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METHOD OF COMPLETE DECOMPOSITION OF BRIDGE CONNECTIONS IN CONNECTIVITY ANALYSIS PROBLEMS OF STRUCTURALLY COMPLEX BIPOLAR NETWORKS

The paper considers the method of complete decomposition of bridge (cross) connections of structurally complex bipolar networks whose content is the generalization algorithm of the well-known Moore-Shannon's decomposition formula for analysis of connectivity of a ladder bridge bipolar network. The offered method allows for considerably reducing a number of analyzed states as compared to known combinatory methods, e.g. an exhaustion method of elements' states [2].

Keywords: *analysis, probability, connectivity, decomposition, Moore-Shannon's decomposition formula, system, random graph, bipolar network, bridge connections, binominal coefficient, combination.*

1. Introduction

1.1. Terms and definitions

Bridge connection (BC) is a connection between two adjoining vertices v_i and v_j, $i \neq j$, belonging to upper and lower "independent frames" respectively [2].

Connectivity is a property of bipolar networks to detain upstate and recover it during acceptable time in case of random and parametric failures, physical damages as well as deliberate and unintentional disturbances.

A graph is taken as random if its elements are either in upstate with the probability p, or in downstate with the probability $q=1-p$, where p is the coefficient of availability of a random graph's element (RG) [2].

BN is a bipolar network; RG BN is a random graph of a bipolar network (see detailed definition of RG BN in [2]); MSDF is Moore-Shannon's decomposition formula; EMES is an exhaustion method of elements' states; MCDBC is a method of complete decomposition of bridge connections; CP is connectivity probability; BC is binominal coefficient; C is a combination; PBC is a vertex of a boundary couple; SCS is structurally complex systems; TCR is a theory of combinatory reliability.

1.2. Brief analysis of topicality and state-of-the-art of the problem

Ladder bridge bipolar networks (whose number of bridge connections will be designated as m^M) are widely used in telecommunication networks, power supply and transport networks, arrangement of public announcement systems etc. [3].

Availability of ladder bridge connections in BN increases the efficiency of their functioning while making it more difficult and time-consuming to analyze connectivity of such networks [2,4,5]. This is where the problem of connectivity analysis of RG BN with m^M>1 bridge (cross) connections lies.

The problem of analysis of RG BN with bridge (cross) connections has been covered in a lot of works, whose inexhaustive list can be found for example in works [5,6], which show that along with combinatory methods of connectivity analysis of such RG BN [2,3] logical probabilistic methods of reliability analysis of structurally complex systems have got widely used and developed [5]. Therefore, there is no common analytical approach to solve this class of hard solved problems [5].

There is an opinion that a universal approach to solve this class of hard solved problems is to use computers with special software [2,5].

The author of the work [5] comments this situation the following way: "We only have to develop special mathematical software based on a serious theory and approved analytical methods. The unavailability of those forces researchers to do direct and complete search of all system state on PC. Permanent increase of PC productivity supports their hope in this promising research way to prove and invent analytical methods without too much effort". So, what we should have to do is to develop analytical methods that could be used in engineering.

A substantial contribution to solving the problem of reliability analysis of BN with bridge connections was given by the work [1] that offered a decomposition formula for a one-bridge circuit (MSDF). Further works proved the possibility of applying MSDF for BN with m^M>1. However, a one-bridge circuit proposed by C. Shannon was used to illustrate this possibility.

What is the reason of application of such illustrated examples? The reason is that the complexity and time-consuming character of practical implementation of MSDF for m^M>1 bridge connections is defined by increase of the number of all possible states (combinations) of analyzed RG BN with m^M>1 bridge connections in proportion to 2^{m^M}.

Increase of the number of bridge connections m^M brings quite a difficult task of ordering and taking account of all possible combinations (states) of analyzed BN. For example, in work [5] this obstacle was "managed to overcome by means of a table method of SCS reliability calculation". In this paper in order to not "go astray" in the labyrinth of various possible combinations with increase of the number of bridge connections m^M of BN we use the decomposition algorithm for m^M>1, whose basis is the properties of a binominal distribution and its binominal coefficients.

Scientific novelty of the paper is generalization of MSDF application for BN with m^M>1 bridge connections based on the properties of a binominal distribution and its BC [6], defining the formal principle of development of the decomposition algorithm of initial RG BN with m^M bridge connections (m^M>1) in 2^{m^M} conditional parallel sequential RG BN.

