Estimation of the degradation factor of a censored geometrical process

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Abstract. Aim. The article examines the behaviour of renewable objects that are complex systems and generate temporally unhomogeneous failure flows. The objects' dependability is described with a geometrical processes model. The mathematical model of such processes allows considering both the ageing and renewal of a system. In the first case the failure flow rate increases with time. That corresponds with the period of ageing, when the failure rate progressively grows and the system fails more and more frequently. In the second case, the failures that show high rate at the beginning of operation become rare with time. In technical literature, this stage of operation is called the burn-in period. Normal renewal process is a special case of the geometric process model. In real operation conditions not all operation times end with a failure. Situations arise when as part of preventive maintenance a shortcoming is identified in an observed object, that gets replaced as the result. Or, for a number of reasons, a procedure is required, for which the object is removed from service and also replaced with an identical one. The object that was removed from service is repaired, modernized or simply stored. Another situation of unfinished operation occurs when the observation of an object is interrupted. More precisely, the object continues operating at the time the observation stops. For example, it may be known that at the current time the object is in operation. Both of the described situations classify the operation time as right censored. The task is to estimate the parameters of the mathematical model of geometric process using the known complete and right censored operation times that are presumably governed by the geometric process model. For complete operation times, this task was solved for various distributions [11-16]. As it is known, taking into consideration censored data increases the estimation quality. In this paper the estimation task is solved subject to the use of complete and right censored data. Additionally, the article aims to provide an analytical justification of increased estimation quality in cases when censoring is taken into account, as well as a practical verification of the developed method with real data. Methods. The maximum likelihood method is used for evaluation of the parameters of the geometrical process model. The likelihood function takes into consideration right censored data. The resulting system of equations is solved by means of the Newton-Raphson method. Conclusions. The article introduces formulas for evaluation of model parameters according to the maximum likelihood method on the assumption of various distribution laws of the time to first failure. The resulting formulas enable the estimation of the parameters of the geometrical process model involving uncertainty in the form of right censoring. Analytical evidence is produced on increased accuracy of estimation in cases when right censored data is taken into consideration. Parameter estimation was performed based on real operational data of an element of the Bilibino NPP protection control system.

Keywords: equipment degradation, heterogeneous failure flow, geometric renewal process, process numerator, restorable system, method of maximum likelihood process, right censoring.

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Introduction

As it is known, in the course of its operation technical equipment goes through several stages. Depending on the stage of operation, equipment dependability indicators change, as do their calculation methods. Until recently, most attention was given to the normal operation period, at which the failure flow parameter (rate) is a nearly constant value. In this case, the equipment operation process is assumed to be homogenous in time, while the dependability indicators are calculated using conventional methods that are, for instance, presented in [1]. Yet calculations of the dependability indicators must take into consideration two other periods: burn-in and heavy wear, when the failure flow parameter first decreases, then increases in time. Generally, there might be other, more complex time dependencies.

In [1-3], there is a short overview of various mathematical models of failure flow nonhomogeneity. Among the primary event flow nonhomogeneity models in current theories are the nonhomogenous Poisson flows, gamma processes, trendrenewal processes, flows based on the normalizing function model, and finally geometric recovery processes.

Geometric processes are described with one of the simplest models of inhomogeneous (in time) recovery processes. The model of these processes appeared quite recently [4-10] and is not vet as popular as the models of conventional recovery processes. That is primarily due to the fact that many theoretical matters related to the properties of such processes, as well as some matters of estimation of parameters of geometric recovery process model under different input data are still poorly studied. Thus, [11-16] set forth and examine some estimation methods (primarily, maximum likelihood method) of the degradation coefficient (denominator) of the geometric recovery process subject to availability of complete statistical information on the failures. In [16], the non-parametric method of confidence interval construction for the geometric process denominator is shown that allows verifying the statistical hypothesis of the presence of one or another geometrical process.

This paper aims to construct an estimation based on the method of maximum likelihood of model parameters in situations when statistical data contains uncertainty in the form of unfinished time between failures. Let us define such operation time as right censored time between failures. Additionally, the paper aims to prove the fact of increasing accuracy of evaluation of the parameters of the examined model if censored data is taken into consideration.

The input data for the required calculations are complete and right censored times between failures of a set of homogeneous elements. For the purpose of this paper, homogeneity is understood as the identity of equipment, identical operating conditions, roughly same age, etc. The operation times have equal dimensions.

