliable we have the following probability of a failure of a crane during 1 year:

$$P_K(1 \text{ год}) = P_{5, \text{max}}(1 \text{ год}) = 0,85^5 = 0,443.$$

Thus a mean time to failure of a crane as a whole shall be:

$$P_K(1 \text{ год}) = \exp\left(-\frac{2,3 \cdot 10^3}{T_K}\right) = 0,443 \Rightarrow$$

$$\Rightarrow T_K = -\frac{2,3 \cdot 10^3}{\ln(0,443)} \approx 2,83 \cdot 10^3.$$

Failure rate of a crane as a whole will be:

$$\omega_K = \frac{t}{T_K} = \frac{2,3 \cdot 10^3}{2,83 \cdot 10^3} \approx 0,81 \text{ 1/year.}$$

For overhead hook electric cranes of general purpose with a lifting capacity up to 50 t in the operation mode "H", the parameter of the flow of sudden failures as per [17] is  $12 \cdot 10^{-3}$  1/hrs, i.e. we can tolerate 12 failures of a crane for 1000 hrs, or 28 failures per a year. The obtained value  $\omega_K = 0,81 < 28$  i.e. not more than 1 failure per a year, is more than acceptable.

## Conclusion

1. Probability of reliable operation of a lifting gear during 1 year of operation is 0,85.

2. A crane as a whole will have not more than one failure per a year. It is much lower than the tolerable value that is why we shall consider the reliability index of the lifting gear rather high.

3. Calculated probability of reliable operation of a crane is so high due to the number of reasons:

It was determined by a lifting gear which is the most reliable of all gears, then the result was distributed to the whole crane;

The elements of a lifting gear were chosen from the catalogue with a certain "margin";

Degradation (wearout) failures were not taken into account, since they are anticipated during the phase of technical maintenance and repair;

Failures caused by the violations of [16] refer not to the crane failures, but to the failures of other systems.

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**Table 3. Results of calculation of reliability indices of an overhead crane's lifting gear**

	Element	Most probable failures	Failure occurrence	$P_i$ or $T_i$
1	Electric motor	Turn-to-turn short circuit	Heat, smell	$T_1 = 9,5 \cdot 10^4$ hrs
2	Gear box	Wear out of bearings and gear teeth	Noise, vibration	$T_2 = 6,35 \cdot 10^4$ hrs
3	Brake electromagnet	Turn-to-turn fault of a coil	Pulley not released	$T_3 = 4,7 \cdot 10^4$ hrs
4	Flexible bolt coupling and brake cheeks	Tearing up of a pulley Cheek wear out	Long breaking distance	$T_4 = 4,7 \cdot 10^4$ hrs
5	Drum, bearing block	Fatigue breakdown	Slap	$P_5 = 0,99$
6	Drum axle	Fatigue breakdown	Slap and drum freezing	$P_6 = 0,99$
7	Drum axle bearing and a joint	Wear out	Noise, vibration	$P_7 = 0,99$
8	Load cable	Breakage	Kinking	$P_8 = 0,99$
9	Hook assembly	Wear out, bearing freezing	Noise, locking	$P_9 = 0,99$

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### About the authors

**Vladimir A. Ermolenko**, Associate professor of the chair “Machine elements and lifting and transport equipment”, Kaluga branch of the Bauman Moscow State Technical University, Kaluga, Russia, e-mail: tvermolenko@rambler.ru

**Pavel V. Vitshuk**, Associate professor of the chair “Machine elements and lifting and transport equipment”, Kaluga branch of the Bauman Moscow State Technical University, Kaluga, Russia, e-mail: zzzVentor@ya.ru

## Approach to ensuring reliability of complex systems based on parameter optimization of reliability diagrams

**Aleksander S. Tolstov**, Scientific and Research Test Institute of the Academy of FPS of Russia,  
e-mail: A.Dick2001@yandex.ru

**Dmitry V. Pantyukhov**, Scientific and Research Test Institute of the Academy of FPS of Russia,  
e-mail: gospamme@yandex.ru



Aleksander S.  
Tolstov



Dmitry V.  
Pantyukhov

**Abstract. Aim.** Fulfillment of the requirements for the reliability indices of complex technical products and systems is one of the priority tasks to be solved along the stages of development and testing. It is advisable to define the parameter values of the elements of the complex system diagram at the design stage, optimally, in terms of the minimum of an efficiency/cost criterion, ensuring the fulfillment of the requirements for the system reliability. **Methods.** The main problem that impedes to solve the task of parameter optimization of a model of the reliability diagram is a significant instability of estimation of probability of reliable operation using a Monte-Carlo method (a significant dependence of the rate of estimation error of time). In such conditions an optimization search task could be solved on provision of a stepwise determination of the number of model experiments, which ensures the required accuracy of the estimation of probability of the system reliable operation, necessary for stable operation of the parameter optimization algorithm. The studies of characteristics of estimation of the system reliable operation allowed determining the interrelation of the estimation of reliable operation and the rate of estimation error, offering its approximation in form of a simple formula. The number of model experiments that ensure the required estimation accuracy, is defined using the developed formula determining the interrelation of the estimation of reliable operation and the rate of estimation error, and the known formulas determining the rate of error of the sum  $N$  of equally distributed independent random values. Use of the obtained formulas makes it possible to organize the work of the parameter optimization algorithm of the system reliability model by determining its parameters with the required accuracy using minimum computer resources in the context of instability of estimation of probability of the optimizable system reliable operation. **Results.** Efficiency of the offered approach to realize parameter optimization of a statistical model of the reliability diagram is shown on the sample of estimation of optimal parameters of the system reliability diagram variant, for which there is an analytical solution for the estimation of reliable operation probability. And the results of parameter optimization with the use of analytical value of the probability of reliable operation are the basis for estimation of the accuracy of the algorithm of parameter optimization of the system reliability model operating with the use of a Monte-Carlo method. It has been shown that the offered approaches ensure the convergence of the search algorithm and the required accuracy in estimation of the parameters of the system reliability diagram that optimally ensure the fulfillment of the requirements for the system reliability. **Conclusions.** The results described in the article confirm technical feasibility and economic viability of determination of optimal values of the system reliability parameters at the design stage. Obtained estimations are the basis for the system integration with required elements, or for the requirements to be set to their reliability, if the development of new elements is necessary. In case there are no elements with design characteristics of reliability, the required reliability of the system can be ensured by special technical redundancy measures and (or) by the creation of the system of technical maintenance and repair.

**Keywords:** reliability, reliability model, reliability diagram, reliable operation, optimization, optimization algorithm, convergence of a search algorithm.

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Fulfillment of requirements to reliability indices of complex technical products and systems is one of the priority tasks to be solved along the stages of development and testing. If necessary it is possible to improve reliability either by improved reliability of system elements, or by taking technical measures on their redundancy and (or) on the crea-

tion of a maintenance system [1]. Technical measures lead to increase in system cost. The measures to be taken should be chosen with an efficiency/cost criterion. And efficiency of the measure taken is defined as an obtained growth of system reliability, and cost is defined as a respective growth of system cost.

Availability of such tool as a model of system reliability allows to formalize the development of recommendations to ensure the required reliability of the system under study. Reference [1] offers to solve the indicated task on the basis of development of search algorithms of optimal synthesis of reliability diagrams of systems with redundancy. With no loss of entity the variant of uninterrupted power supply (UPS) configuration is further described. Its reliability diagram is shown in Figure 1 [1].

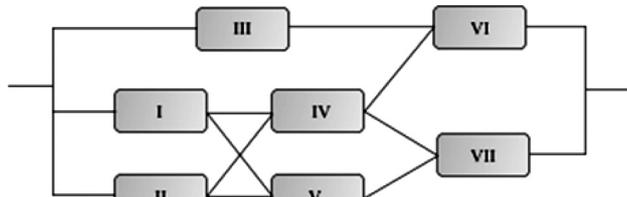


Fig. 1 – Variant of uninterrupted power supply (UPS) with redundancy

This article offers the approach to ensure reliability of complex systems based on parameter optimization of reliability diagrams.

The state of the system is defined by the state of its elements and by the logic of their interrelations in a diagram. The formula to define the state (*fault-free* – 1, *faulty* – 0) of the system in Figure 1 (states of the elements are specified by symbol  $S$  (state) with the respective indices) is as follows:

$$S_{\Sigma} = \{[(S_1 \vee S_2) \wedge S_4 \vee S_3] \wedge S_6\} \vee [(S_1 \vee S_2) \wedge (S_4 \vee S_5) \wedge S_7]. \quad (1)$$

A logical formula to estimate the system state is defined by its structure, it does not depend on the type of law of probability of fault-free state of the elements, and formulation causes no difficulties. Analytic solution could be obtained for reliability diagrams with simple structure, therefore complex systems are analyzed based on statistical computation. A model of reliability of the system built on the basis of a logical formula under certain reliability indicators of system elements is functioning as follows. On the  $i$ -th modeling step, corresponding to the time interval  $t_i$   $N$  of model experiments are done, and in each experiment the system state  $S_{\Sigma ij}$  is defined in accordance with (1). Probability of reliable operation of the system  $\hat{P}_i$  is assessed by averaging of the results of  $N$  of model experiments:

$$\hat{P}_i = \frac{1}{N} \sum_{j=1}^N S_{\Sigma ij}, \quad (2)$$

The obtained assessment of probability of system reliability corresponds to the results of analytic estimations  $P(t)$  and contains the error of estimation  $\Delta P(t)$ :

$$\hat{P}(t) = P(t) + \Delta P(t). \quad (3)$$

Standard deviation (SD) of estimation error  $\sigma_{\Delta P}(t)$  is defined by the number of model experiments  $N$  and SD of the event stream  $S_{\Sigma}$ :

$$\sigma_{\Delta P}(t) = \frac{\sigma_{S_{\Sigma}}(t)}{\sqrt{N}} \quad (4)$$

The results of statistical modeling by the estimation  $\hat{P}_i(2)$  and SD  $\sigma_{S_{\Sigma}}(t)$  of the event stream  $S_{\Sigma}$  under exponential law are shown in Figure 2. For modeling, the average time of reliable operation of elements is accepted as  $T_{\text{iso}} = 5000$  h,  $N = 10000$ .

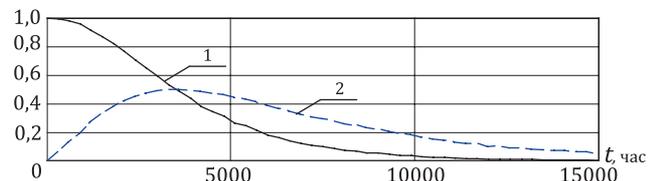


Fig. 2. Dependence of estimation of probability of reliable operation  $\hat{P}(t)$  (1) and SD of the event stream  $\sigma_{S_{\Sigma}}(t)$  (2) on time

The character of the processes  $\hat{P}(t)$  and  $\sigma_{S_{\Sigma}}(t)$ , predefinition of their values in the row of points on the time axis point at a possible functional interdependency of  $\sigma_{S_{\Sigma}}(t)$  and  $\hat{P}(t)$ , which can be defined by a grapho-analytical method. Dependency graph  $\sigma_{S_{\Sigma}} = f(\hat{P})$  made in accordance with Figure 2 is shown in Figure 3. Viable regression function  $\sigma_{S_{\Sigma}} = f(\hat{P})$  for the dependency in Figure 3 is a circular curve equation  $\sigma_{S_{\Sigma}}^2 + (\hat{P} - 0.5)^2 = 0.5^2$ . In reference to a desired variable  $\sigma_{S_{\Sigma}}$  this equation takes the form:

$$\sigma_{S_{\Sigma}} = \sqrt{0.25 - (\hat{P} - 0.5)^2} \quad (5)$$

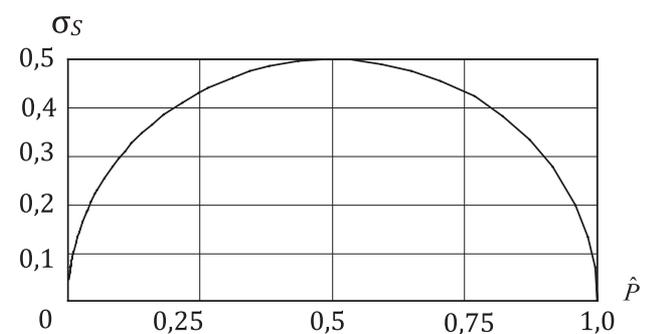


Fig.3 Dependency of SD of the system state stream  $\sigma_{S_{\Sigma}}$  on estimation of probability of reliable operation  $\hat{P}$

Based on (5), with consideration of (4), the formula for a stepwise determination of the number of model experiments necessary to ensure a required value of estimation error SD  $\sigma_{\Delta P_{\text{req}}}$  at the  $i$ -th point of modeling is as follows:

$$N_{\text{requi}} = \frac{\hat{P}_i}{\sigma_{\Delta P_{\text{req}}}} * (1 - \hat{P}_i) \quad (6)$$

Modeling in accordance with (6) makes it possible to estimate the system reliability with the defined accuracy at all points of the modeling time interval.

As stated above, one of the main tasks to be solved during the development of complex/complicated technical systems is the task to fulfil reliability requirements which is formulated as follows:

$$P(t_{req}) \geq P_{req} \quad (7)$$

It means that for the defined time interval the system shall assure reliability not worse than the required reliability. In case the requirement is not fulfilled, it is necessary to change the system to improve reliability of the elements in service (average time of their reliable operation).

Determination of minimum required values of average time of reliable operation of the elements assuring the implementation of (7) can be formalized and realized based on parametric identification of systems [2].

For parametric identification of a reliability model, quality can be indicated by the functional which is a measure of concordance of reliability probability estimation obtained on the model with this parameter vector with the required probability value:

$$Q(w) = [P_{req} - \hat{P}(w, t_{req})]^2, \quad (8)$$

where  $w$  is a parameter vector of a reliability model, whose components are average time intervals of reliable operation of the reliability diagram elements.

Minimization of functional (8) by vector  $w$  under the conditions of its high dimensions can be organized using the first-order gradient-based algorithms. In this case stepwise minimization  $Q(w)$  is done. Every step of minimization is made after its direction is defined which is set by derivative  $Q(w)$  for the parameter vector  $w$ :

$$w_i = w_{i-1} + \Delta_i \cdot \mu_i \quad (9)$$

where  $i \in [1 \dots n]$ ;

$\mu_i$  is a unit vector in the direction of derivative  $\frac{dQ(w)}{dw}$ , defined at the  $i$ -th step;

$\Delta_i$  is a value of the minimization step; it is chosen equal to the step of parameter modification at the calculation of derivative  $\frac{dQ(w)}{dw}$ ; a value of the last step  $\Delta_n$  is calculated by a straight-line interpolation of the interval from the correlation between  $\hat{P}_{n-1}(t_{req})$ ,  $P_{req}$  and  $\hat{P}_n(t_{req})$ .

Components of the vector  $\mu$  determinate sensitivity (8) to the changes of values of parameter vector elements and are calculated by the formula:

$$\mu_{ki} = \frac{\hat{P}_{i-1, w_k}^*(t_{req}) - \hat{P}_{i-1}(t_{req})}{C_{ik}}, \quad (10)$$

where  $\hat{P}_{i-1, w_k}^*(t_{req})$  is the estimation of probability of fault-free state of the system at the moment  $t_{req}$  at the increase of the element  $w_k$  by the modification step value  $\Delta$ ;

$\hat{P}_{i-1}(t_{req})$  is the estimation of probability of a fault-free state of the system at the moment  $t_{req}$  at the previous step of optimization;

$c_{ik}$  is the estimation coefficient of the cost of modification of the  $k$ -th element at the  $i$ -th step (increasing with the increment of average time of reliable operation).

Convergence of a search algorithm that means the penetration of parameter vector estimation at the last step of the algorithm to the permissible area of their optimal values can be provided under respective accuracy of determination of the vector  $\mu_i$  (10), which is defined by the estimation accuracy of probability of a fault-free state of the system.