The practical value is characterized by the possibility of using the offered method in applied engineering tasks of connectivity analysis of RG BN with m^M>1 bridge connections.

2. Initial data and problem statement

Let structurally complex BN be set by graph G [7]:

$$G = \{V, L, \Phi\}, \qquad (1)$$

where $V = \{v_i\}, i = \overline{1, m_V}$ is a set of graph vertices whose number equals to $m_V = |V|$ – the power of the set of graph poles (number of elements of some set or some aggregate of elements is generally called its power);

$L = \{l_{i,j}\}, i, j = \overline{1, m_V}$ $(i < j, i \neq j)$ is a set of graph poles with the power $m_L = |L|:(i, j)$ – the numbers of vertices of a boundary couple (VBC) of the pole $(l_{i,j})$;

$\Phi(l_{i,j}) = v_i$ & v_j is the representation of incidence and adjacency of graph elements such that if the pole $l_{i,j}$ connects the vertices v_i and v_j, then it is considered incident to the vertices of a boundary couple v_i and v_j; if the vertex v_i is connected by the pole $l_{i,j}$ with the vertex v_j, then these vertices are adjacent to each other by the pole $l_{i,j}$.

The vertices of a graph connected to each other by poles make up a specific structure of a graph that could be both simple and complex and represents a graph's capability to transmit information from its vertex of S-source to the vertex of t-drain, see fig. 2.

The poles of RG BN can have partial or through numeration [7].

For partial numeration of poles we use their VBC v_i and v_j such that the order of numeration of poles is formally represented in the form

$$L = \{l_{i,j}\}, i < j \;\&\; i \neq j \text{ end } i, j = \overline{1, m_V}. \qquad (2)$$

For through numeration vertices and poles of a graph will have the following numeration respectively:

$$V = \{v_i\}, i = \overline{1, m_V} \text{ end } L = \{l_\xi\}, \xi = \overline{m_V + 1, m_V + m_L}, \qquad (3)$$

where ξ is the designation of the sequence number of a pole.

Through numeration of a graph's elements is (Fig. 1) is done in accordance with the following guidelines: from the vertex source S to the vertex drain t and from upper vertices to the lower ones.

For the multiplicity of notation of vertices $v_i, i = \overline{1, m_V}$ and poles l_ξ $\xi = \overline{m_V + 1, m_V + m_L}$ in RG BN we will sometimes use only corresponding sequence numbers assigned to these elements $i = \overline{1, m_V}$ and $\xi = \overline{m_V + 1, m_V + m_L}$.

Also, to reduce the difficulty of connectivity analysis of RG BN we will take a non-principal assumption that RG BN vertices are absolutely reliable (in Fig. 2 this assumption is designated as bold circles) and poles have the reliability equal to

$$P_\xi = \overline{m_V + 1, m_V + m_L} = p. \qquad (4)$$

For the above initial data the problem is to offer a method for applying the properties of a binominal distribution and its binominal coefficients to develop a decomposition algorithm of RG BN with m^M>1 bridge connections; to exemplify a practical implementation of this algorithm for decomposing initial RG BN with m^M>1 bridge connections into conditional parallel serial RG BN. Also, we will show that application of the method of decomposition allows for reducing the number of analyzed states by several tens (hundreds) times in relation to existing combinatory methods, e.g. MCDBC of analyzed RG BN.

3. Application of the properties of a binominal distribution and its binominal coefficients for decomposition of bridge connections in bipolar networks

Let each bridge connection $l_\xi^M \in m^M$ of RG BN can be in one of the two states: up state – l_ξ^M with the probability $p(l_\xi^M) = p$ or down state – $\overline{l_\xi^M}$ with the probability $q\left(\overline{l_\xi^M}\right) = 1 - p\left(l_\xi^M\right) = q$. In this case the total number of all possible contradictory combinations for decomposing m^M (m^M>1) bridge connections 2 modulo will be equal to 2^{m^M}, and each of them will include i down states and $m^M - i$ up states. The failure event of any bridge connection $\overline{l_\xi^M}$ does not depend on the state of other bridge connections making up a combination from m^M bridge connections via i, i.e. $C^i_{m^M}$. Note that each combination $C^i_{m^M}$ is a binominal coefficient (BC_i).

