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MATHEMATICAL MODEL OF THE RELIABILITY OF A PRODUCT SUBJECT TO IMPACT STRESS

The paper considers a mathematical model of reliability in view of a product's load accumulation caused by cyclic impact loads. Study of the process when accumulated stress crosses a random level of endurance under repeated impact disturbances allowed us to obtain a mathematical model that makes it possible to calculate reliability and durability parameters. The obtained final relations for reliability parameters have a simple form, which can be used in practical calculations.

Keywords: *damage accumulation, impact load, distribution of sums of random variables, reliability parameter, durability, estimated mean, impact, crossings of the level.*

Introduction

During the process of maintenance of products, aging of material occurs under the influence of factors such as increased temperature and pressure, cyclic loads, presence of corrosive environment, which leads, for example, to fatigue crack growth. By now, physical processes leading to equipment failure has been well studied by fracture mechanics and mechanics of materials.

[1] states that so far the behavior of structural materials under operating conditions has not been studied well enough to provide the basis for model justification of cumulative damages according to the fundamental physical laws. Obviously, these problems can be solved without quantitative assessment of reliability using the experience of maintenance of such devices in the same or similar production areas. However, such solution of the task without a rigorous mathematical analysis of operation processes and quantitative estimates of reliability and durability by no means can be considered well founded.

A product's operation process in question, which is affected by impact loads, can be regarded as a process with aging. [2] introduced the concept of aging based on time behavior of a failure rate function. In particular, it identified an increasing rate function IRF and a decreasing rate function DRF. It was also shown that if a system consists of independent elements with IRF time-to-failure distribution, then a distribution emerges that has a rate function increasing in the mean RFIM.

Often one can get information about a growing aging of devices considering the dynamics of certain parameters. Knowledge of indicators of the parameter changing process allows us to find the distribution of a device's MTBF, which in turn makes it possible to determine the time to stop a device's maintenance until its complete destruction and recovery of its properties. Naturally, during operation of such systems, the issues of quantitatively assessing reliability and durability parameters and afterwards the task of scheduling the time to stop a product's maintenance arise [3].

To date, mathematical models of the reliability of products, which are opposed to static loads, and the methods of obtaining relevant indicators have been satisfactorily developed [4, 5]. If, in addition to static loads, the product is opposed to the load, which has a random amplitude and random oscillation period, mathematical models for describing the operation of such products are very cumbersome, and to obtain quantitative values for corresponding reliability and durability parameters based on their analysis is generally impossible. The process of operation of such products is described by Ito's stochastic differential equation with a discrete constituent [6]. [7] considered the asymptotic method for calculating life indicators of a product, which is opposed to impact loads and showed that such process is a process with independent increments.

Studies of processes of load accumulation are given in [8], where also the optimization of preventive maintenance according to the criterion of minimum expected cost is analyzed.

This paper will study the mathematical model of operation reliability of a product, which is affected by impact loads provided that endurance is a fan random process.

Let us immediately come to the definition of the mathematical problem of determining dependability and durability parameters, using a mathematical model of a product's evolution based on the theory of random processes of accumulation.

Problem definition

We shall consider a product subject to wear due to a number of applied pulse impacts. These can be jolts, shocks, temperature and pressure pulsations, vibrations, etc. From now on, without specifying the physical nature of impacts on a product, we will call pulse actions as impact (cyclic) loads. Let the change in parameters of a product be caused by impact loads (jolts, shocks, pulses), occurred at times t_0, t_1, $t_2, \cdots$, $t_{k+1} \geq t_k$, $k \geq 1$. Denote $\tau_i = t_{i+1}$-t_i, $i > 0$, $t_0 = 0$, where τ_i is random variables equal to the length of time intervals between successive impact applications to a product. Random variables τ_i, $i = 2, 3, 4, \cdots$ are independent and distributed with the same distribution function $F(t)$, where $F_i(t) = P(\tau_i \leq t)$.

It should be noted that if a random variable is equal to $\tau_1 = t_1$, it is possible that $F_1(t) \neq F(t) = P(\tau_1 \leq t)$ will be true for it, therefore, the quantity $\tau_{1\,\mathrm{I}}$ is distributed differently than all other values τ_i. Thus, the sequence of non-negative mutually independent random variables $\{\tau_i, i \geq 1\}$ is completely characterized by distribution functions $F(t)$ and $F_1(t)$.

As at time points $t_i, i \geq 1$ the product is opposed to shock effects (impacts), at the same moments there is a stepwise change of damage manifested in the abrupt increase of load. Each such change will be denoted as θ_i, a non-negative random variable equal to increment (increase) of load value (product wear) as a result of the effect of the *i*-th impact disturbance, $i = 1, 2, 3, \cdots$.

