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APPLICATION OF GENERAL SOLUTION FOR A SYSTEM OF LOGICAL EQUATIONS IN DEPENDABILITY TASKS

The article proposes a new method of solving logical equations with one or more unknown variables and systems of logical equations that use a modified truth table. This method allows finding all general solutions. Solvability conditions of equation systems have been found. The theory has been illustrated by examples. Technical applications have been shown.

Keywords: *dependability, logical model of dependability, logical and probabilistic analysis of dependability, general solution of logical equation systems, modified truth table, indicator, solvability conditions.*

1. Introduction

For systems with a complex structure that is not brought down to serial-parallel networks, a formal description of operability conditions is required. Application of the procedure of system availability logical functions (SHLF) generation to this end by listing shortest paths of successful functioning (SPSF) and corresponding disjuncts results in notation of SHLF in disjunctive normal form (DNF). This method is acceptable for systems with low complexity, but is hardly usable in the case of systems with even medium complexity (if the number of elements exceeds ten). Thus, according to [1], in an electric power system of 15 elements the number of SPSF reaches several hundreds. For more complex systems, the notation of SHLF as BTA becomes practically impossible.

The solution is simplified if the operability conditions are listed with logical equation systems (LES). The solution of LES by means of special methods results in a multi-bracket equation and a significantly more compact SHLF formula. Currently, there are a number of solutions of logical equation systems. For non-homogenous linear LES with constant or variable coefficients, the determinant method is used [2], which allows for the following specific solution:

$$y=f(X)y_0,$$

where y_0 is the indicator of constant terms of a non-homogenous equation system.

For $y_0 = 0$ the specific solution corresponds to homogenous LES and is zero. In [3] the authors set forth three more methods of deducing specific LES solution: substitution, reduction to one equation and matrix method. Ways of successfully using the determinant and other method of deducing specific solutions are shown in [4, 5].

A specific solution of non-homogenous LES in a number of cases is insufficient or causes an incorrect reflection of all conditions of successful operation of a technical system. Accordingly, attempts have been made to eliminate the drawbacks of the specific solution and find the general LES solution. In [3] the authors describe two methods of deducing the general solution of non-homogenous LES, namely the substitution method and method of reduction to one equation with n unknown values. The general solution looks as follows:

$$y = f(X) y_0 \vee g(X) y_c, \qquad (1)$$

where y_0 is the vector of unknown logic functions; f and g are known functions that depend on equation coefficients; y_c is the arbitrary function of logical algebra; V is for disjunction operation. If $y_0 = 0$, homogenous system solution is found. Homogenous LES are solved by substitution or transformation into non-homogenous LES.

Upon generation of a general solution of the form (1) certain difficulties arise. First, there are no recommendations as to the choice of random LAF y_c, although the results of dependability evaluation depend on that choice. Also, according to [3], there is only one solution of type (1). In fact, there may be a number of general solutions. But only one of them can fulfill special conditions of system operation. In order to find it, the full spectrum of general solutions must be available. There are now methods to do so. Insufficiently elaborate are also the methods of solving one equation with several unknown values that can also have several general solutions.

Upon generation of a general solution one more difficulty arises. For LES, the solution has the form

$$y_i = f_{1i}(X) \vee f_{2i}(X) y_c, i = 1,...n.$$

If we suppose that here y_c are identical for all functions y_i, we will only deduce one general solution. If they are different for different y_i, then the question arises as to how to choose one.

Finally, there is the problem of logical correctness of the notation of the equation system and its solvability. In papers dedicated to the methods of LES solution this issue is generally avoided and it is by default assumed that a solution always exists and the system is solvable. In reality, that is not always the case. Equations and equation systems may not have a single general solution. Attempts to formally use known methods in such cases result in non-interpretable or simply absurd results. Therefore, at first it must be made sure that the LES has at least one general solution and the conditions of its solvability must be identified.

There is one more problem. For some classes of technical systems both specific and general solutions [3] can be unacceptable because they do not reflect some significant features of their operation.

In particular, that happens if the system includes feedback loops that are typical to some technical systems. Such loops are often used in information, electrical power, process, transportation, telecommunication and other systems. The common feature of all the above systems is that they perform transmission (transportation) or transformation of substances (information, electric power, energy material, etc.) and contain a "feeding" element (source of information, generator, energy carrier collector). Therefore, successful operation of such systems ensuring the operability of a certain group of components is not enough. Conditions must be created for successful transportation of or transformation of the substance for its delivery form input (the "feeding" component) to the output of the system via a direct channel. The feedback loop is one of the means of ensuring such conditions. In information and telecommunication systems there are loops for transmission of service information from receiver to transmitter. In electrical energy systems there are loops for own consumption of power. In closed-looped process systems with wasteless use of carrier, substances create the loop of used carrier (e.g., steam) transformation and delivery of recovered carrier to the system's input. Other special conditions of operation (SCO) can be formulated, for instance, the presence of at least on "feeding" component in the SPSF.

If a specific LES solution is used, an undesired absorption of the feedback loop components by direct channel components may take place. That causes the distortion of real conditions of operation and incorrect evaluation of dependability. If general solution [3] is used, the "feeding component" is often lost.

In [7] the authors set forth an analytical method of deducing the general solution of Boolean equations. That method is universal, but it does not suggest any constructive rules, general solution algorithms and selection of the alternative solution that full corresponds to the physical essence of the technical system reflected in the logical dependability model.