Requirements for the accuracy of probability estimation can be determined by a statistical model on the basis of analytical solution available for the reliability diagram shown in Figure 1 [3]:

$$P(t) = P_3(t) \cdot P_6(t) + (P_1(t) + P_2(t) - P_1(t) \cdot P_2(t)) \cdot \{P_4(t) \cdot [(1 - P_3(t)) \cdot P_6(t) + (1 - P_6(t)) \cdot P_7(t)] + (1 - P_3(t) \cdot P_6(t)) \cdot (1 - P_4(t)) \cdot P_5(t) \cdot P_7(t)\} \quad (11)$$

Analytical reliability model (11) used as the object of a search algorithm excludes stochastic character of the constituents used to calculate derivative (10), but it provides unconditional convergence of a search algorithm. The obtained values of the reliability model parameter vector bringing the minimum of (8) are the basis for the estimation of convergence of the search algorithm of optimization, built above the statistical reliability model. Besides, operating with an analytical reliability model helps to estimate an acceptable level of determination errors  $\mu_{ki}$  (10) providing with the required parameters of convergence of the statistical algorithm. In this regard (10) is extended with a summand simulating an error of calculation of the vector of the derivative  $\mu$ :

$$\mu_{kiu} = \mu_{ki} \cdot (1 + \sigma_u \cdot \xi_{nki}), \quad (12)$$

where  $\xi_{nki}$  is a number from the sample of normal random numbers with a zero mathematical expectation and unit variance;

$\sigma_u$  is a specified rate (SD) of the introduced relative error of the calculation of the derivative vector components.

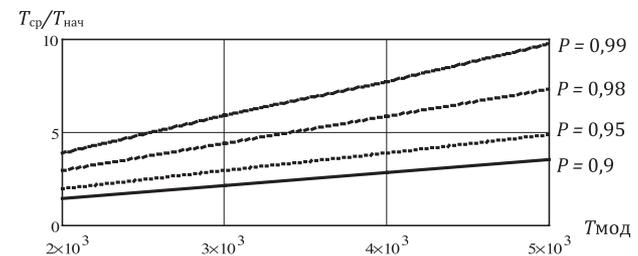


Fig. 4 Dependency of the required level of the system reliability on the specified duration and probability of reliable operation

**Table 1. Results of calculation of optimal parameters of reliability of the elements ensuring reliability of the system and initial data for the determination of allowable errors in calculation of derivatives.**

No. of element	T = 2000			T = 3000			T = 4000			T = 5000		
	P = 0,9	P = 0,95	P = 0,99	P = 0,9	P = 0,95	P = 0,99	P = 0,9	P = 0,95	P = 0,99	P = 0,9	P = 0,95	P = 0,99
1	1,29	1,7	3,18	1,86	2,51	4,82	2,44	3,32	6,33	3,03	4,14	7,97
2	1,29	1,7	3,18	1,86	2,51	4,82	2,44	3,32	6,33	3,03	4,14	7,97
3	1,42	1,98	3,84	2,2	3,03	5,88	3	4,09	7,74	3,81	5,15	9,77
4	1,39	1,89	3,59	2,08	2,84	5,46	2,77	3,79	7,18	3,47	4,73	9,05
5	1,25	1,62	3	1,74	2,33	4,52	2,24	3,06	5,92	2,75	3,79	7,45
6	1,76	2,57	5,47	2,75	3,93	8,35	3,73	5,27	10,95	4,72	6,63	13,82
7	1,61	2,32	4,95	2,4	3,43	7,51	3,16	4,53	9,81	3,9	5,63	12,35
$T_{av}$	1,43	1,97	3,89	2,13	2,94	5,91	2,83	3,91	7,75	3,53	4,89	9,77
<b>Maximum deviation of parameter from an optimal value, %</b>												
$\Sigma_n = 30\%$	4,6	3,3	4,3	4,1	4,9	1,1	4,6	4,9	4,3	5,0	4,2	3,4
$\Sigma_n = 50\%$	10,1	10,4	9,3	12,5	13,9	6,6	3,9	8,2	7,7	12	10,5	5,4
<b>Minimum values of components of the vector of derivatives <math>\mu_k</math> (<math>\cdot 10^{-3}</math>)</b>												
$\mu_k \min$	3,8	1,86	0,31	3,35	1,52	0,2	3,84	1,27	0,15	2,63	0,97	0,12

The value  $\sigma_n$  leading, for instance, to not more than 5 % level of error of determination of the reliability model parameters in relation to their optimal values defines the requirements for the accuracy of estimations made to check the probability of fault-free state of the system.

Figure 4 and Table 1 show the dependency of the required level of the system reliability (relative to the value  $T_{init} = 5000$  h) on the specified duration and probability of reliable operation, obtained with an analytical model of optimization.

Table 1 also represents the results of estimation of the effect of noise contamination of the derivative vector in accordance with (12) on the results of parameter estimation by a search algorithm.

Analysis of the results listed in Table 1 shows that the acceptable relative level of error of estimation of the derivative components (10) made with a statistical reliability model leading to not more than 5 % of deviations in the determination of the reliability model parameters is the level  $\sigma_n = 30\%$ .

Absolute level of estimation error is defined from the formula

$$\sigma_{\Delta P} = \mu_{kmin} * \sigma_n \quad (13)$$

Keeping in mind that SD of the constituents of (10)  $\hat{P}_{i-1, w_k}^*(t_{req.})$  and  $\hat{P}_{i-1}(t_{req.})$  are practically equal, the acceptable SD value of error  $\mu_k$  shall exceed acceptable values  $\sigma_{\Delta P}(t)$  approximately by 1,4.

Figure 5 shows the graph  $\sigma_{\Delta P_{\text{доп}}}(P)$  obtained with an analytical model based on the allowable value  $\sigma_n$  with consideration of the  $\mu_k$  behavior depending on the estimation of probability of reliable operation of the system, and its approximation built on the basis of (5) on the probability interval  $P \geq 0.75$  and the polynomial  $aP^2 + bP + c$  ( $a = -1.25$ ;  $b = 1$ ;  $c = 0.17$ ) on the interval  $P < 0.75$ , providing the required accuracy within the probability interval  $P \leq 0.99$ .

The formula for approximation of dependency  $\sigma_{\Delta P_{\text{доп}}}(P)$  on the probability interval  $P \geq 0.75$  is as follows:

$$\sigma_{\Delta P_{\text{доп}}}(P) = \sigma_s \cdot (1-P) \cdot 10^{-2} \quad (14)$$

Based on (14) and (4), the formula for a stepped determination of number of the experiments  $N_{\text{req}}$  required to ensure the convergence of a search algorithm operating with a statistical model on the interval  $P \geq 0.75$  is as follows:

$$N_{\text{req}} = 10^4 / (1 - \hat{P}_i) \quad (15)$$

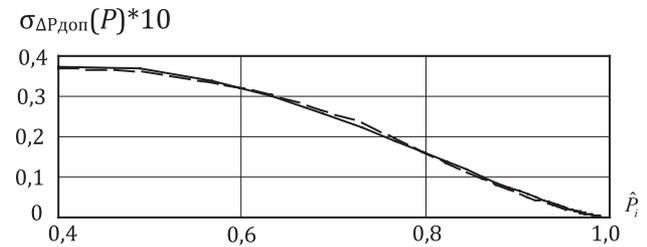


Fig. 5 Dependency of the value of an allowable error in the estimation of reliability probability (full line) on the estimation of probability of reliable operation of the system and its approximation (dashed line)

The executed estimation of optimal parameters of the system, whose reliability model corresponds to Figure 1, using a statistical model corresponding to formula (1) showed that at the determination of  $N_{\text{req}}$  in accordance with (15) there is a convergence of a search algorithm and estimation of the system parameters with the accuracy not lower than  $\pm 5\%$ . In the experiment carried out, an average error of parameter determination in relation to the results listed in Table 1 is 3,03 %. The obtained formulas are determined by the characteristics of the state stream and they do not depend on complexity of the system reliability diagram.

The values listed in the table show the minimum required value of average time of reliable operation of the diagram

elements (Figure 1) to ensure the selected reliability requirements (in relation to initial values  $T_{in} = 5000$  h). The results of optimization algorithm generally comply with the results expected based on the location of elements in the reliability diagram. Maximum deviations of optimal values of average time of reliable operation of the elements in relation to their average values ( $T_{av}$ ) are 21 – 41 % depending on time requirements and probability of the system reliable operation. It shows economic and technical viability of determination of optimal (minimum required) values of the system reliability parameters at the design stage. In case there are no elements with design characteristics of reliability, the required reliability of the system can be ensured either by special technical redundancy measures and (or) in case of operational and technical feasibility by the creation of the system of technical maintenance and repair.

Application of the offered approach makes it possible at the design stage to define the parameter values of the elements in the scope of reliability diagrams of complex systems served to ensure optimal fulfillment of requirements for the system reliability. The obtained formulas to define the convergence parameters are the basis for computational stability and efficiency of an optimization algorithm operating with a statistical model of the reliability diagram.

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## About the authors

**Aleksander S. Tolstov** – Research Assistant, Scientific and Research Test Institute of the Academy of FPS of Russia. Russia, 302034, Orel, Str. Priborostroiteley, 35, e-mail: A.Dick2001@yandex.ru

**Dmitry V. Pantyukhov** – Research Assistant, Scientific and Research Test Institute of the Academy of FPS of Russia. Russia, 302034, Orel, Str. Priborostroiteley, 35, e-mail: gospamme@yandex.ru

## Geometrical method for the operational control of the distributed solution of information-computing tasks in computer networks

**Sergey A. Zhurbin**, the 4th CSRI, Ministry of Defense of the Russian Federation, Korolev, Moscow Region, Russia, e-mail: serdgo4@yandex.ru

**Gennady V. Kazakov**, the 4th CSRI, Ministry of Defence of the Russian Federation, Korolev, Moscow Region, Russia, e-mail: kgv.64@mail.ru



Sergey A. Zhurbin



Gennady V. Kazakov

**Abstract. Aim.** Some of the main performance indicators of ACS application are the operational efficiency and stability of the control of the above mentioned systems. The wide application of computing techniques in ACS as well as the organization of computer networks on this basis stipulate the necessity of effective control of distributed computation processes to ensure the required level of operational efficiency and stability while solving the specified tasks. The existing methods used to organize the computation process (method of dynamic programming, branch and bound method, sequential synthesis, etc.) may turn out to be bulky or less accurate in certain situations. These methods help to find a solution in the mode of interactive choice of an optimal variant to organize a computation process, i.e. consecutive approach to the required result and do not allow getting an a priori estimation of the time of computation process in a network. Application of the specified methods when solving research tasks in the course of design of computer networks presents itself as quite difficult. This article offers the application of a geometrical method that allows estimating the minimum time necessary to solve the set of information-computing tasks as well as ensuring their optimal assignment in a computing system. Besides, the method allows finding a full set of possible variants for the organization of a computation process in a network with an a priori estimation of time of the decision for each variant. The principle of the method is to represent the sets of all possible distributions of tasks by workstations in form of a broken hypersurface. To solve the indicated task the criterion and conditions of the optimality of the time spent to solve information-computing tasks have been introduced. **Results and conclusions.** This article describes many variants of realization of a computation process for homogeneous and non-homogeneous computing environments. Solution algorithm for a homogeneous computing environment is quite simple and makes it possible to define a minimum time necessary for a computation operations. It is based on a geometrical representation of the distribution of tasks by workstations in form of the hyperplane constructed in orthonormal space whose basis vectors are computation capacities of workstations. Besides, the algorithm for homogeneous computing environment can be successfully used for an approximate estimation of the minimum time necessary to solve a set of tasks in a network, for non-homogeneous computing environment as well. Minimum time necessary to solve functionally different tasks in a non-homogeneous computing environment is defined using a piecewise linear hypersurface that slightly complicates the algorithm, though in general, with consideration of computation capabilities of moderns computers, it is still simply realized. The estimations carried out in the course of preliminary researches, allowed concluding about the application of a geometrical method in a computer network for a large amount of workstations and information-computing tasks. The possibility of an a-priori estimation of the minimum time necessary to solve a set of tasks in the computer network allows using the offered method to solve research tasks at the stage of design of a computer network to estimate such indicators as operational efficiency, reliability, stability and etc. The possibility of an aprioristic assessment of the minimum time of the solution of a complex of tasks in the computer network allows to use, offered in work, a method in the solution of research tasks at a design stage of the computer network for an assessment of her such indicators as efficiency, reliability, stability, etc.

**Keywords:** automated workstation, computer network, quality indicators of computation process organization, control of computation process.

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Currently the notion “computing systems” is no longer the innovation. Trends of the technology development are largely determined by the concerns to improve the performance of the current computing systems and the systems

under development [1, 3, 4] by deployment of technical and technological novelties. Capacity of the first computers, as well as their functional reliability were not high enough, that is why they did not make it possible to solve many ap-

plication tasks, or it took much time to solve them in view of all recovery processes required after failures. Just after the first computers appeared, different methods to combine several computers into one system were developed to improve the performance, functional reliability, and to reduce the time necessary to solve the tasks. The idea was simple: if the capacity of one computer was not enough to solve the tasks, then it was necessary to parallelize the whole set of tasks  $\Omega$  among computers, and then each computer would solve its sub-set of the tasks  $\Omega_i$  ( $i = 1, L$ ), where  $L$  is the amount of computers in a computing system (CS). The same aim was pursued at the development of multiprocessor systems or, as they say, the systems with multi-core processors. With the advent of technical and technological possibility of information exchange between computers, a strong impetus was given to the development of the concept of CS construction which is a logical result of the evolution of computer technologies. So the computers were able to exchange information, they were provided with computation capacity and user-friendly interface, there was a little left to do – to teach them to control the computation process. The methods of distributed computation process, such as, for instance, the dynamic programming, the branch and bound method, sequential synthesis, etc. [1, 2, 3, 4] started their active development. These flexible generic approaches and methods became widely applicable in the efficient use of computing resources while solving a wide range of tasks: information, computing, technological and many other tasks. However, if we consider some one type of tasks, for example, computing tasks when it is necessary to distribute a certain set of one-type tasks in a multiprocessor or in a multi-core processor space, the generic methods may turn out to be bulkier or less accurate.

This article offers a geometrical method that allows estimating the time necessary to solve the tasks in homogeneous and non-homogeneous environments as well as ensuring their optimal assignment in CS. For a homogeneous environment, all computation processes in CS are assumed to be linear for all types of tasks, for a non-homogeneous environment – the linearity is observed only for the tasks of the same type, and there is no linearity for the set of different-type tasks. Linear and nonlinear relations mentioned above shall be described in more detail below.

Let there be a CS whose nodes are computing machines generally of different technical characteristics (the speed of a processor and a front side bus, random access memory capacity, etc.). The computer network nodes are automated workstations (WS), solving a certain set of the entered information-computing tasks (ICT). ICTs solved in CS are independent of each other from the point of view of the pooled input and output data. The task of distribution of the whole variety of ICTs to be solved in CS is laid on a certain control center CS (CSCS), which can perform the computation process in CS in automatic or automated mode. In case of the high level of technical facilities, and if CS is provided with data communication channels, a high end server performing automatic control of CS may serve

as a CSCS. If the level of technical facilities is insufficient or low, and in case there are no required communication channels, total control is taken by a human, and information exchange is carried out by means of courier service. This paper describes the CSs of the high technical level with data communication channels of high performance. The sample of an unspecified CS is shown in Figure 1.

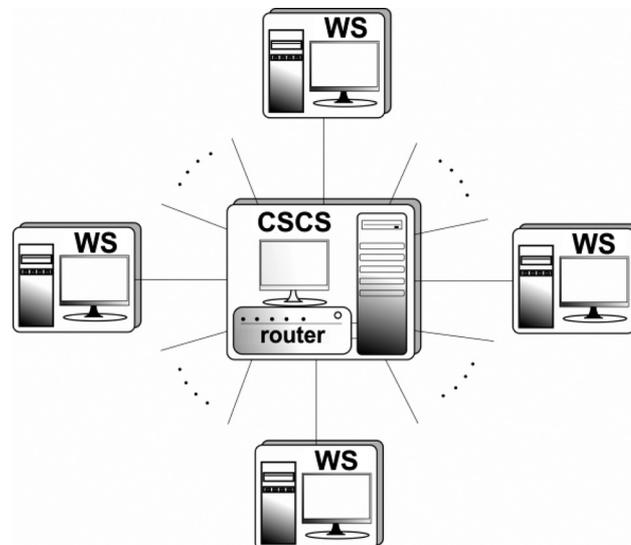


Fig. 1. Computer network. General view

The CS architecture may be rather diverse, but the CS itself should have several main properties:

WS of the network have the same rights and priorities in relation to the ICT solution;

The whole set of tasks solved in the CS can be solved at any WS;

WS, at which ICTs are solved, operate in parallel;

all WSs of the CS are thoroughly reliable.