As regards the random variables θ_i, it is natural to assume that they are also independent, as well as they are distributed with the same distribution function $G(y)$, and therefore, we have the following condition for them $G_1(y) = G_2(y) = \cdots = G(y)$, where $G_i(y) = P(\theta_i \leq y)$. We suppose that between times of two successive shocks the value of the load applied to the product will not change. Note that the value θ_0 is independent on the sequence of random variables $\{\tau_i, i \geq 1\}$. The study [3] notes that the process of product operation described above is defined by RFIM distribution, which is characterized by a rate function increasing in the mean.

Now let us denote the endurance value at time point *t* as χ_t. We shall represent the random process $\{\chi_t\}_{t\geq 0}$ of endurance changing as a monotonically decreasing linear random function in the following form

$$\chi_t = \chi_0 - X_t ,$$

where $\{\chi_t\}_{t\geq 0}$ is a stochastic process with the property $t=0$, χ_0 is the initial value, which may be not random. Suppose that $X_t = Vt$, V is the speed of endurance changing, then the expectation and the dispersion of a linear random function will be defined as follows

$$M\chi_t = M(\chi_0 - tV) = M\chi_0 - tMV\ ,\ M(\chi_t - M\chi_t) = 0\ ,$$

$$M(\chi_t - M\chi_t)^2 = D\chi_0 + t^2 DV - 2tM\chi_t MV\ ,$$

$$M(\chi_t - M\chi_t)^3 = M(\chi_0 - M\chi_0)^3 + t^2 M(V - MV)^3 ,$$

where $M\chi_0$, MV, $D\chi_0$, DV are expectations and dispersions of the initial value and the speed of endurance changing, respectively. Hereafter, we will assume that the random variables are independent. A one-dimensional distribution function of the process $\{\chi_t\}_{t\geq 0}$ will be denoted as

$$F_\chi^t(x) = P(\chi_t \leq x)\ .$$

The random process described above is called as a fan process, and all its implementations have a common random point $(M\chi_0, 0)$ [9]. Regarding the function $F_\chi(y)$, we shall require that it should meet all of the properties of a distribution function.

If we consider the traditional model of reliability "load – endurance", then the probability of failure-free operation of a product is independent from time, since it is assumed that the product is subject to static impacts in the process of operation.

Our objective is to obtain reliability and durability parameters of a product, which is opposed to random disturbances during its operation, with its random endurance.

Formalization and solution of the problem

The dynamics of a key parameter of product availability operating under conditions of impact loads can be represented graphically (Fig. 1). It is assumed that a product stops working as soon as the accumulated damages exceed the specified level of endurance, the value of which is limited by the random function $\chi_t = \chi_0 - X_t$, (see Fig. 1).

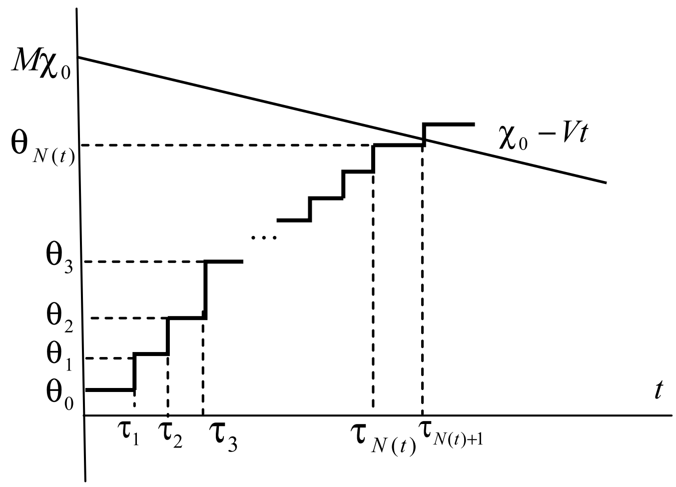

Fig.1. Graphical representation of operation process under conditions of impact loads

It is the accounting of random actions that allows us to introduce time dependence in this model. As a result, a traditional static model of reliability "load – endurance" becomes a dynamic model.

