Novozhilov E.O.

GUIDELINES FOR CONSTRUCTION OF A RISK MATRIX

The paper deals with the issue of selecting parameters that secure the construction of a risk matrix. Main aspects affecting the error of presentation of risk matrix results are identified. The construction of a risk matrix with minimum error for given conditions is shown by means of a particular example. Directions for further research are outlined.

Keywords: *risk, risk matrix, risk estimation error, undesirable event frequency, severity of consequences of an undesirable event.*

In respect to technical systems, the risk is considered as a combination of probability (or frequency) of an undesirable event and degree of severity of its consequences [1]. The measure of a risk is a risk level which is determined by certain functionality linking the probability (or frequency) of an undesirable event and the expected value of the severity of consequences (damage) of this event [1]. Typically, this functionality represents the multiplication of the undesirable event frequency by the average amount of its consequences [2, 3]:

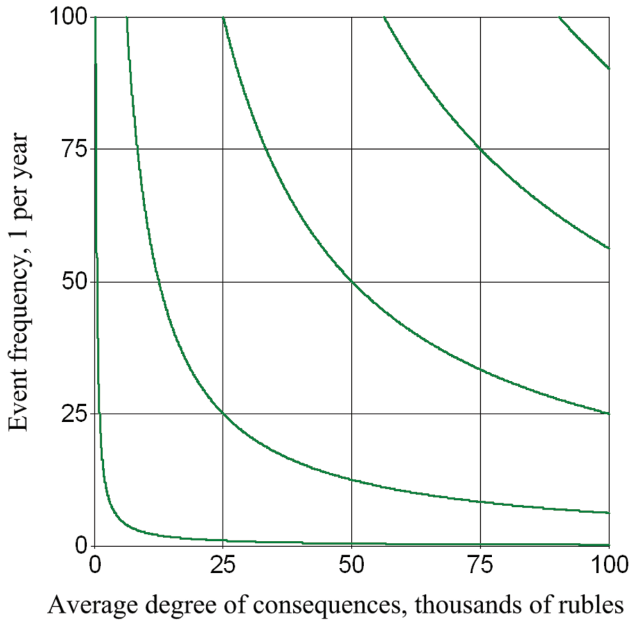
$$R = F \times C.$$

For the aims of risk treatment, however, it is not only a risk level that is of interest, but also proportion of its components –frequency and consequences. Two risks, for example, may be of the same level, but the first one can be related to high event frequency and low degree of consequence severity, and the second one can refer to rare events with serious consequences. For the first case the risk treatment can involve the event frequency reduction, and for the second case it is possible to apply for risk transfer (insurance).

Due to the necessity of representing the risk value as well as the combination of its components, two means have got a wide application: a risk graph in the coordinates of «frequency-consequences» and a risk matrix. The risk matrix is considered as the most practical and illustrative tool used for decision making support in risk management systems. It has a form of cell table that represents the combination of the frequency of an undesirable event and the severity of its consequences and makes it possible to provide authorized decision-makers with visual information on risk levels for event in question. The parameters of a matrix depend on the field of its application.

A risk matrix has the following main parameters:

- number N of categories (value intervals) of risks. References recommend to use 3 [2, 4] or 4 [1, 2, 5, 6] risk categories. In particular GOST R 54505 [1] introduces 4 risk categories (*negligible, acceptable, undesirable, unacceptable*). Consequently, for N of risk categories it is required to set N-1 of boundary risk levels (boundaries of the far regions are generally not limited): R_1; R_2;…; R_{N-1}. Their values are determined based on maximum and minimum risk level values indicated by the matrix, as well as depending on the selected risk scale (linear, logarithmic or other nonlinear scale). The main boundary level is normally an acceptable risk level (risk values above this level are considered as inadmissible). This level is set and remains the main level when constructing a risk scale. A risk category on a matrix is normally