Inhomogeneity of geometric type failure flows

The name of the process is directly associated with the concept of geometric progression. Geometric processes are a generalization of renewal processes. Unlike the normal renewal process that models ideal repair, geometrical process can be used in modelling, for example, of imperfect repair, when the resulting process cycle durations are not distributed evenly. Nevertheless, compared to other inhomogeneous processes the model is quite bare, as the cycle durations are "governed" by the same parameters. Geometric processes (in the context of the dependability theory) were defined in [4-7].

Definition. The random value (r.v.) ξ is equal to the r.v. η in distribution, if their distribution functions are identical: $F_{\xi}(x) = F_{\eta}(x)$. Equality in distribution is denoted as follows.

$$\boldsymbol{\xi}^{\boldsymbol{a}} = \boldsymbol{\eta}. \tag{1}$$

Definition. The sequence of nonnegative (e.g. lifetime) of independent r.v.'s $\{\Delta_k; k=1,2,...\}$ forms a geometric process (GP), if equality in distribution is satisfied



Figure 1. Set of homogenous geometric processes

$$\Delta_{k+1} \stackrel{d}{=} \gamma \Delta_k, k = 1, 2, \dots,$$
(2)

where $\gamma > 0$ is the real number constant that, by analogy with the geometric progression, is called the denominator of geometric process. Under values below 1 let us call denominator γ the degradation coefficient.

Input information

Let us assume that observation covers k of single-type immediately recoverable objects each of which has a realization of times between failures (Figure 1). In other words, we can observe k homogenous independent geometric processes. Homogeneity is understood in the way that each process has the same denominator γ .

Also, let us assume that information is available on incomplete (not ended with failure) times between failures. Thus, in Figure 1, for the first object the complete operation times are the first time, $\Delta_{1,1}$, third time, $\Delta_{1,3}$. The forth time is incomplete, as the geometric process did not end with a failure by the time the observation of this process interrupted. The second time is also incomplete. In real-life conditions that corresponds to a situation when the object under observation was replaced for some reason, e.g. if preventive maintenance identified a serious defect. Obviously, $\Delta_{1,2}^*$ cannot be considered a complete operation time. Thus, the operation time that did not end with failures will be called right censored and we will assume that that could be causes by at least two reasons: interruption of observation or equipment replacement.

Let us denote such incomplete operation times $\Delta_{i,j}^*$ and define the associated geometric process as a right censored geometric process.

Additionally, let us assume that generally the data table may have gaps, e.g. the third object is missing information on the third and forth operation times, while the fifth and subsequent ones are present. This is also true for the incomplete operation times.

Let us transpose the data table, i.e. let us group the information in accordance with the number of operation time. Let the last observed operation time have the number s. Let us represent the input information as follows:

$$\left(\left(\Delta_{1,1},...,\Delta_{n_{1},1}\right),\left(\Delta_{1,1}^{*},...,\Delta_{m_{1},1}^{*}\right)\right) \text{ is the realization}$$
of the first operation times, $\left(\Delta_{1},\Delta_{1}^{*}\right);$ (3)

$$\left(\left(\Delta_{1,2},...,\Delta_{n_{2},2}\right),\left(\Delta_{1,2}^{*},...,\Delta_{m_{2},2}^{*}\right)\right) \text{ is the realization}$$
of the second operation times, $\left(\Delta_{2},\Delta_{2}^{*}\right);$ (4)

$$\left(\left(\Delta_{1,s},...,\Delta_{n_{s},s}\right),\left(\Delta_{1,s}^{*},...,\Delta_{m_{s},s}^{*}\right)\right) \text{ is the } s^{\text{th}} \text{ operation}$$

times between failures, $\left(\Delta_{s},\Delta_{s}^{*}\right)$. (5)

In virtue of formula (2) *i*+1 time between failures is related with the first operation time with the following relation in distribution: $\Delta_{i+1}^{d} = \gamma^{i} \Delta_{1}, i = 0, 1, ...$ Thus, the distribution function $F_{i+1}(x)$, dependability function (PNF) $P_{i+1}(x)$, distribution density *i*+1 of operation time $f_{i+1}(x)$ will be defined based on the respective functional characteristics of the first operation time $F_{1}(x), P_{1}(x)$ and $f_{1}(x)$ similarly to the method set forth in [14]:

$$F_{i+1}(x) = F_1(\gamma^{-i}x), P_{i+1}(x) = P_1(\gamma^{-i}x), f_{i+1}(x) = \gamma^{-i}f_1(\gamma^{-i}x).$$
(6)