It is easy to see that since a product is opposed to an increasing load L_t $t \geq 0$, it can be defined by the following equality

$$L_t = \begin{cases} \sum_{i=1}^{N_1(t)} \theta_i, & N_1(t) = 1, 2... \\ 0, & N_1(t) = 0 \end{cases}. \tag{1}$$

The value L_t means the accumulated damage to a product during operation *t.* A stochastic process defined in this way is called cumulative. Here, the summing is made for all impact loads, which have occurred up to the time *t* inclusive. We shall denote as $N_1(t) = N_t$ the number of renewals of recovery counting process $\{N_1(t)\}_{t>0}$, corresponding to the process $M\chi_t = M(\chi_0 + tV) = M\chi_0 + tMV$. Therefore, $N_1(t) = N_t$ is a random number of impact loads over time $(0, t]$ or is the number of cycles of recovery process $\{\tau_i, i \geq 1\}$, the expectation of which is a function of recovery $H_1(t) = MN_1(t)$.

In view of the fact that the process $\{L_t\}_{t>0}$ is stepwise increasing and its implementations are stepwise functions, then specifying the acceptable limit (endurance), in our case $\chi_t = \chi_0 - X_t$, we can calculate the appropriate reliability and durability indices.

Then we shall introduce additional, but necessary notations and definitions for a further study without going over description of the evolution of a product subject to periodic random effects.

We shall assume that impact disturbances are random, i.e. they are applied to a product at random time points and have random amplitude. Each disturbance leads to decrease in endurance, or it can be assumed that every action leads to increase of load on some random value described by a relevant distribution function. Thus, the random process of accumulation, in which load change is a stepwise increasing stochastic process, and a product's failure occurs only when this process crosses the border (see Fig. 1), which is a random variable, and moreover, a random process (fan shaped).

We shall use the results obtained for the case of deterministic endurance of a product in the calculation of the characteristics of product reliability and durability [10].

Further we shall consider that the process $\{L_t\}_{t>0}$ is a simple process of recovery generated by distribution functions $F(t)$, $F_1(t)$. This simplifying assumption is easily generalized to the case of a delayed recovery. It is easy to receive evidence that to obtain the probability of failure-free operation of a product over time *t*, one of the recorded relations for the probability $P(L_t \le \mathrm{x})$ can be used [3]. Using the condition of a product failure, and the total probability formula we shall calculate the conditional probability that the cumulative load does not exceed the endurance of the product during operation, which we write as

$$P\left(L_t \le x\right) = MJ_{\sum\limits_{i=1}^{N_1(t)}\theta_i \le x} = \sum_{k=0}^{\infty}\left(MJ_{\sum\limits_{i=1}^{N_1(t)}\theta_i \le x}\Big|_{N_1(t)=k}\right) P(N_1(t)=k) = \sum_{k=0}^{\infty}\left(MJ_{\sum\limits_{i=1}^{k}\theta_i \le x}\right) P(N_1(t)=k) =$$

$$= \sum_{k=0}^{\infty} G^{*(k)}(x) P(N_1(t)=k) = \sum_{k=0}^{\infty} G^{*(k)}(x)\left(F^{*(k)}(t) - F^{*(k+1)}(t)\right).$$

Since the endurance is a random variable in the considered model, then using the formula of conditional expectation of a random function $P(L_t \le \mathrm{x})$, we obtain the probability of failure-free operation of a product over time. Then

$$P(t) = \int_0^{\infty} P(L_t \le \mathrm{x})\, dF_{\chi_t}(x) = \int_0^{\infty}\sum_{k=0}^{\infty} P(N_1(t)=k) G^{*(k)}(x)\, dF_{\chi_t}(x) =$$

$$= \sum_{k=0}^{\infty} P(N_1(t)=k)\int_0^{\infty} G^{*(k)}(x)\, dF_{\chi_t}(x) = \sum_{k=0}^{\infty} P(N_1(t)=k) C(k) = \sum_{k=0}^{\infty} C(k) P_k(t) = MC(N_1(t)), \tag{2}$$

where $C(k) = \int_0^{\infty} G^{*(k)}(x)\, dF_{\chi_t}(x) = MG^{*(k)}(\chi_t)$.

It should be noted that relation (2) is a distribution function of the accumulated load over time *t*.

Similarly, we introduce the second recovery process $\{Z_x\}_{x>0}$ associated with the time function of a product by the following formula (see Fig. 1)

$$Z_x = \begin{cases} \sum\limits_{i=1}^{N_2(x)} \tau_i + \tau_t, & N_2(x) = 1, 2, \cdots \\ 0, & N_2(x) = 0 \end{cases}$$

where Z_x is a random time to failure for a given acceptable load *x*. Here $N_2(x) = N_x$ is the number of cycles of recovery until the exhaustion of endurance resources by the process of load accumulation. The expectation of a random variable $N_2(x)$ is also a function of recovery, which we denote as $H_2(x) = MN_2(x)$. The quantity $\sum\limits_{i=0}^{N_2(x)} \tau_i + \tau_t$ is a random mean time between failures of a product before crossing the endurance level χ_t by accumulated load, where τ_t is the reverse residual time, that is the time during which a product operates properly after the last impact. A conditional distribution function of a product's MTBF can be written using the formula of total probability