Let the CS with  $m$  of WSs take  $M$  of ICTs including  $n$  types of tasks, i.e.

$$M = \sum_{j=1}^n \xi_j,$$

where  $\xi_j$  is the amount of tasks of the  $j$ -th type. The types of tasks differ, for instance, in scope and content of input and output data. Let for any pair of WSs the following equation hold true

$$\frac{t_{i,1}}{t_{j,1}} = \frac{t_{i,2}}{t_{j,2}} = \dots = \frac{t_{i,n}}{t_{j,n}}, \quad (1)$$

where  $t_{i,k}$  is the time to solve one task of the  $k$ -th type at the  $i$ -th WS;  $t_{j,k}$  is the time to solve one task of the  $k$ -th type at the  $j$ -th WS ( $i = 1, m; j = 1, m; i \neq j; k = 1, n$ ).

Condition (1) specifies the fact that the time to solve an ICT at WS linearly depends only on the WS computation capacity, the time of bringing the initial information is not considered, as its scope is quite little and the times for information exchange between WSs are also negligible. For example, if the WS1 has a higher speed response than the WS2 by  $\varphi$  times, then for all types of tasks we can write



**Table 1. Results of calculation of optimal parameters of reliability of the elements ensuring reliability of the system and initial data for the determination of allowable errors in calculation of derivatives.**

No. of element	T = 2000			T = 3000			T = 4000			T = 5000		
	P = 0,9	P = 0,95	P = 0,99	P = 0,9	P = 0,95	P = 0,99	P = 0,9	P = 0,95	P = 0,99	P = 0,9	P = 0,95	P = 0,99
1	1,29	1,7	3,18	1,86	2,51	4,82	2,44	3,32	6,33	3,03	4,14	7,97
2	1,29	1,7	3,18	1,86	2,51	4,82	2,44	3,32	6,33	3,03	4,14	7,97
3	1,42	1,98	3,84	2,2	3,03	5,88	3	4,09	7,74	3,81	5,15	9,77
4	1,39	1,89	3,59	2,08	2,84	5,46	2,77	3,79	7,18	3,47	4,73	9,05
5	1,25	1,62	3	1,74	2,33	4,52	2,24	3,06	5,92	2,75	3,79	7,45
6	1,76	2,57	5,47	2,75	3,93	8,35	3,73	5,27	10,95	4,72	6,63	13,82
7	1,61	2,32	4,95	2,4	3,43	7,51	3,16	4,53	9,81	3,9	5,63	12,35
$T_{av}$	1,43	1,97	3,89	2,13	2,94	5,91	2,83	3,91	7,75	3,53	4,89	9,77
<b>Maximum deviation of parameter from an optimal value, %</b>												
$\Sigma_n = 30\%$	4,6	3,3	4,3	4,1	4,9	1,1	4,6	4,9	4,3	5,0	4,2	3,4
$\Sigma_n = 50\%$	10,1	10,4	9,3	12,5	13,9	6,6	3,9	8,2	7,7	12	10,5	5,4
<b>Minimum values of components of the vector of derivatives <math>\mu_k</math> (*10<sup>-3</sup>)</b>												
$\mu_k$ min	3,8	1,86	0,31	3,35	1,52	0,2	3,84	1,27	0,15	2,63	0,97	0,12

bution of ICTs by WSs. In Figure 2  $T_1, T_2, T_3$  is the time necessary to solve several ICTs distributed to the 1-st, 2-nd and 3-d WS respectively taking (3) into account.

Let us go back to the optimality criterion related to the time necessary to solve ICTs in the CS. Let us assume the time necessary to solve a set of ICTs in CS to be the time interval between the start of a computation process and its termination at all the WSs. Let us write this definition in form of the following mathematical equation

$$T_s = \max \{T_i\}_{i=1}^m, \tag{11}$$

where  $T_s$  is the total time necessary to solve ICTs in CS;  $T_i$  is the time necessary to solve the ICTs distributed to the  $i$ -th WS;  $m$  is the amount of CS WSs. Then taking (11) into account, the minimum time necessary to solve ICTs in CS shall be achieved when  $\forall T_i (i = 1, m)$ , the equation will be followed

$$T_1 = T_2 = \dots = T_m. \tag{12}$$

Based on (10) and (12) the minimum time necessary to solve ICTs in CS will be

$$T_{min} = \frac{1}{\sum_{i=1}^m \frac{1}{T_{tot}^i}}. \tag{13}$$

Equation (13) holds true when the linearity condition is satisfied (1). Let us consider the case then the linearity condition will be satisfied within one type of ICTs, and not satisfied for all types of ICT. It becomes possible when the time necessary to solve ICT at WS includes the time of bringing of the initial information, or when not only computing tasks are being solved, but also graphic and information tasks, i.e. the tasks which are essentially different.

In this case the following equation holds true

$$\frac{t_{i,1}}{t_{j,1}} \neq \frac{t_{i,2}}{t_{j,2}} \neq \dots \neq \frac{t_{i,n}}{t_{j,n}}, \tag{14}$$

where  $t_{i,k}$  is the time necessary to solve one task of the  $k$ -th type at the  $i$ -th WS;  $t_{j,k}$  is the time necessary to solve one task of the  $k$ -th type at the  $j$ -th WS ( $i = 1, m; j = 1, m; i \neq j; k = 1, n$ ). Thus we see that the linearity was observed in relation to all types of tasks in (1), but in (14) it found only in relation to one arbitrary type of tasks and not found in relation to all the tasks. Such CS shall be considered as a non-homogeneous computing environment. Let us show it on a graphical example for two WSs with the numbers  $i$  and  $j$ .

The relations expressed by (1) and (14) are the slope ratio of the straight lines [7], specifying the functional dependence of times  $T_i$  and  $T_j$  of the variants of distribution of tasks between the  $i$ -th and the  $j$ -th WSs, for instance, in Figure 2 it is a segment, connecting the points  $T_{tot}^i, T_{tot}^j$  with  $i = 1, 3; j = 1, 3; i \neq j$ . Then let us write (14) as follows

$$\text{tg } \alpha_1 \neq \text{tg } \alpha_2 \neq \dots \neq \text{tg } \alpha_n,$$

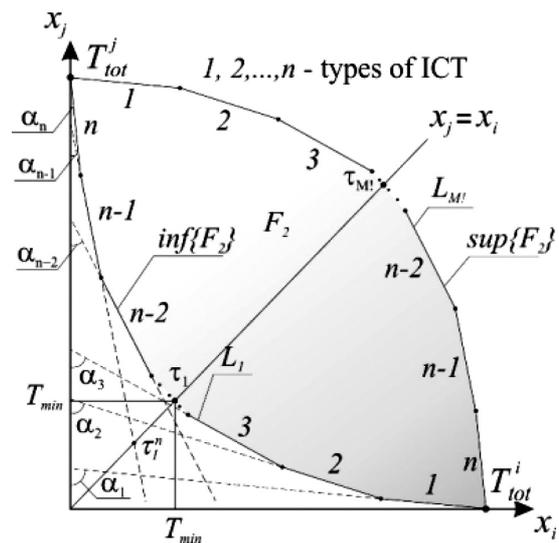


Fig. 3. Graph of change of the time necessary to solve ICTs distributed between the  $i$ -th and the  $j$ -th WSs depending on sequence of the task transfer

where  $\alpha_k$  is a slope of the  $k$ -th straight line to the axis  $x_j$ . Let the control of the computation process in the network be such, so that all  $M$  of tasks of  $m$  types are assigned to the  $i$ -th WS, after which they are transferred in a certain sequence to the  $j$ -th WS. Then the dependence of time necessary to solve the whole set of tasks distributed between the  $i$ -th and the  $j$ -th WSs shall be represented by a broken line that connects the points  $T_{\text{tot}}^i$  and  $T_{\text{tot}}^j$ , and the form of this broken line will depend on the sequence in which the tasks will be transferred from the  $i$ -th WS to the  $j$ -th WS. Geometric interpretation of the latter thesis is shown in Figure 3.

It is clear that the amount of variants for the distribution of ICTs between WSs (amount of broken lines) shall be equal to the number of shifts of the total amount of tasks, i.e.  $M!$  that will be inside the area  $F_2$  (Figure 3) ("2" in  $F_2$  is the area dimensions). Broken lines  $L_1$  and  $L_{M!}$  (Fig.3) are the lower and the upper bounds of the area  $F_2$ , i.e.

$$\begin{aligned} L_1 \{ |\text{tg } \alpha_1| > |\text{tg } \alpha_2| > \dots > |\text{tg } \alpha_{n-1}| > |\text{tg } \alpha_n| \} &= \inf \{ F_2 \} \\ L_{M!} \{ |\text{tg } \alpha_1| < |\text{tg } \alpha_2| < \dots < |\text{tg } \alpha_{n-1}| < |\text{tg } \alpha_n| \} &= \sup \{ F_2 \} \end{aligned} \quad (15)$$

where  $L_1 \{ |\text{tg } \alpha_1| > |\text{tg } \alpha_2| > \dots > |\text{tg } \alpha_{n-1}| > |\text{tg } \alpha_n| \}$  and  $L_{M!} \{ |\text{tg } \alpha_1| < |\text{tg } \alpha_2| < \dots < |\text{tg } \alpha_{n-1}| < |\text{tg } \alpha_n| \}$  are the broken lines consisting of the variety of segments  $\{l_i\}_{i=1}^n$  for which the following inequations hold true

$$\begin{aligned} &|\text{tg } \alpha_1| > |\text{tg } \alpha_2| > \dots > |\text{tg } \alpha_{n-1}| > |\text{tg } \alpha_n| \quad \text{и} \\ &|\text{tg } \alpha_1| < |\text{tg } \alpha_2| < \dots < |\text{tg } \alpha_{n-1}| < |\text{tg } \alpha_n| \end{aligned}$$

where  $\text{tg } \alpha_j$  is a slope ratio of the straight line, on which the segment  $l_j$  lies.

We can see from Figure 3 that for any  $L_i \in F_2$ ,  $i = \overline{1, M!}$  there is a point  $\tau_i$ , which shall fulfill condition (12), but it is evident that

$$T_{\min} = \tau_1 \in \inf \{ F_2 \}. \quad (16)$$

Therefore  $\inf \{ F_2 \}$  will determine the sequence of ICTs transfer between two WSs, under which the condition (16) will be valid. Let us consider the algorithm of evaluation of  $T_{\min}$  in accordance with criterion (12) for  $\inf \{ F_2 \}$ , as it is this broken line that will set an optimal in sense of criterion (12) and condition (16) functional dependence of change of the time necessary to solve ICTs at their distribution between WS $i$  and WS $j$ . Without loss of generality let us consider the numbers of the tasks' types corresponding to the segments  $l_j$ , to correspond to the numbers  $\alpha_j$  in definition (15).

Let all the ICTs entering the system be allocated at the WS $i$ , then the time necessary to solve them shall be  $T_{\text{tot}}^i$ . Let us start to transfer the ICT of the 1<sup>st</sup> type from the WS $i$  to the WS $j$ . Then the dependence of change of the time neces-

sary to solve ICTs at the redistribution between WSs will be expressed by the segments  $l_1$ , lying on the straight line represented by the equation

$$f(x_i) = x_j = a \cdot x_i + b,$$

where  $a = \text{tg } \alpha$ ;  $\alpha$  is a slope of the straight line in relation to the coordinate axis;  $b$  is an absolute term, whose value depends on the values of the coordinates of the points  $x_i$  and  $x_j$  on the plane crossed by a straight line.

For the segment  $l_1$   $a = \text{tg } \alpha_1$  and  $x_i = T_{\text{tot}}^i = (T_n^i + T_{n-1}^i + \dots + T_2^i + T_1^i)$ ,  $x_j = 0$ , then for  $l_1$  the following equation holds true (see Figure 3)

$$\begin{aligned} f_1(x_i) = x_j &= \text{tg } \alpha_1 \cdot (x_i - T_{\text{tot}}^i) = \\ &= \text{tg } \alpha_1 \cdot (x_i - (T_n^i + T_{n-1}^i + \dots + T_2^i + T_1^i)), \end{aligned}$$

where  $T_j^i$  is the time necessary to solve all tasks of the  $j$ -th type at the WS $i$ . Then for an arbitrary segment  $l_k \in \inf \{ F_2 \}$  it is not difficult to form a straight line for which  $a = \text{tg } \alpha_k$ ,  $x_i = (T_n^i + T_{n-1}^i + \dots + T_k^i)$  and  $x_j = (T_1^j + T_2^j + \dots + T_{k-1}^j)$ , we shall have the following expression of the straight line

$$\begin{aligned} f_k(x_i) = x_j &= \text{tg } \alpha_k \cdot x_i + \\ &+ ((T_1^j + T_2^j + \dots + T_{k-1}^j) - \text{tg } \alpha_k \cdot (T_n^i + T_{n-1}^i + \dots + T_k^i)), \end{aligned}$$

where  $(T_1^j + T_2^j + \dots + T_{k-1}^j)$  is a total time necessary to solve the tasks from the 1-st to the  $(k-1)$ -th type at the WS $j$  (with  $k=1$   $(T_1^j + T_2^j + \dots + T_{k-1}^j) = 0$ );  $(T_n^i + T_{n-1}^i + \dots + T_k^i)$  is a total time necessary to solve all the tasks from the  $k$ -th to the  $n$ -th type at the WS $i$ .

According to criterion (12) for each straight line  $f_k(x_i)$  let us find the point  $\tau_1^k$  ( $k = \overline{1, n}$ ), which is a crossing of the straight line  $f_k(x_i)$  and the straight line  $x_j = x_i$

$$\tau_1^k = \frac{(T_1^j + T_2^j + \dots + T_{k-1}^j) - \text{tg } \alpha_k \cdot (T_n^i + T_{n-1}^i + \dots + T_k^i)}{1 - \text{tg } \alpha_k}.$$

Further on for all values of  $\tau_1^k$  ( $k = \overline{1, n}$ ) it is necessary to choose the maximum values corresponding to  $T_{\min}$ , i.e.

$$T_{\min} = \max \{ \tau_1^k \}_{k=1}^n. \quad (17)$$

Figure 3 shows that for two WSs, the equation  $T_{\min}$  divides the whole set of tasks into two subsets. The first subset are the tasks to be solved at the WS $i$ , the second subset includes the tasks to be solved at the WS $j$ . By analogy with the computation process at two WSs, the task of estimation of the value  $T_{\min}$  for  $m$  WS is reduced to the construction of  $\inf \{ F_m \}$ , which shall be a piecewise linear hypersurface. Analytically, as in the case of (10), it is difficult to represent this hypersurface, that is why it is offered to use a geometrical method to construct it.