Let us now consider the matter of estimation of unknown parameters of the geometric process model on the assumption of the following laws of distribution of the first time between failures:

$$P_1(x) = \exp(-\lambda x^{\beta}), \tag{7}$$

$$P_1(x) = \exp(-\lambda_1 x - \lambda_2 x^2). \tag{8}$$

The dependability function (7) pertains to the Weibull-Gnedenko distribution, (8) describes the distribution with linearly increasing failure rate (see [1]). It is obvious that both (7) and (8) generalize the exponential distribution that is obtained if $\beta = 1$ and $\lambda_1 = \lambda$, $\lambda_2 = 0$ respectively.

Obviously, the distributions (7) and (8) can be generalized by the following distribution:

$$P_{1}(x) = \exp\left(-\sum_{l=1}^{p} \lambda_{l} x^{\beta_{l}}\right).$$
(9)

If $\beta_1 = \beta$, $\lambda_1 = \lambda$, $\lambda_2 = ... = \lambda_p = 0$ we deduce (7), if $\beta_1 = 1$, $\beta_2 = 2$, $\lambda_3 = ... = \lambda_p = 0$ we deduce the distribution (8) and, finally, if $\beta_1 = 1$, $\lambda_1 = \lambda$, $\lambda_2 = ... = \lambda_p = 0$ we deduce the exponential distribution.

Now let us consider the maximum likelihood method as the primary method for estimation of the unknown model parameters.

Method of maximum likelihood

We will define the model parameter estimators by means of the standard method of maximum likelihood. Let us write the likelihood function by means of (6) and using $\vec{\theta} = (\vec{\lambda}, \vec{\beta})$ to denote the vector of (possibly) unknown parameters of the distribution law:

$$L(\gamma, \vec{\theta}) = \prod_{i=1}^{s} \prod_{j=1}^{m_{i}} f_{i}(\Delta_{j,i}) \prod_{k=1}^{m_{i}} P_{i}(\Delta_{k,i}^{*}) =$$
$$= \prod_{i=1}^{s} \prod_{j=1}^{m_{i}} \gamma^{-i+1} f_{1}(\gamma^{-i+1}\Delta_{j,i}) \prod_{k=1}^{m_{i}} P_{1}(\gamma^{-i+1}\Delta_{k,i}^{*}).$$

We can naturally expect that the parameters $\overline{\theta}$ may be partially or completely known.

The log-likelihood function (LLF) will be as follows

$$l(\gamma, \vec{\theta}) = \sum_{i=1}^{s} \left(\sum_{j=1}^{n_{i}} \left[(-i+1) \ln \gamma + \ln f_{1}(\gamma^{-i+1}\Delta_{j,i}) \right] + \sum_{k=1}^{m_{i}} \ln P_{1}(\gamma^{-i+1}\Delta_{k,i}^{*}) \right) \right)$$

It is not complicated to deduct a simplified form of the LLF:

$$l(\boldsymbol{\gamma}, \vec{\theta}) = \sum_{i=1}^{s} \sum_{j=1}^{n_i} \ln f_1(\boldsymbol{\gamma}^{-i+1} \Delta_{j,i}) + \sum_{i=1}^{s} \sum_{k=1}^{m_i} \ln P_1(\boldsymbol{\gamma}^{-i+1} \Delta_{k,i}^*) - N_1 \ln \boldsymbol{\gamma}, \quad (10)$$

where
$$N_1 = \sum_{i=1}^{s} (i-1)n_i = n_2 + 2n_3... + (s-1)n_s.$$
 (11)

By substituting here the distribution density (9) we deduct formulas for model parameters estimation. After a number of substitutions and simplifications the LLF of the generalized distribution is as follows:

$$l(\boldsymbol{\gamma}, \vec{\boldsymbol{\theta}}) = \sum_{i=1}^{s} \sum_{j=1}^{n_{i}} \ln\left(\sum_{l=1}^{p} \lambda_{l} \beta_{l} \boldsymbol{\gamma}^{-(i-1)(\beta_{l}-1)} \Delta_{j,i}^{\beta_{l}-1}\right) - \sum_{l=1}^{p} \lambda_{l} \sum_{i=1}^{s} C_{i}(\beta_{l}) \boldsymbol{\gamma}^{-(i-1)\beta_{l}} - N_{1} \ln \boldsymbol{\gamma}.$$
 (12)

