Dzirkal E.V.

N.O. DEMIDOVICH'S SEQUENTIAL CONTROL METHOD

Our late colleague Nikolai Olegovich Demidovich developed and gave us a great method of sequential control of dependability (and other similar properties of products) that is significantly superior to the "classic" Wald's method (featured in all textbooks). Not many people can now recognize Demidovich's method in GOST R 27.402-95 and IEC 61124. His first articles date back to the 1960s. The method that uses computers and formulas allows choosing the sequential control boundaries that ensure the accuracy of specified risks values. N.O. Demidovich's boundaries can create indecision regions of any shape (including closed ones) and do not require truncation. The purpose of this article is to reestablish N.O. Demidovich's (and Russia's) priority in sequential control and replace Wald's method (paying the latter the well-deserved tribute of course).

Keywords: *Demidovich's method, sequential control, vendor and buyer risks.*

1. Introduction. General methodology of sequential control

The sequential dependability control (as well as of other similar parameters) consists in the following: at each particular moment a certain value is compared to two boundaries, acceptance and rejection. Between those boundaries is the indecision region (if the result falls within it, the tests continue). During dependability tests, the summarized operating times and failures are plotted on the test plan graph in a stepped line (failure process implementation line). Such graph for Wald's classic sequential method is shown in fig. 1. The tests are carried out until the failure process implementation line first reaches the acceptance boundary (lower line in fig. 1) or the rejection boundary (upper line).

Wald's method that is described in all textbooks, does not involve any limitations of testing time. As soon as the Wald indecision region is truncated, the supplier and the customer risks immediately and significantly increase (by an unknown magnitude). No one knows at what level the region should be truncated. Usually, it is done arbitrarily, which represents the primary disadvantage of Wald's method.

N.O. Demidovich has developed a method that allows defining the boundaries that enable the sequential control to precisely identify the risks. N.O. Demidovich's boundaries can create indecision regions of any shape, including closed, that does not require truncation (he adopted a triangular shape). The method is recognized by ISO/IEC experts, authorized in Russia by the GOST R 27.402-95 standard and is included in the second edition of the draft international standard IEC 61124 that is being prepared to issue.

It should be noted that even before Demidovich's plans there were other plans developed in the USSR by N.E. Yarlykov that provided the same advantages (they were featured in GOST 27.410-87 but left unnoticed by IEC/ISO for a number of reasons).

Demidovich's plans are obviously superior to "classic" Wald's plans and the latter should be replaced in all standards and textbooks. N.O. Demidovich's first article from the *Nadiozhnost i kontrol kachestva* (Dependability and Quality Control) journal as of 1990 was reprinted in the Dependability journal (No. 3, 2013). Below is the text of another of his articles (written as an annex to a GOST) where the method is described in greater detail. There are some explanations added by E.V. Dzirkal.

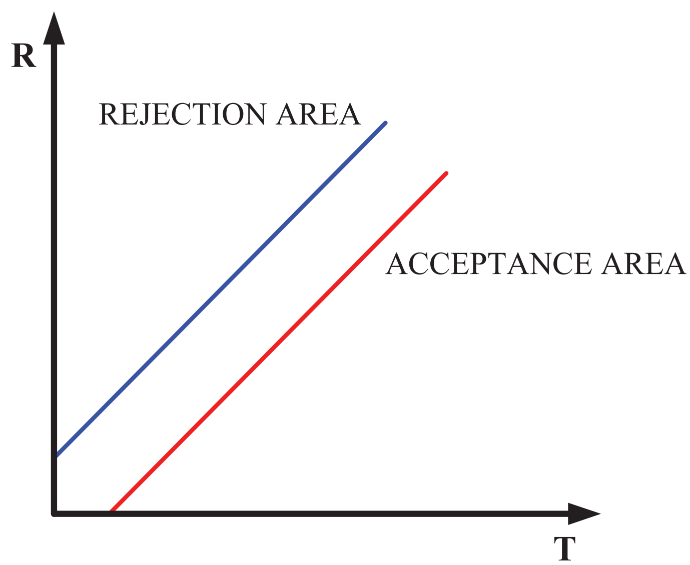


Fig. 1

2. N.O. Demidovich's approach

The work by N.O. Demidovich was performed in two stages. First, he developed the method that uses the computer to calculate all parameters (testing time, both risks, etc.) for plans of any shape. At the second stage, he used that method to identify (by selection) the optimal parameters for the plan of the chosen shape, the criterion being the duration of testing. He identified as many optimal plans for different combinations of input data as was required for inclusion in the Russian, then international standards.