Let us consider the algorithm of  $\inf \{ F_m \}$  on the example for  $m=3$  and  $n=4$ . Let us assume all ICTs to be distributed

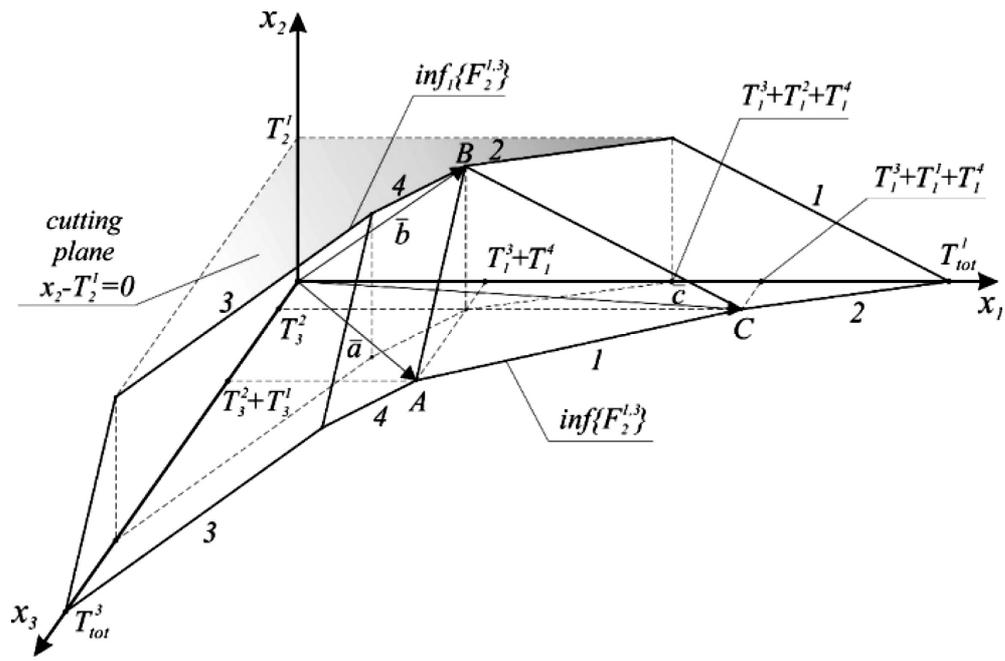


Fig. 5. Fragment of the piecewise linear surface  $Q$

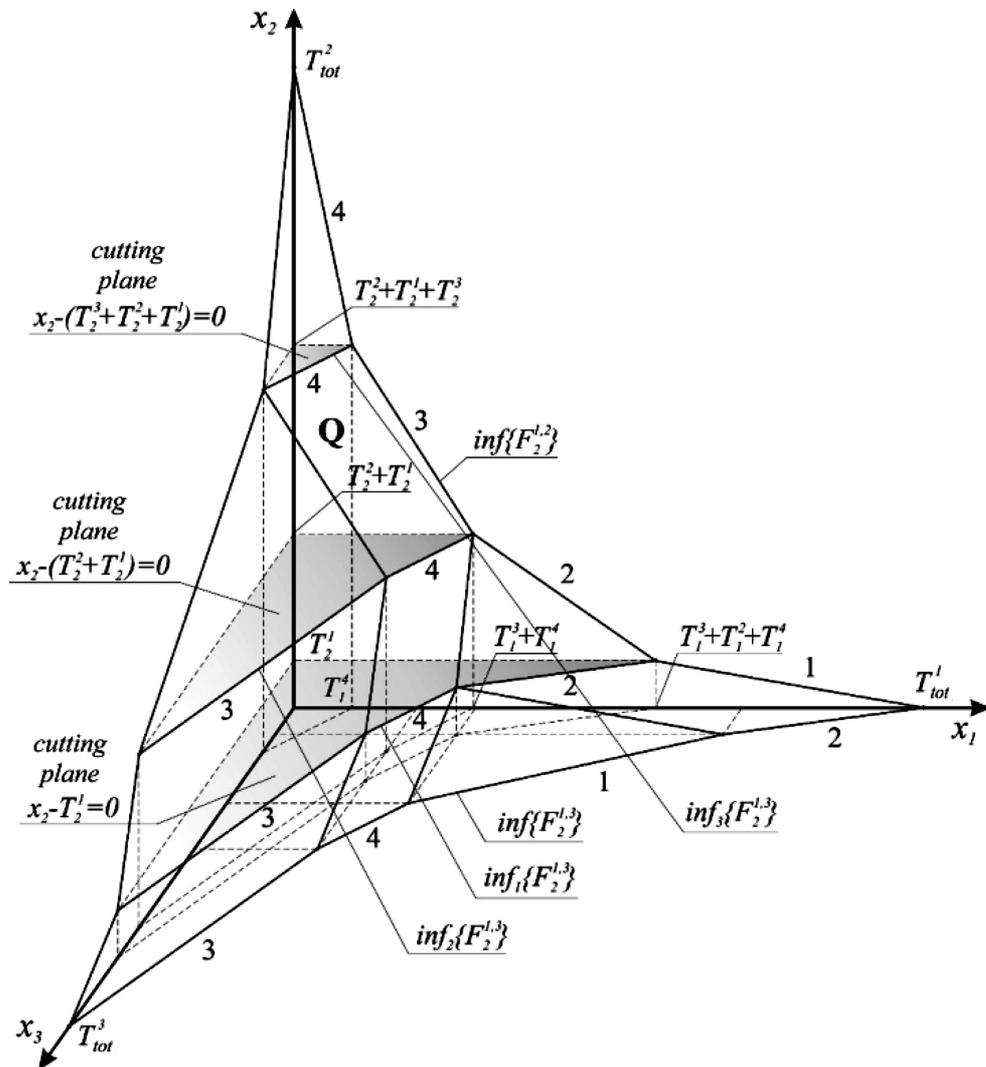


Fig. 4. Piecewise linear surface  $Q$  describing the variety of possible variants for the distribution of ICTs between three WSs

to the WS1. Then the time necessary to solve all the ICTs at the WS1 shall be equal to  $T_{\text{tot}}^1$ . There is a potential to transfer the ICTs from the WS1 to the WS2 and WS3, and the dependence of the change of the solution time while transferring the ICTs from the WS1 to the WS2 will correspond to  $\inf\{F_2^{1,2}\}$ , and while transferring the tasks from the WS1 to the WS3 –  $\inf\{F_2^{1,3}\}$ . In general the sequences of ICT transfer that determine  $\inf\{F_2^{1,2}\}$  and  $\inf\{F_2^{1,3}\}$ , may be different. Without loss of generality let us accept that the sequences of ICT transfer that determine  $\inf\{F_2^{1,2}\}$  and  $\inf\{F_2^{1,3}\}$ , are such as it is shown in Figure 4.

Let us transfer the ICTs of the 1<sup>st</sup> type from the WS1 to the WS2 in accordance with the sequence defined by  $\inf\{F_2^{1,2}\}$ .

Then the dependence of the change of time necessary to solve the ICTs of the 1<sup>st</sup> type, distributed between WS1 and WS2, will be expressed by a linear function  $x_2(x_1)=l_1$ , where  $l_1$  is the segment 1 in the plane  $x_1, x_2$ , connecting the points  $(T_{\text{tot}}^1, 0)$ ,  $(T_1^3 + T_1^2 + T_1^3, T_2^1)$  (Figure 4), i.e. we can write that

$$l_1 \in \inf\{F_2^{1,2}\}.$$

As the result there will be the ICTs of the 2<sup>nd</sup>, 4<sup>th</sup> and 3<sup>rd</sup> types left, allocated in the sequence that fulfills condition (15) and specifies  $\inf_1\{F_2^{1,3}\}$  (Figure 4).

It is evident that  $\inf\{F_2^{1,3}\}$  is a trace of the piecewise linear surface  $Q$  by the plane  $x_2=0$  and  $\inf_1\{F_2^{1,3}\}$ ,  $\inf_2\{F_2^{1,3}\}$ ,  $\inf_3\{F_2^{1,3}\}$  are traces of the piecewise linear surface  $Q$  by the planes  $x_2 - T_2^1 = 0$ ,  $x_2 - (T_2^2 + T_2^1) = 0$  and  $x_2 - (T_2^3 + T_2^2 + T_2^1) = 0$  respectively, parallel to the plane  $x_2=0$  (Figure 4). Figure 4 shows that the surface  $Q$  between the traces, for instance between  $\inf\{F_2^{1,3}\}$  and  $\inf_1\{F_2^{1,3}\}$ , are the intercrossing planes formed by parallel or by crossing segments (coplanar vectors). By the example of the plane  $ABC$ , lying between  $\inf\{F_2^{1,3}\}$  and  $\inf_1\{F_2^{1,3}\}$  (Figure 5), let us consider the construction of an arbitrary plane forming a piecewise linear surface  $Q$ .

We know [5, 6], that in orthonormal space the plane is definitely set by a normal vector  $\bar{N}$  and by the point lying on the plane. In our case let us choose the point  $A$ , referring to the plane, with the coordinates  $(T_1^3 + T_1^4, 0, T_3^2 + T_3^1)$  (Figure 5).

Let us consider three vectors  $\bar{a}$ ,  $\bar{b}$ ,  $\bar{c}$ , whose coordinates coincide with the points  $A, B, C$  respectively (Figure 5). As the indicated points are on the plane, any linear combination  $\bar{a}$ ,  $\bar{b}$ ,  $\bar{c}$  will represent the set of coplanar vectors [5, 6], i.e.

$$\overline{AB} = \bar{b} - \bar{a}, \quad \overline{AC} = \bar{c} - \bar{a}.$$

The normal to the plane is known to be the result of a vector product of a pair of coplanar vectors [5, 6], therefore, we can write

$$[\overline{AB}, \overline{AC}] = \bar{N}, \quad (18)$$

where  $[\overline{AB}, \overline{AC}]$  is a vector product of the vectors  $\overline{AB}$  and  $\overline{AC}$ .

Let  $x_1, x_2, x_3$  be an orthonormal basis of the vector space, then let us rewrite the expression (18) in a coordinate form.

$$\begin{aligned} \bar{a} &= (T_1^3 + T_1^4, 0, T_3^2 + T_3^1); \quad \bar{b} = (T_1^3 + T_1^4, T_2^1, T_3^2); \\ \bar{c} &= (T_1^3 + T_1^4 + T_1^4, 0, T_3^2); \quad \overline{AB} = \bar{b} - \bar{a} = (0, T_2^1, -T_3^1); \\ \overline{AC} &= \bar{c} - \bar{a} = (T_1^1, 0, -T_3^1); \\ \bar{N} &= [\overline{AB}, \overline{AC}] = \begin{vmatrix} x_1 & x_2 & x_3 \\ 0 & T_2^1 & -T_3^1 \\ T_1^1 & 0 & -T_3^1 \end{vmatrix} = x_1 \cdot \begin{vmatrix} T_2^1 & -T_3^1 \\ 0 & -T_3^1 \end{vmatrix} - \\ & - x_2 \cdot \begin{vmatrix} 0 & -T_3^1 \\ T_1^1 & -T_3^1 \end{vmatrix} + x_3 \cdot \begin{vmatrix} 0 & T_2^1 \\ T_1^1 & 0 \end{vmatrix} = \\ & = -x_1 \cdot T_2^1 T_3^1 + x_2 \cdot T_3^1 T_1^1 - x_3 \cdot T_2^1 T_1^1. \end{aligned}$$

Therefore, normal vector to the plane surface  $ABC$  shall be as follows

$$\bar{N} = (-T_2^1 T_3^1, T_3^1 T_1^1, -T_2^1 T_1^1).$$

Canonical expression of the plane in the space with an orthonormal basis  $x_1, x_2, x_3$ , going through the point  $M^0(x_1^0, x_2^0, x_3^0)$  and having the normal vector  $\bar{N} = (A, B, C)$  is as follows

$$A(x_1 - x_1^0) + B(x_2 - x_2^0) + C(x_3 - x_3^0) = 0.$$

Then the expression of the plane  $ABC$  with the normal vector  $\bar{N} = (-T_2^1 T_3^1, T_3^1 T_1^1, -T_2^1 T_1^1)$  passing through the point  $A(T_1^3 + T_1^4, 0, T_3^2 + T_3^1)$  will be as follows

$$\begin{aligned} -T_2^1 T_3^1 (x_1 - (T_1^3 + T_1^4)) + T_3^1 T_1^1 x_2 - \\ -T_2^1 T_1^1 (x_3 - (T_3^2 + T_3^1)) = 0. \end{aligned}$$

Similar way is used to construct the rest planes forming a piecewise linear surface  $Q$ . Then for each plane of the surface  $Q$  in accordance with the criterion (12) we shall find the point of crossing of the plane with a straight line  $x_1=x_2=x_3$ . The amount of planes forming the surface  $Q$  can be found using equation [7]

$$L = \frac{2 + (n-1)}{2} \cdot n,$$

where  $L$  is the amount of the planes forming the surface  $Q$ ;  $n$  is the amount of types of ICTs distributed between WSs. Let us denote the point of crossing of the straight line  $x_1=x_2=x_3$  with the  $k$ -th plane  $\tau^k$ . Then by analogy with (17)

$$T_{\min}^{\text{WS1}} = \max\{\tau^k\}_{k=1}^L,$$

where  $T_{\min}^{\text{WS1}}$  is the value of minimum time necessary to solve ICTs provided all ICTs are potentially allocated at the WS1. The value  $T_{\min}^{\text{WS1}}$  was obtained provided that the distribution of ICTs between WSs started with the WS1,

i.e. all ICTs were virtually allocated at the WS1 and then distributed between the WS2 or WS3. If to form the surface  $Q$  provided that the distribution of ICTs starts with the WS2 of the WS3, we will have the values  $T_{\min}^{WS2}$  and  $T_{\min}^{WS3}$ , for which the following inequation will generally be fulfilled

$$T_{\min}^{APM1} \neq T_{\min}^{APM2} \neq T_{\min}^{APM3}. \quad (19)$$

It is evident that for a complete solution of the task it is necessary to choose the minimum value from the obtained values (19). I.e., for  $m$  WS we will have a minimax task

$$T_{\min} = \min \left\{ T_{\min}^{WSi} \right\}_{i=1}^m = \min \left\{ \max \left\{ \tau_{WSi}^k \right\}_{k=1}^L \right\}_{i=1}^m. \quad (20)$$

Therefore, having obtained (20)  $T_{\min}$ , which is the point of an absolute minimum of the task, through the coordinates of this point  $T_1=T_2=\dots=T_m$  corresponding to the axes  $x_1, x_2, \dots, x_m$ , we will get the best (in terms of the criterion (12) variant of the organization of a computation process in CS.

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## About the authors

**Sergey A. Zhurbin** – senior researcher in the 4th CSRI, Ministry of Defense of the Russian Federation. 13-224, Pushkinskaya street, Korolev, Moscow region, 141092, Russia. Phone +7 (910) 408-97-04; e-mail: serdgo4@yandex.ru.

**Gennady V. Kazakov** – Ph.D. in engineering, associate professor, director of the 4th CSRI, Ministry of Defence of the Russian Federation. 17-103, Pushkinskaya street, Yubileinyi, Moscow region, 141092, Russia. Phone +7 (910) 467-71-66; e-mail: kgv.64@mail.ru.

# Determination of probability of nominal mode of main product pipeline operation with consideration of ageing of pumping units

**Anatoly V. Karmanov**, the chair "Automation of production processes", Gubkin Russian State University of Oil and Gas, Moscow, Russia, e-mail: abkar2007@yandex.ru

**Dmitry A. Roslyakov**, JSC Transneftproduct, Moscow, Russia, e-mail: karter.diman@yandex.ru

**Anton S. Telyuk**, the chair "Automation of production processes", Gubkin Russian State University of Oil and Gas, Moscow, Russia



Anatoly V.  
Karmanov



Dmitry A. Roslyakov



Anton S. Telyuk

**Abstract. Aim.** The article provides a method and a formula for calculation of probability of nominal operating mode for main product pipeline (MPP) – further as the text goes, MPP availability function – with consideration of ageing of its pumping units which are periodically maintained in accordance with a normative service strategy. This availability function is determined in the following assumptions: 1. MPP is composed of two basic parts: passive part – high reliable line part; and active part including pump stations which ensure nominal operating mode for the product's pumping-over. MPP may contain any finite number of pump stations. 2. Each pump station includes the system of main pumping units (MPU system) which are active elements of the station, instrumentation and control, pipeline accessories and shutoff valves, as well as other essential technological equipment. MPU system is the part of pump stations ensuring nominal conditions for the oil products pumping-over and which is usually consists of four homogeneous MPUs. 3. MPU arrangement makes it possible to bring each working unit into standby, and substitute it with any standby unit. 4. A required nominal mode for MPP operation is determined by hydraulic and cost calculations as the result of which a required operating mode is indicated for each pump station. For each station the number of MPU is indicated which must be in a working order, and the rest MPU shall be either in standby, or under restoring repair performed in accordance with a normative service strategy. Thus, nominal mode of MPP operation is ensured by the respective modes of pump stations, which with regard to pumping units are determined by the number of active MPUs. Analysis of statistics related to the failures of pumping units maintained in accordance with a normative service strategy makes it possible to define the units' failure rate in each interval between overhauls. In particular, failure rates are increasing on the respective intervals which means the ageing of units with their operation. Then the method for calculation of availability function for any pumping unit within the scope of MPP is offered. Initial conditions and differential equations are written to find an availability function for each MPU system at pump stations, obtained using the "death and reproduction" scheme. Basic results of calculations per each of three sequential intervals between overhauls are represented in form of graphs that show the influence of ageing of the units on the values of MPU availability function at a pump station: values of derivatives of availability function are sequentially decreasing for the respective times counted from the start of each recurrent overhaul. The expression to calculate availability function of MPP with several pump stations is also provided. The results of calculation of the availability function can serve as the grounds for modernization of a normative periodic strategy on order to increase the probability of MPP nominal mode, as well as other technical and economic performance indicators of MPU systems, in particular, energy efficiency indicators. In particular, it is pointed out that certain types of non-periodic service strategies, built on the basis of a normative strategy may significantly increase the values of indicated.