Let us represent the LLF for specific distributions: Weibull-Gnedenko distribution.

$$l = N_2 \cdot (\ln \lambda + \ln \beta) - \beta N_1 \ln \gamma + + (\beta - 1) C_0 - \lambda \sum_{i=1}^{s} C_i (\beta) \cdot \gamma^{-(i-1)\beta}, \qquad (13)$$

where
$$N_2 = \sum_{i=1}^{s} n_i = n_1 + \dots + n_s$$
, (14)

Distribution with linearly increasing rate.

$$l = \sum_{i=1}^{s} \sum_{j=1}^{n_{i}} \ln \left(\lambda_{1} + 2\lambda_{2}\gamma^{-(i-1)}\Delta_{j,i} \right) - \sum_{l=1}^{2} \lambda_{l} \sum_{i=1}^{s} C_{i} \left(\beta_{l} \right) \gamma^{-(i-1)\beta_{l}} - N_{1} \ln \gamma.$$
(15)

Exponential distribution.

$$l = N_2 \ln \lambda - N_1 \ln \gamma - \lambda \sum_{i=1}^{s} C_i (1) \cdot \gamma^{-(i-1)}, \qquad (16)$$

In formulas from (12) to (15) the following designations are introduced:

$$C_{0} = \sum_{i=1}^{s} \sum_{j=1}^{n_{i}} \ln\left(\Delta_{j,i}\right), \ C_{i}\left(\beta\right) = \sum_{j=1}^{n_{i}} \left(\Delta_{j,i}\right)^{\beta} + \sum_{k=1}^{m_{i}} \left(\Delta_{k,i}^{*}\right)^{\beta}. \ (17)$$

Substantiation of censoring consideration

Let us examine the degradation coefficient estimation under LLF (16), as it is the simplest one. If the rate of the exponential law is know, the estimation of parameter γ will be the solution of the equation:

$$\frac{\partial l}{\partial \gamma} = -\frac{N_2}{\gamma} + \lambda \left(\frac{C_2(1)}{\gamma^2} + \frac{2C_3(1)}{\gamma^3} + \dots + \frac{(s-1)C_s(1)}{\gamma^s} \right) = 0.$$
(18)

Thus, the estimation $\hat{\gamma}$ is the solution of the equation $\phi(\hat{\gamma}) = \frac{N_2}{\lambda}$, where

$$\varphi(\gamma) = \frac{C_2(1)}{\gamma} + \frac{2C_3(1)}{\gamma^2} + \dots + \frac{(s-1)C_s(1)}{\gamma^{s-1}}$$

As $C_i(\beta) \ge 0$, then $\varphi(\gamma)$ is a monotonically decreasing function with a range space in the form of half line $(0, +\infty)$, and equation (18) has an unambiguous solution. Let us calculate the second LLF derivative:

$$\frac{\partial^2 l}{\partial \gamma^2} = \frac{N_2}{\gamma^2} - \lambda \left(\frac{2C_2(1)}{\gamma^3} + \frac{6C_3(1)}{\gamma^4} + \dots + \frac{(s-1)sC_s(1)}{\gamma^{s+1}} \right).$$

The second LLF derivative at the bending point $\hat{\gamma}$ will be defined by the formula:

$$\frac{\partial^2 l}{\partial \gamma^2}\Big|_{\gamma=\hat{\gamma}} = -\frac{\lambda}{\hat{\gamma}^2} \left(\frac{C_2(1)}{\hat{\gamma}} + \frac{4C_3(1)}{\hat{\gamma}^2} + \dots + \frac{(s-1)^2 C_s(1)}{\hat{\gamma}^{s-1}} \right) < 0.$$
(19)

The last inequation once again proves that the bending point $\hat{\gamma}$ is the maximum point. Yet the most important deduction out of (19) is that taking into account censored operation times according to (17) increases the $C_i(1)$ coefficients. That increases the "steepness" of the LLF and, consequently, the accuracy of the resulting evaluations.

Practical application

In order to demonstrate the capabilities of the geometric processes model let us consider an example of its practical application in statistical analysis of failure data of compensated neutron chambers (KNK-56) of the reactor control and protection system (CPS) of the Bilibino NPP. Earlier (e.g. see [17]) a similar analysis established that a number of CPS elements, including KNK-56, generate temporally unhomogeneous failure flows (Figure 2).

