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EQUIPMENT DEPENDABILITY TEST MANAGEMENT

A higher economic efficiency of equipment dependability testing can be achieved by means of reduced testing time or smaller quantity of test samples. The reduction of testing time increases sample censoring, while a lower quantity of test samples decreases the volume of equipment operating times sample. Test parameters may be reduced only if information processing methods ensure the validity of the calculated dependability indicators.

As the result of the tests, small censored samples of equipment mean times to failure are generated. Dependability calculation using such samples is performed through the maximum likelihood method. The article presents the findings of experimental studies of precision of maximum likelihood parameter estimation of exponential law over small singly right censored samples. The studies were performed by means of computer simulation of censored samples similar to those generated as the result of equipment dependability testing. These experimental data show that most maximum likelihood estimates obtained over small singly right censored samples have significant deviations from true values.

This paper features regression models establishing dependence between deviation of maximum likelihood estimates from true values and parameters defining the sample structure. They allow calculating and introducing corrections to maximum likelihood estimates. Experimental studies of their application efficiency were conducted. The accuracy of maximum likelihood estimates significantly increased upon application of the developed models and correction of maximum likelihood estimates. Software for application of regression models was developed.

Keywords: *computer simulation, information processing, equipment testing, dependability, censored samples, maximum likelihood method.*

Dependability parameters are controlled at all stages of equipment design and operation. To evaluate the dependability parameters of newly developed equipment its is put to tests.

The optimization of equipment dependability testing system is aimed at reducing the associated costs. One of the ways to reduce the costs is shortening the testing time. If test samples are expensive, a lower cost can be achieved through reduction of their number.

As a result of testing, samples of times to failure of the tested equipment are generated. Shorter testing times and reduced number of samples result in small right censored samples of times to failures.

The method of maximum likelihood is the primary mathematical method used for parametric estimation of dependability over right censored samples.

In the context of reduced testing time and number of tested items, validation study of the maximum likelihood (ML) estimates is becoming more relevant.

A number of fully developed testing plans exist. This paper examines the *[N, U, T]* plan that is presented in standard [1]. According to this plan, N objects are tested simultaneously. The objects that failed during the test are neither recovered nor replaced. The tests are stopped upon the expiry of the observation time or achievement of time to failure T for each of the nonfailed objects. The more recent and currently valid standard [2] also describes a similar test plan. It is called an observation time-limited plan.

Tests performed as per plan *[N, U, T]* generate singly censored samples of times to failure. This paper presents the findings of experimental studies of ML accuracy estimation of exponential distribution law over small singly right censored samples.

For the purpose of the research, an algorithm and subroutine for simulation of computer failures that occur during tests as per plan *[N, U, T]* were developed and included in the software for equipment failure simulation and data processing that is described in [7].

The following algorithm of singly censored samples was used:

1. A random variables t distributed according to an exponential distribution law calculated using formula [6] was generated

$$z = -\frac{1}{\lambda}\ln R,$$

where R is a random variable uniformly distributed over the interval (0,1).

2. The obtained random variables are compared with the specified testing time T. If $t>T$, the random variable t corresponding to the time to failure is added to the simulated sample. If $t>T$, the random variable T corresponding to the time to censoring is added to the simulated sample.

3. Simulation process continues until the number of obtained random variables becomes equal to the specified number of sample members N (sample size).

Singly right censored samples of random variables with size $N = 5, 10, 15, 20$ were computer-simulated. Generation of samples was performed under the following restrictions:

$$6 \le N < 10,\ q \ge 0{,}5$$
$$10 \le N < 20,\ q \ge 0{,}3$$
$$20 \le N \le 50,\ q \ge 0{,}2\ ,$$

where q is the degree of sample censoring. Restrictions were adopted as per the recommendations of [7].

The number of generated samples V for each variable N is 3000. For each sample, through the method of maximum likelihood, exponential distributions and their relative deviations δ from the true values that are used in sample generation were estimated

$$\delta = \frac{\lambda - \lambda_{ОМП}}{\lambda}, \qquad (1)$$

where λ is the true value of exponential distribution,

$\lambda_{ОМП}$ is the maximum likelihood estimate of the exponential distribution.

The results of the simulation are reflected in histograms of relative deviations of maximum likelihood estimates of the exponential distribution. The Y axis shows the percentage of estimates from the total number caught in the given interval. The results are shown in fig. 1.