**Keywords:** main product pipeline, pumping unit, reliability, ageing, availability function, probability of nominal operating mode.

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## 1. Introduction

Main oil product pipeline (MPP) is a complex developing technical system for an uninterrupted and scheduled supply of oil products to consumers during the whole time of a product pipeline operation. MPP is composed of two basic

parts: passive part, i.e. a pipeline line part, and active part including pump stations which ensure nominal operating mode for the product's pumping-over.

Each pump station includes the system of main pumping units (MPU system) which are active elements of the station, instrumentation and control, pipeline accessories and shutoff

valves, as well as other essential technological equipment. MPU system is the main active part of pump stations ensuring nominal conditions for the oil products pumping-over, it consists of  $n$  of homogeneous MPUs with the respective pipeline arrangements, where  $n$  is normally equal to 4. MPU arrangement makes it possible to bring each working unit into standby, and substitute it with any standby unit. A required nominal mode for MPP operation is determined by hydraulic and cost calculations as the result of which a required operating mode is indicated for each pump station. For each  $v$ -th station the number  $m_v$  of MPU is indicated which must be in a working order, and the rest  $(n - m_v)$  MPU shall be either in standby, or under restoring repair, where  $v = 1, \dots, N$ ,  $N$  is the number of pump stations within MPP. Then, nominal mode of MPP operation is ensured by the respective modes of pump stations, and this mode of MPP operation in part related to the working units at the stations is determined by the following set of parameters:

$$(m_1, \dots, m_N). \quad (1)$$

Each MPU is a set of two main coupled parts:

1) main line pump (MLP); 2) motor driver (MD), which is a MLP drive. MLP and MD are ageing equipment [1, 2], specified during the period of operation by wear of its constituents and damage accumulation where the term “damage” is understood [2] as “the event when the state is not fault-free anymore, though the equipment is still serviceable”. MPU ageing is exerted in both, degradation of MPU reliability indices, and in degradation of its technical and economic characteristics [3].

To prevent from negative developments of a random process of ageing a unit periodically goes through maintenance. MPU maintenance includes 1) diagnostic monitoring and inspections, as well as 2) different types of preventive maintenance (PM). This maintenance is performed in accordance with a certain rule approved earlier – PM strategy. It is clear that the ageing of MPU greatly depends on the type and scope of PM strategy. All types of repairs within the scope of PM strategy largely but not fully eliminate negative effects of the MPU ageing. That is why the method of estimation of the probability  $k(m_1, \dots, m_N, t)$  of nominal mode of MPP operation in each moment of time  $t$  under MPU ageing is of practical and scientific interest. And we shall consider that the MPP mode of operation is specified by the set (1).

This paper describes one analytical method of estimation of probability  $k(m_1, \dots, m_N, t)$ , which shall be further called the availability function of MPP.

## 2. Ageing process affecting a failure rate of MPU serviced by the MP periodic strategy

Periodic strategy  $s$  of MPU PM can be represented in the following form:  $s = (s_{mlp}, s_{md})$ , where  $s_{mlp}$  is a PM strategy of the main line pump (MLP),  $s_{md}$  is a PM strategy of the motor driver (MD). And the preventive maintenance for

MLP and MD by the strategies  $s_{mlp}$  and  $s_{md}$  are carried out consistently in time, i.e. when routine repairs are conducted for MLP, similar repair is carried out for MD, etc. Each constituent  $s_{mlp}, s_{md}$  of the strategy  $s$  specifies the periodic sequence (cyclic recurrence) of diagnostic inspections, routine, intermediate repairs and overhauls, as well as the events when the emergency restoring repairs (ERR) are immediately carried out, and some other characteristics of service of the respective MPU equipment. For instance, it may contain time limits for all types of the repairs, it may sometimes indicate the specified lifetime, after which the equipment shall be written off, or the events when the equipment is placed into a standby (from a standby) and other essential details. Formal transcripts of standard periodic (cyclic) strategies  $s_{mlp}, s_{md}$ , currently used for the maintenance of MLP and MD, are listed in the papers [4, 5] and have the following form:

$$s_{mlp} = \{\theta_{or}, n_{or}, n_{ir}, n_{rr}^{mlp}, n_{di}, C_{or}^{mlp}, C_{ir}^{mlp}, C_{rr}^{mlp}, C_{err}^{mlp}\},$$

$$s_{md} = \{\theta_{or}, n_{or}, n_{rr}^{md}, C_{or}^{md}, C_{rr}^{md}, C_{err}^{md}\},$$

where  $\theta_{or}$  is an overhaul repair cycle (OR) equal to the time period between the nearest overhauls, which is defined by an MPU operating time,  $n_{or}$  is the amount of overhaul repair cycles,  $n_{ir}$  is the amount of cycles of intermediate repairs (IR) “inside” the OR cycle,  $n_{rr}^{mlp}$  is the amount of cycles of routine repairs (RR) of MLP “inside” the IR cycle,  $n_{di}^{mlp}$  is the amount of cycles of diagnostic inspection of MLP “inside” the RR cycle,  $n_{rr}^{md}$  is the amount of cycles of routine repairs of MD “inside” the OR cycle,  $C_{or}^{mlp}, C_{ir}^{mlp}, C_{rr}^{mlp}, C_{err}^{mlp}, C_{or}^{md}, C_{rr}^{md}, C_{err}^{md}$  are the rules and scope of the works carried out at any type of the repair of MLP and MD respectively. In particular,  $C_{err}^{(i)}$  is the rule of emergency restoring repair, in the rule  $C_{err}^{(i)}$  it is necessary to indicate the scope of works to be carried out in case of the MLP emergency shutdown, as well as other essential details, for example, the regulations for ERR. Sometimes the strategy  $s = (s_{mlp}, s_{md})$  contains the following normative standard indicators: 1) calendar time  $t_p$  till the MPU write-off; 2) time limit  $\theta_p$  for MPU operating time till its write-off.

For instance, major parameters of the strategy  $s$  may have the following values:

$$\Theta_{or} = 6 \cdot 10^4 \text{ h}, n_{or} = 2, n_{ir} = 4, n_{rr}^{mlp} = 1,$$

$$n_{rr}^{md} = 10, \theta_p = 3 \cdot \theta_{or}, t_p = 2 \cdot \theta_p. \quad (2)$$

And the calendar time period  $t_{or}$  from the start of MPU operation up to its nearest overhaul, or between the nearest overhauls is equal to  $2 \cdot \theta_{or}$ .

Statistical analysis of the impact of the ageing on the MLP and MD reliability performance is given in papers [4, 5]. Particularly, they contain the tables with the values of MLP and MD failure rates per each time interval between the nearest restoring repairs carried out in accordance with the strategy  $s$ . These tables also show that due to the ageing process an average failure rate of MPU  $\lambda_i$  in each interval  $J_i =$

$[(i-1) \cdot t_{or}, i \cdot t_{or}]$ , where  $i = 1, 2, 3$  increase monotonously. And the variables  $\lambda_i$ ,  $i = 1, 2, 3$  have the following values:

$$\lambda_1 = 3,51 \cdot 10^{-5}, \lambda_2 = 1,41 \cdot 10^{-4}, \lambda_3 = 6,50 \cdot 10^{-4}, \quad (3)$$

where the dimension of the given variables is 1/h.

Below is the modification of the famous mathematical model represented in [6] used to calculate the availability function  $k_v(t)$  of the MPU system at the  $v$ -th pump station with consideration of ageing of the pumping units serviced by the periodic strategy  $s$  with the parameters defined by the equations (2).

### 3. Determination of the availability function of a pump station and MPP

Paper [6] contains the description and substantiating of the mathematical model used to calculate the availability function  $k(t)$  of the MPU system of one pumping station where  $k(t)$  is a probability of the system with 4 identical MPUs being in the time moment  $t$  in the state when two and more MPUs are in operable condition. The strategy  $s$  is considered to be fixed in the period  $[0, t_p)$  of MPU operation, and MPU is not getting aged, i.e. its failure rate remains constant through the whole period of operation. This mathematical model is a homogeneous Markov process, which has  $n+1$  of states where  $n = 4$ , and which is set by a graph of the “death and reproduction” scheme [7].

In this case the calendar period  $[0, t_p)$  of the MPU operation for each  $v$ -th pump station is a sum of three sequential intervals  $J_i = [(i-1) \cdot t_{or}, i \cdot t_{or})$ , in which the failure rates  $\lambda^{(i)}$  of MPU are different, where  $i = 1, 2, 3$ ; and the amount of units being in operable condition on each  $v$ -th pump station should be not less than  $m_v$  in order to ensure the nominal mode of MPP operation. Considering the mentioned peculiarities the Markov process described in [6] becomes an inhomogeneous process  $\eta_v(t)$ ,  $(t \in [0, t_p), v = 1, \dots, N)$ , which is defined in each interval  $J_i$ ,  $i = 1, 2, 3$  by a homogeneous Markov process  $\zeta_v^{(i)}(x)$ , where  $x \in [0, t_{or})$ . The process  $\zeta_v^{(i)}(x)$  is set by the graph of the “death and reproduction” scheme shown in Figure 1.

In Figure 1 each  $j$ -th node of the graph corresponds to the MPU system state when  $j$  pcs. of MPU are in non-operable condition,  $j = 0, 1, \dots, n$ . And  $\mu$  is the MPU repair rate with one repair team,  $\lambda_i$  is the MPU failure rate in the interval  $J_i$ ,  $b_j \cdot \lambda_i$  is the rate of transition from the state ( $j$ ) to the state ( $j+1$ ) of the MPU system where  $b_j = m_v$ , if  $j = 0, 1, \dots, n-m_v$ , и  $b_j = m_v - k$ , если  $j = n-m_v+k$ ,  $k = 1, \dots, m_v-1$ .

The components of the vector  $a^{(i)} = [a_0^{(i)}(0), \dots, a_3^{(i)}(0)]$

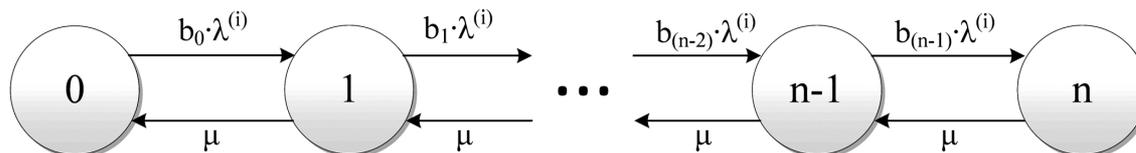


Fig. 1. Graph of the “death and reproduction” scheme for the process  $\zeta_v^{(i)}(x)$

of the initial distribution of the process  $\zeta_v^{(i)}(x)$  are defined by the following equation:

$$a_0^{(i)}(0) = p_0^{(i)}(0, v) = 1, a_j^{(i)}(0) = p_j^{(i)}(0, v) = 0, \quad (4)$$

where  $i = 1, 2, 3, j = 0, 1, \dots, n, p_j^{(i)}(x, v)$  is the probability of the process  $\zeta_v^{(i)}(x)$  being in the state  $j$  in the time moment  $x \in [0, t_{or}), v = 1, \dots, N$ .

For any  $t \in [0, t_p)$  there is such an interval  $J_i = [(i-1) \cdot t_{or}, i \cdot t_{or})$ , where  $t \in J_i$ , where  $i = 1, 2, 3$ . Then  $t$  can be represented as  $t = (i-1) \cdot t_{or} + x$ ,  $x \in [0, t_{or})$ . And the probability

$$\mathbf{P}^{(i)}(x, v) = p_0^{(i)}(x, v) + p_1^{(i)}(x, v) + \dots + p_{n-m_v}^{(i)}(x, v), \quad (5)$$

is the availability function  $k_v(t)$  of the MPU system of the  $v$ -th pump station, i.e. the following equation holds true:

$$k_v(t) = k_v((i-1) \cdot t_{or} + x) = \mathbf{P}^{(i)}(x, v). \quad (6)$$

For each  $i = 1, 2, 3$  the probabilities  $p_j^{(i)}(x, v)$  are defined by the solution of the following differential equation system:

$$\begin{aligned} dp_0^{(i)}(x, v)/dt &= -b_0 \cdot \lambda^{(i)} \cdot p_0^{(i)}(x, v) + \mu \cdot p_1^{(i)}(x, v), \\ dp_j^{(i)}(x, v)/dt &= \mu \cdot p_{j+1}^{(i)}(x, v) - (b_j \cdot \lambda^{(i)} + \mu) \cdot p_j^{(i)}(x, v) + \\ &\quad + b_{j-1} \cdot \lambda^{(i)} \cdot p_{j-1}^{(i)}(x, v), j = 1, \dots, n-1, \\ dp_n^{(i)}(x, v)/dt &= b_{n-1} \cdot \lambda^{(i)} \cdot p_{n-1}^{(i)}(x, v) - \mu \cdot p_n^{(i)}(x, v). \end{aligned} \quad (7)$$

The initial condition for system (7) is the initial distribution vector, whose components are defined by equation (4).

Let any  $v$ -th pump station have  $m_v = 2$  and  $n = 4$ , where  $v \in (1, \dots, N)$ . Then the calculation of the availability function  $k_v(t)$  in the time interval  $[0, t_p)$  with initial data defined in equations (2), (3) and with  $\mu = 0,5 \cdot 10^{-2}$  (1/h) will give the result shown in Figure 2.

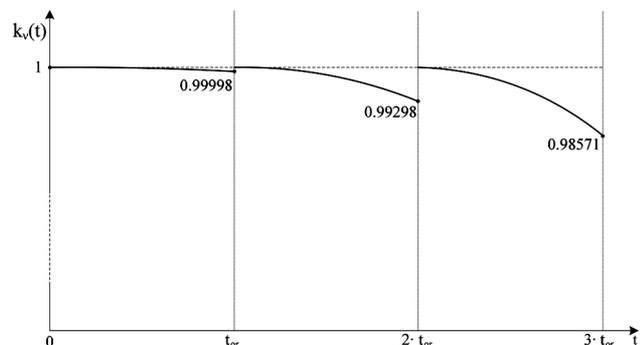


Fig. 2. Graph  $k_v(t)$  in the time interval  $[0, t_p)$

The availability function  $k(m_1, \dots, m_N, t)$  of MPP, whose nominal mode is set by (1), is defined (in case statistically

independent operation of pump stations is provided) by the equation:

$$k(m_1, \dots, m_N, t) = k_1(t) \cdot k_2(t) \cdot \dots \cdot k_N(t). \quad (8)$$

#### 4. Conclusion

The availability function  $k_v(t)$  of MPU system of each  $v$ -th pump station where  $v = 1, \dots, N$  within MPP is a function monotonously decreasing with time in each  $i$ -th interval  $J_i$ , where  $i = 1, 2, 3$ . Accordingly, in each interval  $J_i$  a decreasing function is  $k(m_1, \dots, m_N, t)$  which is a probability of the nominal mode of operation of the whole MPP including  $N$  of pump stations. And for any  $v \in (1, \dots, N)$  a decrease of function  $k_v(t)$  grows in the operation interval  $J_{i+1}$  in comparison to the interval  $J_i$ , where  $i = 1, 2$ , which is caused by the ageing (accumulation of failures) of the main line pumping units serviced in accordance with a periodic PM strategy  $s$ . In particular, this fact may denote that non-periodic PM strategies mitigating the ageing effect can be more effective than the current periodic (cyclic) strategies.