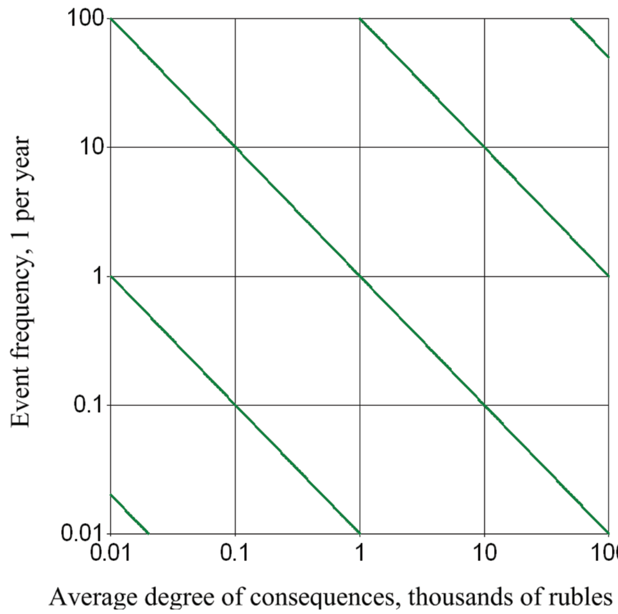


Fig. 1. Risk graphs

denoted with a particular color assigned to the cell relative to this category;

- number m of value intervals of event frequencies (corresponds to the number of matrix cells vertically). GOST R 54505 [1] introduces 6 intervals: the event may be *incredible, improbable, remote, occasional, probable, frequent*. For m of frequency scale intervals it is required to set m+1 of boundary values, which shall be determined by given minimum and maximum values of the event frequency, and by the selected frequency scale range;

- number n of value intervals of event consequences (corresponds to the number of matrix cells horizontally). GOST R 54505 [1] introduces 4 intervals: consequences may be *negligible, insignificant, critical, catastrophic*. For n of consequence scale intervals it is required to set n+1 of boundary values, which shall be determined by given minimum and maximum values of event consequences, and by the selected consequence scale range.

Let us consider risk graphs in linear (Fig. 1a) and logarithmic (Fig. 1b) «frequency-consequences» coordinates assuming that the risk is estimated as multiplication of the undesirable event frequency by the degree of its consequences. The graphs show the lines corresponding to constant risk levels R = const.

To change the graph into the risk matrix, it is required to array matrix cells on the coordinate plane and to assign the cells to a certain risk category.

According to Fig. 1 graphs, the logarithmic scale is preferable due to the following reasons:

- in linear scale the R = const lines are hyperbolic, but in logarithmic scale they go right;

- logarithmic scale allows to use much broader range of values for frequency and consequences.

Therefore, it is reasonable to consider the risk graph «frequency-consequences» in logarithmic coordinates and its reference to the cells of a risk matrix where the risk scale as well as the scales of frequency and consequences are presented in logarithmic format.

It should be noted that the risk matrix does have a number of disadvantages, and the main one is a high degree of error of risk estimation [7]. Thus the aim of this paper is to analyze the approaches for construction of a risk matrix enabling to reduce the error of results presentation under the given matrix parameters.

The matrix cell parameters in logarithmic coordinates correspond to the following formula (Fig. 2):

$$\log_k \frac{r_2}{r_1} = \log_k \frac{f_2}{f_1} + \log_k \frac{c_2}{c_1},$$

where r_1, r_2 are minimum and maximum risk levels represented in a cell;

f_1, f_2 are minimum and maximum event frequencies represented in a cell;

c_1, c_2 are minimum and maximum consequence degree represented in a cell.

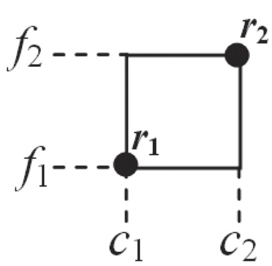


Fig. 2. Parameters of a risk matrix cell

Let us take the relative array pitch of a risk cell in logarithmic coordinates as being equal to entity:

$$\log_k \frac{r_2}{r_1} = 1.$$

Then the relative array pitches of the cell of the scale of frequency and consequences will respectively be equal to:

$$\alpha = \log_k \frac{f_2}{f_1}; \beta = \log_k \frac{c_2}{c_1}.$$

In this case the following condition must be met

$$\alpha + \beta = 1. \quad (1)$$

In case under consideration, the relative array pitch of the risk cell shall be equal to k, and the relative array pitches of the cell of the scale of frequency and consequences shall be k^α and k^β respectively. Meanwhile the range of frequencies, the range of consequences and the range of risks of the matrix shall be equal to:

$$k^{\alpha m}; k^{\beta n}; k^{(\alpha m+\beta n)}. \quad (2)$$

The obvious problem when overlapping the cells of the risk matrix onto the graph is that the right lines dividing the coordinate plane into several ranges (intervals) of risk values do have a slant. As the result, several matrix cells will be divided by these lines into parts. Such cells will cover two (or more) ranges of values, which leads to difficulties when assigning these cells to a certain risk category. In the most unfavorable case, a cell may be divided into two segments of equal space, this preventing us from precisely defining what range of risk values most of points allocated inside this cell belong to.

Consequently, the first task is to define such values of R = const slope ratio, that will conduce to no matrix cells divided in half (or very few of them).

The analysis has lead to a hypothesis that the slope ratio with no divided cells is formulated as the ratio of natural numbers 1:2; 2:3, 1:4; 3:4; 2:5; 4:5, etc., and the ratio of two odd numbers is not useful as it implies the cells divided in half. This hypothesis has been checked by means of computer simulation modeling. The half line going from the O point on the γ angle runs through the cells located on the plane (100×100 square cell matrix with the side equal to 1 was used), and several cells are divided by the half line (Fig. 3). The slope ratio changed with a pitch of 0.001 within the range from 0 to 1. As the estimation criterion we used the largest space areas of all divided cells, from which the worst (the least) value was chosen.

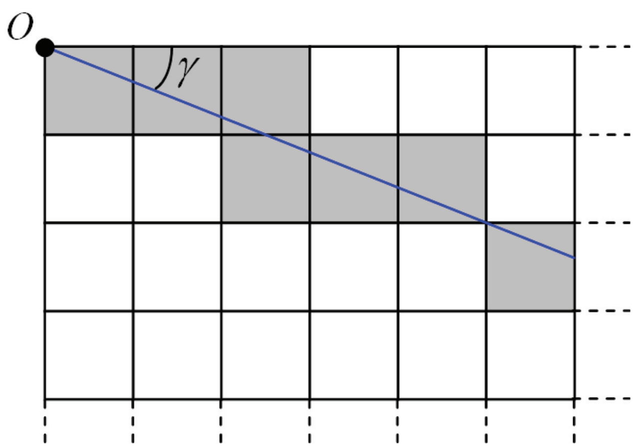


Fig. 3. Division of cells by a half line running at different angles

As a result, we received the values given in Table 1 (these are the results with the worst value of space area of the divided cells largest segment exceeding 0.599).

Table 1. The worst value of space area of the cell largest segment of slope ratio

Slope ratio	The worst value of space area of the largest segment
-1/2 or -2	0.75
-2/3 or -1.5	0.6667
-1/4 or -4	0.625
-3/4 or -4/3	0.625
-2/5 or -2.5	0.6
-4/5 or -1.25	0.6

As it follows from the results listed in Table 1, the most favorable cell division goes with the slope ratio of tg γ = –0.5 or tg γ = –2. For further consideration these values shall be used as the values of the highest precision of the results presentation.

The second task lies in selection of array pitch between several right lines running on the cell plane and dividing the plane into ranges of values. The pitch can be measured both horizontally (d_β, Fig. 4) and vertically (d_α).

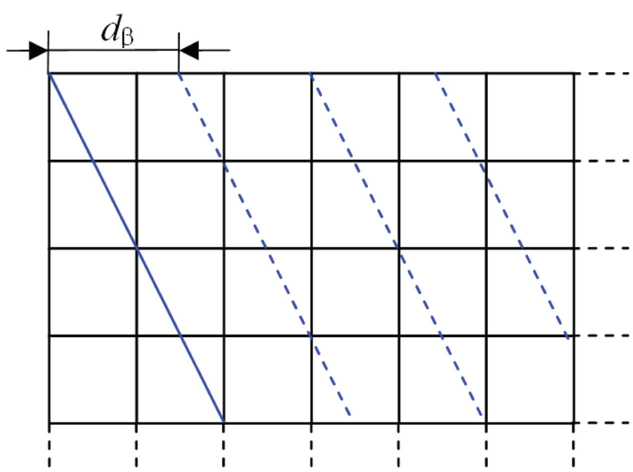


Fig. 4. Horizontal pitch between R = const lines

The analysis showed that probability p of reference of any point inside a cell to the required range follows the law:

- for tg γ = -2: $\begin{cases} p = d_\beta, & 0 \le d_\beta \le 0,5; \\ p = 1 - \dfrac{1}{4d_\beta}, & d_\beta > 0,5; \end{cases}$

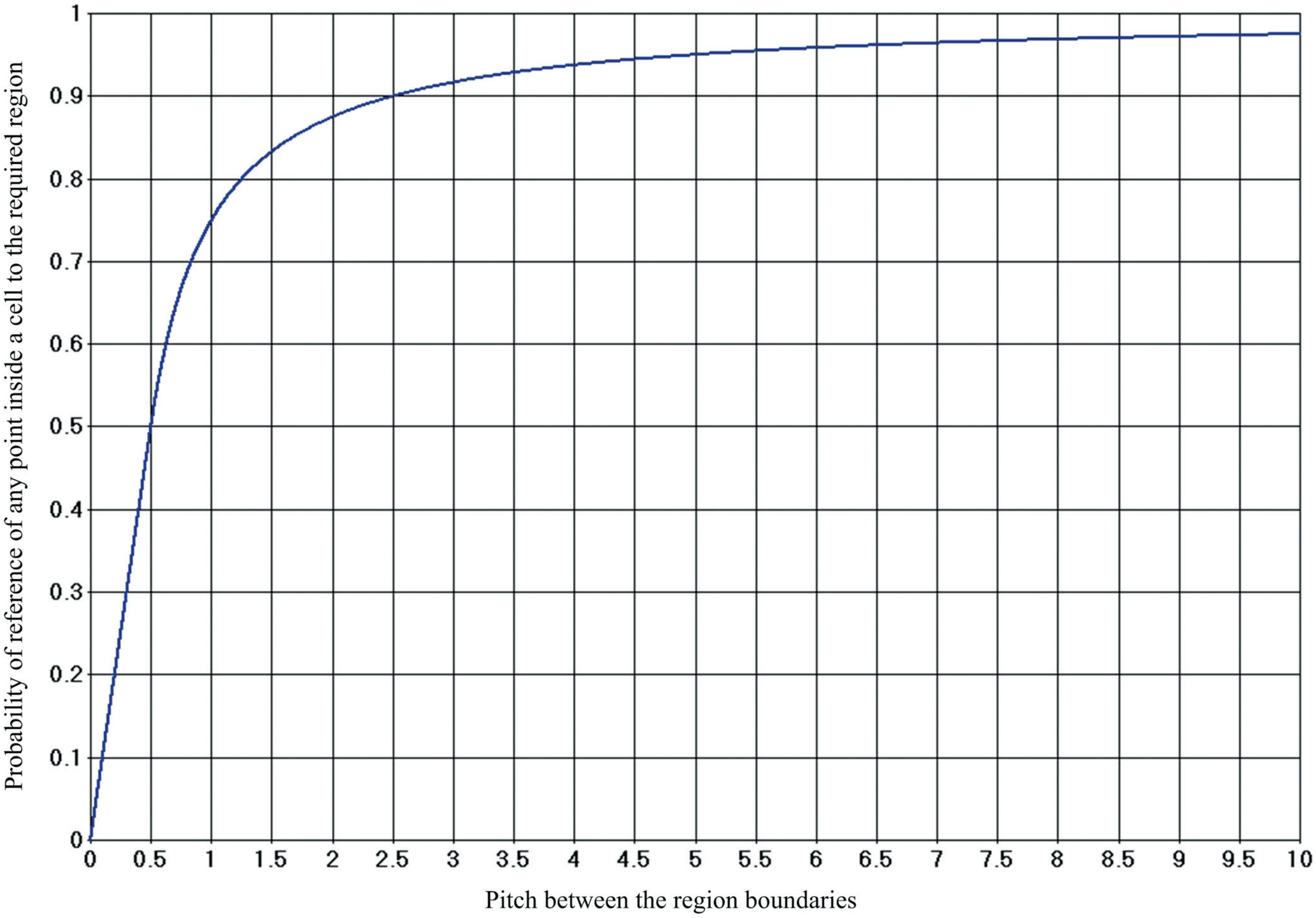


Fig. 5. Correlation between the probability of the point reference to the required region and the pitch between the region boundaries

- for tg γ = -0,5: $\begin{cases} p = \dfrac{d_\beta}{2}, & 0 \le d_\beta \le 1; \\ p = 1 - \dfrac{1}{2d_\beta}, & d_\beta > 1. \end{cases}$

The graph of this dependency for tg γ = –2 is shown in Fig. 5.