The results set forth below are based on the use of modified truth table and allow partially overcoming the above difficulties.

2. General solution of equation with one unknown value

$$A_1(X) \vee A_2(X) y \vee A_3(X) y' = \\ = A_4(X) \vee A_5(X) y \vee A_6(X) y', \qquad (2)$$

where $X = (x_1, x_2, \ldots, x_n)$ is the vector of independent Boolean variable, indicators of system components operability; $A_i(X)$ are the known functions of the vector argument; y' is the negation of y.

Let us transform (2) into canonical form without loss of roots. In order to do that, we must transit from Boolean basis to Zhigalkin's basis. By orthogonalizing the summands in (2) and replacing the disjunction operation with exclusive OR (module 2 addition) we deduce

$$A_1 \oplus A_1' A_2 y \oplus A_1' A_3 y' = A_3 \oplus A_4' A_5 y \oplus A_4' A_6 y'. \qquad (3)$$

By adding from left and right the right part of (3) and using the the equality $y \oplus \oplus y = 0$ we deduce

$$(A_1 \oplus A_4) \oplus (A_1' A_2 \oplus A_4' A_5) y \oplus (A_1' A_3 \oplus A_4' A_6) y' = 0.$$

Searching through various combinations of A_i we deduce a modified truth table for function y that consists of three groups of lines: 1) y is completely defined, 2) y is not defined, i.e. its value is indifferent.

(0 V 1), 3) there is no solution, as (2) is not realized in case of any value of y (table 1). Values A_i are not independent, as their are a function of vector X. Therefore, strictly speaking, some combinations of values A_i (of the vector in table 1) may prove to be impossible at any value of vector X.

Table 1. Modified truth table

№ Item	$A_1 \ldots A_6$	Form of the equation	y	Group	№ Item	$A_1 \ldots A_6$	Form of the equation	y	Group
0	000000	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2	32	100000	$1 \oplus 0y \oplus 0y' = 0$	No	3
1	000001	$y' = 0$	1	1	33	100001	$1 \oplus y' = 0$	0	1
2	000010	$y = 0$	0	1	34	100010	$1 \oplus y = 0$	1	1
3	000011	$y \oplus y' = 0$	No	3	35	100011	$1 \oplus y \oplus y' = 0$	0 V1	2
4	000100	$1 \oplus 0y \oplus 0y' = 0$	No	3	36	100100	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
5	000101	$1 \oplus 0y \oplus 0y' = 0$	No	3	37	100101	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
6	000110	$1 \oplus 0y \oplus 0y' = 0$	No	3	38	100110	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
7	000111	$1 \oplus 0y \oplus 0y' = 0$	No	3	39	100111	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
8	001000	$y' = 0$	1	1	40	101000	$1 \oplus 0y \oplus 0y' = 0$	No	3
9	001001	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2	41	101001	$1 \oplus y' = 0$	0	1
10	001010	$y \oplus y' = 0$	No	3	42	101010	$1 \oplus y = 0$	1	1
11	001011	$y = 0$	0	1	43	101011	$1 \oplus y \oplus y' = 0$	0 V1	2
12	001100	$1 \oplus y' = 0$	0	1	44	101100	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
13	001101	$1 \oplus y' = 0$	0	1	45	101101	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
14	001110	$1 \oplus y' = 0$	0	1	46	101110	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
15	001111	$1 \oplus y' = 0$	0	1	47	101111	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
16	010000	$y = 0$	0	1	48	110000	$1 \oplus 0y \oplus 0y' = 0$	No	3
17	010001	$y \oplus y' = 0$	No	3	49	110001	$1 \oplus y' = 0$	0	1
18	010010	0⊕0y⊕0y'=0	0 V1	2	50	110010	$1 \oplus y = 0$	1	1
19	010011	$y' = 0$	1	1	51	110011	$1 \oplus y \oplus y' = 0$	0 V1	2
20	010100	$1 \oplus y = 0$	1	1	52	110100	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
21	010101	$1 \oplus y = 0$	1	1	53	110101	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
22	010110	$1 \oplus y = 0$	1	1	54	110110	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
23	010111	$1 \oplus y = 0$	1	1	55	110111	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
24	011000	$y \oplus y' = 0$	No	3	56	111000	$1 \oplus 0y \oplus 0y' = 0$	No	3
25	011001	$y = 0$	0	1	57	111001	$1 \oplus y' = 0$	0	1
26	011010	$y' = 0$	1	1	58	111010	$1 \oplus y = 0$	1	1
27	011011	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2	59	111011	$1 \oplus y \oplus y' = 0$	0 V1	2
28	011100	$1 \oplus y \oplus y' = 0$	0 V1	2	60	111100	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
29	011101	$1 \oplus y \oplus y' = 0$	0 V1	2	61	111101	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
30	011110	$1 \oplus y \oplus y' = 0$	0 V1	2	62	111110	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2
31	011111	$1 \oplus y \oplus y' = 0$	0 V1	2	63	111111	$0 \oplus 0y \oplus 0y' = 0$	0 V1	2

Table 1 data shows that out of 64 combinations of values A_i in 24 combinations function y is defined (group 1), including as one in 12, in 28 the combination is not defined (group 2) and in 12 the system (2) is unsolvable (group 3).

