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PROBABILISTIC ASPECTS OF RELIABILITY OF ELECTRICAL INSULATION

The mathematical model of electrical insulation failure is proposed. The model is based on the superposition of uniform and exponential probability distributions. Relation for time dependence of the insulation failure rate is obtained. Application of this relation provides correct analytical description of insulation reliability on a long time interval.

Keywords: *reliability, failure rate, electrical insulation*

Introduction

At present, the Weibull-Gnedenko probabilistic distribution is used for quantitative assessment of electrical insulation reliability [1 – 3]. Yet, this distribution can only be applied to separate periods of insulation service (break-in period, normal service or final wear), while the period of normal service has a time-independent failure rate uncharacteristic for "aging" objects.

To estimate the reliability of insulation on a long time interval, the superposition of single laws of distributions is used. Application of compound distributions leads to increase of the number of its parameters, whose definition is usually related to considerable mathematical difficulties. Owing to this, of vital importance is to construct adequate mathematical models [4] based on application of optimum probabilistic laws of distributions ensuring a close analytical description of insulation reliability on a long time interval. Below we propose an alternative model of failure of electrical insulation based on the combination of the uniform and exponential laws of probability distributions.

Theoretical analysis and its results

Technically considering an insulator as an object that functions till the first failure (breakdown), we can characterize the reliability of insulation as the function of failure rate [5, p. 10]

$$F(t) = \frac{\phi(t)}{1 - \Phi(t)}, \quad (1)$$

where t is the time, $\phi(t)$ is the density of failure rate probability distribution, $\Phi(t)$ is the function of failure rate probability distributions.

The value $F(t)dt$ is the probability of the system element with time to failure t failing on the time interval $[t;t+dt]$. Practically failure rate is defined as the relation of the number of elements failed by the moment t per time unit to the total number of such elements remained in upstate till the moment t.

The failure of electrical insulation will be considered as the superposition of a sudden failure and a gradual failure as a result of electrical wear. Other factors of insulation aging will not be taken into account here assuming that it functions under normal conditions.

The ideal sudden failure features "memory lack" – prior use of a device doesn't impact the residual time of its failure-free service. While there is an abrupt degradation of insulation properties in time caused by severe influence of strong destructive factors. In case of even wear characterized by gradual degradation of insulation properties, the probability of failure uniformly increases in time. The above assumption simulates the failure of electrical insulation by superimposing these two event flows: actual failure (in case of voltage surge or other random event) is the more probable, the more worn the insulation is. Therefore, functions $\Phi(t)$ and $\phi(t)$ in formula (1) in case under consideration will express themselves by linear combinations of respective functions related to probabilistic distributions of sudden failures and even wear.

Ideal sudden failures are described by an exponential distribution [6, pp. 133 – 134]:

$$\phi_1(t) = t_0^{-1} e^{-t/t_0}; \ \Phi_1(t) = 1 - e^{-t/t_0}, \tag{2}$$

where t_0 is the mathematical expectation of time to sudden failure under specified conditions.

Even wear is described by a uniform (rectangular) distribution [6, p. 209]:

$$\phi_2(t) = \begin{cases} \dfrac{1}{b-a}, & a \le t \le b; \\ 0, & t \notin [a;b]; \end{cases} \quad \Phi_2(t) = \begin{cases} 0, & t < a; \\ \dfrac{t-a}{b-a}, & a \le t \le b; \\ 1, & t > b. \end{cases} \tag{3}$$

In our case a=0 that corresponds to the moment of putting insulation into service, b=t_E is the partial time of total wear of insulation in the absence of sudden failures.

Superposition of probability laws (2) and (3) gives:

$$\phi(t) = \frac{c_w}{t_E} + (1 - c_w)\frac{e^{-t/t_0}}{t_0};$$

$$\Phi(t) = \frac{c_w t}{t_E} + (1 - c_w)(1 - e^{-t/t_0}), \tag{4}$$

where multiplier c_w defines the share contribution of even wear under specified conditions of service. Substituting expressions (4) in formula (1), we finally have relation

$$F(t) = \frac{c_w + (1-c_w)\dfrac{t_E}{t_0} e^{-t/t_0}}{t_E[1 + (1-c_w)(e^{-t/t_0} - 1)] - c_w t}, \tag{5}$$

that allows simulating actual failures of electrical insulation in service. Taking into account that $F(t)\big|_{t \to \bar{t}} \to \infty$, by using expression (5) we can define the time to failure of insulation:

$$\bar{t} = t_E \left(1 + \frac{1 - c_w}{c_w} e^{-\bar{t}/t_0} \right).$$

For $\bar{t}/t_0 \ge 8$ with accuracy better than 1.5 per cent, we can assume that $\bar{t} \approx t_E$.