The graph of Figure 5 shows that the optimal pitch value d_β will be within the range from 1 to 5, as the p values which are less than 0.5 do not allow accurately relating the cell with the region of risk values, and the p values from 0.5 to 0.75 do not provide the presentation of results in an accurate way. On the other hand, if the values of the range d_β do exceed 5, the function passes into saturation and the p value increases insignificantly (it makes no sense to choose large d_β values).

It is also worth mentioning that the higher the pitch value d_β is, the bigger risk matrix shall be required to «cover» the given number of risk categories (so that there would be at least one cell of every risk category in the matrix). Therefore, it is the d_β value that determinates the correlation of the number of cells of different categories in the matrix.

The maximum d_β value under which the m x n matrix shows the given number N ($N > 2$) of risk categories for the slope ratio -0.5 and -2, is formulated by:

- for tg γ = -2: $d_\beta \le \dfrac{0,5 \cdot m + n - 2}{N - 2}$; (3)

- for tg γ = -0,5: $d_\beta \le \dfrac{2m + n - 4}{N - 2}$.

For proper interpretation of risk levels it is necessary to observe the correlation

$$\alpha \cdot d_\alpha = \beta \cdot d_\beta,$$

where $\alpha \cdot d_\alpha = \beta \cdot d_\beta$ is the conditional pitch of scale of matrix risks.

The relative pitch of a risk matrix is found from the condition:

$$K = k^{\beta d_\beta} = k^{\alpha d_\alpha}. \quad (4)$$

As $d_\alpha = -\text{tg}\,\gamma \cdot d_\beta$, then it follows that

$$tg\gamma = -\frac{\beta}{\alpha}. \quad (5)$$

It should also be noted that the relative pitch K between the boundaries of risk categories (regions of values) in general can differ from the relative pitch k of a cell of the risk scale, and it depends on the d_β value.

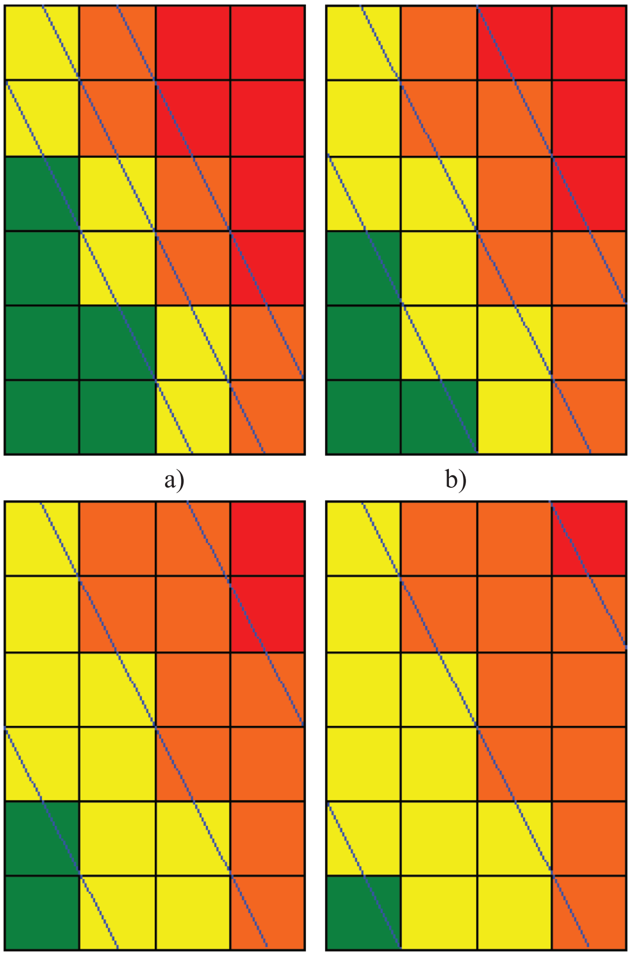
A separate task consists in the assignment of values to the scales of frequencies and consequences attached to the matrix cell boundaries. And the main requirement here is the provision of necessary ranges of frequencies and consequences.