Using the modified truth, a solution is generated based on the following rules:

1. To each set A_i, where y=1, constituent 1 (total of up to 12 constituents) is put into correspondence.

2. To each set of group 2 constituent 1 is put into correspondence. The constituent is multiplied by indicator R that can take the values 0 or 1.

3. Sets of group 3 are not taken into consideration.

Thus, the general solution can include only up to 40 constituents 1 with or without indicator

$$y = (\vee_{i \in M_1} K_i) \vee (\vee_{i \in M_2} K_i R_i), \tag{4}$$

where M_1 = (1, 8, 19, 20, 21, 22, 23, 26, 34, 42, 50, 58), M_2 = (0, 9, 18, 27, 28, 29, 30, 31, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63). Constituents 1 with and without indicator, in an explicit form, an be represented with the following formulas

$$K = A_1^{\alpha 1}(X) A_2^{\alpha 2}(X) \ldots A_6^{\alpha 6}(X),$$

$$K^c = KR = R(A_1^{\alpha 1}(X) A_2^{\alpha 2}(X) \ldots A_6^{\alpha 6}(X)). \tag{5}$$

They are some function of vector X and can be represented in DNF or perfect DNF (PDNF). Indicator R in (5) is considered to be a vector, while product KR a scalar product. Then, (5) can be represented as

$$K^c(X) = k_c^1 r_1 \vee k_c^2 r_2 \vee \ldots \vee k_c^m r_m,$$

where k_c^j is constituent 1 relating to vector X; r_j is an indicator function; m is the number of constituents 1 in SDNF.

By searching through the possible values of indicator functions we deduce 2^m various possible solutions each of which must be verified as fake root are possible. Verification is performed by means of substitution into (2).

Logical function (4) is minimizable. It is not difficult to identify that out of 40 constituents 27 constituents 1 are not simplifiable, while in 13 of them a generalized gluing is possible after a certain grouping, as shown below: K_0 V K_1, K_8 V K_9^c , K_{18}^c V K_{19}, K_{20} V K_{21} V K_{22} V K_{23}, K_{19} V K_{23} , K_{26} V K_{27}^c , K_{34} V K_{42} V K_{50} V K_{58}. After gluing the minimal DNF (MDNF) contains 34 summands (instead of 40 in PDNF). The simplification rate that is evaluated based on the proportion of letters in MDNF and SDNF is 0.925. The problem-solving algorithm using MDNF includes the following stages:

1. Based on the notation of the equation (2) explicit expressions for $A_i(X)$ are established.
2. $A_i(X)$ is substituted into MDNF and y is set down in explicit form form using indicator functions.
3. In each disjunct K_i^c K_i is transformed into SDNF set down scalar product K_iR with indicator variables r_j.
4. All possible values of r_j are searched through and all possible solutions are formulated.
5. Each possible solution is verified by substituting into the initial equation.
6. After the selection a set of solution is generated, one of which is specific, and all the others are general, including one according to [4] with indicator functions identical or equal to 1.

In order to find the solution of equation

$$(x_1 \vee x_2)y \vee y' = 1, \qquad (6)$$

it must be taken that $A_1 = A_6 = 1$, $A_2 = A_3 = A_4 = 0$, $A_5 = x_1$ V x_2 . The available sets 33 and 35 fall into groups 1 and 2. As in set 33 the value y=0, then the range M_1 in formula (4) is empty and the solution is

$$y = K_c = KR = x_1x_2'r_1 \vee x_1x_2r_2 \vee x_1'x_2r_3,$$

where $K = A_5 = x_1$ V $x_2 = x_1x_{2'}$ V x_1x_2 V $x_{1'}x_2$. For eight values of vector R there are eight values: 0, $x_{1'}x_2$, x_1x_2, x_2, $x_1x_{2'}$, $x_{1'}x_2$ V $x_1x_{2'}$, x_1, x_1 V x_2 , including the zero solution. That is the specific solution, while all the others are general, one of which corresponds to [3]. Substituting those values in (6) shows that all of them are equation roots. The following probabilities correspond to the general solutions $P_c = P\{y = 1\}$: q_1p_2, p_1p_2, p_2, p_1q_2, $q_1p_2+p_1q_2$, p_1, $1–q_1q_2$. However, not all roots are always true. For example, if instead of (6) we take the equation

$$(x_1x_2)y \vee x_2'y' = 1, \qquad (7)$$

we deduce four constituents 1, two of which are part of solution (4):

$$y = K_{34} \vee K_{35} = A_5A_6' \vee A_5A_6, A_5 = x_1x_2, A_6 = x_2'.$$

As we can see, there is no set X, where $K_{35} = 1$, while root $A_5A_{6'} = x_1x_2$ is wrong. That is the reason equation (7) has no solution.