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#### About the authors

**Anatoly V. Karmanov** – Doctor of physico-mathematical sciences, professor of the chair “Automation of production processes”, Gubkin Russian State University of Oil and Gas, 119991, Moscow, Leninsky prospect, 65, building 1, tel.: +7 (915) 366-51-18, e-mail: abkar2007@yandex.ru

**Dmitry A. Roslyakov** – Chief specialist of Department of operation of Joint-Stock Co. “AK Transneftproduct”, 115184, Moscow, Vishnyakovsky street, 2/36, building 1, tel. +7 (909) 937-50-64, e-mail: karter.diman@yandex.ru

**Anton S. Telyuk** – PhD in engineering, assistant of the chair “Automation of production processes” Gubkin Russian State University of Oil and Gas, 119991, Moscow, Leninsky prospect, 65, building 1

## Estimation of quality of a small sampling biometric data using a more efficient form of the chi-square test

**Berik B. Akhmetov**, International Informatization Academy (IIA), Turkestan, Kazakhstan, e-mail: berik.akhmetov@ayu.edu.kz

**Aleksander I. Ivanov**, Laboratory of biometric and neural network technologies, JSC Penza Research Electric and Technical Institute, Penza, Russia, e-mail: ivan@pniei.penza.ru



Berik B. Akhmetov



Aleksander I. Ivanov

**Abstract. Aim.** The purpose is to increase the power of the Pearson's chi-square test so that this test will become efficient on small test samplings. . It is necessary to reduce the scope of a test sample from 200 examples to 20 examples while maintaining the probability of errors of the first and the second kind. Selection of 20 examples of biometric images is considered by users to be a comfortable level of effort. The need to select more examples is perceived by users negatively.

**Methods.** The article offers one more (the second) form of the Pearson test that is much less sensitive to the scope of data in a test sampling. It is shown that a traditional form of the chi-square test is more sensitive to the scope of a test sampling than the Cramer-von Mises test. The offered (second) form of the chi-square test is less sensitive to the scope of a test sampling than a classical form of the chi-square test and less sensitive than the Cramer-von Mises test as well. This effect is achieved by the transition from the space of frequency of occurrence of events and probabilities of a group of similar events occurring in the space of more accurately evaluated junior statistical moments (mean and standard deviation). The fractal dimension of the new synthetic form of chi-square test coincides with the fractal dimension of the classical form of the chi-square test. **Results.** The offered second variant of the chi-square test is presumably one of the most powerful of all existing statistical tests. The analytical description of correlation of standard deviations of a classical form of the chi-square test and a new form of the chi-square test is given. The standard deviation of the second form of the chi-square test decreases by half on retention of a statistical expectation on samplings of the same scope. The latter is equivalent to a four-time reduction of the requirements to the scope of a test sampling within the interval from 16 to 20 examples. Power gain as the result of the application of a new test is growing with the growth of a test sampling scope. **Conclusions.** When creating a classical chi-square test in 1900, Pearson was guided by limited computing opportunities of the existing computer facilities, and he had to rely on the analytical relations that he found. Today the situation has changed and there are no more restrictions in relation to the engaged computing resources. However we continue to rely on those created with computing resources of 1900 by inertia. Probably, we should try to consider modern opportunities of computer facilities and to build more powerful options of statistical tests. Even if new tests will require a search of large number of possible states (they will have big tables calculated in advance instead of analytical relations), it is not a constraining factor today. When data is insufficient (in biometrics, in medicine, in economy) a computing complexity of statistical tests does not play a special role if the result of estimations is more accurate.

**Keywords:** multivariate statistical analysis, chi-square test, small samplings of test biometric data.

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### Introduction

An information-oriented society requires active use of Internet resources. State and private organizations create personal user accounts on their web-sites. Unfortunately, the current practice of password security of the access to personal user accounts is quite vulnerable. Users are not able to remember long randomly chosen passwords. An owner of an information resource cannot be sure that it will be exactly his host who will get access to the personal account. A password may be intercepted by a

backdoor. Besides it is quite easy to spoof an IP address of an Internet-user.

To protect the access to user accounts, the technologies of personal biometric authentication are currently being developed by means of transformation of personal biometric data into a cryptographic key of a person, or into a long randomly chosen password. The following biometric images are used: an image of finger mark [1], an image of eye iris [2], voice password [3], hand-written password [4], an image of blood vessels of an eye ground or a palm [5]. Naturally, biometrics-code transformers cannot be ideal, they have

the probabilities of errors of the first and the second kind. It becomes necessary to test the errors of the first and the second kind on real biometric data. Moreover, when setting the “indistinct extractors” [1, 2, 3] and training the neural network transformers [4, 5] it is necessary to control the absence of gross errors in biometric data. Basically, on a small number of examples of a biometric image it is necessary to control the indicator of relationship of the biometric data distribution to the multivariate normal law [6]. Formally, for this purpose we can use a simple univariate Pearson chi-square test [7, 8], but such approach is far from the best one. In this article we will try to show that a classic form of the Pearson chi-square test is by far not the only one, i.e. it is possible to set the task on searching of the most effective Pearson’s functionalities, considering different peculiarities of their practical application.

### Occurrence of quantization noise at the statistical processing of small samplings

Let us consider the simplest situation, when a test or a learning sampling is represented by 9 examples of the “Self” image. Since a continuous function of probability  $P(x)$  of the first biometric parameter  $v_1$  is a small sampling function, we have to describe it by a step monotonously increasing function  $\tilde{P}(x)$ , as it is shown on the left part of Figure 1.

To construct a step monotonously increasing approximation  $\tilde{P}(x)$ , it is necessary to sort biometric data in its ascending order:

$$x_i = \text{sort}(v_{1,i}) \text{ for } i = 0, 1, 2, \dots, n, \quad (1)$$

where  $n$  is a dimension of a test sampling, or a number of quanta of approximation of a monotone function of probability.

In this case a monotonously increasing step function will be described by the following piecewise constant approximation:

$$\tilde{P}(x_i) = \frac{i}{n}. \quad (2)$$

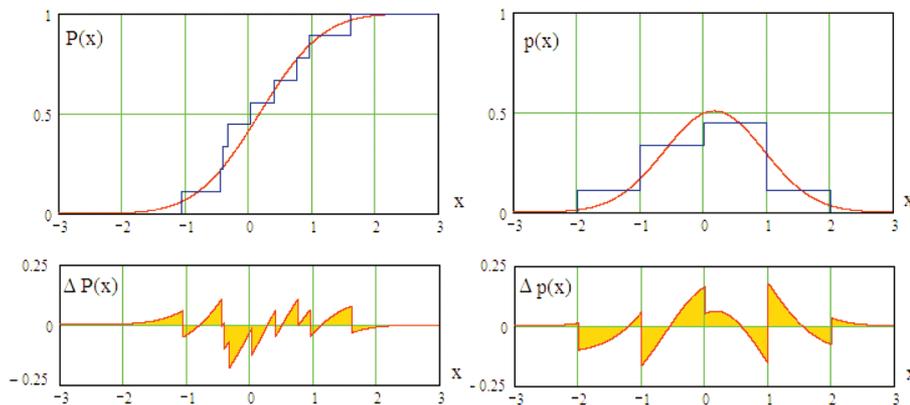


Fig. 1. Effects of quantization of a continuous probability of values distribution and a continuous density of values distribution by 9 examples that cause continuous noise of a quantization error

An approximation error or a quantization noise is found as a difference of a continuous probability function and its step approximation:

$$\Delta P(x) = P(x) - \tilde{P}(x). \quad (3)$$

The lower part of Figure 1 shows the functions of the quantization error or quantization noise caused by small test samplings.

In the context of mentioned above, the Kolmogorov-Smirnov test [7] should be considered as a search of the maximum value of the module of approximation error:

$$\sup_{-\infty < v < +\infty} |P(x) - \tilde{P}(x)| = \max |\Delta P(x_i)| \quad (4)$$

or a choice of the biggest from local maximums of the quantization noise.

From the same perspective, the Cramer-von Mises test [7] is the estimation of standard deviation of the quantization noise of the continuous probability function:

$$\begin{aligned} \int_{-\infty}^{\infty} \{P(x) - \tilde{P}(x)\}^2 \cdot dx &= \int_{-\infty}^{\infty} \{E(\Delta P(x)) - \Delta P(x)\}^2 \cdot dx = \\ &= \int_{-\infty}^{\infty} \{\Delta P(x)\}^2 \cdot dx = \sigma^2(\Delta P(x)), \end{aligned} \quad (5)$$

if the condition of a zero statistical expectation of a quantization noise is fulfilled  $E(\Delta P(x))=0$ .

It should be emphasized that the Kolmogorov-Smirnov test (4) always has a lower power in comparison to the Cramer-von Mises test (5). The Kolmogorov-Smirnov test (4) is a point test, and the Cramer-von Mises test (5) is integral.

It is evident that with the increase of  $n$  of a test sampling, both these statistical tests are getting power of estimations, however, the estimation by an integral test is always more reliable than the point estimation.

### Classic variant of the Pearson chi-square test

General practice of check of statistical hypotheses in most sectors of industry is reduced to the construction of

histograms of the available data (right part of Figure 1) and to the calculation of the classic chi-square test:

$$\chi^2 = \sum_{i=1}^k \left\{ \frac{\left( \frac{n_i}{n} - P_i \right)^2}{P_i} \right\} = \sum_{i=1}^k \left\{ \frac{(\tilde{P}_i - P_i)^2}{P_i} \right\}, \quad (6)$$

where  $n_i$  is the number of samples, occurring in the  $i$ -th column of the histogram,  $P_i$  is the probability of occurrence in the  $i$ -th column of the histogram of the theoretical distribution,  $k$  is the number of columns of the histogram.

Wide application of chi-square test is determined by the fact that for this test we know the analytical description of distribution density:

$$p(\chi^2, m) = \frac{1}{m} \left\{ \frac{1}{2^{\frac{m}{2}} \cdot \Gamma\left(\frac{m}{2}\right)} \left\{ x^{\frac{m}{2}-1} \cdot \exp\left(-\frac{x}{2}\right) \right\} \right\}, \quad (7)$$

where  $\Gamma(\cdot)$  is a gamma-function,  $m$  is the number of degrees of freedom.

The number of degrees of freedom  $m$  can be set in different ways [8]. For instance, it can be defined through the scope  $n$  of a test sampling:

$$m = \sqrt{n} - 3 = k - 3, \quad (8)$$

if the number  $k$  of the histogram columns is chosen by the rounding off up to the nearest integer of the value  $\sqrt{n}$ :

$$k = \text{round}(\sqrt{n}). \quad (9)$$

Let us note that the value of the number  $k$  of the histogram columns and the value of the number  $m$  of the degrees of freedom for a classic chi-square test always turns out to be much smaller in comparison to the scope  $n$  of a test sampling. So, the error of step approximation of density of distribution of the values  $\Delta p(x) = p(x) - \tilde{p}(x)$  (right part of Figure 1) is always more than the error of approximation of a probability function (3). So in the left part of Figure 1 the approximation of the probability function is constructed

using 9 steps, whereas the function of approximation of probability distribution is constructed using just 4 steps in the right part of Figure 1. The quantization noises of the Cramer-von Mises test turn out to be less than the quantization noises of the classic chi-square test (6).

It means that the power of the Cramer-von Mises test is always higher than the power of the classic Pearson chi-square test (3).

### Comparison by power of the Cramer-von Mises test with the classic chi-square test

We shall proceed from the fact that biometric data for each of the parameters under control is distributed normally. Then the quality of data of one parameter can be estimated by both, the Cramer-von Mises test, and the chi-square test [7, 8]. To compare the tests let us use the data distribution by the uniform law as an alternative. The results of the numerical simulation for the samplings of 9 examples are shown in Figure 2.

When making a decision, a match threshold plays an important role. Each match threshold gives its probability value  $P_1$  for the errors of the first kind and probability value  $P_2$  for the errors of the second kind. To exclude uncertainty of a match threshold, let us compare the results in the point with equal probability of errors  $P_1 = P_2 = P_{EE}$ .

Figure 2 shows that the distribution of data received by the Cramer-von Mises test gives the value  $P_1 = P_2 = P_{EE} = 0.306$ . Under the same conditions the chi-square test gives the value of equally probable errors  $P_1 = P_2 = P_{EE} = 0.327$ . The results are approximately 9% worse. It means that the chi-square test requires the sampling of 10 examples, whereas for the Cramer-von Mises test only 9 examples are required. The relief in the requirements to the dimensions of a test sampling is explained by the fact that the quantization error of the probability function  $P(x)$  turns out to be smaller than the quantization error of the distribution density  $p(x)$  (see Figure 1).

Calculation procedure of the Cramer-von Mises test is approximately  $\sqrt{n}$  times more effective for the suppression

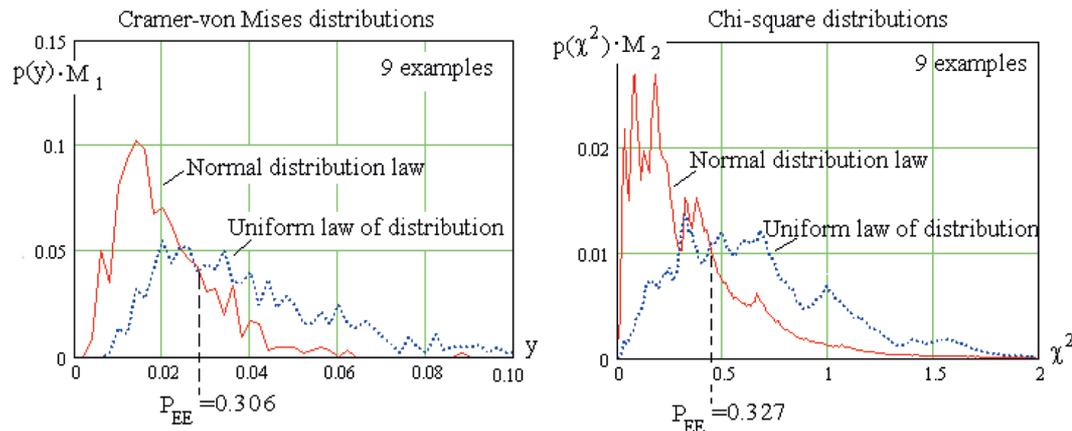


Fig. 2. Distributions of the values of the Cramer-von Mises test and chi-square test for the normal distribution law, and for its alternative in form of the uniform law of distribution for the samplings of 9 examples

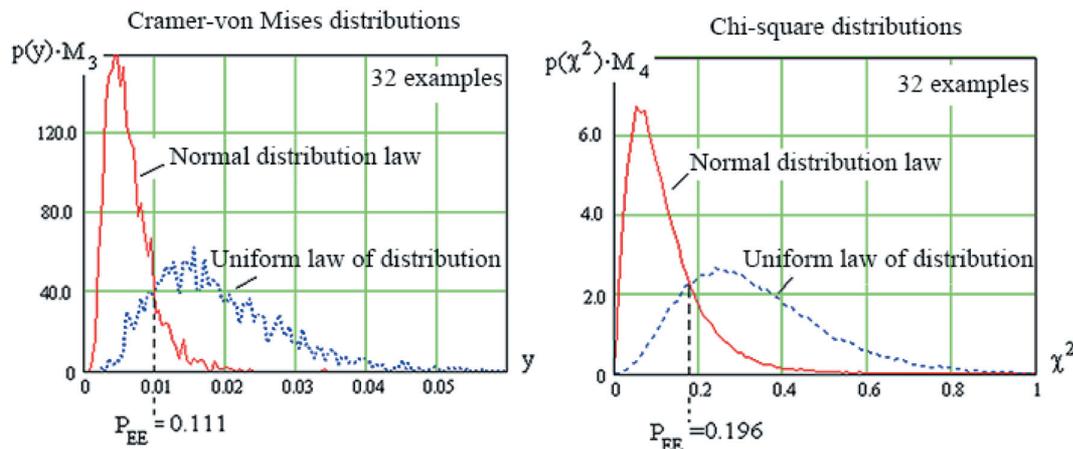


Fig. 3. Distributions of the values of the Cramer-von Mises test and chi-square test for the normal distribution law, and for its alternative in form of the uniform law of distribution for the samplings of 32 examples

of quantization noises in comparison to the data calculation by the chi-square test. The more a test sampling is, the stronger the effect of a more intense suppression of quantization noises is. Figure 3 shows the simulation data for the sampling of 32 examples.

Figure 3 shows that for the sampling of 32 examples, the Cramer-von Mises test gives  $P_{EE} = 0.111$ , which is 43% less than the chi-square test gives:  $P_{EE} = 0.196$ . In the first approximation we may expect about 40% decline of the scope of the test sampling if to pass form the chi-square test to the Cramer-von Mises test.