Let us consider the application of the above guidelines exemplified by the construction of a typical risk matrix [1] with 4 risk categories and size of 6×4 ($N = 4$, $m = 6$, $n = 4$).

When selecting the slope ratio for R = const lines it is should be taken into account that in accordance with (2) the relative ranges of frequency and consequences of a matrix are equal to $\alpha \cdot m$ and $\beta \cdot n$.

As per (5), for the slope ratio of –0.5 the correlation $\alpha = 2 \cdot \beta$ will be observed. Considering (1) we get that in this case $\alpha = 2/3$ and $\beta = 1/3$, and relative ranges of frequency and consequences are – 4 and 1.333.

In case the slope ratio has been chosen as being equal to –2 we have: $\alpha = 1/3$, $\beta = 2/3$, relative ranges of frequency and consequences are – 2 and 2.667. The second variant is more preferable as the relative ranges of the scales differ less notably. That is why for further calculations we will use tg $\gamma = -2$.



a) b) c) d)

Fig. 6. Cells of 6×4 risk matrix with tg $\gamma = -2$ and d_β = 1; 1.5; 2; 2.5

With the application of (3) let us find the maximum value of the pitch d_β, with which the matrix of given dimensionality covers 4 risk categories:

$$d_\beta \le \frac{0,5 \cdot m + n - 2}{N - 2} = \frac{0,5 \cdot 6 + 4 - 2}{4 - 2} = 2,5.$$

The minimum value of d_β shall be taken as being equal to entity.

Figures 6 a-d show the matrix cells with the given parameters with $d_\beta = 1$, $d_\beta = 1.5$, $d_\beta = 2$ and $d_\beta = 2.5$ respectively. According to (2), relative pitches of the risk scales for the present matrices are equal to 0.667; 1; 1.333 and 1.667 respectively. As it appears from the graph in Fig. 5, precision of the results presentation (estimated by the probability of the point hitting the respective region of values) is 0.75; 0.833; 0.875 and 0.9 (see Fig. 5).

For the convenience of practical use the matrix shown in Fig. 6 b is more preferable. Distribution of the cells by the risk value ranges is steadier, and besides, as per (4), the relative pitch K of the matrix risk scale coincides with the relative pitch k of the cell risks. Thus for further application let us assume $d_\beta = 1.5$ and $K = k$.

Let us consider the assignment of numerical values to the scales of frequency and consequences for the matrix with the selected parameters provided that on the risk scale we set only R_{add} value, corresponding to the boundary between the risk categories «undesirable» and «unacceptable» (other limit values of risk categories are received with the use of K coefficient) (Figure 7).

To assign numerical values to the ranges of a matrix we need the samples of values of event frequencies and total amounts of consequences gained by several intervals of observation. It will give the opportunity to draw points on this matrix that correspond to a risk level for every selected interval of observation.

Relative range A of frequencies (considering margin factors a_{max} and a_{min}) is calculated by the formula:

$$A = \frac{a_{max} \cdot F_{max}}{\dfrac{F_{min}}{a_{min}}},$$

where F_{min}, F_{max} are the minimum and maximum values of frequency of an undesirable event I a sample.

Relative range B of consequences (considering margin factors b_{max} and b_{min}) is calculated by the similar formula:

$$B = \frac{b_{max} \cdot C_{max}}{\dfrac{C_{min}}{b_{min}}},$$

where C_{min}, C_{max} are the minimum and maximum values of the total amount of consequences of an undesirable event in a sample.