Then for the purpose of ordering the process of generating all possible combinations from m^M via i and their accounting for the connectivity analysis of RG BN with m^M bridge connections (m^M>1) it is reasonable to use a binominal distribution that is expressed by formula [6]

$$P_{i,m^M} = C^i_{m^M} p^{(m^M - i)} q^i, \qquad (5)$$

where P_{i,m^M} is the probability that as a result of decomposition of m^M bridge connections in combination $C^i_{m^M}$ there will be i downstate bridge poles;

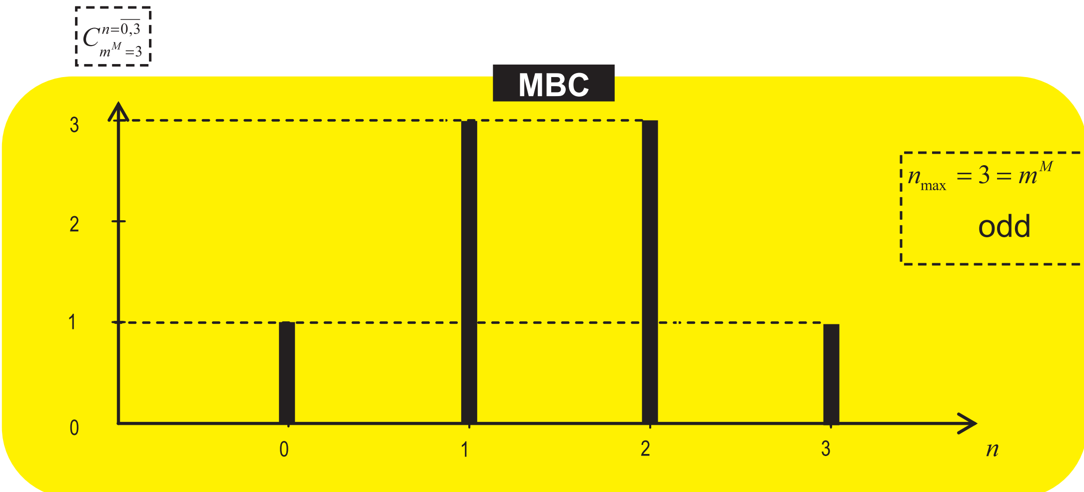
$C^i_{m^M}$ is a binominal coefficient characterizing a number of combinations (states) which can be taken from m^M bridge connections via i (here m^M is the parameter of a combination (PaC), and i is a variable combination (VaC));

cofactor $p^{(m^M - i)} q^i$ is the probability of obtaining $БК_i = C^i_{m^M}$;

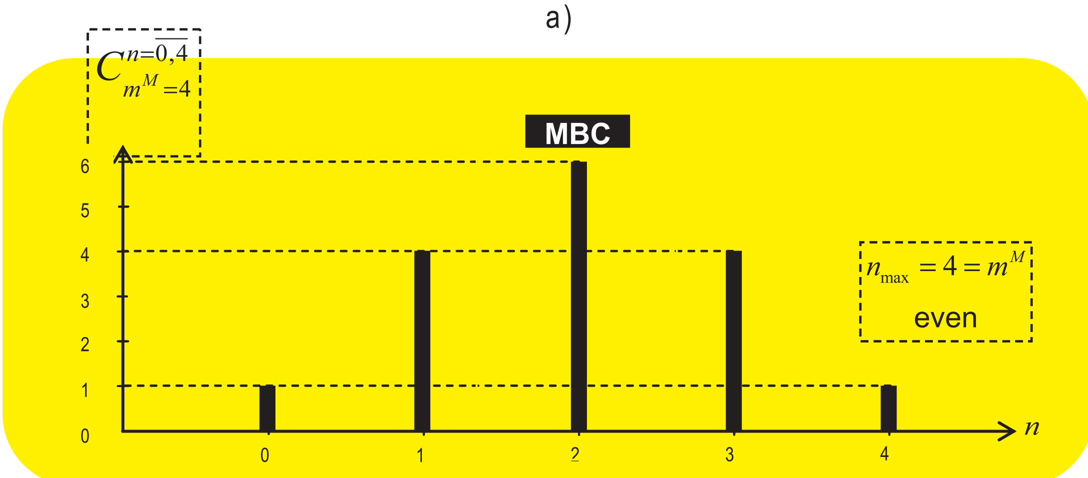
$p^{(m^M - i)}$ is the probability of finding a subset of bridge poles $L^M = \{l_\xi^M\} \in m^M$ in up state in BC_i;

q^i is the probability of finding a subset of bridge poles $\overline{L^M} = \{l_\xi^M\} \in m^M$ in down state in BC_i.