In order to solve the equation: $y = x_1x_2$ V $x_{2'}x_3\,y$, we must take $A_1 = A_3 = A_6 = 0$, $A_2 = 1$, $A_4 = x_1x_2$, $A_5 = x_{2'}x_3$. According to table 1, to these A_i correspond four constituents 1 with numbers 16, 18, 20 and 22. According to table 1, in set 16 the function equals to zero. The other three are part of solution of form (4)

$$y = K_{18}R_{18} \vee K_{20} \vee K_{22} = x_1x_2' \vee x_2'x_3R_{18}.$$

After transition to K_{18} to SNDF and multiplication by R_{18} we deduce:

$$y = x_1x_2 \vee x_1'x_2'r_1 \vee x_1x_2'x_3r_2.$$

By searching through indicator values we will find four solutions: $y_1 = x_1x_2$, $y_2 = x_1x_2$ V $x_{1'}x_{2'}x_3$, $y_3 = x_1x_2$ V x_1x_3, $y_4 = x_1x_2$ V $x_{2'}x_3$. Verification shows that there are now false roots. Solution y_1 is specific, solution y_4 according to [3]. Two more general solutions complement the full spectrum of solutions. The following probabilities correspond to those solutions:

$$P_{c1} = p_1p_2, P_{c2} = p_1p_2 + q_1q_2p_3, P_{c3} = p_1(1 - q_2q_3),$$
$$P_{c4} = p_1p_2 + q_1p_3.$$

Of practical interest is finding the solvability (or insolvability) conditions of an equation system. According to table 1, there are 12 sets of values A_i numbered 3, 4, 5, 6, 7, 10, 17, 24, 32, 40, 48, 56 that do not correspond to a solution. Ten out of them contain two or three ones. Five types of unsolvable equations correspond to those

$$By \vee Gy' = 0, B \vee Gy' = 0, B \vee Gy =$$
$$= 0, B \vee Gy \vee Cy' = 0, Gy' \vee By = 0. \qquad (8)$$

The first equation corresponds to constituents 1 numbered 3 and 24, the second numbered 5 and 40, the third numbered 6 and 48, the forth numbered 7 and 56, the fifth numbered 10 and 17. The equation is not solvable if functions $B(X)$, $C(X)$ and $G(X)$ under certain sets X can simultaneously take on the value of 1. If that is not the case, then those equations can have solutions too.

Out of (8) can be deduced the solvability condition. After performing a generalized gluing in the first equation (8) based on y we deduce:

$$By \vee Gy' \vee BG = 0.$$

For the first equation of (8) to execute, orthogonality of functions B and G is required in all sets of argument values

$$B(X)G(X) = 0. \qquad (9)$$

Out of the second, third and fifth equations of (8) we also deduce condition (9). Out of the forth equation by means of three generalized gluing operations we deduce

$$B \vee Gy \vee Cy' = B \vee Gy \vee Cy' \vee BG \vee BC \vee CG.$$

Therefore, the solvability condition

$$B(X)G(X) \vee B(X)C(X) \vee C(X)G(X) = 0.$$

This condition absorbs condition (9). Thus, in order for the equations of type (8) to y solvable, functions $B(X)$, $C(X)$ and $G(X)$ must be orthogonal in all sets X.

3. General solution of one equation with n unknown values

For all unknown values the equation is of the form

$$F(X, y_1, y_2) = B(X, y_1, y_2).$$

Let us select in F and B the part that is not connected to y_1 and cut by y_1 in the remaining part

$$C_1(X, y_2) \vee C_2(X, y_2)y_1 \vee C_3(X, y_2)y_1' =$$
$$= C_4(X, y_2) \vee C_5(X, y_2)y_1 \vee C_6(X, y_2)y_1'. \qquad (10)$$

This equation is identical in form with (2) if $A_i = C_i(X, y_2)$ и $y = y_1$. Deeming (10) solvable for y_1, we deduce a solution according to the rules of the previous class:

$$y_1 = F(X, R, y_2). \quad (11)$$

Let us express (11) as

$$y_1 = G_0(X, R_0) \vee G_1(X, R_1) y_2 \vee G_1(X, R_2) y_2^{'}), \quad (12)$$

where R_0, R_1, R_2 are vectors of indicator functions. As in (12) the second and the third summands are orthogonal, the disjunction operation can be replaced with a modulo 2 addition, and then perform an orthogonalization of the first and remaining summands and proceed to a Zhegalkin's basis

$$y_1 = G_0 \oplus G_0^{'}(G_1 y_2 \oplus G_2 y_2^{'}). \quad (13)$$

Using (13) let us generate a modified truth table (table 2). In sets 0, 3, 4, 5, 6 and 7 value y_1 is defined unambiguously (group 1), in sets 1 and 2 functions y_1 and y_2 have two alternative values.

Table 2. Modified truth table

№ Item	$G_0G_1G_2$	Form of the equation	y_1	y_2	Group
0	000	$0 \oplus 0 \oplus y_1 = 0$	0	0V1	1
1	001	$y_{2'} \oplus y_1 = 0$	0 1	1 0	2
2	010	$y_2 \oplus y_1 = 0$	0 1	0 1	2
3	011	$1 \oplus y_1 = 0$	1	0V1	1
4	100	$1 \oplus y_1 = 0$	1	0V1	1
5	101	$1 \oplus y_1 = 0$	1	0V1	1
6	110	$1 \oplus y_1 = 0$	1	0V1	1
7	111	$1 \oplus y_1 = 0$	1	0V1	1

By combining alternatives we deduce four solutions according the truth tables (table 3).