The required relative pitch of the risk scale is deduced from the provision that the region of this matrix includes the points corresponding to the minimum and maximum risk

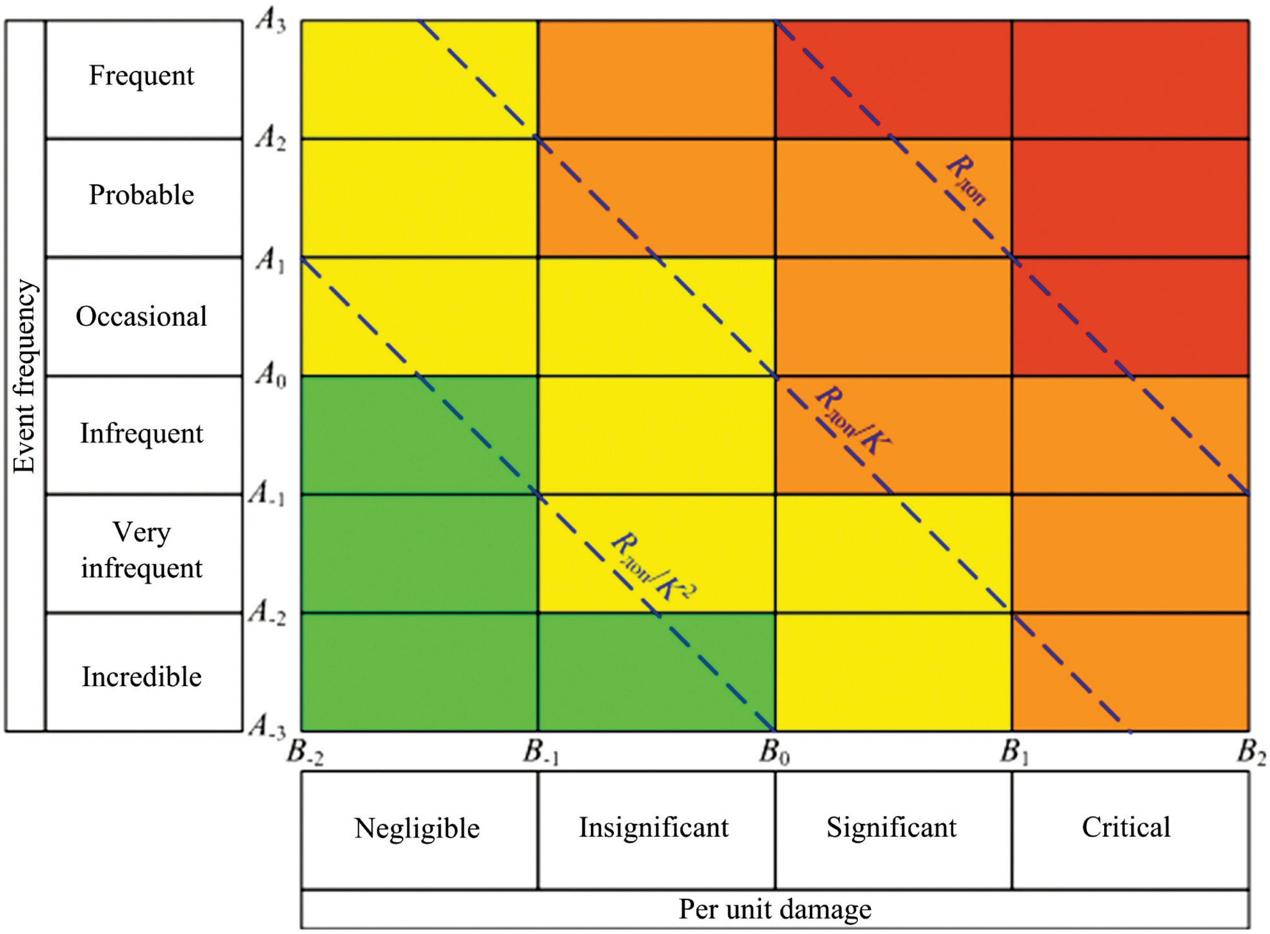


Fig. 7. 6×4 risk matrix with tg $\gamma = -2$ and $d_\beta = 1.5$

levels for the assigned samples of the values of frequencies and total amounts of consequences:

$$\begin{cases} K \geq \left(\dfrac{f_{max} \cdot C_{max} \cdot a_{max} \cdot b_{max}}{R_{äîï}} \right)^{\frac{1}{2\alpha+\beta}} \\ K \geq \left(\dfrac{f_{min} \cdot C_{min}}{R_{äîï} \cdot a_{min} \cdot b_{min}} \right)^{\frac{1}{-4\alpha-3\beta}} \end{cases},$$

where R_{add} is the acceptable risk level;

$\alpha = 1/3$ is the relative pitch of a frequency scale;

$\beta = 2/3$ is the relative pitch of a consequence scale.

