Original article
https://doi.org/10.21683/1729-2646-2019-19-4-8-11

Algorithm of prompt detection of dependability characteristics variation

Dmitry S. Repin[1*], **Gennady F. Filaretov**[2]

[1]Institute of Information Technology, Federal State Autonomous Educational Institution for Further Vocational Education Center for Implementation of the National Educational Policy and Information Technology, Moscow, Russian Federation;
[2]National Research University Moscow Power Engineering Institute (MPEI), Moscow, Russian Federation
[*]r_d_s@inbox.ru


Dmitry S. Repin


Gennady F. Filaretov

Abstract. *The Aim of the paper is to develop an algorithm of prompt detection of the moment of dependability characteristics variation in a system that consists of a set of homogeneous elements, assuming that failures of such elements occur at random moments in time, are a Poisson flow of events and, consequently, the time intervals between them are an exponential probability distribution. In order to solve the problem, it is suggested using one of the classical algorithms of detection of "imbalance" of a discrete random process, i.e. spontaneous change of one of its probabilistic characteristics. As such a characteristic, the exponential distribution parameter* θ *was chosen, that is uniquely associated with the mean time between failures* T_{mn}*:* $\theta = 1/T_{mn}$*. It is believed that the imbalance consists in the discontinuous variation of parameter* θ *from the initial steady state* $\theta = \theta_0$ *to the level of minimal (expected, maximum allowable, critical) imbalance, when* $\theta = \theta_1 > \theta_0$*. In this paper, the imbalance is detected using the cumulative sum algorithm (CUSUM) as it has certain optimal properties and is widely used in practice. For this algorithm, the required design ratios, descriptions of its properties and features are provided. The paper proposes a procedure for synthesizing the control algorithm with desired properties, in the course of which, based on the user-selected values of desired mean time between false alarms* $\overline{T}_{\mathrm{FA}}$*, initial basic level* θ_0 *and nominal imbalance* $\theta_1 > \theta_0$*, the value of decision boundary H is identified, the speed of algorithm action is estimated trough the calculation of the average lag in the detection of nominal imbalance* $\overline{T}_{lag}$*, along with its efficiency* $E_d = \overline{T}_{\mathrm{FA}} / \overline{T}_{\mathrm{lag}}$ *for various values of d, that quantitatively characterize the value of imbalance:* $d=\theta_1/\theta_0$*. For the purpose of practical implementation of the synthesis procedure, the paper cites reference data, that was obtained by means of simulation and that ensures the development of the control algorithm with required characteristics. It is noted that the presented synthesis procedure can, in principle, also be used for cases of gradual (continuous) change of parameter* θ*. However, the statistical properties of the control procedure will remain unclear as they require sufficiently intense additional research.*

Keywords: *system dependability; detection of dependability characteristics variation; detection of discrete random process imbalance; cumulative sum algorithm; control algorithm synthesis.*

For citation: *Repin DS, Filaretov GF. Algorithm of prompt detection of dependability characteristics variation. Dependability 2019; 4: 8-11 p. https://doi.org/10.21683/1729-2646-2019-19-4-8-11*

Received on 17.10.2019 / Revised on: 16.11.2019 / For printing 14.12.2019

The paper examines the problem of prompt detection of the moment of dependability characteristics variation in a system that consists of a set of homogeneous elements. It is assumed that failures of such elements occur at random moments in time t_1, t_2, …, t_i-1, t_i, t_{i+}1, … and are a Poisson event flow. As it is known [1], in this case time intervals $\tau_i = t_i - t_i$-1 follow the exponential distribution of the form

$$f(\tau) = \theta \cdot e^{-\theta\tau}; \theta > 0,\ \tau > 0, \qquad (1)$$

where $\theta = 1/T_{mn}$, T_{mn} is the mean time between failures.

Let us assume that in the initial steady state parameter $\theta = \theta_0$. As system elements age (wear out), this parameter will obviously change. The research of such non-steady situations is certainly of significant interest [2]. This paper deals with prompt (real-time) detection of variations of value $\theta > \theta_0$, when such variations become significant. Essentially, this is a well-known problem of detection of the so-called "imbalance" of random processes [3].

According to the classical imbalance problem definition, it is assumed that such imbalance is discontinuous in its nature. In the context of dependability, this definition is hardly realistic. However, it can be considered as a tentative application of this approach in the context of real-time supervision of dependability characteristics of complex systems.

