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Efficient estimation of mean time to failure

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Abstract. *Product testing plan of type NMT has been chosen as the subject of research plan. This plan's time between failures is subject to the exponential law where N is the number of same-type tested products; T is the time to failure (same for each product); M is a feature of the plan meaning that after each failure the working condition of the product is recovered over the course of the test. In this case, the time to failure is defined according to formula* $T_{01} = NT/\omega$*, where* ω *is the number of observed failures,* $\omega > 0$*, that occurred within the time T. This estimate is biased. Besides that, if it is required to solve a problem that involves achieving a point estimation of mean time to failure (*T_0*) of products based on tests that did not produce any failures, estimate* T_{01} *cannot be used. If over the time of testing the number of observed failures is small (the number does not exceed several ones), the estimate can contain a significant error due to the bias. In order to solve the above problem, it suffices to find an unbiased efficient estimate* T_{0ef} *of the value* T_0*, if such exists, in the class of consistent biased estimates (the class of consistent estimates that includes all estimates generated by method of substitution, of which the maximum likelihood method, contains estimates with any bias, including those with a fixed one, in the form of function of parameter or constant). In general, there is currently no rule for finding unbiased estimates, and their identification is a sort of art. In some cases, the generated unbiased efficient estimates are quite lengthy and have a complex calculation algorithm. They are also not always sufficiently efficient in the class of all biased estimates and not always have a considerable advantage over simple yet biased estimates from the point of view of proximity to the estimated value.* **The aim** *of the article is to find the estimate of value* T_0 *that is simple and more efficient in comparison with the conventional one and negligibly inferior to the estimate* T_{0ef}*, if such exists, in terms of proximity to* T_0 *when using the NMT plan.* **Methods.** *In obtaining an efficient estimate integral characteristics were used, i.e. total relative square of the deviation of expected realization of estimate* T_0 *from various values* T_0 *per various failure flows of the tested product population. A sufficiently wide range of class estimates was considered and a functional built based on the integral characteristic, of which the solution finally allowed deducing a simple and efficient evaluation of mean time to failure for the NMT plan.* **Conclusions.** *The achieved estimate of mean time to failure for the NMT plan is efficient within a sufficiently wide range of estimates and is not improvable within the considered class of estimates. Additionally, the achieved estimate enables point estimation of mean time to failure based on the results of tests that did not have any failures.*

Keywords: *mean time to failure, exponential law, efficient estimation.*

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In modern manufacturing of complex high-dependability products, it has become fairly common when it is required to produce a point estimate of product dependability indicator based on tests that did not result in failure. In compliance with [1], mean time to failure is chosen as the indicator that characterizes reliability as a dependability property of complex restorable itemsT_0. From managerial and economic points of view, the *NMT* plan is optimal for testing restorable (replaceable) products (hereinafter referred to as products) under condition that time between failures is subject to the exponential law, where *N* is the number of same-type tested products; *T is* the time to failure (same for each product); *M* is a feature of the plan meaning that after each failure the working condition of the product is recovered over the course of the test [2]. In this case, the mean time to failure is defined according to formula $T_{01} = NT/\omega$, where $\omega > 0$ is the number of observed failures that occurred within the time *T*. This estimate is biased [2]. Besides that, estimate T_{01} cannot be used for the purpose of solving the above problem. If over the time of testing the number of observed failures is small (the number does not exceed several ones), the estimate can contain a significant error due to the bias.

The above entails the problem of defining the value of relative confidence error δ of the evaluated value T_0, as the solution requires the results of point and confidence estimation [3, 4]. Solving this problem is impossible by using T_{01} if the tests did not produce any failures. In case the number of observed failures is small, the solution has a significant bias of error δ. Therefore, conventional estimation must be used in order to eliminate said shortcomings.

In order to solve the above problem, it suffices to find an unbiased efficient estimate T_{0ef} of the value T_0, if such exists in the class of consistent biased estimates (the class of consistent estimates that also includes all estimates generated by method of substitution, of which the maximum likelihood method, contains estimates with any bias, including those with a fixed one, in the form of function of parameter or constant [5]). In general, there is no rule for finding unbiased estimates, and their identification is a sort of art. In some cases, the generated unbiased efficient estimates are quite lengthy and have a complex calculation algorithm [6-8]. They are also not always sufficiently efficient in the class of all biased estimates and not always have a considerable advantage over simple yet biased estimates from the point of view of proximity to the estimated value [9].

The purpose of the article is to find the estimate of value T_0 that is simple and more efficient in comparison with the conventional one and negligibly inferior to the estimate T_{0ef}, if such exists, in terms of proximity to T_0 when using the *NMT* plan.