In the figure we can note a relatively high failure rate in the 1980s followed by a low failure rate. The fact of inhomogeneity was evidenced by a number of related statistical criteria [17].



Figure 2. KNK-56 failure flow parameter

It can be expected that this behaviour of the rate will somehow correspond with the geometric process model, whereby with the denominator $\gamma > 1$. This conclusion enabled a preliminary analysis of sufficiently representative failure statistics that became available recently (Table 1).

Table 1 shows selected times between failures of the first five out of 16 (four for each of the 4 BNPP units) KNK-56 elements. With the operation times $\Delta_{j,i}$ is given the «data completeness indicator» $\delta_{i,i}$ that equals 1 if the respective

operation time is complete, and 0 if, among other things, it is right censored.

Let us set forth the calculated parameters of models (12) to (15) from Table 2. It must be noted that the calculations using the model with linearly increasing rate (14) matched the results of the exponential law calculations (15), which indicates that the second parameter, the rate λ_2 under such initial conditions is redundant. Therefore, it was decided to use the generalized model (9) with the number of summands

$$p=2: P_1(x) = \exp(-\lambda_1 x^{\beta_1} - \lambda_2 x^{\beta_2}).$$

Judging by the maximum LLF value in Table 1, the most appropriate distribution law model is the generalized model (9) and the Weibull-Gnedenko distribution. Importantly, in each case the geometric process parameter was larger than 1. I.e. the hypothesis regarding this parameter suggested above was presumably correct. A substantiated decision regarding the adequacy of the geometric process model is possible based on either the confidence interval, or the statistical test comparable to [16]. Respective research is to be conducted in the future.

In conclusion, let us take a look at Figures 3 and 4. The former shows the frequency diagram of time between the

Element	un. 1 – IK1		un. 1 – IK10		un. 1 – IK18		un. 1 – IK9		un. 2 – IK1	
Operation time <i>i</i>	$\Delta_{1,i}$	$\delta_{1,i}$	$\Delta_{2,i}$	δ _{2,i}	$\Delta_{3,i}$	$\delta_{3,i}$	$\Delta_{4,i}$	$\delta_{4,i}$	$\Delta_{5,i}$	δ _{5,i}
1	5.900	1	0.481	1	0.381	1	2.833	1	1.772	0
2	7.022	0	6.036	1	7.075	0	3.556	1	9.192	1
3	0.469	1	5.700	1	1.058	0	0.042	1	3.931	0
4	0.589	0	0.658	1	0.003	0	1.050	1	0.003	1
5	14.494	0	0.047	0	0.014	1	0.622	1	0.003	1
6	13.608	0	0.608	1	0.058	1	12.817	0	0.450	0
7			28.183	1	28.028	0	15.742	0	12.289	1
8			0.369	0					3.844	0
9									6.019	1
10									3.586	0

Table 1. Times between failures of KNK-56

Table 2. MLM evaluations of model parameters

Law	Evaluations								
Waibull Cnadanka	γ	λ	β	LLF- <i>l</i>					
welduii-Ghedeliko	1.226	0.316	0.647	-179.784					
Linearly increasing	γ	λ_1	λ_2	β_1	β_2	LLF- <i>l</i>			
rate	1.156	0.153	0	1	2	-189.927			
Generalized model	γ	λ ₁	λ ₂	β1	β ₂	LLF- <i>l</i>			
(9)	1.218	0.202	0.101	0.747	0.447	-179.675			
Ermonoutial	γ	λ	LLF- <i>l</i>						
Exponential	1.156	0.153	-189.927						

first and the second failure. The latter shows the dependability function (PNF) of the first five KNK-56 operations. We can notice a tendency to progressive improvement of dependability indicators.



Figure 3. Various distribution models of time between the first and the second failure



Fig. 4. Dependability functions of the first five times to failure

Conclusion

The paper presents the geometrical model of renewal processes for the purpose of calculating dependability characteristics of objects that generate temporally unhomogeneous failure flow. The maximum likelihood method is used for evaluation of the model parameters. The paper continues the research of parameter evaluation in geometrical process model. The key feature of the presented research is the capability to take into consideration right censored data. Such uncertainty occurs when non-failed equipment is replaced or in the case of interrupted observation. The authors analytically demonstrate that such provisions improve the accuracy of evaluation. They provide calculations of parameters of various distribution law models based on the operational data of KNK-56 of the Bilibino NPP RCSS.

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