There are quite many known algorithms of imbalance detection [4]. The quality of their operation can be described with a set of such *probabilistic* characteristics as the average value of the time between false alarms $\overline{T}_{FA}$, i.e. the mean time between warnings of imbalance in the absence of such, and mean lag $\overline{T}_{lag}$ in the detection of minimal (expected, maximum allowable, critical) imbalance, when $\theta = \theta_1 > \theta_0$.

As of late, the *cumulative sum algorithm* (CUSUM) proposed by Page back in 1954 [5] has been the most commonly applied. This algorithm, as it was later shown, has certain optimality properties in terms of maximization of the efficiency indicator of detection algorithm $E = \overline{T}_{FA} / \overline{T}_{lag}$. The popularity and considerable capabilities of the algorithm are demonstrated by the bibliometric analysis [6] that shows an exponential growth of the number of associated publications since 1964, as well as examples of various modifications of the original CUSUM algorithm [7-10].

CUSUM is based on a slightly modified sequential Wald analysis. In both cases the likelihood ratio statistic is used in the decision function. In the present case it will take the following form:

$$g_i = \max\{0; g_{i-1} + z_i\},\ i = 1, 2, ...;\ g = 0, \qquad (2)$$

where

$$z_i = \ln\left[f(\tau_i, \theta_1) / f(\tau_i, \theta_0)\right]. \qquad (3)$$

The zero value in formula (2) acts as a sort of an absorbing barrier by not allowing decision function to shift towards the area of negative values.

The decision functions are calculated each time a failure signal arrives. The control procedure lasts until, at a certain step *n,* the following inequality is fulfilled:

$$g_n \geq H, \qquad (4)$$

where H is the decision boundary. In this case an imbalance warning is issued. In reality though, there might be no imbalance, i.e. there is a situation of false alarm.

Subject to (1), formula (3) can be specified:

$$z_i = \ln(\theta_1/\theta_0) - (\theta_1 - \theta_0)\cdot\tau_i = \ln d - (d-1)\cdot(\tau_i\cdot\theta_0);$$
$$d = \theta_1 / \theta_0. \qquad (4)$$

Let us note that the mathematical expectation z_i is in the general case equal to:

$$M\{z_i\} = \ln d - (d-1)\theta_0 M\{\tau_i\}. \qquad (5)$$

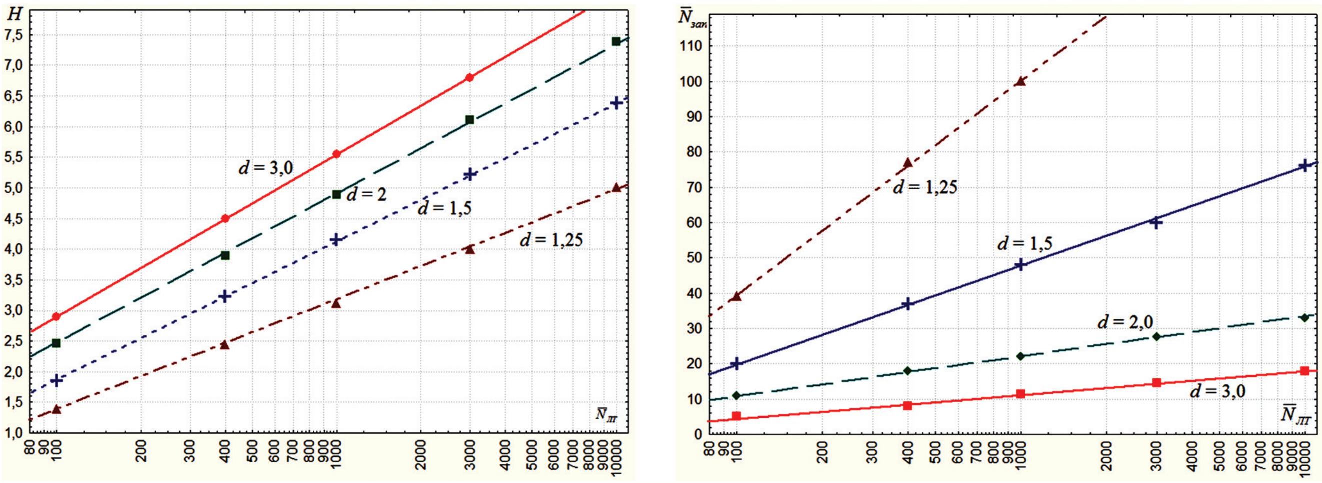
That means that, if there is no imbalance, when $M\{\tau_i\}=1/\theta_0$, the mathematical expectation $M\{z_i\}=\ln d-(d-1)<0$, which impedes the growth of the value of the decision function and results in sufficiently high average values of the time of hitting boundary H, i.e. sufficiently high values of $\overline{T}_{FA}$. Under nominal imbalance, when $M\{\tau_i\}=1/\theta_1$, we will have $M\{z_i\}=\ln d-(d-1)/d=\ln d-(1-1/d)$. In this case $M\{z_i\}>0$, which causes a quick increase of the decision function up to the threshold H, the attainment or crossing of which is the indication of imbalance.