2.1. Input data

A test plan with randomly defined boundaries is shown in fig. 2. It is limited by the maximum total operating time T_{max} and limit (rejection) number of failures *R*.

Notes

a) In the figure, the limit number of failures *R*=5, but in general cases it may be any whole positive number.

b) The boundaries of the test plan are shown with a continuous line, but the boundary values only have a meaning under integer values of the discrete Y-axis.

2.2. General algorithm of test plan characteristics calculation

Step 1. Through the points of intersection of boundaries and horizontal levels; = 0, 1, 2, vertical sections are drawn as shown in fig. 3.

Note: horizontal level *R* = 0 is the X-axis.

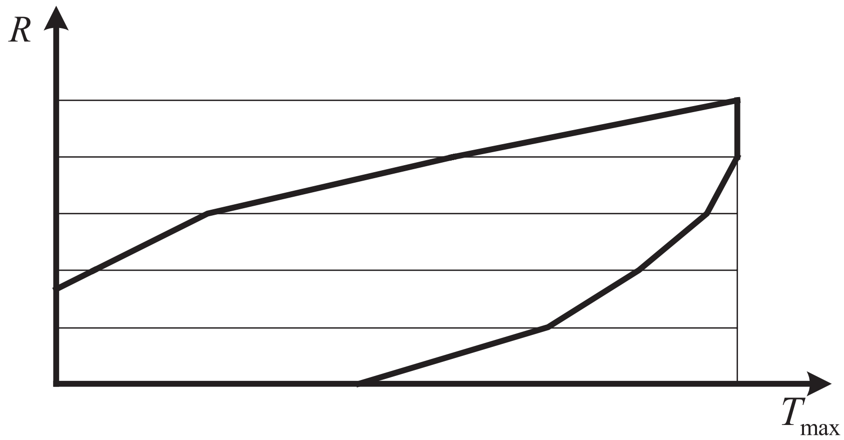
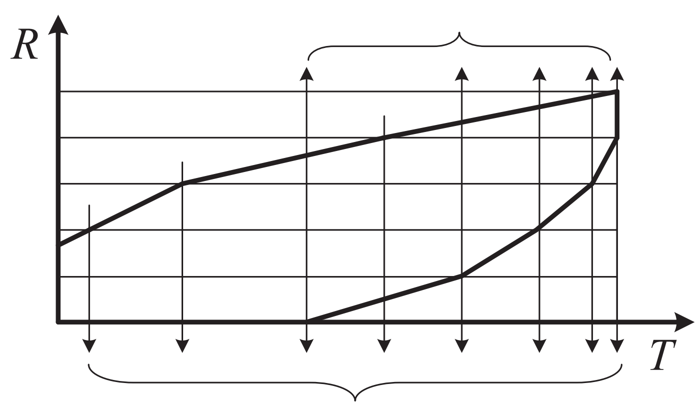


Fig. 2



1 - sections crossing the acceptance boundary
2 - sections crossing the acceptance and rejection boundaries

Fig. 3

Step 2. Points of intersection of horizontal levels and vertical sections within the test plan boundaries are marked along with the acceptance boundary points, as shown in fig. 4. Points within the test plan boundaries in fig. 4 are marked with light-colored circles, while the acceptance boundary points are marked with semi-bold circles.

All possible failure process implementation lines between two points in adjacent sections are shown with one arrow as also shown in fig. 4.