Table 3. Truth table

№ Item	$G_0G_1G_2$	Solution 1		Solution 2		Solution 3		Solution 4	
		y_1	y_2	y_1	y_2	y_1	y_2	y_1	y_2
0	000	0	0 V 1	0	0 V 1	0	0 V 1	0	0 V 1
1	001	0	1	0	1	1	0	1	0
2	010	0	0	1	1	0	0	1	1
3	011	1	0 V 1	1	0 V 1	1	0 V 1	1	0 V 1
4	100	1	0 V 1	1	0 V 1	1	0 V 1	1	0 V 1
5	101	1	0 V 1	1	0 V 1	1	0 V 1	1	0 V 1
6	110	1	0 V 1	1	0 V 1	1	0 V 1	1	0 V 1
7	111	1	0 V 1	1	0 V 1	1	0 V 1	1	0 V 1

Let us find those solutions by introducing for y_2 in 6 sets indicator functions P_i :

$$y_1 = G_0 \vee G_1G_2, y_2 = \\ = K_0P_0 \vee K_1 \vee K_3P_3 \vee K_4P_4 \vee K_5P_5 \vee K_6P_6 \vee K_7P_7, \quad (14)$$

$$y_1 = G_0 \vee G_1G_2, y_2 = \\ = K_0P_0 \vee K_1 \vee K_2 \vee K_3P_3 \vee K_4P_4 \vee K_5P_5 \vee K_6P_6 \vee K_7P_7, \quad (15)$$

$$y_1 = G_0 \vee G_1G_2, y_2 = \\ = K_0P_0 \vee K_3P_3 \vee K_4P_4 \vee K_5P_5 \vee K_6P_6 \vee K_7P_7, \quad (16)$$

$$y_1 = G_0 \vee G_1 \vee G_2, y_2 = \\ = K_0P_0 \vee K_2 \vee K_3P_3 \vee K_4P_4 \vee K_5P_5 \vee K_6P_6 \vee K_7P_7. \quad (17)$$

In order to find the solution of equation:

$$x_1^{'}y_1 \vee x_1y_1^{'} \vee x_2y_1y_2^{'} = 0,$$

let us transform the equation into (10):

$$y_1(x_1^{'} \vee x_1y_1^{'}) \vee x_1y_1^{'} = 0. \quad (18)$$

Therefore $C_1 = C_2 = C_3 = C_4 = 0$, $C_5 = x_{1'} \vee x_2y_{2'}$, $C_6 = x_1$. According to table 1, out if four constituents 1 (K_0, K_1, K_2 , K_3) are eliminated, as for K_2 we have: y_1=0, and K_3 is part of group 3. For K_0 and K_1 we have

$$y_1 = A_5^{'}A_6^{'}R_0 \vee A_5^{'}A_6 = (x_1^{'} \vee x_1y_1^{'})^{'}(x_1^{'}R_0 \vee x_1). \quad (19)$$

After a simple transformation (19) is reduced to

$$y_1 = x_1(x_2^{'} \vee y_2). \quad (20)$$

By comparing (20) and (12) we deduce: $G_0 = x_1x_{2'}$, $G_1 = x_1$, $G_2 = 0$, $K_1 = K_3 = K_5 = K_7 = 0$.

According to (14)

$$y_1 = G_0 \vee G_1G_2 = x_1x_2^{'}, y_2 = x_1^{'}P_0 \vee x_1x_2^{'}P_6. \quad (21)$$

For verification, let us substitute (21) into (18).

$$(x_1^{'} \vee x_2y_2^{'})x_1x_2^{'} \vee x_1(x_1x_2^{'})^{'} = x_1x_2 \neq 0.$$

The left part of the equation is not equal to the right one. Therefore, the first solution is not the root of the equation.

According to (15) solution 2 is as follows:

$$y_1 = x_1, y_2 = y_2 = x_1x_2 \vee x_1^{'}x_2^{'}P_1 \vee x_1^{'}x_2P_2 \vee x_1x_2^{'}P_3.$$

Here, it is required to search through the possible values of indicators P_i and deduce eight solutions for y_2:

$$y_2 = (x_1x_2, x_1, x_2, x_1 \vee x_2, x_1 \oplus x_2, x_1 \vee x_2^{'}, x_1^{'} \vee x_2, 1). \quad (22)$$

The verification shows that all solutions are roots of equation (18).

According to (16) solution 3 for y_1 is identical to solution (21). Therefore, it is not the root of the equation. Solution 4 is identical to solution 2. Finally, we have: y_1=x_1, and y_2 we take from (22). Solution (22) corresponds to probability

$$P_2 = P\{y_2 = 1\} = \\ = (p_1p_2, p_1, p_2, 1 - q_1q_2, p_1q_2 + q_1p_2, 1 - q_1p_2, 1 - p_1q_2, 1).$$

Let us consider the general case when one equation has n unknown values:

$$F_1(X, y_1, ...y_n) = 0. \quad (23)$$

Cutting in (23) by y_n gives

$$y_nF_1(X, y_1, ...y_{n-1}, 1)K_0P_0 \vee K_1y_n^{'}F_1(X, y_1, ...y_{n-1}, 0) = 0. \quad (24)$$

After comparing (24) with (8) we see that (24) falls in the first type of unsolvable equations. To make it solvable, condition (9) must be fulfilled:

$$F_1(X, y_1, ...y_{n-1}, 1)F_1(X, y_1, ...y_{n-1}, 0) = \\ = F_2(X, y_1, ...y_{n-1}) = 0. \quad (25)$$