### One more variant of the chi-square test making the best use of the limited scope of the test sampling

Basically, both the Cramer-von Mises test and the Pearson chi-square test are the schemes of the suppression of quantization noises. I.e. we can try to amplify the property of these statistical functionalities to suppress the quantization noises. For example, we can use a supplementary digital-data filter configured to smooth the rises of piecewise constant approximation of the function of value distribution density [9, 10].

One more way is to check other possible variants of the calculation of the chi-square test. In particular, biometrics has been using the so called Pearson functionalities (networks of Pearson functionalities [11]) for the data preliminary normalized by a standard deviation:

$$\chi_2^2 = \frac{1}{n} \sum_{i=1}^n \left\{ \frac{(E(x) - x_i)^2}{\sigma(x)} \right\}, \quad (12)$$

where  $E(x)$  is a statistical expectation of data of the test sampling,  $\sigma(x) \approx 1$  is a standard deviation of the preliminary normalized data of the test sampling.

It should be noted that at the processing of biometric data, a preliminary normalizing of data is usually made by its standard deviation, in attempt to fulfill the condition

$\sigma(x) = 1$ . However, on small samplings, this condition cannot be fulfilled. For instance, a relative error of the calculation of standard deviation on small samplings of 20 examples is random and can comprise up to  $\pm 30\%$ , at the smaller samplings an error may be even more. To compensate normalizing errors in formula (12) there occurs the term close to, but always different from entity.

Let us note that the equation (12) makes the summing up of squared deviations by all calculations of the test sampling, whereas the classic chi-square test (6) sums up the squared deviations only by the number of histogram columns. As  $n > k$ , then we can expect a higher power of the chi-square test (12) in comparison to the similar classic chi-square test (6).

### Comparison by power of two variants of the chi-square test

The Cramer-von Mises test turns out to be more powerful than the classic Pearson chi-square test due to the fact that it presses upon the quantization noises of a smaller amplitude (let us compare the left and the right parts of Figure 1). However, for the Cramer-von Mises test there is no analytical description, and that is its huge disadvantage. That is why we shall further compare only the powers of two modifications of the chi-square tests (6) and (12).

The results of simulation modeling with 9 and 32 examples in the test sampling for the test (12) are provided in Figure 4.

If to compare the crossing of the distributions on the left graph of Figure 4 that gives  $P_{EE} = 0.106$ , and the analogous crossing on the right graph of Figure 2 that gives  $P_{EE} = 0.327$ , then we will get approximately a three-time profit in power between two variants of tests (confidence in the solutions based on them). With a growth of the scope of the test sampling, the profit of the second form of the chi-square test increases. If to compare the data of crossing of the distributions on the right graph of Figure 4, providing  $P_{EE} = 0.007$ , with the data on the

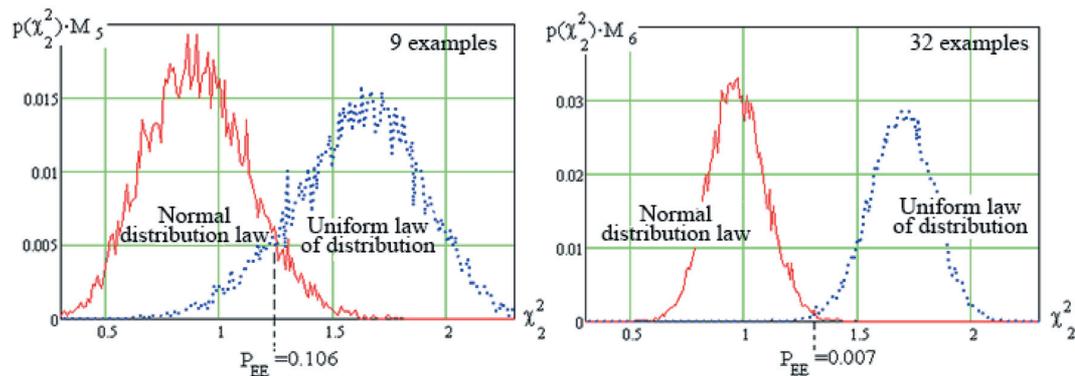


Fig. 4. Histograms of the values distribution of the second, more effective form of the chi-square test

right graph of Figure 3, providing  $P_{EE} = 0.196$ , then we will get a 28 times profit.

It turns out that both, the Cramer-von Mises test and the second form of the chi-square test are more powerful than the classic chi-square test due to the calculation of these two tests by the whole sampling. The classic chi-square test loses out to these two tests because it sums up a squared error by number of columns of the empirical histogram. It is quite easy to make sure that the Cramer-von Mises test is in its power within the interval between two forms of the chi-square tests.

### Analytical description of the second form of the chi-square test

The essential property of the second form of the chi-square test is that for independent data its statistical properties are very well described by normal distribution laws. And the statistical expectation for the distribution of formula (12) with an absence of a normalizing factor  $1/n$  is close to the scope of the test sampling:

$$E(\chi_2^2) \approx n - 0,778. \quad (13)$$

This statement is illustrated by the positions of maximums of unbroken curved lines of Figure 5. I.e. the value of statisti-

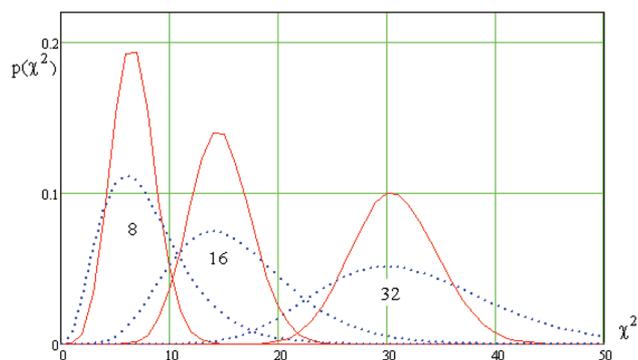


Fig. 5. Distributions of values of the second form of the chi-square test (unbroken curved lines) for  $n = 8, 16, 32$  and classic chi-square distributions, when the number of degrees of freedom coincides with the scope of a sampling (dotted curves)

cal expectation of the second form of the chi-square test is almost always taken (with accuracy to the correction 0.778) from the classic chi-square distribution with the number of degrees of freedom  $m = n$  (dotted curves in Figure 5).

Figure 5 shows that standard deviations of normal distribution laws are always smaller than standard deviations of classic Pearson chi-square tests. At statistical calculations with an engineering accuracy standard deviations are described by the following equation:

$$\sigma(n, \chi_2^2) \approx \sqrt{\frac{E(\chi_2^2)}{2}} \approx \sqrt{\frac{n - 0,778}{2}} \quad (14).$$

It should be underlined that the accuracy of approximation (14) increases with a normalizing of values distribution of the second form of the chi-square distributions. So for  $n = 8, 16, 32$  a relative error of approximation of standard deviation shall be  $\Delta\sigma = 3.10\%, 1.70\%, 0.75\%$ , which is quite acceptable for an engineering practice of statistical processing of biometric data.

### Conclusion

We are used to the fact that for reliable statistical estimations the samplings with hundreds of examples are required. Only in the case, when we have a large sampling and rely on standardized recommendations [8], there is a confidence in the quality of the performed statistical analysis. This is the current technical practice.

This article showed that the power of the chi-square test can be essentially increased, i.e. reliable estimations can be obtained on much smaller data samplings. It is very important for practice, especially if a destructing testing of costly products is performed. Already existing practice of statistical processing of biometric data proves true of this important stipulation.

Essential estimation resources are not a problem nowadays. We can make a statistical data processing more complicated, for instance, making it multivariate [11]. Today we have a technical capability of multiply complicating the applied methods of statistical analysis. In the last century we used to take one test and had to be satisfied with its results, but today we can use dozens of well-known statistical tests and, if necessary, create new tests especially for a certain practical task.

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## About the authors

**Berik B. Akhmetov** – PhD., Member of the International Informatization Academy (IIA), Vice-president of the International Hoca Ahmet Yesevi Turkish-Kazakh University.

29 B.Sattarkhanov Avenue, Bld. of Rectorate, Turkestan, 161200, Kazakhstan, tel.: +7 (72533) 3-35-77, e-mail: berik.akhmetov@ayu.edu.kz

**Alexander I. Ivanov** – Dr. Sci., Associate Professor – Head of Laboratory of biometric and neural network technologies, JSC Penza Research Electric and Technical Institute.

9 Sovetskaya Str., Penza, 440000, Russia, tel.: +7 (841-2) 59-33-10, e-mail: ivan@pniei.penza.ru

## Risk assessment of a system with diverse elements

**Valentin A. Gapanovich**, JSC Russian Railways, Moscow, Russia

**Igor B. Shubinsky**, CJSC IBTrans, Moscow, Russia, igor-shubinsky@yandex.ru

**Alexey M. Zamyshlyayev**, JSC NIIAS, Moscow, Russia, A.Zamyshlaev@vniias.ru



Валентин А.  
Гапанович



Игорь Б.  
Шубинский



Алексей М.  
Замышляев

**Abstract.** A measure of the safety of a system's object can be the value of an associated risk which is based on the risks of its constituent factors (elements). The main task of the paper is the definition of the integral risk of an object and a system as a whole. This is as follows. Summing up of risks of all elements is not acceptable, since they may have, for example, different measures (the number of fatalities during a certain period of time is a social risk, and the cost of losses is an economic one). We need some other methodological tool that can transform different measures of safety of objects (elements) into a certain single integral measure of a system's risk. Such tasks occur in medicine, food industry, in transport sector, etc. The paper offers a method to define the integral risk of a system based on the processing of a common field of the results of decisions taken on the level of risks of a system's elements. The results further probable decisions, for example, one of four decisions: intolerable risk level, undesirable level, tolerable and negligible risk level. Digitalization of these decisions of constituent elements with consideration of nonlinear growth of danger of the risk approaching to the intolerable level is made using a power function. It helps to define a numerical value equivalent to a component risk level, and then to find a weighted mean resulting numerical value equivalent to a risk level for all system components and solve an inverse task of definition of the integral risk of a system. This article describes an example of how this method could be used to solve the task of the investment priority for the works on technical maintenance of railway track. This task is limited to the ranking of track sections by priority of overhaul performance depending on the level of risks of the following factors: number of defective and flawed rails per 1 track km.; number of defective clamps per 1 track km.; number of pumping sleepers per 1 track km.; number of faulty wooden sleepers per 1 track km.; number of places of temporary repair; defects of roadbed; failure rate. Based on the risk matrices constructed by the method described above in relation to each of the listed factors, an integral risk matrix is formed for the list of sections, and based on the integral estimation each section gets a priority of an overhaul performance. The given example is indicative of the efficiency and practicability of the method offered.

**Keywords:** safety, risk, risk assessment, risk matrix, risk color, digitalization of risk color, color weight, element, system, integral estimation of a system's risk.

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### Introduction

In different sectors of industry and transport in asset management one seeks to establish balance between expenses, possibilities, risks and a required assets productivity, in accordance with ISO 55000 [1]. For instance, on railway transport the implementation of URRAN project [2, 3] provides for consistent application of criteria of safety, technical and economic rationale ensuring when taking decisions on replacement (repair) or extension of a system's service life. Such algorithm of management of railway transport maintenance is realized in two stages. The first stage is to analyze capabilities, safety, reliability and productivity of a system based on processing of current data, and to make a decision on whether it is reasonable (or not) to invest in technical maintenance. If it is decided to be reasonable then the second stage of management takes place when one identifies a system's objects that are of greatest concern from the point of view of safety and require investment in the first place.

A measure of the safety of a system's object can be the value of an associated risk which is based on the risks of its constituent factors (elements). The main task of the paper is the definition of the integral risk of an object and a system. This is as follows. Summing up of risks of all elements is not acceptable, since they may have, for example, different measures (the number of fatalities during a certain period of time is a social risk, and the cost of losses is an economic one). We need some other methodological tool that can transform different measures of safety of objects (elements) into a certain single integral measure of a system's risk. Such tasks occur in medicine, food industry, in transport sector, etc.

### Problem statement

Let system  $A$  generally consist of a finite set of diverse elements  $A = \{a_1, a_2, \dots, a_i, \dots, a_j, \dots, a_k\}$ . And there may be a possibility of equivalence of separate constituent elements  $a_i \leftrightarrow a_j$ . Safe operation of each system element is estimated

by a certain risk value  $a_i \rightarrow R_i$ . Risk is understood as the combination of frequency (probability) of a hazard and its consequences  $R=F \wedge C$  [4]. For the illustrative purpose, the explanation will be restricted to a special case of risk determination in form of  $R=F \cdot C$ . Risks are formalized with the use of a risk matrix tool. The mathematical basis of construction of a risk matrix is described in works [5, 6]. Generally, a risk matrix contains  $m$  lines and  $n$  columns. Each line corresponds to a certain frequency of a hazard  $f_1, f_2, \dots, f_m$ . Columns correspond to possible consequences (damage)  $c_1, c_2, \dots, c_n$ . A measure of consequences depends on the object of analysis. It could be a price (in relation to economic, technical or anthropogenic risks), fatality in relation to social risks, number of negative consequences or negative occurrence of a hazardous event (in relation to moral risks) etc.

It is supposed that the frequencies of hazardous events and their consequences are estimated by a posteriori data. This let us define safety risks of all system elements as respective lines and columns cross. Risks for diverse elements are not equal among themselves, for example,  $R_1 \neq R_i$  (risks of equivalent elements are equal  $R_i = R_j$ ). Risk is assessed based on ALARP principle (Risk is as low as reasonably practicable) [4]. This principle includes four assessment levels (Figure 1): two unequivocal and two in-between levels.

The unequivocal levels are the levels of assessment of *intolerable risk* (above the red bar in Figure 1) and *negligible risk* (below the green bar in Figure 1). Areas of these risks are usually marked with red and green colors respectively. The in-between levels are the levels of ALARP area. Above

the broken line there is a level of *undesirable risk*. The area of this risk is usually marked with orange color. Below the broken line there is a level of *tolerable risk*. The area of this risk is usually marked with yellow color.

The task is to assess the level of risk of a system based on the results of assessment of risks of its constituent diverse elements. Risks of the elements are supposed to be mutually independent.

### Assessment of a system's risk

In many cases the system under study consists of diverse objects that differ in scales of consequences and types of risks (for example, technological or social ones). At present one can neither sum up the risks of constituent objects, nor form a common scale of consequences. To assess a system's risk by the risks of constituent diverse elements, it is necessary to have at least one common measure for all risks. If to consider risks in reference to the scales of measurement of  $f$  and  $c$ , such common measure is not available. A measure of consequences may be different. It also applies to the rates of hazardous events that can be different for elements  $a_i$  and  $a_j$ . However, under close examination of the constructed matrices of risks of a system's elements, we find a common measure of risk assessment that is contained in the levels of decision making.