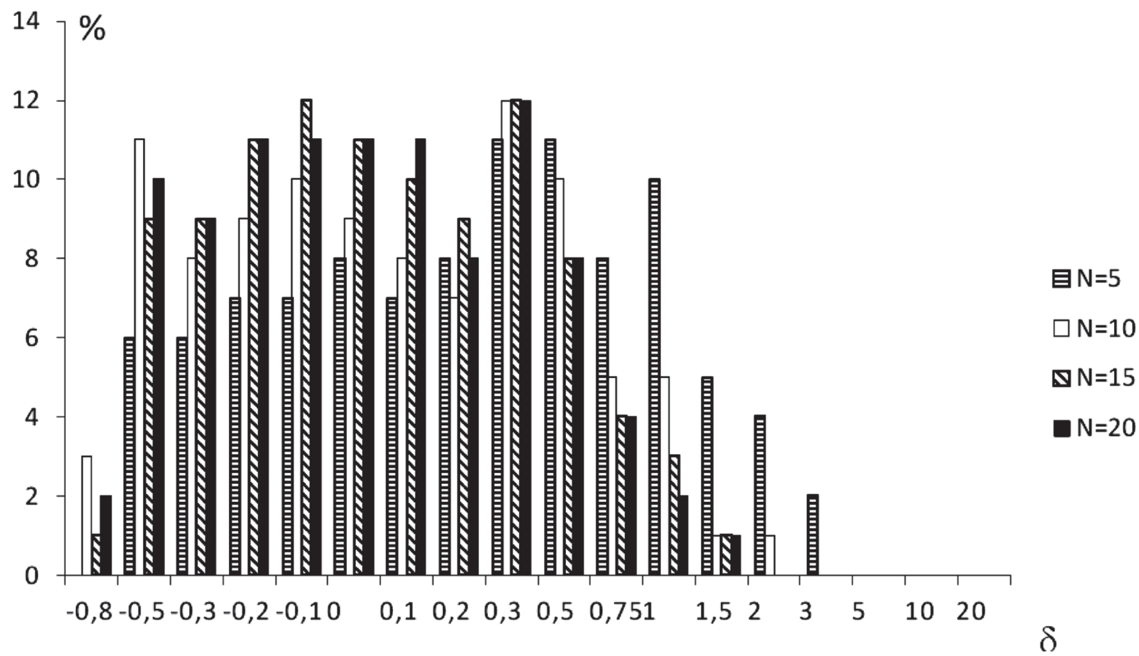

Fig. 1. Relative deviations of the maximum likelihood estimate

These experimental data show that the majority of the maximum likelihood estimates obtained from small, singly right censored samples have significant deviations from the true values. For example, 2 % of the exponential distribution estimates for $N = 5$ show relative deviations from 3 to 5; 4 % from 2 up to 3; 5 % from 1,5 up to 2. As sample size N increases the accuracy of estimations improves. For N =20 the relative deviations of exponential distribution law estimations do not exceed 1,5. Despite that 4 % of estimations show relative deviations from 0,75 to 1; 8 % from 0,5 to 0,75; 12 % from 0,3 to 0,5.

It can be concluded that the accuracy of the maximum likelihood method for variables N<20 is low. The relative deviation of estimates from the true values can be 5 or higher, while half of all estimates show deviations above 0,3 depending on the sample size.

Paper [7] suggests a technique to improve the accuracy of the maximum likelihood estimates for small multicensored samples generated in the equipment failure data collection system in the course of its operation. The described studies were aimed to verify the applicability of this method in equipment dependability tests as per plan *[N, U, T]*.

The task was solved in five stages:

1. Computer simulation of singly right censored samples of random variables distributed exponentially typical to test plan *[N, U, T]* and the calculation of sample parameters defining its structure.

The structure of the generated sample of random variables was describer using standard parameters and their derivatives:

- the degree of censoring

$$X_1 = q = \frac{k}{N},$$

where k is the number of complete random variables, N is the number of sample members.

- coefficient of variation

$$X_2 = \frac{S}{\overline{Z}},$$

where S is the estimation of the mean-square deviation of all random variables in the sample;
$\overline{Z}$ is the mathematical expectation of all members in the sample.
- coefficient of variation of complete random variables

$$X_3 = \frac{S_П}{\overline{Z}},$$

where $S_П$ is the estimation of the mean-square deviation of random variables.

- empirical coefficient of skewness [4]

$$X_4 = \widetilde{A} = \frac{\overline{(z-\overline{Z})^3}}{\left(\sqrt{\overline{(z-\overline{Z})^2}}\right)^3}.$$

- coefficient of excess

$$X_5 = \widetilde{E} = \frac{\widetilde{\mu}_4}{S^4} - 3,$$

where $\widetilde{\mu}_4$ is the fourth central moment.