A practical application of the algorithm would require an appropriate control procedure to be synthesized. Synthesis is understood as the definition of the decision boundary H based on user-selected values of $\overline{T}_{FA}$, initial base level θ_0 and nominal imbalance $\theta_1 > \theta_0$. Additionally, synthesis normally involves the estimation of the algorithm's speed of action through the calculation of $\overline{T}_{lag}$ and its efficiency E_d for various values of d.

Such calculation must be preceded by finding the general formulas that associate the above characteristic with each other. They are obtained using simulation. In the course of

Table 1. Estimated formulas for the definition of the decision boundary *H* and average lag $\overline{N}_{lag}$.

d	Formula for calculating H	Formula for calculating $\overline{N}_{lag}$
1.25	$H = -2.26 + 1.81 \cdot \log \overline{N}_{FA}$	$\overline{N}_{lag} = -83.01 + 61.17 \cdot \log \overline{N}_{FA}$
1.5	$H = -2.68 + 2.27 \cdot \log \overline{N}_{FA}$	$\overline{N}_{lag} = -35.38 + 27.71 \cdot \log \overline{N}_{FA}$
2.0	$H = -2.12 + 2.31 \cdot \log \overline{N}_{FA}$	$\overline{N}_{lag} = -10.82 + 10.985 \cdot \log \overline{N}_{FA}$
3.0	$H = -2.38 + 2.64 \cdot \log \overline{N}_{FA}$	$\overline{N}_{lag} = -8.62 + 6.64 \cdot \log \overline{N}_{FA}$



a) b)

Figure 1. The dependence of the decision boundary H (a) and average lag $\overline{N}_{\text{lag}}$ (b) on $\overline{N}_{\text{FA}}$

the simulation, intervals τ_i were considered as values of a discrete time series on value grid i, therefore both the mean time between false alarms and the mean lag were defined as the average number of samples $\overline{N}_{\text{FA}}$ and $\overline{N}_{\text{lag}}$ respectively; the transition from discrete to real time can be easily performed using obvious formulas $\overline{T}_{\text{FA}} = \overline{N}_{\text{FA}} / \theta_0$ and $\overline{T}_{\text{lag}} = \overline{N}_{lag} / \theta_1$.

The simulation helped find the dependences of boundary H from $\overline{N}_{\text{FA}}$ under various d from the typical set $d = 1.25$; $d = 1.5$; $d = 2.0$; $d = 3.0$, where $d = \theta_1/\theta_0$ and dependences $\overline{N}_{\text{lag}}$ on $\overline{N}_{\text{FA}}$. As it turned out, if such dependences are considered as the function $\log \overline{N}_{\text{FA}}$, they are closely approximated by linear models as follows: $H = a + b \cdot \log \overline{N}_{\text{FA}}$; $\overline{N}_{\text{lag}} = c + d \cdot \log \overline{N}_{\text{FA}}$. The corresponding calculation formulas are shown in Table 1, while the models themselves in graph form are shown in Figures 1 a) and b).

The efficiency indicator E_d of the control procedure can be calculated using the following formula:

$$E(d) = \frac{\overline{T}_{\text{FA}}}{\overline{T}_{\text{lag}}} = \frac{\overline{N}_{\text{FA}} / \theta_0}{\overline{N}_{\text{lag}} / \theta_1} = \frac{\overline{N}_{\text{FA}}}{\overline{N}_{\text{lag}}} \cdot d \qquad (6)$$

Expected values of efficiency indicator E_{d} for various d and $\overline{N}_{\text{FA}}$ are shown in Table 2.