According to ALARP principle, there are four levels of risk severities. A common field for combining the results is the colors of decisions (risk levels) for each of the objects. These levels are marked with green, yellow, orange, and finally with red color as their importance grows. The green

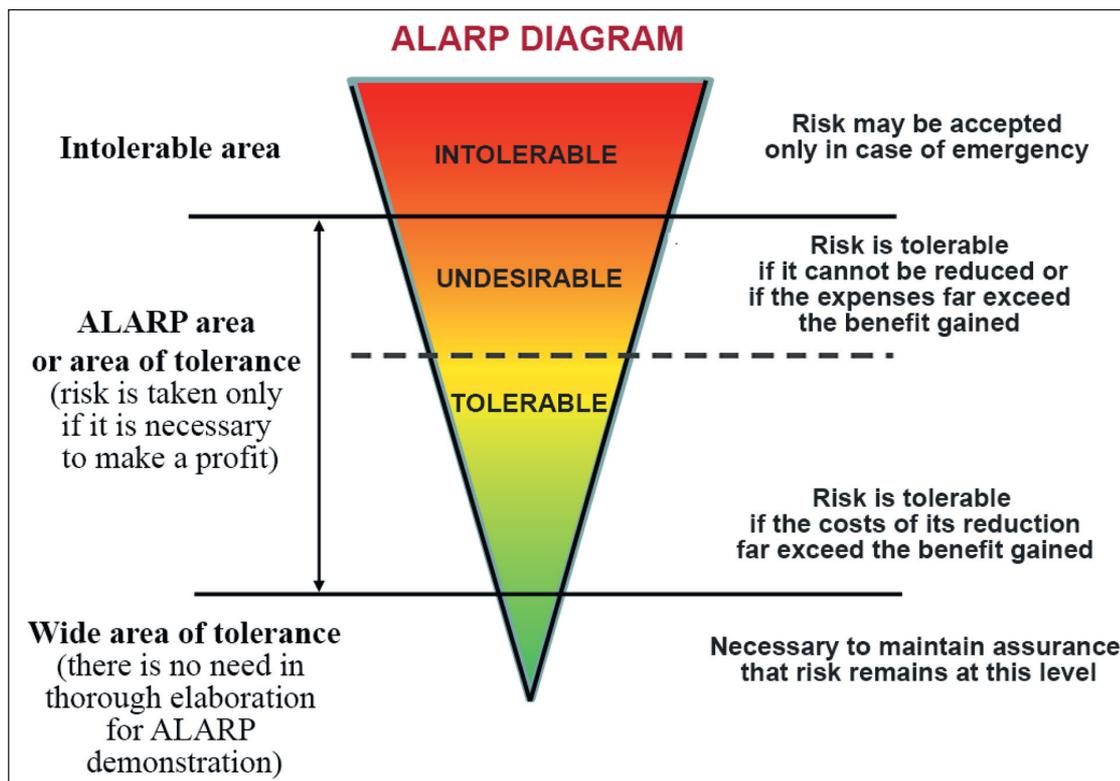


Fig.1. Risk assessment based on ALARP principle

color of a decision means that risk is so negligible that it can be discounted. The function of importance in green cells of a matrix should have low values (from zero up to a certain insignificant value). In addition to that, the orange color and especially the red color mean the highest degree of severity, and the function of importance in these matrix cells should have the maximum high values. There is a possibility of three strategies to construct the functions of the importance of decisions on a risk level in accordance with the accepted colors: 1 – linear; 2 – power; 3 – logarithmic. Figure 2 provides a conceptual representation of the functions of importance.

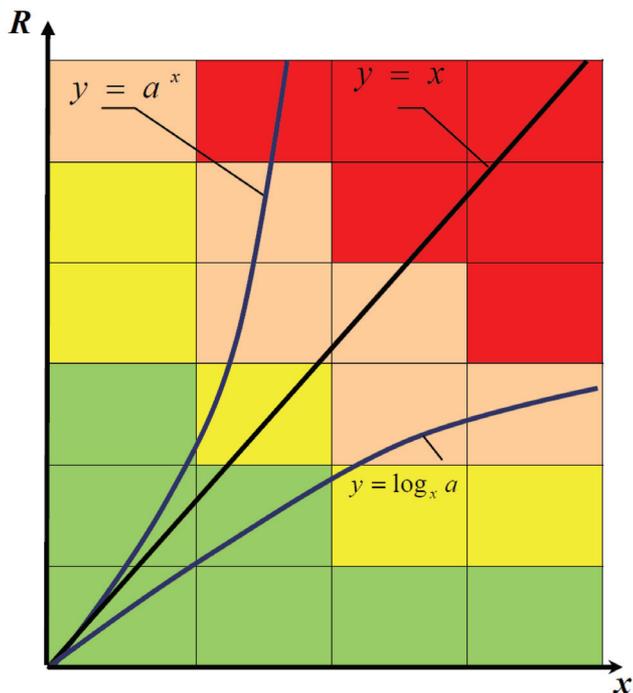


Fig. 2. Functions of importance of decisions on a risk level (concept)

Strategy 1 specifies an indifferent attitude to a change of importance of a decision color.

Strategy 2 specifies a responsible attitude to a change of importance of a decision color.

Strategy 3 should be considered as an irresponsible attitude to a decision taken on an object's risk level, as in this case the function of importance mitigates a severity of red color that reflects an intolerable risk level.

Therefore, to digitize the results of assessment of objects' risks expressed by one of the indicated colors, it is reasonable to use a power function (Figure 2). However, the degree of an importance function corresponding to one of four colors takes only integer values. That is why a power function itself should have a stepwise character based on  $a > 1$  (for instance,  $a = 1.1; 1.5; 2; 3; \dots$ ).

Figure 3 shows step functions of importance with the above indicated bases with four integer values of the degree of a function ( $n = 0, 1, 2, 3$ ).

Step functions with base  $1 \leq a < 2$  do not provide a quick response to a change of the importance of a decision color

(Figure 4), especially in the field of high risk levels. However, with base  $a > 2$  there is an unreasonably quick response to an undesirable level and especially to an intolerable risk level and almost a neglect of the importance of a tolerable risk level (Figure 3). A compromise solution is to choose base 2 of the step function of importance of colors of decisions taken on a risk level.

A color weight is generally defined by formula (1)

$$w_n = \frac{m_n}{\sum_{n=0}^3 m_n} \quad (1)$$

where a digitized value of a risk level color, for instance *negligible* (marked with green color in Figure 3), with importance function  $m_n = 2^n$  is equal:  $m_0 = 2^0 = 1$ .

Integral assessment of system risk can be calculated by formula (2)

$$R = \frac{\sum_{n=0}^3 k_n m_n w_n}{\sum_{n=0}^3 k_n m_n} \quad (2)$$

where  $k_n$  is a number of a system's elements with a risk level of the  $n$ -th coloration;

$m_n$  is a function of the importance of risk coloration;

$w_n$  is a color weight.

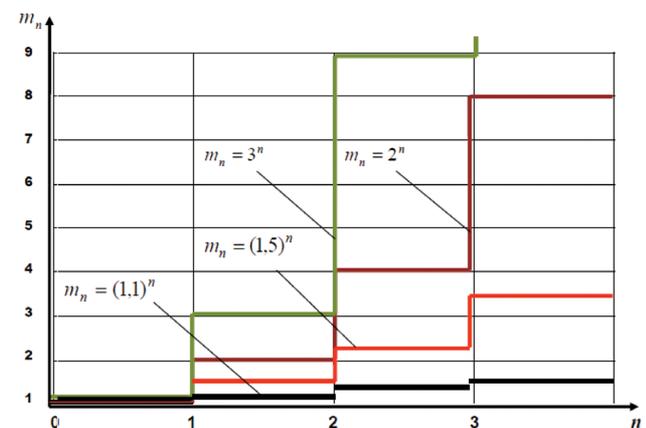


Fig.3. Step functions of the importance of risk level decision colors

Table 1 summarizes weights of colors of typical risk matrix cells as well as values of assessment of decisions taken on integral risk.

Within the limits of each of the second and the third risk levels the ranking of integral risks is made by a linear law in order of increase.

## Example

Let us consider the assignment of works on technical maintenance of railway track. An assignment algorithm can be divided into two blocks. Within the first block using logical inequations, we compare real and control values of

Table 1

	Index, $n$	Digitized value, $m_n$	Color weight, $w_n$	Values of estimation of a decision taken on integral risk
	0	1	0.067	$w_0=R$
	1	2	0.133	$w_1 \leq R < w_2$
	2	4	0.266	$w_2 \leq R < w_3$
	3	8	0.533	$W_3=R$

the following indices: speed of passenger trains; speed of freight trains; direct expenses on running maintenance of 1 km of track; handled tonnage, mil. gross. t.; residual life of railway track.

Thus, within the first block we analyze such parameters as track group, track category, track class, handled tonnage after overhaul, estimation of residual life of railway track. For different groups depending on the type of underrail base, real state of track bed structure and engineering structures, speeds for freight and passenger trains are established. Infrastructure restrictions are checked for. Real direct expenses on running maintenance of 1 km of track are analyzed with consideration of cost of failure elimination on 1 km of track, cost of train delay due to failure, data about wages of track servicemen, cost of materials and cost of machine operation. Other expenses have been considered as conditional-constant or insignificant and are not taken into account.

The analysis results in identifying the reasons for speed restrictions for freight and passenger trains caused by poor status of engineering structures, roadbed and other track elements that require an overhaul. When a decision is made on impossibility of review of design speeds towards reduction, an investment request is formed and proceeding to the second block is made.

If all logical inequations are positively fulfilled, the algorithm is completed, running maintenance works are assigned and there is no need to proceed to the second block.

The second block of the algorithm is a family of risk matrices for the ranking of track sections by priority of overhaul performance depending on the level of risks of the following factors:

- 1 – number of defective and flawed rails per 1 track km.;
- 2 – number of defective clamps per 1 track km.;
- 3 – number of pumping sleepers per 1 track km.;
- 4 – number of faulty wooden sleepers per 1 track km.;
- 5 – number of places of temporary repair;
- 6 – failure rate;
- 7 – defects of roadbed.

Based on the constructed risk matrices by the method described above, an integral risk matrix is formed (Fig-

ure 4) for the list of sections, and based on the integral estimation each section gets a priority of an overhaul performance.

Reference number of a factor	Section 1	Section 2	Section 3	Section 4
1	0.13	0.53	0.13	0.07
2	0.53	0.13	0.13	0.07
3	0.07	0.27	0.07	0.13
4	0.13	0.27	0.27	0.07
5	0.27	0.53	0.13	0.07
6	0.27	0.13	0.07	0.07
7	0.07	0.53	0.13	0.07
Mean value for a section	0.32	0.43	0.14	0.09
Priority	2	1	3	4

Fig.4. Integral risk matrix

In this example the assessment of integral risks is carried out for four track sections. Despite a rather low risk of traffic disruption caused by track failures (factor 6), there are still active risks for section 2 that are related to defective and flawed rails (factor 1), defects of roadbed (factor 7) and number of places of temporary repair (factor 5), and for section 1 factor 1 is a problem (risk related to defective clamps). Assessment of integral risks has shown that for the first three sections they have an undesirable level, and an integral risk of the fourth section is close to the level that may be neglected. Ranking of risks of the first three sections shows that the priority of investment should be passed to the second section, then – to the first one and afterwards it could probably be passed to the third section if there is such possibility, since an integral risk of the third section is close to a tolerable level.

### Conclusion

It is reasonable and convenient to estimate the safety of technical systems using risks. In practice the ALARP principle is widely applied. It helps to assess the rationality and sufficiency of economic expenses spent to reduce risks of violation of system safety. A convenient tool used to realize this principle and to support decision making is a risk matrix. A risk matrix is constructed for each element of a system. A system normally has diverse elements – frequency of negative consequences may have different

scales, and the consequences may be defined by different physical values. It does not allow combining the risks of elements by addition to determine the integral risk of a system. The paper offers a method to define the integral risk of a system based on the processing of a common field of risks of a system's elements, i.e. a field of the results of decisions taken on risks that are expressed by colors. Digitalization of the colors of risks of constituent elements with consideration of nonlinear growth of danger of the risk approaching to the intolerable level helps to define its digitalized value, and then the integral risk level. The given example is indicative of the efficiency and simplicity of application of the method offered.

This method could be applied in other sectors, for example, in medicine, in assessment of integral risks in food industry, in assessment of efficiency of complex technical systems, etc.

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## About the authors

**Valentin A. Gapanovich** – PhD, chief engineer, senior vice president JSC RZD, Moscow, Russia, tel. +7 (495) 262-16-43

**Igor B. Shubinsky** – Dr.Sci., professor, director of CJSC IBTrans, Moscow, Russia, tel.: +7 (495) 786-68-57, e-mail: igor-shubinsky@yandex.ru

**Alexey M. Zamyshlyayev** – Dr.Sci., Deputy director general JSC NIIAS, Moscow, Russia, tel.: +7 (495) 967-77-02, e-mail: A.Zamyshlaev@gismprs.ru



**Tarasov Aleksander Alekseevich  
(1958 – 2016)**

Overnight into May 7 Tarasov Aleksander Alekseevich, a member of editorial board of the journal “Dependability” died suddenly in the pride of his years. He was an excellent scientist, a science facilitator, teacher, friend, a wonderful person and a family man. Aleksander Alekseevich got excellent education. In 1976 he entered Kiev Higher Engineering Radio-Technical School PVO, where leading scientists of the time taught: academicians B.V.Gnedenko and I.N.Kovalenko, an associate member B.N.Malinovsky and others. In 1978 he passed to a famous F.E. Dzerzhinsky Military Academy (today Peter the Great Military Academy) and graduated with distinction with a specialization in Electronic computing techniques. This knowledge served as the basis for successful research career where Aleksander Alekseevich made a great progress – he defended PhD and doctoral theses, he became one of the leading Russian scientists in the field of functional reliability of information systems. Professor Tarasov was characterized in the desire to open up something new in science, research the most

crucial processes. He was among the first scientists who studied the problem of information security of mission-critical objects. His papers on information security made him very popular in this country.

Aleksander Alekseevich was engaged in a considerable pedagogical work. He had held classes in Research Nuclear University MEPhI for many years, he was a head of Institute of information sciences and safety technologies of the Russian state university for the humanities. Professor Tarasov was also a member of several dissertation councils, he was actively involved in the training of educational research specialists. His talent of a teacher was combined with high requirements to a research work of candidates. His commitment to principles contributed to the increase of a scientific level of dissertation councils.

A wonderful person, friend and a family man passed away. We shall keep a loving memory of him in our hearts forever.

*Editorial board  
of the journal “Dependability”*



### **Cherkesov Gennady Nikolaevich (1937 – 2016)**

Gennady Nikolaevich Cherkesov, a leading Russian theorist of reliability of technical systems, passed away.

Gennady Nikolaevich was born on August 9, 1937 in the city of Bezhitsa in Bryansk Region. In 1954 he finished Kromskaya secondary school and entered Leningrad Polytechnic Institute (LPI). In 1962 he graduated from LPI radio engineering faculty with a degree in “Mathematical and computing devices and equipment”, and was hired as an engineer of Experimental design bureau under LPI that was engaged in the development of the systems of control of space vehicles and manned spacecrafts.

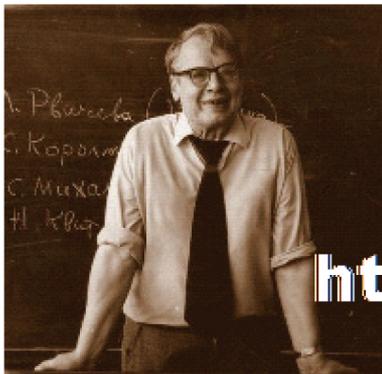
In 1964 he entered an LPI PhD program in the department “Information and control systems”. After a successful defense of a thesis he stayed at the Institute as an assistant (1967-1970), and as associate professor (1970-1982). In 1982 he defended a doctoral thesis and ever since he worked as the professor of LPI, and then as the professor of St. Petersburg State Polytechnic University (SPbSPU), holding a professor academic title.

In reliability theory he discovered a new research area – theory of time redundancy. His monograph published in 1974 with the description of the theory foundations, became one of

the first books on time redundancy in the world scientific and research literature, stimulating an intensive development of this area in reliability theory. Applied researches related to the use of theoretical results belong to different spheres of technology (energy systems, heat supply systems, technological systems, nuclear power plants, underwater sound systems, etc.).

Gennady Nikolaevich is an author of more than 200 scientific works, including 162 articles, 17 monographs, 7 study guides, 20 methodological works, 15 algorithms and programs, 2 inventions. In 2005 he published the book “Reliability of hardware and software systems”, which was recommended by the Ministry of education and science as a study guide for the students of “Informatics and computer engineering”. Over the last years Gennady Nikolaevich obtained very important results of reliability analysis of technical systems with spare parts, he started active work on the creation of the theory of system survivability. He also was a head of technical committee TC 119 “Reliability of technical systems”. To the very last moment professor G.N. Cherkesov was committed to the main devotion of his life – reliability.

*Editorial board  
of the journal “Dependability”*



<http://Gnedenko-Forum.org/>

**Dear colleagues!**

In 2005 the informal Association of Experts in Reliability, Applied Probability and Statistics (I.G.O.R.) was established with its own Internet website GNEDENKO FORUM. The site has been named after the outstanding mathematician Boris Vladimirovich Gnedenko (1912-1995). The Forum's purpose is an improvement of personal and professional contacts between experts in the mathematical statistics, probability theory and their important branches, such as reliability theory and quality control, the theory of mass service, storekeeping theory, etc.

Since January 2006, the Forum has published a quarterly international electronic magazine

***“Reliability: Theory and Applications”.***

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