Five more parameters are mathematical expressions composed of standard sample characteristics:
- the ratio of the mathematical expectation of complete random variables to that of all members of the sample

$$X_6 = \frac{\overline{Z_П}}{\overline{Z}}.$$

- the ratio of the mathematical expectation of censored random variables to that of all members of the sample

$$X_7 = \frac{\overline{Z_Ц}}{\overline{Z}}.$$

- the relative departure of the mathematical expectation from the middle of the range

$$X_8 = \frac{\frac{R}{2} - \overline{Z}}{\overline{Z}},$$

where $R = Z_{max} - Z_{min}$ is the range, $Z_{max,}$ Z_{min} are respectively the maximum and minimum values of the random variables.

- the ratio of the median to the mathematical expectation of the random variables

$$X_9 = \frac{\widetilde{M}e}{\overline{Z}},$$

where $\widetilde{M}e$ is the median.

- the ratio of the mode to the mathematical expectation

$$X_{10} = \frac{\widetilde{M}o}{\overline{Z}},$$

where $\widetilde{M}o$ is the mode.

All parameters are measured in relative units and do not depend on absolute values of random variables. That will allow applying the obtained equations to equipments with different mean time to failure values.

2. Calculation of maximum likelihood estimates.

3. Calculation of the dependent parameter of regression model, deviation of the maximum likelihood estimate from the true value using the following formula:

$$Y = \frac{\lambda}{\lambda_{OM\Pi}},$$

4. Development of regressions. The study resulted in the generation of regression models that establish a connection between the deviation of ML estimates from the true value and the parameters defining the structure of the sample. For each sample size N, its own regression equation was designed.

Mathematical models were developed as linear regression equations as shown below:

$$\overline{y}(x) = b_0 + b_1 x_1 + \cdots + b_{10} x. \quad (2)$$

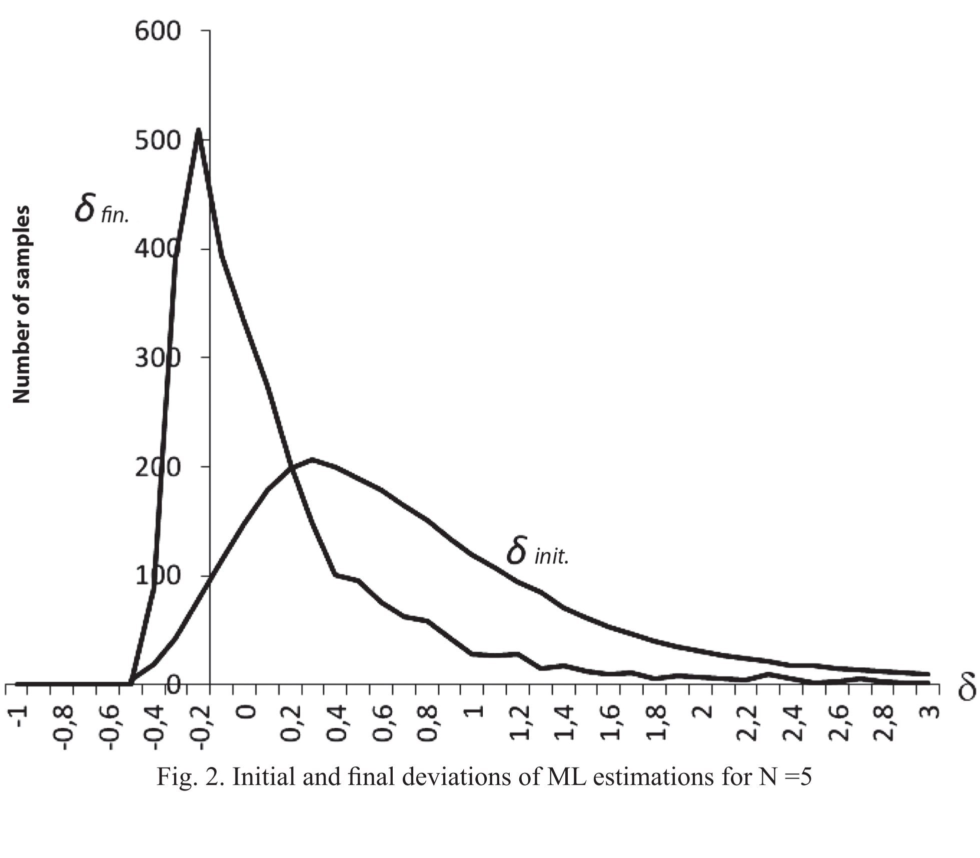

Fig. 2. Initial and final deviations of ML estimations for N =5

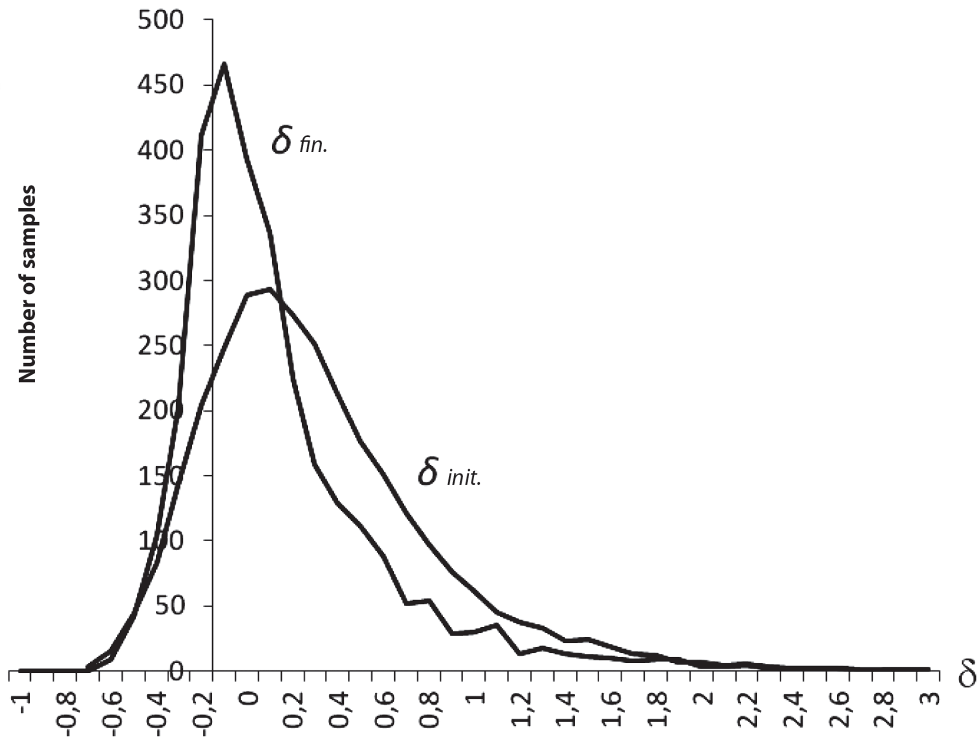

Fig. 3. Initial and final deviations of ML estimations for N =10

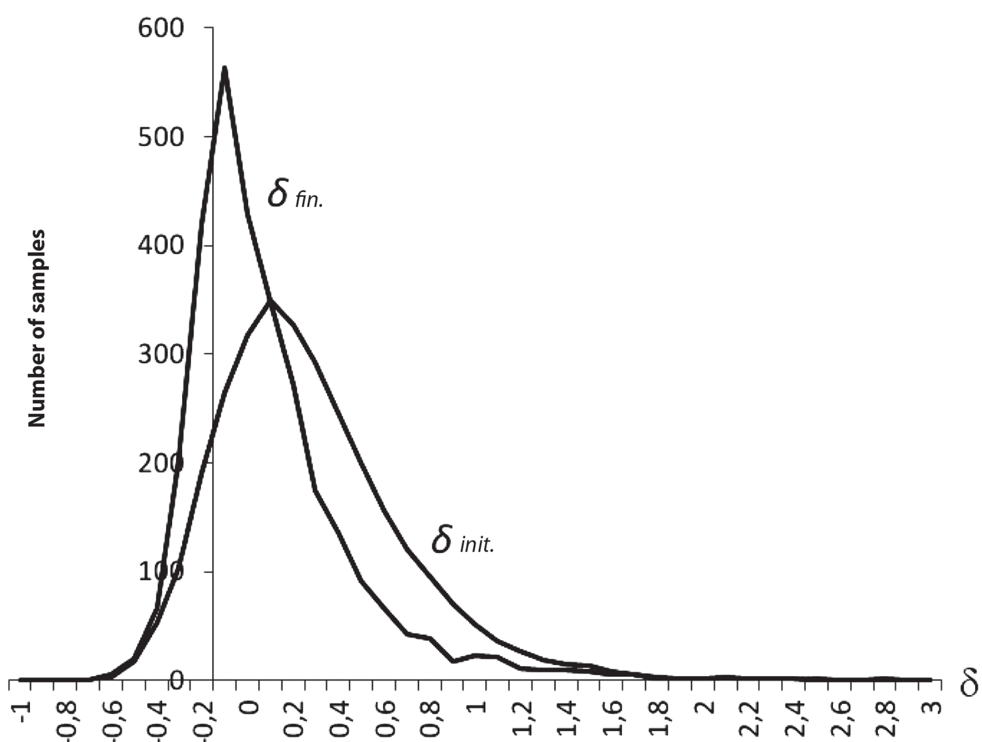

Fig. 4. Initial and final deviations of ML estimations for N =15

The resulting regression equations enable higher accuracy of maximum likelihood estimation by introducing correction data $\bar{y}(x)$ to the ML assessment using the following formula:

$$\lambda_{KOH} = \lambda_{OMП} \cdot \bar{y}(x), \qquad (3)$$

where λ_{KOH} is the final estimation of the distribution parameter.

5. The effectiveness of the developed regression equations was evaluated as part of the study. For each sample generated using regression equations (2), the correction data for ML estimation and the final distribution parameter estimate were calculated using expression (3).

The results of application effectiveness study of the developed regression equations for the exponential distribution law are shown in fig. 2 – fig. 5.