A regular series of binominal coefficients $БК_i\left(i = \overline{0, m^M}\right)$ represents the well-known Newton's binomial formula (NB) [6], which has the property of symmetry in relation to maximum binominal coefficients (MBC); $БК_{i=0}$ characterizes a combination when subset $L^M = \{l_\xi^M\} = |m^M|$ of bridge



a)



b)

Fig. 1.

connections of graph G is in upstate; $БК_{i=m^M}$ characterizes a combination when a subset of bride connections is in downstate, i.e. $\overline{L^M} = \left\{ \overline{l_\xi^M} \right\} = \left| m^M \right|$.

If PaC has an odd value, then MBC will be only two (fig. 1, a). If PaC has an even value, then MBC will be one (fig. 1,б).

Fig. 1 evidently shows that first, BC in NB proceed symmetrically in relation to MBC owing to the fact that for set m^M it is always $\frac{m^M!}{i_1!(m^M-i_1)!} = \frac{m^M!}{i_2!(m^M-i_2)!}$ provided that $0 \le i_{1(2)} \le m^M$, $i_1 \ne i_2$ end $i_1+i_2=m^M$.

Second, all BC are a combination of the type $C_{m^M}^{i=\overline{0,m^M}}$, and third, the total sum of all possible combinations is absolutely equal to $2^{m^M} = \sum_{i=0}^{m^M} \frac{m^M!}{i!(m^M-i)!} = \sum_{i=0}^{m^M} C_{m^M}^i$. For example, see fig. 1, a:

$$2^{m^M=3} = 8 = (БК_{i=0} = 1) + (БК_{i=1} = 3) + \\ +(БК_{i=2} = 3) + (БК_{i=3} = 1) = 8 = 2^{m^M=3}.$$

Technically, the procedure of composition of all possible combinations from m^M via $0 \le i \le m^M$ only for $БК_i\,(i = \overline{0,m^M})$ we will write down in the following way:

$$БК_i = K_{m^M}^i = \left\{ k_{\omega=\overline{1,C_{m^M}^i}} \right\} = \sum_{\omega=1}^{C_{m^M}^i} \prod_{\substack{\varepsilon=1 \\ \varepsilon\in k_\omega}}^{i} \bullet \overline{l_{\xi_\varepsilon}^M} \prod_{\substack{\varepsilon=1 \\ \varepsilon\in k_\omega}}^{m^M-i} \bullet l_{\xi_\varepsilon}^M \qquad (6)$$

where ω is the number of the current combination k_ω; $\sum_{\omega=1}^{C_{m^M}^i}$ – is the combination sum (CS), making up exactly $C_{m^M}^i$ $\omega - x$ formed combinations from m^M variable via i; $\prod_{\substack{\varepsilon=1 \\ \varepsilon\in k_\omega}}^{i} \bullet$ is the combination product (CP), combining $(\bullet)$ exactly i downstate bridge connections in one; $\prod_{\substack{\varepsilon=1 \\ \varepsilon\in k_\omega}}^{m^M-i} \bullet$ is CP, combining exactly $m^M - i$ upstate bridge connections in one ω-th formed combination.

For example, let it be that the analyzed RG BN lists three bridge connections with the numbers l_{11}^M, l_{14}^M and l_{17}^M, see fig. 2. Let them be positioned in the field of variables in the following order: $\Omega = \left\{ l_{\xi=11_{\varepsilon=1}}^M, l_{\xi=14_{\varepsilon=2}}^M, l_{\xi=17_{\varepsilon=3}}^M \right\}$. Then for $m^M = 3$ combinations via (i=2) in PC will according to (6) look like:

$$K_{m^M=3}^{i=2} = \left\{ k_{\omega=1} = \left\{ \overline{l}_{\xi=11_{\varepsilon=1}}^M \bullet \overline{l}_{\xi=14_{\varepsilon=2}}^M \bullet l_{\xi=17_{\varepsilon=3}}^M \right\} + k_{\omega=2} = \\ = \left\{ \overline{l}_{\xi=11_{\varepsilon=1}}^M \bullet l_{\xi=14_{\varepsilon=2}}^M \bullet \overline{l}_{\xi=17_{\varepsilon=3}}^M \right\} + k_{\omega=3} = \left\{ l_{\xi=11_{\varepsilon=1}}^M \bullet \overline{l}_{\xi=14_{\varepsilon=2}}^M \bullet \overline{l}_{\xi=17_{\varepsilon=3}}^M \right\} \right\}.$$