Function graph (5) looks like trough (see Fig. 1). It distinctly shows three time intervals: break-in period I, normal service period II and final wear period III.

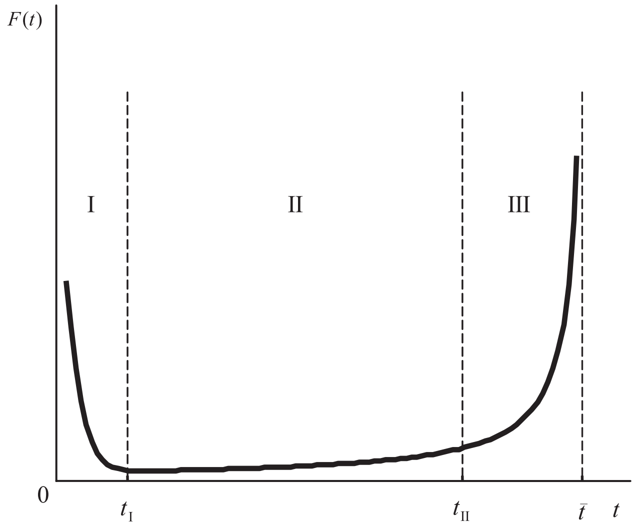

Fig. 1. Function of failure rate

On interval I failure rate abruptly decreases in time. The duration of this period is short and is defined by the quality of insulation manufacturing [7]. In the majority of cases this period is not realized in practice due to the fact that insulation break-in can only be related to the "burn-in" phenomenon, i.e. discarding initially defective samples of insulation that have admittedly lower longevity compared to samples remained in service [3, p. 54].

Interval II corresponds to normal service and is characterized by a feebly increasing failure rate. At the beginning of service F(t) is practically constant, while as t increases, the failure rate grows, which implies the development of gradual failures caused by natural ageing.

Time interval III corresponds to the end of life time of electrical insulation: its failure rate radically grows in time. Abrupt degradation of insulation physical and chemical properties is stipulated by intensification of ageing and wearing processes. Insulation service is not acceptable at this stage.

Compared to the classical Weibull-Gnedenko distribution, the distribution proposed here has two advantages.

1) The entire life time of insulation service is covered.

2) The period of normal service has a gradually increasing failure rate natural for "normally ageing" objects.

In Fig. 2 dependences F(t) calculated under formula (5) for mid-voltage power cables are correlated with experimental data [8, 9].

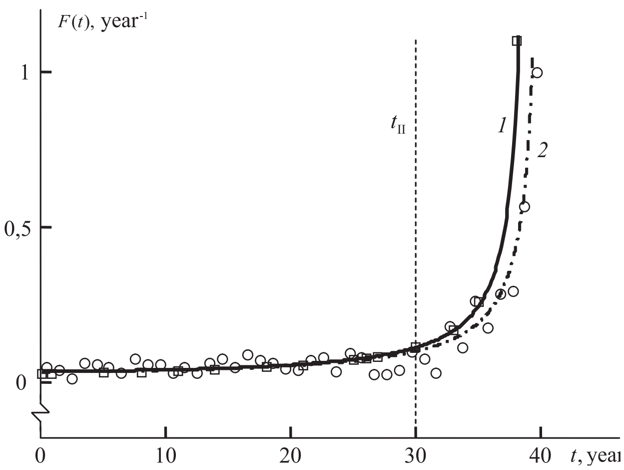

Fig. 2. Time dependences of failure rate function: □□□ – power cables [8]; ○○○ – insulation of power cables [9]

Calculations were made for the following values: c_w = 0,89, t_0 = 10 years, t_E = 39 years, $\overline{t} = 39{,}097$ years (for dependency *1*) and c_w = 0,872, t_0 = 13 years, t_E = 40 years, $\overline{t} = 40{,}265$ years (for dependency *2*). Maximum deviation of theoretical calculations from experimental data in the first case was 0.135 per cent, in the second case – 6.344 per cent.

Comparatively high values of parameter c_w indicate the prevalence of even wear of insulation over sudden failures. In both cases there is no break-in period, while the period of normal service is 30 years. The average failure rate of cables for the period of normal service is 0.05 year^{-1}.

Conclusion

The paper proposed a mathematical model of electrical insulation failure based on the superposition of uniform and exponential probability distributions. The model gave analytical relations for calculating key qualitative parameters of insulation reliability: functions of failure probability distributions, densities of this function and functions of failure rates. Application of this relation provides correct analytical description of insulation reliability on a long time interval.

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