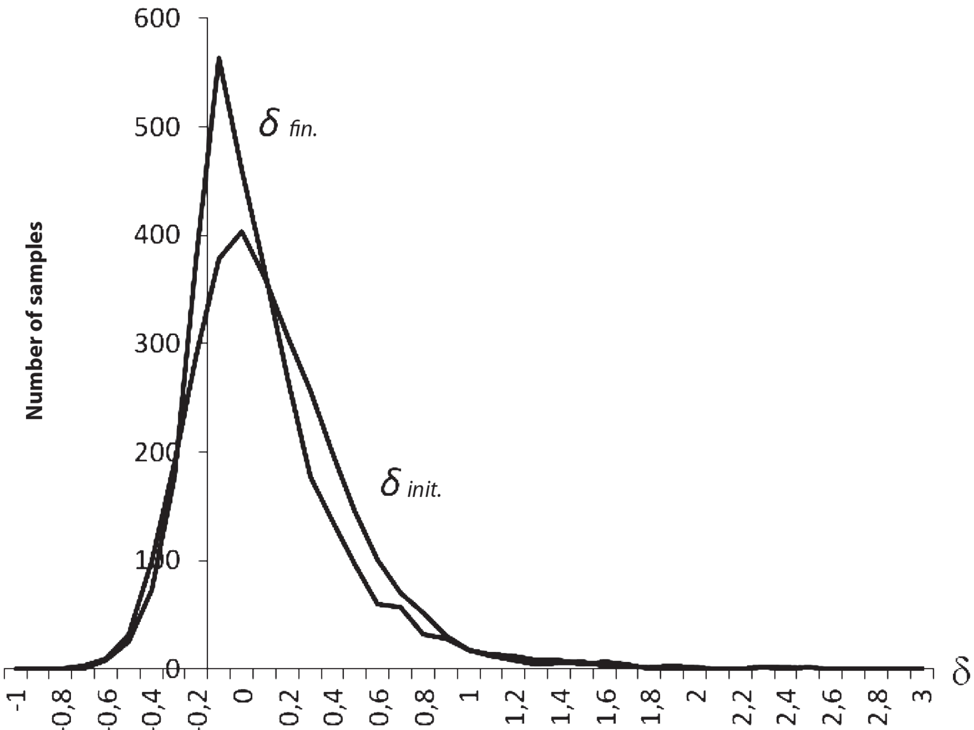

Fig. 5. Initial and final deviations of ML estimations for N =20

The diagrams given in fig. 2 – 6 show that the accuracy of ML estimation significantly improves with the application of the developed models and introduction of the correction data. Upon introducing the correction data, the relative deviations of estimations from true values of distribution parameters, depending on sample size *N*, do not exceed 0,3 – 0,5, while initial deviations at small values of *N* can be more than 3. The maximum effect of the correction data introduction to the maximum likelihood estimate of the exponential distribution is achieved when the number of members of the sample is $N = 5$. It can be concluded that the correction data introduction allows improving the accuracy of maximum likelihood 1,5 – 3 times depending on the sample size.

In order to ensure calculation of electrical equipment dependability using the suggested method computer software was developed.

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