In order to find the efficient estimate we will use integral characteristics [10, 11]. Let us use the total relative square of the deviation of expected realization of estimate $T_{0\omega}$ from various values T_0 per various failure flows of the tested product population [11]:

$$AT_{0\omega} = \int_0^{\infty} 1/T_0^2 \left\{\Theta T_{0\omega} - T_0\right\}^2 \partial\Delta, \quad (1)$$

where Δ indicates the Poisson failure flow with parameter NT/T_0 [12], while $\Theta T_{0\omega}$ is the expectation of the suggested estimate. By using the properties of the Poisson flow with parameter NT/T_0 [12] we will deduce

$$\Theta T_{0\omega} = \sum_{\kappa=0}^{\infty} T_{0\omega\kappa} E^{-\Delta} \Delta^{\kappa}/\kappa!. \quad (2)$$

Let us represent estimate T_{01} as

$$T_{01} = \frac{NT}{\omega+1} + \frac{NT}{\omega(\omega+1)}. \quad (3)$$

Given that ω is sufficient statistic [13] let us consider the estimate class $T_{0\omega}$ that can be represented as (3), i.e.

$$T_{0\omega} = \frac{NT}{\omega+1} + NTf(\omega). \quad (4)$$

This estimate class includes the efficient estimate from [11].

The expectation of estimates of class (4) according to (2) will be expressed by formula

$$\Theta T_{0\omega} = T_0\left(1-E^{-\Delta}\right) + T_0 E^{-\Delta} \sum_{\kappa=0}^{\infty} \Delta f(\kappa)\Delta^{\kappa}/\kappa!. \quad (5)$$

Let us denote $B = \sum_{\kappa=0}^{\infty} \Delta f(\kappa)\Delta^{\kappa}/\kappa!$. After substituting (5) into (1) we deduce

$$AT_{0\omega} = \int_0^{\infty} E^{-2\Delta}(B-1)^2 \partial\Delta = B_2 - 2B_1 + 1/2, \quad (6)$$

where $B_2 = \sum_{\iota=0}^{\infty}\sum_{\kappa=0}^{\infty} f(\iota) f(\kappa) 0,5^{\iota+\kappa+3}(\iota+\kappa+2)!/(\iota!\kappa!)$,

$$B_1 = \sum_{\kappa=0}^{\infty} f(\kappa) 0,5^{\kappa+2}(\kappa+1).$$

Let us identify the lower limit of the functional (6) for which purpose let us assume

$$B_2 = \sum_{\iota=0}^{\infty} f(\iota) 0,5^{\iota+1} \sum_{\kappa=0}^{\infty} f(\kappa) 0,5^{\kappa+2}(\kappa+1)(\kappa+2)\ldots(\kappa+\iota+2)/\iota!$$

and note that

$$\begin{aligned}(\kappa+2)\ldots(\kappa+\iota+2)/\iota! &= (\kappa/2+1)\cdot(\kappa/3+1)\cdot\ldots\cdot \\ &\cdot(\kappa/(\iota+2)+1)\cdot 2\cdot 3\cdot\ldots\cdot(\iota+1)\cdot(\iota+2)/\iota! = \\ &(\kappa/2+1)\cdot(\kappa/3+1)\cdot\ldots\cdot(\kappa/(\iota+2)+1)\cdot 2\cdot 3\cdot\ldots\cdot \\ &\cdot(\iota+1)\cdot(\iota+2) \geq 2(\iota+1).\end{aligned}$$

Therefore

$$B_2 \geq 2\cdot 0,5\sum_{\iota=0}^{\infty} f(\iota) 0,5^{\iota+1}(\iota+1) 2\cdot B_1 = 4B_1^2.$$

By substituting the right side of the inequality into (6) we deduce $AT_{0\omega} \geq 4\cdot B_1^2 - 2B_1 + 1/2$. By deriving the right side with respect to B_1 and equating it to zero we deduce the lower limit $AT_{0\omega} \geq 0.25$.

Let us identify the composite estimate that belongs to the class under consideration, i.e.: $T_{02} = 2NT$ if $\omega = 0$ and $T_{02} = NT/(\omega+1)$ if $\omega > 0$. Out of general $T_{0\omega}$ follows $f(\omega) = 1$ if $\omega = 0$ and $f(\omega) = 0$ if $\omega > 0$. As is easy to see, in this case $B_2 = 0.25$ and $B_1 = 0.25$. By substituting the resulting values into (6), we deduce $AT_{02} = 0.25$, i.e. the estimate T_{02} affords to the functional $AT_{0\omega}$ a minimum equal to 0.25. Given the above deduced estimate of the lower limit of the functional $AT_{0\omega}$ we can suppose that the estimate T_{02} is not improvable within the considered class of estimates.

Therefore T_{02} is the desired estimate. Let us use it to solve the above problems.

Example. Over a 1000-hour reliability testing of 50 modules no failures were observed. Based on the test results, point estimation of parameter T_0 must be performed.

Solution. For $\omega = 0$ we deduce $T_{02} = 2NT = 2\cdot 50\cdot 1000 = 100000$ h.

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