As should be, it is evident that exactly three combinations are formed for the case $C_{m^M=3}^{i=1} = \frac{3!}{2!\cdot 1!} = 3$ (see fig. 1, and the third column left for i=2).

Technically, the procedure of ordered composition of all possible combinations from m^M via $i = \overline{0,m^M}$ for all $БК_i$ is almost the same as in (6). There is only one CS for controlling i added. In general the procedure looks like:

$$БК_i = \overline{0,m^M} = K_{m^M}^{i=\overline{0,m^M}} = \left\{ \sum_{i=0}^{m^M} k_{\omega=\overline{1,C_{m^M}^i}}^i \right\} = \\ = \sum_{i=0}^{m^M} \sum_{\omega=1}^{C_{m^M}^i} \prod_{\substack{\varepsilon=1 \\ \varepsilon\in k_\omega}}^{i} \bullet \overline{l_{\xi_\varepsilon}^M} \prod_{\substack{\varepsilon=1 \\ \varepsilon\in k_\omega}}^{m^M-i} \bullet l_{\xi_\varepsilon}^M. \qquad (7)$$

Since all possible combinations make up the entire group of contradictory events, then the sum of all probabilities P_{i,m^M} is equal to unity: $\sum_{i=0}^{m^M} P_{i,m^M} = 1$.

4. Solution of stated problems

The offered method is in essence as follows. Let's assume that it is required to determine the connectivity probability of structurally complex RG BN in which each ξ-th element can be found in one of two states: up state (designated as l_ξ) or down state (designated as $\overline{l_\xi}$) with the probability $q(\overline{l_\xi}) = 1 - P(\overline{l_\xi})$, set of graph poles with the power $m_L = |L|$.

Then using the exhaustion method of elements' states (EMES) for definition of connectivity probability of the specified vertex pair (poles) S and t in analyzed structurally complex RG BN it is required to analyze 2^{m_L} of all possible states [3]. Obviously at increase in the number of structural elements in RG BN the quantity of analyzed states and, consequently, labor input of application of this method increases proportionally to the quantity 2^{m_L}.

Similar complexity and labor input are presented by other combinatory methods of complete search of all possible states of analyzed structurally complex RG BN.

Therefore, the problem of complexity and labor input reduction in combinatory methods of connectivity probability (CP) analysis can be solved based on reduction of number of the analyzed states describing initial RG BN.

The solution of this problem is possible based on application of Moore-Shannon's decomposition formula (MSDF) not only for RG BN single-bridge circuit, as shown in [5], but also for some subset of bridge connections $L^M = \{l_\xi^i\} = \left| m_L^M \right|$, as part of analyzed structurally complex RG BN.

With such approach the number of analyzed states of initial RG BN is reduced proportionally to the following relation

$$\gamma = K/K^M, \qquad (8)$$

where $K = 2^{m_L}$ is the number of all possible states (combinations) of elements describing as a whole the structure of RG BN;

K^M = is the number of all possible states (combinations) of bridge connections (shared edges) in the structure of analyzed RG BN, $K \gg K^M$.

Below is offered the algorithm of MSDF application for any number of bridge edges in analyzed RG BN structures, based on the properties of binominal distribution and its binominal coefficients (5).