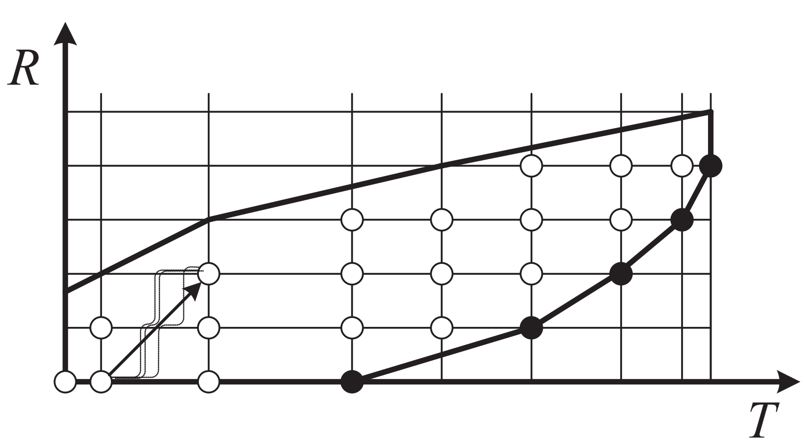


Fig. 4

Each failure process implementation line from the origin of coordinates to the intersection with the rejection boundary or attainment of the acceptance boundary can be represented as a series of points and connecting arrows.

In order to calculate the test plan characteristics it suffices to consider the marked points only.

All possible failure process implementation lines before the acceptance boundary represent a total of arrows connecting the points in adjacent sections as shown in fig. 5.

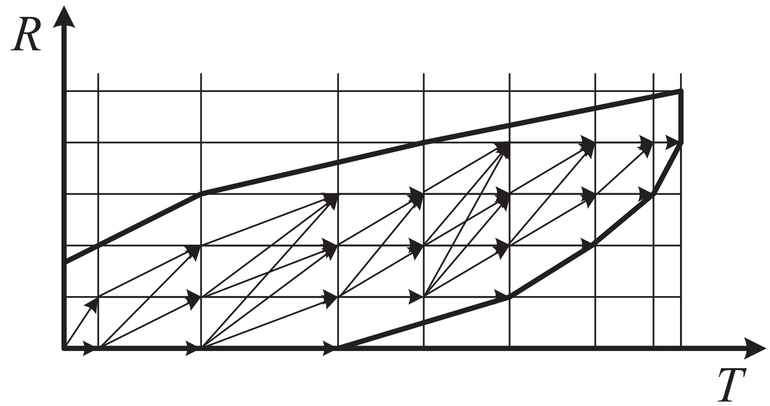


Fig. 5

Step 3. Successively (along vertical sections beginning with the first one and from bottom to top along the points of each sections) the probability value is calculated of the failure process implementation line passing through this inner point and probability value of the line reaching this point of the acceptance boundary. Then, the operating characteristic, vendor risk, buyer risk and expected total operating time before acceptance decision are calculated.

The probability value of the failure process implementation line passing through the inner point for each inner point of the test plan is calculated according to the formula

$$q_l^{(k)} = e^{-\lambda\Delta_k} \sum_{i=a_{k-1}}^{\min(l,b_{k-1})} q_i^{(k-1)} \frac{(\lambda\Delta_k)^{l-i}}{(l-i)!}, \tag{1}$$

where $q_l^{(k)}$ is the probability of failure process implementation line crossing the inner point at the l^{th} level in the k^{th} section;

k is the number of vertical section, k=1, 2,…, s;

$\lambda = 1/T$ is the failure rate of tested products;

T is the true (unknown) value of mean time to failure;

i is the summation index in the k^{th} section;

l *is* the fixed number of horizontal level, $l = 0, 1, \dots, R-1$;

m is the summation index in the $(k+1)^{\text{th}}$ section;

Δ_k is the interval of total operating time between adjacent k^{th} and $(k-1)^{\text{th}}$ sections;

a_k is the number of the lower inner point in the k^{th} section; b_k is the number of the higher inner point in the k^{th} section.

The probability value of the failure process implementation line reaching the point of acceptance boundary for each point of the acceptance boundary is calculated according to the formula

$$p_j \equiv q_{l=a_{k-1}}^{(k)} = q_{a_{k-1}}^{(k-1)} e^{-\lambda\Delta_k}. \tag{2}$$

Note: equation (2) is a special case of equation (1).

