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SOLVING THE TASK OF RESOURCES ALLOCATION FOR CRITICAL INFRASTRUCTURE PROTECTION AGAINST TERRORIST ATTACKS BASED ON SUBJECTIVE EXPERT ESTIMATES

The paper studies the stability of the algorithm for solving the problem of optimal resources allocation for critical infrastructure protection against possible terrorist attacks based on subjective expert estimates. The results for different types of estimates (optimistic, moderate, pessimistic ones) are compared. Such analysis makes it possible to define what measures should be taken for each level of protection (or accepted level of vulnerability) and generally limited resources allocated for these measures.

Keywords: *critical infrastructure, protection, optimization, resources allocation, expert estimates.*

1. Introduction

The solution of the problem of optimal resources allocation for critical infrastructure prevention against possible terrorist attacks is naturally based on subjective estimates made by experts in the field. Relying on expert estimates is inevitable in this case: there is no other possibility to get input data for the system resilience analysis. There is no such phenomenon like “collecting real data,” as well as there are no “homogenous samples” for consistent statistical analysis of observations, since any case is unique and non-reproducible. Nevertheless, a quantitative analysis of necessary level of protection has to be performed.

What are the bullets of such expertise? In our opinion they should include the following:

- possibility of terrorist attacks on some object or group of objects;
- possible time of such attack;
- expected consequences of an attack and possible losses;
- possible measure of protection and related expenses.

Since expert estimates of such complex things are generally fuzzy due to the lack of common understanding of the same actions and counter-actions within a group of experts, the question arises: is it possible at all to make any reasonable prediction and, moreover, speak about “optimal allocation of protection resources”?

First of all, we should underline that the concept of “optimal solution” relates only to mathematical models. In practice, unreliable (and even inconsistent) data and inevitable inaccuracy of the model (i.e. the difference between a model and reality) allow us to speak only about “rational solutions”.

However, since the problem exists, in any particular case it has to be solved with or without using mathematical models. Our objective is to analyze the stability of solutions of optimal resources allocation under fuzziness of expert estimates.

2. Analysis of solution stability

First, let us analyze how the variation of expenses estimating influences the solution of the problem on the level of a single object, which has to be protected against a possible terrorist attack. For the transparency of explanation, we avoid to consider the protection influence on the Federal and State levels.

Let us consider some nominal object (Object 1) which can be subject to a terrorist attack. It is assumed that there may be three different types of enemy's actions (Act 1, Act 2 and Act 3). The defending side can choose several specific protection measures against each type of action $\{M(i, j)\}$, where i corresponds to the type of action, and j corresponds to the type of an undertaken protective measure respectively.

Let us assume that we have three variants of estimates of protection costs: the lower, the middle and the upper one as presented in Table 1-3. The lower estimates are about 20% lower than the corresponding middle estimates, and the upper ones are respectively about 20% higher.

There are three groups of expert estimates: "optimistic", "moderate" and "pessimistic." The first one assumes that success in each situation can be reached by lower expenses for protective measures; the last group requires larger expenses for protection in the same situation; and the middle group shows expenses in between.

How should the data in Tables 1-3 be interpreted?

Let us consider the possibility of Act 1 against the object. With no protection at all, the object's vulnerability equals to 1 (or 100%). If we spend ΔE = 0.8 of conditional units of protection cost (c.u.p.c.) and undertake measure $M(1, 1)$, the object's vulnerability decreases to 0.25. If we are not satisfied with such level of protection, we apply next protective measure $M(1, 2)$, which leads to decrease of the object's vulnerability from 0.25 to 0.3 and, respectively, costs 2 c.u.p.c.

Now let us consider the actions of all the three possible terrorist attacks. In advance no one knows what kind of action will be undertaken against a particular object. In this