From the analysis of MSDF content [5] it is possible to notice, that it determines total probability of connectivity of poles (vertices) S and t in single-bridge RG BN which we shall express in the following form:

$$P_{s,t} = p(k_0^M)p(G_{s,t}/k_0^M) + q(k_1^M)p(G_{s,t}/k_1^M), \quad (9)$$

where combinations k_o^M *and* k_1^M form a complete group of disjoint states of bridge (shared) connection l_ξ^M:$k_o^M \sim l_\xi^M$; $k_1^M \sim \bar{l}_\xi^M$, where l_ξ^M and are designations of up state and down state of the shared edge respectively, $p(k_o^M) + q(k_1^M) = 1$;

$p(G_{s,t}/k_o^M)$ is the connectivity probability of poles S and t in RG BN $G_{s,t}$ provided that the state of bridge edge l_ξ^M is in up state, and therefore there is a joining of adjacent vertices by this edge;

$p(G_{s,t}/k_1^M)$ is the connectivity probability of poles S and t in RG BN $G_{s,t}$ provided that the state of bridge edge l_ξ^M is in down state, and therefore there is a disjoining of adjacent vertices by down state edge l_ξ^M.

In view of the specified definitions the formula (9) will take the following form

$$P_{s,t} = \sum_{i=0}^{1} p(k_i^M)p(G_{s,t}/k_i^M). \quad (10)$$

Let's assume that arbitrary RG BN $G_{s,t}$ is characterized by some subset $L^M = \left\{ l_\xi^M / \xi = \overline{1, m_L^M} \right\}$ of shared edges forming a complete group of disjoint states (combinations):

$$K^M = 2^{m_L^M} = \left\{ k_i^M / i = \overline{0, I} \right\}, \quad (11)$$

where I is the designation of a complete group of disjoint (contradictory) states (combinations). Since all possible combinations (11) are contradictory and have a complete group, then the probability $P_{s,t}$ of connectivity of initial structurally complex RG BN $G_{s,t}$ according to (10) will be defined as

$$P_{s,t} = \sum_{i=0}^{I} p(k_i^M)p(G_{s,t}/k_i^M), \quad (12)$$

where $p(G_{s,t}/k_i^M) = p(G_{s,t}^i)$ is the connectivity probability of RG BN $G_{s,t}^i$, obtained as a result of transformation of the initial RG BN $G_{s,t}$ provided that the state of bridge connections (shared edges) corresponds to combination k_i^M. In line with this definition, formula (12) will be expressed in the following way:

$$P_{s,t} = \sum_{i=0}^{I} p(k_i^M)p(G_{s,t}^i). \quad (13)$$

Formula (13) means a complete probability of connectivity of poles S and t for analyzed RG BN $G_{s,t}$, in whose structure there is some subset of shared bridge edges $L^M = \left\{ l_\xi^M \right\} = \left| m_L^M \right|$.

Calculation of probabilities of combination $P(k_i^M)$ is made according to the above equalities (6), as well as (7) and (8).

As conditional graph RG BN $G_{s,t}^i$ is formed as a result of decomposition of shared edges' subset $L^M = \{l_\xi^M\} = m_L^M$ of initial RG BN $G_{s,t}$ into up states or down states and as a result of it there is accordingly a joining or disjoining adjacent vertices in RG BN $G_{s,t}$ over the aggregate of the shared edges m_L^M,then in the structures of conditional RG BN $G_{s,t}^i$ bridge connections are eliminated. By virtue of it calculation of connectivity probability of poles S and t in analyzed RG BN is made under formulas of series-parallel (parallel-serial) connection of elements.

For illustration of application of the offered method we shall determine probability of connectivity RG BN in whose structure there are three shared bridge edges l_{11}^M, l_{14}^M and l_{17}^M, see fig. 2.

Let's accept that vertices $\nu_{\xi=\overline{1,8}}$ of analyzed RG BN are absolutely unfailing, and the probability of staying of RG BN edges in up state $p\,(l_{\xi=9,19}) = p = 0{,}9$.

We shall determine the complete group of disjoint states (combinations) of shared edges at their decomposition 2 modulo as follows, according to equalities (6), as well as (7) and (8):

$$2^3 = \sum_{i=0}^{3} C_3^i = \sum_{i=0}^{3} \frac{3!}{i!(3-i)!} = 8, \quad (14)$$

including:

$$БК_{i=0} = C_3^0 = 1,\ БК_{i=1} = C_3^1 = 3,$$
$$БК_{i=2} = C_3^2 = 3,\ БК_{i=3} = C_3^3 = 1.$$