Principles for defining margin factors should be considered separately. For test calculations the values $a_{max} = b_{max} = 2$ and $a_{min} = b_{min} = 1.5$ were used.

If coefficient K is selected by the first of two above mentioned conditions, the matrix scales are assigned beginning with maximum levels A_3 and B_2:

$$A_3 = F_{max} \cdot a_{max} \cdot \left(\frac{R_{доп} \cdot K^{(\alpha+\beta)}}{F_{max} \cdot C_{max} \cdot a_{max} \cdot b_{max}} \right)^{\alpha},$$

$$B_2 = C_{max} \cdot b_{max} \cdot \left(\frac{R_{доп} \cdot K^{(\alpha+\beta)}}{F_{max} \cdot C_{max} \cdot a_{max} \cdot b_{max}} \right)^{\beta}.$$

When choosing K under the second condition the assignment is done starting with minimum levels A_{-3} and B_{-2}:

$$A_{-3} = \frac{F_{min}}{a_{min}} \cdot \left(\frac{a_{min} \cdot b_{min} \cdot R \cdot K^{(-4\alpha-3\beta)}}{F_{min} \cdot C_{min}} \right)^{\alpha},$$

$$B_{-2} = \frac{C_{min}}{b_{min}} \cdot \left(\frac{a_{min} \cdot b_{min} \cdot R \cdot K^{(-4\alpha-3\beta)}}{F_{min} \cdot C_{min}} \right)^{\beta}.$$

Afterwards the other scale levels are defined by formulas:

$$A_{i-1} = A_i \cdot K^{(-\alpha)};\ A_{i+1} = A_i \cdot K^{\alpha};$$

$$B_{i-1} = B_i \cdot K^{(-\beta)};\ B_{i+1} = B_i \cdot K^{\beta}.$$

Linear coordinates of the point with frequency f and consequences c, in the form of displacement measured from the bottom left corner of matrix cells (see Figure 7), are calculated by formulas (width and height of the cell are taken as being equal to 1):

$$a = \frac{1}{\alpha} \log_k \left(\frac{f}{A_{-3}} \right);\ b = \frac{1}{\beta} \log_k \left(\frac{c}{B_{-2}} \right),$$

where *a* is the displacement on the frequency axis (vertically);

b is the displacement on the consequence axis (horizontally)

Therefore, the risk matrix gets constructed that helps to present the risk in a quantitative way with a controllable error.

The paper has investigated the main aspects of constructing a risk matrix affecting the error of results presentation. The application of these approaches has been exemplified by a particular case. Their extension to matrices with different parameters, the specification of the rules for construction of risk matrices and criteria for selection of their parameters are the subjects for further research.

References

1. GOST R 54505-2011 Functional safety. Risk management on railway transport.

2. **Pickering A., Cowley S.P.** Risk Matrices: implied accuracy and false assumptions // Journal of Health & Safety Research & Practice. – Volume 2, issue 1. October, 2010.

3. **Cox L. A. Jr., Huber W.** Optimal design of qualitative risk matrices to classify binary quantitative risks [abstract] // Proceedings of the Annual Meeting of the Society of Risk Analysis. Boston, December 7-10, 2008.

4. **Clemens P.L.** Working with the risk assessment matrix (lecture notes). 2nd edition. Tullahoma, TN: Sverdrup Technology, Inc., 1993.

5. A risk matrix for risk managers. – The National Patient Safety Agency. London, January, 2008.

6. Risk assessment in audit planning. A guide for auditors on how best to assess risks when planning audit work // Internal Audit Community of Practice. http://www.pempal.org/data/upload/files/2014/09/rap_guide_eng.pdf.

7. **L.Cox.** Whats wrong with risk matrices? Risk analysis. Vol. 28. №2, 2008. Pp. 497-511.