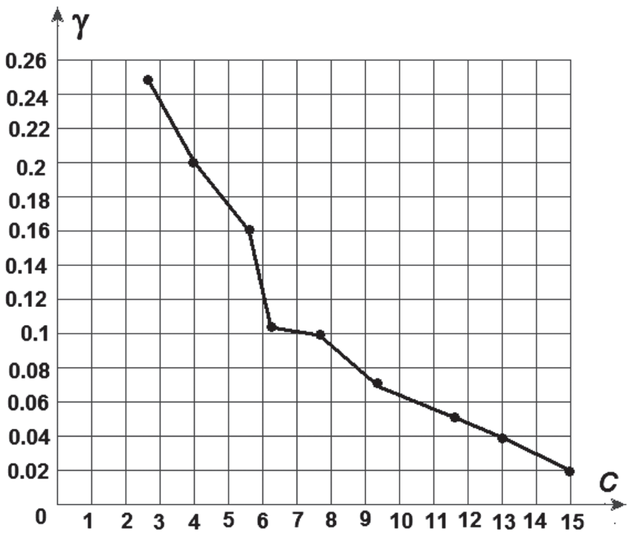

Fig. 1. Dependence of object vulnerability on cost of protection measures (for "optimistic" estimates)

Fig. 2. Dependence of object vulnerability on cost of protection measures (for "moderate" estimates)

Table 1.
Case of optimistic estimates

OBJECT-1		γ_1	C
Act 1	M(1, 1)	0.25	0.8
	M(1, 2)	0.2	2
	M(1, 3)	0.1	2.5
	M(1, 4)	0.01	3.8
Act 2	M(2, 1)	0.2	1.6
	M(2, 2)	0.16	2.8
	M(2, 3)	0.07	3.2
	M(2, 4)	0.02	5.6
Act 3	M(3, 1)	0.11	0.4
	M(3, 2)	0.1	2
	M(3, 3)	0.05	2.4
	M(3, 4)	0.04	3.6
	M(3, 5)	0.01	5.6

Table 2.
Case of moderate estimates

OBJECT-1		γ_2	C
Act 1	M(1, 1)	0.25	1
	M(1, 2)	0.2	2.5
	M(1, 3)	0.1	3
	M(1, 4)	0.01	4
Act 2	M(2, 1)	0.2	2
	M(2, 2)	0.16	3
	M(2, 3)	0.07	4
	M(2, 4)	0.02	7
Act 3	M(3, 1)	0.11	0.5
	M(3, 2)	0.1	2.5
	M(3, 3)	0.05	3
	M(3, 4)	0.04	5
	M(3, 5)	0.01	7

Table 3.
Case of pessimistic estimates

OBJECT-1		γ_3	C
Act 1	M(1, 1)	0.25	1.2
	M(1, 2)	0.2	3
	M(1, 3)	0.1	6
	M(1, 4)	0.01	7.8
Act 2	M(2, 1)	0.2	2.4
	M(2, 2)	0.16	3.2
	M(2, 3)	0.07	4.8
	M(2, 4)	0.02	8.4
Act 3	M(3, 1)	0.11	2
	M(3, 2)	0.1	3
	M(3, 3)	0.05	3.6
	M(3, 4)	0.04	6
	M(3, 5)	0.01	8.4

Table 4. Case of optimistic estimates

Object 1			
Item No.	**Undertaken measures**	**Resulting γ_{object}**	**Total costs, C_{object}**
1	M(1, 1), M(2, 1), M(3, 1)	max {0.25, 0.2, 0.11}=0.25	0.8+1.6+0.4=2.8
2	M(1, 2), M(2, 1), M(3, 1)	max {0.2, 0.2, 0.11}=0.2	2+1.6+0.4=4
3	M(1, 3), M(2, 2), M(3, 1)	max {0.1, 0.16, 0.11}=0.16	2.5+2.8+0.4=5.7
4	M(1, 3), M(2, 3), M(3, 1)	max {0.1, 0.07, 0.11}=0.11	2.5+3.2+0.4=6.1
5	M(1, 3), M(2, 3), M(3, 2)	max {0.1, 0.07, 0.1}=0.1	2.5+3.2+2=7.7
6	M(1, 4), M(2, 3), M(3, 3)	max {0.01, 0.07, 0.05}=0.07	3.8+3.2+2.4=9.4
7	M(1, 4), M(2, 4), M(3, 3)	max {0.01, 0.02, 0.05}=0.05	3.8+5.6+2.4=11.8
8	M(1, 4), M(2, 4), M(3, 4)	max {0.01, 0.02, 0.04}=0.04	3.8+5.6+3.6=13
9	M(1, 4), M(2, 4), M(3, 5)	max {0.01, 0.02, 0.01}=0.02	3.8+5.6+5.6=15