By comparing (25) and (10) we deduce: $C_1 = C_2 = C_3 = C_4 = 0$, $C_5 = F_1(1)$, $C_6 = F_1(0)$. According to table 1 let us write

$$y_n = K_0P_0 \vee K_1 = C_5^{'}(R_0 \vee C_6) = \\ = F_1^{'}(X, y_1, ...y_{n-1}, 1)(R_0 \vee F_1(X, y_1, ...y_{n-1}, 0)). \quad (26)$$

Out of (25) we deduce

$$y_{n-1}F_2(X,y_1,...y_{n-2},1)\vee y'_{n-1}F_2(X,y_1,...y_{n-2},0)=0.$$

Then, solvability condition is generated again. By repeating operations n 3 times we deduce

$$y_3F_{n-2}(X,y_1,y_2,1)\vee y'_3F_{n-2}(X,y_1,y_2,1)=0.$$

From here, as in (26), we have

$$y_3=F'_{n-2}(X,y_1,y_2,1)(R_{n-2}\vee F_{n-2}(X,y_1,y_2,0)). \quad (27)$$

Next

$$F_{n-1}(X,y_1,y_2)= \\ =F_{n-2}(X,y_1,y_2,1)F_{n-2}(X,y_1,y_2,0)=0. \quad (28)$$

Solution (28) has been found earlier and presented with formulas (14)–(17). Knowing that y_1 and y_2, we deduce using formula (27) y_3, then from bottom to top all other unknown values up to y_n. While performing this procedure the possibility of false roots must be taken into consideration. Therefore, at each step of the bottom-up motion verification must be performed by substituting the solution in the respective equation.

4. Logical equation system

Logical equation system has the form of

$$y_k=f_k(X,y_1,y_2,...,y_m),k=1,...,m. \quad (29)$$

Let us transform (29) into the canonical form

$$y_k\oplus f_k=y_kf'_k\vee y'_kf_k= \\ =F_k(X,y_1,y_2,...,y_m),k=1,...,m. \quad (30)$$

The LES solution algorithm (30) consists of $2m$-1 steps.

Step 1. The first equation ($k=1$) of system (30) is solved for y_1 to find the function of X and $y^1=(y_2, y_3, \dots, y_m)$:

$$y_1=W_1(X,y^1),F_k(X,W_1(X,y^1),y_2,...,y_m)= \\ =F_k^{(1)}(X,y^1)=0,k=2,...,m. \quad (31)$$

Step 2. Out of the first equation of (31) solution is found for y_2 and substituted in the rest of the equations

$$y_2=W_2(X,y^2),F_k^{(2)}(X,y^2)= \\ =0,k=3,...,m,y^2=(y_3,y_4,...,y_m). \quad (32)$$

Step m–1. Successive elimination from the equation system of the unknown values $y_1, y_2, \dots, y_{m-1}$ results in system

$$y_k=W_k(X,y^k),k=1,...,m-1,F_m^{(m-1)}(X,y_m)=0. \quad (33)$$

Step m. The last equation (33) is solved using formulas of class 1 and deduce

$$y_m=W_m(X). \quad (34)$$

Step m+1. Solution (34) is substituted in the penultimate equation (33) and deduce

$$y_{m-1}=W_{m-1}(X,W_m(X)).$$

Step $2m$-1. Deduced is $y_1=W_1(X,W_2,W_3,...,W_m)$.

The resulting solutions must be verified.

In order to exemplify the algorithm, let us deduce solution of a system of three logical equations

$$y_1=a_{11}y_1\vee a_{12}y_2\vee\sigma_1y_0,y_2= \\ =a_{21}y_1\vee a_{22}y_2\vee\sigma_2y_0,y_3=a_{31}y_1\vee a_{32}y_2. \quad (35)$$

Specific solution (35) is deduced using the determinant method [5]

$$y_1=(a_{12}\sigma_2\vee\sigma_1)y_0,y_2=(a_{21}\sigma_1\vee\sigma_2)y_0,y_3= \\ =(\sigma_1(a_{31}\vee a_{32}a_{21})\vee\sigma_2(a_{32}\vee a_{31}a_{12}))y_0.$$

If $y_0=0$, system (35) is homogenous and the specific solution is zero. In order to obtain a general solution at step 1, in the first equation of (35) we deduce: $C_1=C_3=C_6=0$, $C_2=1$, $C_4=\sigma_1y_0$ V $a_{12}y_2$, $C_5=a_{11}$. Using table 1 we establish that we must consider four constituents 1 numbered 16, 18, 20 and 22. We then eliminate K_{16}. The rest results in solution

$$y_1=C_4\vee C_5R_1=\sigma_1y_0\vee a_{12}y_2\vee a_{11}R_1. \quad (36)$$

At step 2 let us substitute (36) in the second and third equations of (35)

$$y_2=(a_{21}\sigma_1\vee\sigma_2)y_0\vee a_{21}a_{11}R_1\vee(a_{22}\vee a_{21}a_{12})y_2, \quad (37) \\ y_3=a_{31}\sigma_1y_0\vee a_{31}a_{11}R_1\vee(a_{32}\vee a_{31}a_{12})y_2.$$

The first equation of (37) is solved as

$$y_2=C_4\vee C_5R_2= \\ =(a_{21}\sigma_1\vee\sigma_2)y_0\vee a_{21}a_{11}R_1\vee(a_{22}\vee a_{21}a_{12})R_2. \quad (38)$$