Write down combinations k_ω^i, corresponding to $БК_{i=\overline{0,3}}$ $:БК_{i=0} = C_3^0 = 1 : k_{\omega=1}^{i=0} = l_{\xi=11_{\varepsilon=1}}^M \bullet l_{\xi=14_{\varepsilon=2}}^M \bullet l_{\xi=17_{\varepsilon=3}}^M$ is the initial combination when all shared edges are in upstate;

$$БК_{i=1} = C_3^1 = 3 : k_{\omega=1}^1 = \bar{l}_{\xi=11_{\varepsilon=1}} \bullet l_{\xi=14_{\varepsilon=2}} \bullet l_{\xi=17_{\varepsilon=3}};$$

$$k_{\omega=2}^1 = l_{\xi=11_{\varepsilon=1}} \bullet \bar{l}_{\xi=14_{\varepsilon=2}} \bullet l_{\xi=17_{\varepsilon=3}};$$

$$k_{\omega=3}^1 = l_{\xi=11_{\varepsilon=1}} \bullet l_{\xi=14_{\varepsilon=2}} \bullet \bar{l}_{\xi=17_{\varepsilon=3}};$$

$$БК_{i=2} = C_3^2 = 3 : k_{\omega=1}^2 = \bar{l}_{\xi=11_{\varepsilon=1}} \bullet \bar{l}_{\xi=14_{\varepsilon=2}} \bullet l_{\xi=17_{\varepsilon=3}};$$

$$k_{\omega=2}^2 = \bar{l}_{\xi=11_{\varepsilon=1}} \bullet l_{\xi=14_{\varepsilon=2}} \bullet \bar{l}_{\xi=17_{\varepsilon=3}};$$

$$k_{\omega=3}^2 = l_{\xi=11_{\varepsilon=1}} \bullet \bar{l}_{\xi=14_{\varepsilon=2}} \bullet \bar{l}_{\xi=17_{\varepsilon=3}};$$

$$БК_{i=3} = C_3^3 = 1 : k_{\omega=1}^3 = \bar{l}_{\xi=11_{\varepsilon=1}} \bullet \bar{l}_{\xi=14_{\varepsilon=2}} \bullet \bar{l}_{\xi=17_{\varepsilon=3}};$$

is the resulting combination when all shared edges are in downstate.

To calculate the probability $p\left(k^i_{\omega=\overline{1,C_m^i}}\right)$ of each state (combination) $k^i_{\omega=\overline{1,C_m^i}}$ of analyzed RG BN (Fig.2) we shall use equality (6).

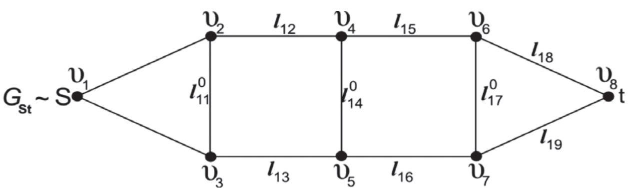


Fig. 2. Three-bridged RG BN

As a result of decomposition of the initial three-bridged structure RG BN (fig. 2), according to binominal coefficients (7), we shall receive conditional RG BN $G^i_{s,t}$ ($i=\overline{0,7}$), whose structures are presented in fig. 3.

In view of RG BN $G^i_{s,t}$ conditional structures (fig. 3) we shall determine the total probability of connectivity of poles S and t for initial RG BN $G_{s,t}$ (fig. 2) under formula (13):

$$P_{s,t} = p^3 \bullet p\,(G^0_{s,t}) + (1\text{-}p) \bullet p^2 \bullet p\,(G^1_{s,t})$$

$$(1\text{-}p) \bullet p^2 \bullet p(G^2_{s,t}) + (1\text{-}p) \bullet p^2 \bullet$$

$$(G^3_{s,t}) + (1-p)^2 \bullet p \bullet p\left(G^4_{s,t}\right) + (1-p)^2 \bullet p \bullet p\left(G^5_{s,t}\right) +$$

$$+ (1-p)^2 \bullet p \bullet p\left(G^6_{s,t}\right) + (1-p)^3 \bullet p \bullet \left(G^7_{s,t}\right), \qquad (15)$$

where

$$p\,(G^0_{s,t}) = (2p - P^2)^4;$$

$$p\,(G^1_{s,t}) = (2p^2 - P^4)(2p - P^2)^2;$$

$$p\,(G^2_{s,t}) = (2p^2 - p^4)(2p - p^2)^2;$$

$$p\,(G^3_{s,t}) = (2p^2 - p^4)(2p - p^2)^2;$$

$$p\,(G^4_{s,t}) = (2p^3 - p^6)\ (2p - p^2);$$

$$p\,(G^5_{s,t}) = (2p^2 - p^4)^2;$$

$$p\,(G^6_{s,t}) = (2p^3 - p^6)\ (2p - p^2);$$

$$p\,(G^7_{s,t}) = 2p^4 - p^8.$$

The connectivity probability of poles S and t in analyzed RG BN for the accepted initial data is equal to $P_{s,t}$ =0,955596.