Table 2. Expected values of efficiency indicator Ed

d	$\overline{N}_{\text{FA}}$				
	100	400	1000	3000	10000
1.25	3.2	6.5	12.5	–	–
1.50	7.5	16.2	31.3	75.0	197.4
2.00	18.2	44.4	90.9	165.0	330.0
3.00	60.0	150.0	260.9	620.7	1667.0

It is obvious that, unfortunately, the efficiency of the control procedure for the most practically interesting small values of d and $\overline{N}_{\text{FA}}$ is relatively low.

In conclusion, it can be noted that the above synthesis procedure, in principle, can also be used for cases of gradual (continuous) change of parameter θ. However, the statistical properties of the control procedure will remain unclear. Their definition for the purpose of obtaining the dependences similar to those shown in Table 1 requires quite intensive additional research.

References

[1] Kapur K., Lamberson L. Reliability in engineering design. Moscow: Mir; 1980.

[2] Baranov L.A., Yermolin Yu.A. Dependability of objects with nonstationary failure rate. Dependability 2017;4:3-9.

[3] Brodsky B.E., Darkhovsky B.S. O zadache skoreyshego obnaruzheniya momenta izmeneniya veroyatnostnykh kharakteristik sluchaynoy posledovatelnosti [On the problem of prompt detection of the moment of change of probabilistic characteristics of a random sequence]. Avtomatika i telemekhanika 1983;10:125131 [in Russian].

[4] Nikiforov I.V. Posledovatelnoe obnaruzhenie izmeneniya svoystv vremennykh ryadov [Sequential detection of temporal series property changes]. Moscow: Nauka; 1983 [in Russian].

[5] Page E.S. Continuous inspection schemes. Biometrika 1954;41(1):100-115.

[6] Shafid A. Bibliometric Analysis of EWMA and CUSUM Control Chart Schemes. ITEE Journal 2018;7(2):1-11.

[7] Vorobeychikov S.E., Konev V.V. Kharakteristiki protsedury obnaruzheniya razladki protsessa avtoregressii s neizvestnym raspredeleniem pomekhi [Characteristics of the procedure of detection of imbalance in an autoregression process with unknown noise distribution]. Avtomatika i telemekhanika 1992;3:68-75.

[8] Chernoyarov O.V., Rashitov M.F. Obnaruzhenie razladki gaussovskogo sluchaynogo protsessa s neizvestnoy intensivnostyu. Chast 1 [Detection of imbalance in a Gaussian random process with unknown intensity. Part 1]. In: Proceedings of the international science and technology conference INTERMATIC 2012;3:11-14 [in Russian].

[9] Filaretov G.F., Chervova A.A. Posledovatelnyy algoritm obnaruzheniya momenta izmeneniya dispersii vremennogo ryada [Sequential algorithm of detection of the moment of change in the temporal series dispersion]. Zavodskaya laboratoriya. Diagnostika materialov 2019;85(3):75-82.

[10] Sivova D.G., Filaretov G.F. Posledovatelnyy algoritm obnaruzheniya momenta izmeneniya kharakteristik vektornykh vremennykh ryadov [Sequential algorithm of detection of the moment of change in a vector temporal sequence characteristics]. Vestnik MEI 2014;2:63-69 [in Russian].

About the authors

Dmitry S. Repin, Candidate of Engineering, Deputy Director, Institute of Information Technology, Federal State Autonomous Educational Institution for Further Vocational Education Center for Implementation of the National Educational Policy and Information Technology, Moscow, Russian Federation, e-mail: r_d_s@inbox.ru; Mobile: +7 9166659242.

Gennady F. Filaretov, Doctor of Engineering, Professor, Professor of the Department of Control and Computer Science, National Research University Moscow Power Engineering Institute (MPEI), Moscow, Russian Federation, e-mail: gefefi@yandex.ru; Mobile: +7 9255176319.

The authors' contribution

Filaretov G.F. Review and analysis of the state of the art of the problem, theoretical aspect of the paper.

Repin D.S. Development of software tools for the simulation experiment, its performance, processing of the results, acquisition of data required for the synthesis of the control algorithm.