At step 3 let after substituting (38) in the second equations of (37) we have

$$y_3=(\sigma_1(a_{31}\vee a_{32}a_{21})\vee\sigma_2(a_{32}\vee a_{31}a_{12}))y_0\vee \\ \vee a_{21}a_{11}(a_{32}\vee a_{31}a_{12})R_1\vee(a_{32}\vee a_{31}a_{12})(a_{22}\vee a_{21}a_{12})R_2. (39)$$

Step 4 is skipped because y_2 in system (37) does not depend on y_3 . At step 5 let us substitute (39) in (36) and deduce

$$y_1=(a_{12}\sigma_2\vee\sigma_1)y_0\vee a_{11}R_1\vee(a_{22}\vee a_{21})a_{12}R_2. \quad (40)$$

The total (38)–(40) gives general solution (35) that depends on indictors R_1, R_2. If $R_1=R_2=1$ we have

$$y_k=A_{1k}y_0\vee A_{2k},k=1,2,3, \quad (41)$$

$$A_{11}=a_{12}\sigma_2\vee\sigma_1,A_{12}=a_{21}\sigma_1\vee\sigma_2,A_{13}= \\ =\sigma_1(a_{31}\vee a_{32}a_{21})\vee\sigma_2(a_{32}\vee a_{31}a_{12}), \\ A_{21}=a_{11}\vee(a_{22}\vee a_{21})a_{12},A_{22}=a_{21}a_{11}\vee a_{22}\vee a_{21}a_{12}, \\ A_{23}=(a_{32}\vee a_{31}a_{12})(a_{22}\vee a_{21}(a_{12}\vee a_{11})).$$

If $y_0=0$ we have a general solution of a homogenous system. Beside general solution (41) two more general solutions exist that correspond to sets (R_1, R_2) = (0,1) and (1,0).

If $p_i=1$–$q_i=P\{\sigma_i=1\}$, $p_{ij}=1$–$q_{ij}=P\{a_{ij}=1\}$, using the formulas (41) by means of method of transition to complete replacement [5] we will deduce the probabilities

$$P_1=P\{y_1=1\}=1-q_1q_{11}(q_{12}+p_{12}q_2q_{21}q_{22}), \quad (42)$$

$$P_2=P\{y_2=1\}=1-q_2q_{22}(q_{21}+p_{21}q_1q_{11}q_{12}), \quad (43)$$

$$P_3=P\{y_3=1\}=p_1(p_{31}+q_{31}p_{32}p_{21})+ \\ +q_1(p_{12}(1-q_{31}q_{32})(1-q_2q_{22}q_{21})+ \\ +q_{12}p_{32}(1-q_2q_{22}(1-p_{21}p_{11})))+p_1p_{32}q_{31}q_{21}(1-q_2q_{22}). (44)$$

For a homogenous system, if $R_1=R_2=1$ the probabilities are deduced from (42) – (44), assuming that $q_1=q_2=1$,

$p_1 = 0$. For the specific solution ($R_1 = R_2 = 0$) probability P_1 is deduced out of (42) where $q_{11} = q_{21} = q_{22} = 1$, probability P_2 is deduced out of (43) where $q_{11} = q_{12} = q_{22} = 1$, while probability P_3 is deduced using formula

$$P_3 = P\{y_3 = 1\} = p_{31}(1 - q_1(q_2 + p_2 q_{32} q_{12})) + \\ + q_{31} p_{32}(p_2 + q_2 p_1 p_{21}).$$

For the general solution where $R_1 = 0$, R_2=1 probabilities P_1, P_2, P_3 are deduced respectively from (42)–(44), assuming that $q_{11} = 1$. For the general solution where R_1= 1, R_2=0 probability P_1 is deduced using formula (42) where $q_{21} = q_{22} = 1$, probability P_2 is deduced using formula (43) where $q_{12} = q_{22} = 1$, while probability P_3 is deduced using formula

$$P_3 = P\{y_3 = 1\} = p_1(p_{31} + q_{31} p_{32} p_{21}) + \\ + q_1(p_{32} + q_{32} p_{12} p_{21})(p_2 + q_2 p_{21} p_{11}) + q_{21} q_{31} p_1 p_{32} p_2.$$

Formalization of the listing of special conditions of operation and possibility to use them for the purpose of choosing the general solution is illustrated with the following example.

Example. A system designed for transforming heat energy into mechanical consists of two subsystems connected with a link (see figure). The subsystem includes turbine generators (components 1 and 8), steam generators (2 and 9), turbines (4 and 11), control equipment (3 and 10), main capacitors (5 and 12), condensate pumps (6 and 13), consumers (7 and 14). For normal subsystem operation a functioning feedback loop is required (components 5 and 6 or 12 and 13) and availability of at least one power source (1, 8) to the consumer. Therefore, the general form of SHLF shall be

$$F = x_p(x_{\text{тг}1} f_1 \vee x_{\text{тг}2} f_2). \qquad (45)$$

Here f_1 and f_2 are logical functions of operability of the part of the system from the output of the respective turbine generator to the consumer.

If F has the form of $F = x_{\text{p}}(x_{\text{тг}1} f_1 \vee x_{\text{tg}2} f_2 \vee f_3)$, then if all components are in working condition (x_i=1) the values of the indicators must be chosen in such a way as to fulfill the conditions:

$$f_1(R_1, R_2, ..., R_k) = f_2(R_1, R_2, ..., R_k) = \\ = 1, f_3(R_1, R_2, ..., R_k) = 0.$$

If that is not so, the third summand absorbs the first two and the operability function can be equal to one even if the turbine generators are absent, which contradicts the physical essence of the system.