Results of the comparative labor input analysis of the method of complete decomposition of bridge connections (MCDBC) as regards to EMES under the accepted parameter (1) are shown in Table 1.

From the presented results it follows, that with complication of analyzed RG BN structure we have:

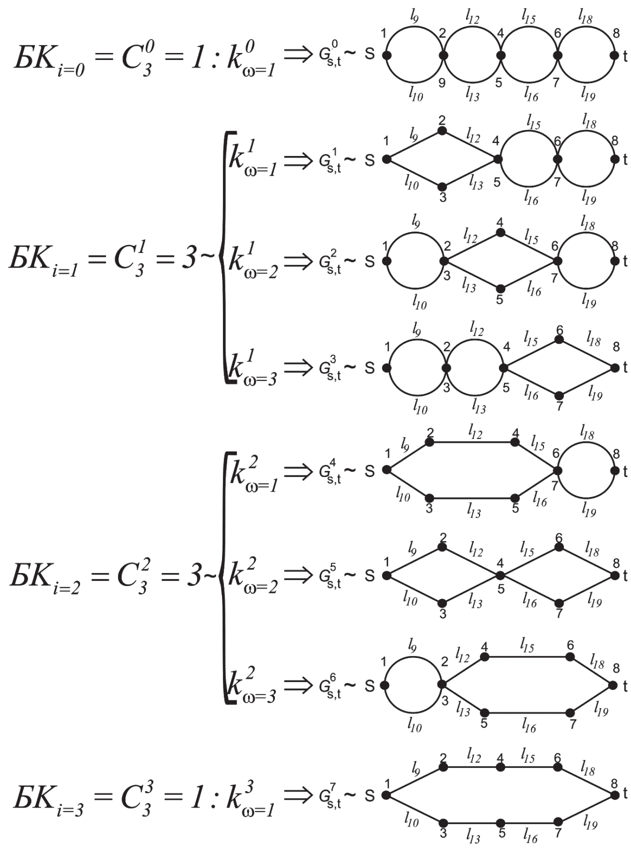


Fig. 3. Conditional RG BN $G^{i=\overline{0,7}}_{s,t}$

Labor input of EMES grows exponentially (see 2-nd column of Table 1)

Labor input of MCDBE relatively to EMES is reduced proportionally to the size $2^{(m_L - m^0_L)}$, where $m_L > m^0_L$.

Thus, the problem of complexity and labor input elimination in exact calculation of connectivity probability of structurally complex RG BN has the analytical solution based on offered method of complete decomposition of bridge connections.

6. Conclusion

The possibility of MSDF application for the case when RG BN has more than one bridge connection has been proved in works [2,5,8].

The goal of the paper has been to consider a combinatory algorithm of practical implementation of the above possibility for solving problems of connectivity analysis of such RG BN variant where solution of choice of bridge connections is not required because it is solved a priori by the structure of analyzed RG BN, see fig. 2.

Table 1. Labor input of MCDBE

The name of analyzed RG BN	Number of analyzed states (hypotheses) EMES: $K=2^{m_L}$	Number of analyzed states (hypotheses) MCDBC: $K^M = 2^{m^M_L}$	Reduction of the number of analyzed states $\gamma = K / K^M$ (times)
One-bridged RG BN	32	2	16
Two-bridged RG BN	256	4	64
Three-bridged RG BN	2048	8	256

However, the problem of defining bridge connections in analyzed structurally complex RG BN is considered fundamental in TCR whose basic provisions are stated in works [2,8].

Therefore, of scientific and practical interest is the solution of a twofold problem: 1) choice of aggregate $L^M = \{l_\xi^M\} = |m^M|$ of bridge connections from their set $L=\{l_\xi\}=|m_L|$, as part of the initial structurally complex RG BN, and 2) their further practical application for calculating the connectivity of analyzed RG BN.

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