The operating characteristic value is calculated according to the formula:

$$L = \sum_{j=0}^{R-1} p_j. \tag{3}$$

True values of vendor and buyer risks are calculated according to the formulas:

$$\alpha_1 = 1 - L(T_\alpha) = 1 - \sum_{j=0}^{R-1} p_j(T_\alpha) \tag{4}$$

$$\beta_1 = L(T_\beta) = \sum_{j=0}^{R-1} p_j(T_\beta). \tag{5}$$

It is to be reminded that the vendor (manufacturer) risk α is the probability of product rejection decision under the condition that the true value of the mean time to failure equals the acceptance level T_α. Buyer (consumer) risk β is the probability of product acceptance decision under the condition that the true value of the mean time to failure equals the rejection level T_β.

The value of the expected total operating time of tested products before acceptance decision is calculated according to the formula:

$$T_O^{(+)} = \frac{\sum_{j=0}^{R-1} \tau_j p_j}{\sum_{j=1}^{R-1} p_j}, \tag{6}$$

where τ_j is the total operating time to the j^{th} section of the acceptance boundary, $\tau_j = \tau_1, \ldots\ldots \tau_{R-1}.$

Step 4. Successively (from the origin of coordinates and from bottom to top along the points of each sections) the probability value is calculated of the failure process implementation line from the specified inner point crossing the rejection boundary between adjacent sections.

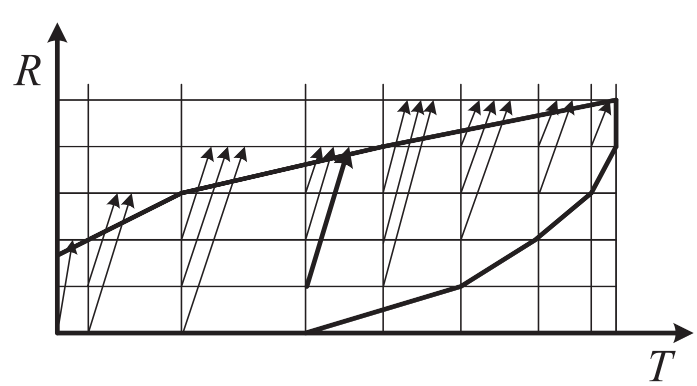


Fig. 6

The probability value of the failure process implementation line from the specified inner point crossing the rejection boundary between adjacent sections for each inner point of the plan is calculated according to the formula:

$$Q_i^{(k)} = 1 - e^{-\lambda\Delta_{k+1}} \sum_{m=0}^{b_{k+1}-i} \frac{(\lambda\Delta_{k+1})^m}{m!}. \quad (7)$$

The corresponding value of the expected total operating time within the interval is calculated according to the formula:

$$\tilde{\Delta}_{k+1} = \frac{b_{k+1} - i + 1}{\lambda Q_i^{(k)}} \left(1 - e^{-\lambda\Delta_{k+1}} \sum_{m=0}^{b_{k+1}-i+1} \frac{(\lambda\Delta_{k+1})}{m!} \right). \quad (8)$$

The value of the expected total operating time is calculated according to the formula:

$$T_O = \sum_{j=0}^{R-1} p_j \tau_j + \sum_{k=1}^{s-1} \sum_{i=\alpha_k}^{b_k} q_i^{(k)} Q_i^{(k)} (t_k + \tilde{\Delta}_{k+1}). \quad (9)$$

Notes

Values q_i; p_j; L; $T_o^{(+)}$; $Q_i^{(k)}$; $\tilde{\Delta}_{k+1}$; T_o are functions depending on the controllable value T.

Initial values of the variables featured in the formulas: $t_O = 0$; $q_O^{(0)} = 1$; $q_i^{(0)} = 0$.

2.3. Optimal plans calculation

Optimal test plans are identified in the following order:

Step 1. Input data D, α, β (D is the relation between the acceptance and rejection boundaries, $D = T_\alpha / T_\beta$) are specified (chosen, set).

Step 2. Test plan type is chosen (e.g. consecutive plan with the boundaries shown in fig. 7. N.O. Demidovich deemed this plan shape to be optimal).