Table 5. Case of moderate estimates

Object 1			
Item No.	**Undertaken measures**	**Resulting γ_{object}**	**Total costs, C_{object}**
1	M(1, 1), M(2, 1), M(3, 1)	max {0.25, 0.2, 0.11}=0.25	1+2+0.5=3.5
2	M(1, 2), M(2, 1), M(3, 1)	max {0.2, 0.2, 0.11}=0.2	2.5+2+0.5=5
3	M(1, 3), M(2, 2), M(3, 1)	max {0.1, 0.16, 0.11}=0.16	3+3+0.5=6.5
4	M(1, 3), M(2, 3), M(3, 1)	max {0.1, 0.07, 0.11}=0.11	3+4+0.5=7.5
5	M(1, 3), M(2, 3), M(3, 2)	max {0.1, 0.07, 0.1}=0.1	3+4+2.5=9.5
6	M(1, 4), M(2, 3), M(3, 3)	max {0.01, 0.07, 0.05}=0.07	4+4+3=11
7	M(1, 4), M(2, 4), M(3, 3)	max {0.01, 0.02, 0.05}=0.05	4+7+3=14
8	M(1, 4), M(2, 4), M(3, 4)	max {0.01, 0.02, 0.04}=0.04	4+7+5=16
9	M(1, 4), M(2, 4), M(3, 5)	max {0.01, 0.02, 0.01}=0.02	4+7+7=18

Table 6. Case of pessimistic estimates

Object 1			
Item No.	**Undertaken measures**	**Resulting γ_{object}**	**Total costs, C_{object}**
1	M(1, 1), M(2, 1), M(3, 1)	max {0.25, 0.2, 0.11}=0.25	1.2+2.4+2=5.6
2	M(1, 2), M(2, 1), M(3, 1)	max {0.2, 0.2, 0.11}=0.2	3+2.4+2=7.4
3	M(1, 3), M(2, 2), M(3, 1)	max {0.1, 0.16, 0.11}=0.16	3+3.2+2=8.2
4	M(1, 3), M(2, 3), M(3, 1)	max {0.1, 0.07, 0.11}=0.11	3+4.8+2=9.8
5	M(1, 3), M(2, 3), M(3, 2)	max {0.1, 0.07, 0.1}=0.1	3+4.8+3=10.8
6	M(1, 4), M(2, 3), M(3, 3)	max {0.01, 0.07, 0.05}=0.07	4+4.8+3.6=12.4
7	M(1, 4), M(2, 4), M(3, 3)	max {0.01, 0.02, 0.05}=0.05	4+8.4+3.6=16
8	M(1, 4), M(2, 4), M(3, 4)	max {0.01, 0.02, 0.04}=0.04	4+8.4+6=18.4
9	M(1, 4), M(2, 4), M(3, 5)	max {0.01, 0.02, 0.01}=0.02	4+8.4+8.4=20.8

Fig. 3. Dependence of object vulnerability on cost of protection measures (for "pessimistic" estimates)

situation the most reasonable strategy provides equal protection levels against all considered types of terrorist attacks, for example as suggested in [1,3]. It means that if one needs to ensure a level of protection equal to γ, then one has to consider only such measures against each action that delivers vulnerability level not less than γ. For instance, as in the considered case, if the required level of vulnerability has to be not higher than 0.1, one has to use simultaneously the following measures of protection against possible terrorist attacks: $M(1, 3)$, $M(2, 3)$ and $M(3, 4)$.

The method of equal protection against various types of hostile attacks appears to be rather natural. While dealing with natural or other unintended impacts, one can speak about the subjective probabilities of impacts of some particular type. However, such approach is not appropriate in the case of an intentional attack from a well informed and prepared enemy. The fact is that as soon as the enemy learns about your assumptions about his possible actions, he takes advantage of this knowledge and chooses the action that you expect least of all.

In the example considered above, if one chooses measures $M(1, 2)$ with $\gamma_1 = 0.2$, $M(2, 3)$ with $\gamma_2 = 0.07$ and $M(3, 4)$ with $\gamma_3 = 0.04$, the guaranteed level of the object protection is equal to

$\gamma_{object} = \max(\gamma_1, \gamma_2, \gamma_3) = \max(0.2; 0.07; 0.04) = 0.2$.

For choosing a required (or needed) level of object protection, one can make up a function reflecting the dependence of vulnerability from protection cost.

This function graph is presented in fig. 1.

For information, we present calculation results without detailed explanations for the cases of "moderate" and "pessimistic" estimates.