The logical equation system for the technical system under consideration has the following form

$$y_2 = x_2 y_6(x_1 \vee y_9), y_3 = x_3 y_2, y_4 = x_4 y_2 y_3, y_5 = \\ = x_5 y_4, y_6 = x_6 y_5, y_7 = x_7 y_4, \qquad (46)$$

$$y_9 = x_9 x_{13}(x_8 \vee y_2), y_{10} = x_{10} y_9, y_{11} = x_{11} y_9 y_{10}, y_{12} = \\ = x_{12} y_{11}, y_{13} = x_{13} y_{12}, y_{14} = x_{14} y_{11}.$$

It is required to find expressions for y_7 and y_{14} taking (45) into consideration. The equation system is homogenous and, therefore, it must have zero specific solution. At the first step out of the first five equations of (46) we deduce

$$y_6 = x_6 x_5 x_4 x_3 x_2 (x_1 \vee y_9) y_6 = B(x_1 \vee y_9) y_6. \qquad (47)$$

Solution (47) according to the method of class 1 has the form of

$$y_6 = B(x_1 \vee y_9) R_1. \qquad (48)$$

At step 2 of (48) of both the first and the second equations of of (46) we deduce

$$y_9 = x_9 x_{13}(x_8 \vee BR_1(x_1 \vee R_2)). \qquad (49)$$

At step 3 of the equations for y_{10}–y_{13} of system (46) we deduce: $y_{13} = x_{13} x_{12} x_{11} x_{10} y_9$.

Let us substitute (49) here ad solve the homogenous equation for y_{13}:

$$y_{13} = AR_3(x_8 \vee BR_1(x_1 \vee R_2)), A = x_9 x_{10} x_{11} x_{12} x_{13}. \qquad (50)$$

Out of (49) and (50) follows that $y_9 = y_{13}$.

At step 4 out of equations for y_{10}, y_{11} and y_{14} of system (46) in view of (50) we deduce:

$$y_{14} = x_{14} AR_3(x_8 \vee BR_1(x_1 \vee R_2)). \qquad (51)$$

In the same manner we deduce:

$$y_7 = x_7 BR_1(x_1 \vee AR_3(x_8 \vee R_2)). \qquad (52)$$

Out of (50) and (51) it follows that specific solution (46) is zero. After comparing (50) and (51) and (45) we see that special conditions are fulfilled only if $R_1 = R_3 = 1$, $R_2 = 0$. Thus, finally we have

$$y_7 = x_7 B(x_1 \vee Ax_8), y_{14} = x_{14} A(x_8 \vee Bx_1).$$

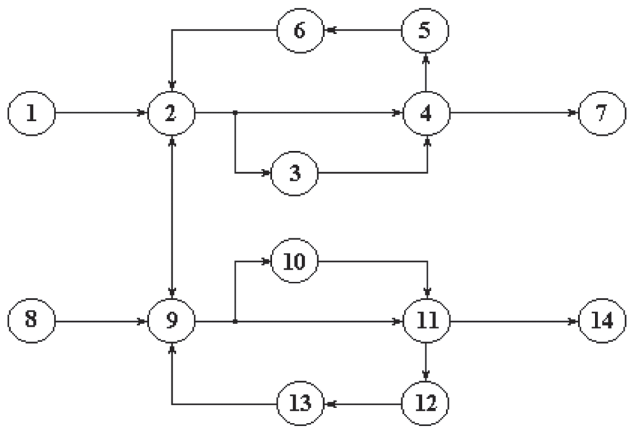

Fig. Technical system structure (1, 8 turbine generators; 2, 9 steam generators; 3, 10 control equipment; 4, 11 turbines; 5, 12 main capacitors; 6, 13 condensate pumps; 7, 14 reducers).

Probabilities are deduced using formulas

$$P_7 = P\{y_7 = 1\} = p_7 P_B(p_1 + q_1 p_8 P_A), P_{14} = \\ = P\{y_{14} = 1\} = p_{14} P_A(p_8 + q_8 p_1 P_B), \\ P_A = p_9 p_{10} p_{11} p_{12} p_{13}, P_B = p_2 p_3 p_4 p_5 p_6.$$

5. Conclusion

Logical and probabilistic analysis of a complex technical system is one of the key components of system design in many industries. Due to significant labour intensity, simply searching through the possible solutions is impractical. Logic equation systems significantly simplifies the analysis process. The table-based method of a general solution of logical equations suggested in this paper extends the capabilities of system developers and enables the evaluation of characteristics adequate to the logical structure. The definition of all general solutions with subsequent choice of one of them allow, using the indicator vector, taking into consideration special conditions of operation, including the use of cross-connections between parallel channels and feedback loops that support or improve system functions. In the presence of a multitude of general solutions in technical solutions, the task consists in the formal choice of the only acceptable solution by listing special logical conditions of system operation.

At the stage of logical analysis it is possible not only to establish the existence of at least one general solution or the fact of insolvability of the equation system, but also find the causes and location of the formula's logical inconsistency. That is quite relevant for algorithmically controlled resources as that enables a quick elimination of the uncovered incorrectness. The control of the system operation logic enables new ways of improving dependability and efficiency. The preferred application areas are energy, automated control and diagnostics systems.

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