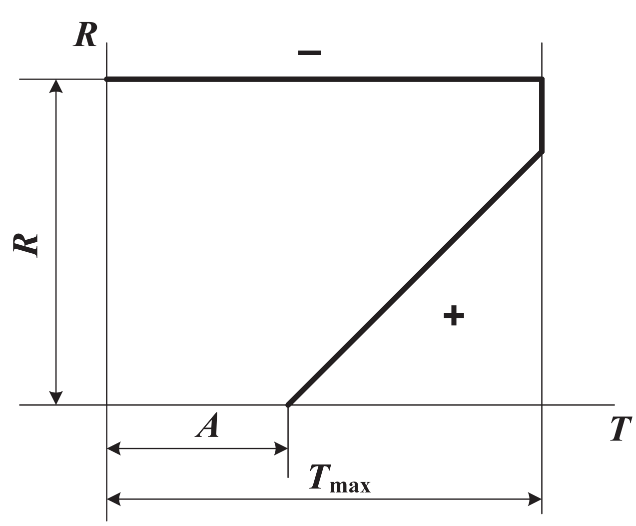


Fig. 7

Step 3. Initial values of the plan's control parameters *A, R,* T_{max} are chosen (see fig. 7).

Step 4. The characteristics of the chosen test plan type with initial values of control parameters are calculated. As the result of the calculations, at this step the first inaccurate and non-optimal test plan is generated.

Step 5. The values of the control parameters are modified, calculations according to step 4 formulas are repeated and the second test plan is obtained. Then, values of the control parameters are modified again, calculations are repeated and the third test plan is obtained, etc. This procedure is repeated iteratively to eventually obtain true values of risks approximating the preset values. When the true values of risks match the preset values with predefined precision the procedure is finished.

As the result of the calculations, at this step the first accurate (but not yet optimal) test plan is generated.

Step 6. The value of the control parameter $T_{\mathbf{max}}$ is modified within set limits, calculations as per steps 4 and 5 are repeated and the second accurate test plan is obtained. The value of the control parameter is modified again, calculations are repeated and the third accurate test plan is obtained, etc. This procedure is repeated iteratively thus reducing the expected total operating time $T_{\mathbf{max}}$ or expected total operating time before acceptance decision $T_0^{(+)}$. When those values reach the minimum with required precision the procedure is finished.

As the result of the calculations, at this step the first accurate optimal test plan is generated.

***Step** 7*. If the value of the control parameter $T_{\mathbf{max}}$ of the first accurate optimal test plan does not exceed the specified limit, the value of the control parameter R is increased by one, calculations as per steps 4, 5, 6 are repeated and the second accurate optimal plan is obtained. If the value of the control parameter $T_{\mathbf{max}}$ of the second accurate optimal test plan does not exceed the set limit either, the value of the control parameter R is again increased by one, calculations are repeated, the third test plan is obtained, etc.

Note: each subsequent test plan is more optimal compared to the previous one.

If the value of the control parameter $T_{\mathbf{max}}$ of the first accurate optimal test plan exceeds the set limit, the vendor and buyer jointly decide upon the choice of the test plan by means of possible changes in the set of input data and limitation of the maximum total operating time.

Formulas (1), (2), (7) and (8) are recurrent (identical for all points of the test plan) and form the recurrent element given in fig. 8.

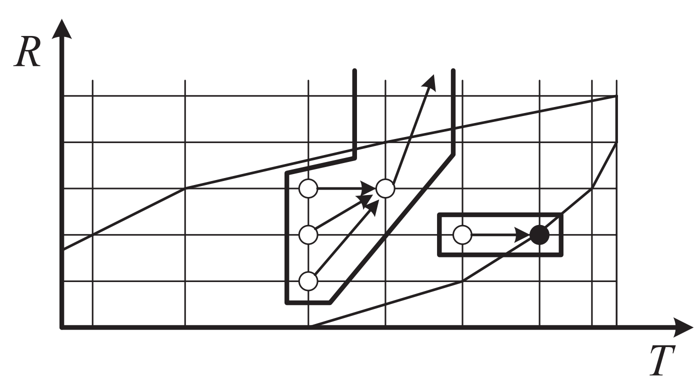


Fig. 8

Risk values and test plans characteristics are calculated using a computer program. Manual calculations are cumbersome and do not allow obtaining accurate optimal test plans.