Data of Table 5 are respectively presented in fig. 2.

Such analysis gives a possibility to find what measures should be undertaken for each required level of protection (or admissible level of vulnerability) and given, generally limited resources.

The final "curve" of "Expenses vs. Vulnerability" relation is presented below.

However, decision makers are interested mostly in the correctness of undertaken measures, rather than in the dif-

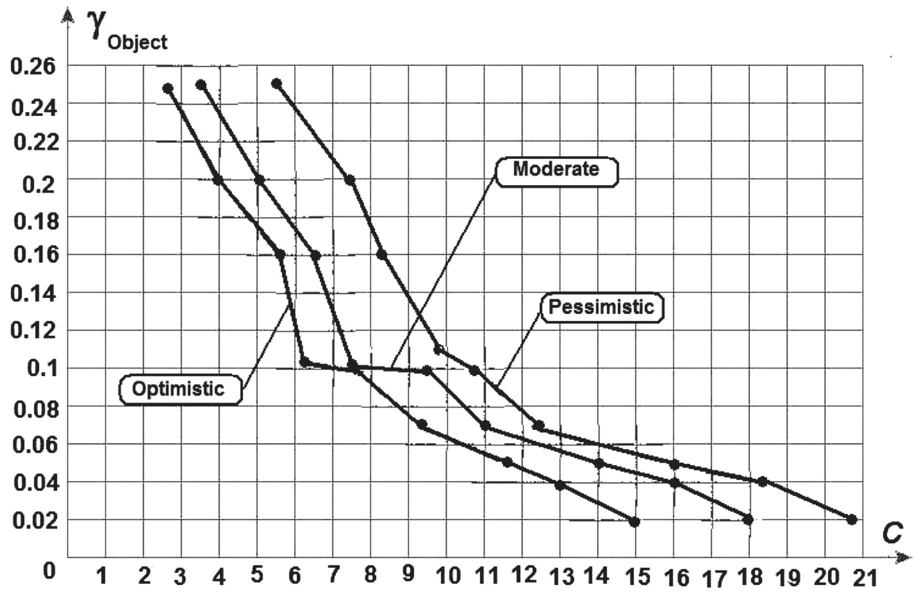

Fig. 4. Comparison of solutions for three types of estimates

Table 7. Comparison of solutions for three types of scenarios

Type of scenario	Undertaken protection measures	
	$\gamma_{Object} \leq 0.1$	$\gamma_{Object} \leq 0.02$
Optimistic	M(1, 3), M(2, 3), M(3, 2)	M(1, 4), M(2, 4), M(3, 5)
Moderate	M(1, 3), M(2, 3), M(3, 2)	M(1, 4), M(2, 4), M(3, 5)
Pessimistic	M(1, 3), M(2, 3), M(3, 2)	M(1, 4), M(2, 4), M(3, 5)

ference in absolute values of the estimated costs of object protection. In other words, they are concerned about how healthy, for example, the solution adopted for the optimistic scenario, will be in case if the situation is in fact better than it was described in the pessimistic scenario.

Using the tables above, consider two solutions of the direct problem with required levels of vulnerability as 0.1 and 0.02.

It is obvious that if the goal is to reach some given level of vulnerability, the vector of solution (i.e. a set of undertaken measures for protecting the object against terrorist attacks) in the frame of considered conditions will be the same, though it will lead to alternative expenses.

The comparison of solutions for an object vulnerability required level of not higher than 0.1 and not higher than 0.02 is presented in Table 7.

As one can see, the solutions for all of the three scenarios coincide for both levels of object protection! Of course, such situation occurs not always, however we should underline that the vectors of solution for minimax criterion $\gamma_{object} = \max(\gamma_1, \gamma_2, \gamma_3)$ is much more stable than the vector for probabilistic criterion $1-\gamma_{объект} = 1-\prod_{1 \leq k \leq n}(1-\gamma_k)$.

3. Conclusion

The presented analysis shows that the offered model of optimal allocation of resources for critical infrastructure protection against possible terrorist attacks is rather stable. The problem of research of solution stability is considered in more detail in [2].

Calculation experiments made with the help of an appropriate computer model will give a possibility to analyze more realistic situations, including random instability of input data. However, it seems that such “biased” expert estimates should lead to more serious errors than random variations of the parameters.

References

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