$$P(Z_x \le t) = MJ_{\sum_{i=1}^{N_2(x)} \tau_i + \tau_t \le t} = \sum_{k=0}^{\infty} \left(MJ_{\sum_{i=1}^{N_2(x)} \tau_i + \tau_t \le t} \Big|_{N_2(x)=k} \right) P\left(N_2(x)=k\right) =$$
$$= \sum_{k=0}^{\infty} P\left(\sum_{i=1}^{N_2(x)} \tau_i + \tau_t \le t \Big|_{N_2(x)=k} \right) P\left(N_2(x)=k\right) = \sum_{k=0}^{\infty} F_t * F^{*(k)}(t) P\left(N_2(x)=k\right).$$

where $F_t(t)$ is the distribution function of the reverse residual time τ_t [3].

Calculating the expectation from a recorded above relation, we obtain the distribution function of a product's MTBF and the distribution of time to a first failure.

$$Q(t) = \int_0^{\infty} P(Z_x \le t) dF_{\chi_t}(x) = \int_0^{\infty} \sum_{k=0}^{\infty} F_t * F^{*(k)}(t) P(N_2(x)=k) dF_{\chi_t}(x) =$$
$$= \sum_{k=0}^{\infty} F_t * F^{*(k)}(t) \int_0^{\infty} (G^{*(k)}(x) - G^{*(k+1)}(x)) dF_{\chi}(x). \quad ((3)$$

The function $Q(t)$ is the probability of a product's failure during its operation time *t,* therefore, $Q(t) + P(t) = 1$. The integrand (3) $G^{*(k)}(x) - G^{*(k+1)}(x) = P\left(\sum_{i=1}^{k} \theta_i < x < \sum_{i=1}^{k+1} \theta_i \right)$ is the probability that a failure has occurred between the *k*-th and the *k +1*-th impacts.

It is evident that (3) and (2) are a mixture of distribution functions $F^{*(k)}(t)$ and $G^{*(k)}(x)$ with weights $g_2(x) = G^{*(k)}(x) - G^{*(k+1)}(x)$ and $g_1(t) = F^{*(k)}(t) - F^{*(k+1)}(t)$, respectively. It is important that the distribution of MTBF belongs a class of RFIM with any distribution function $G(t)$.

It is evident that further analytical transformations of (2), (3) in general form are not possible. In solving problems, where it is necessary to calculate numerous convoluted functions, Laplace transforms or generating functions are commonly used, and then there is a need to convert the resulting transformation. It should be noted that the problem of the conversion of a Laplace transform, as a rule, is of the same order of difficulty as the initial problem. The difficulty of solving the initial problem consists in the fact that to get required probabilities in convenient form it is necessary to calculate successively: first, the expectation of time of the i-th convolution function $G(x)$ or $F(t)$, depending on the problem under consideration, and second, the re-calculation of the expectation from the obtained result.

Let us consider the special case of the cumulative process, i.e. the process of impact loads. Let us assume for this that the random variables τ and $\chi_t = \chi$ are exponentially distributed, i.e. $F(t) = 1 - e^{-\lambda t}$ and $F_{\chi}(x) = 1 - e^{-\nu x}$, where λ is the rate of impact loads, and $\nu = 1/M\chi$ is the rate of endurance changes. Under this assumption, the sequence of random variables forms a Poisson flow of events, and from the assumption that $V = 0$ it follows that the crossing of the endurance level by the determining parameter occurs only at moments of impact loads. Since the random processes $\{L_t\}_{t>0}$, $\{Z_x\}_{x>0}$ are changing simultaneously (see Fig. 1), it allows them to be considered as synchronous processes.

Now we rewrite (2) in view of the assumptions made. Taking into account that the rate of impact loads forms a Poisson process, for which $P(N_t = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$, we have

$$P(t)=\int_0^{\infty}P(L_t\leq x)dF_{\chi_t}(x)=\int_0^{\infty}\sum_{k=0}^{\infty}P(N_t=k)G^{*(k)}(x)dF_{\chi_t}(x)=$$
$$=\sum_{k=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^k}{k!}\int_0^{\infty}G^{*(k)}(x)dF_{\chi_t}(x)=1-\sum_{k=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^k}{k!}\int_0^{\infty}F_{\chi_t}(x)dG^{*(k)}(x)=$$
$$=\sum_{k=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^k}{k!}\int_0^{\infty}\overline{F}_{\chi_t}(x)dG^{*(k)}(x).$$

Noting that the integral of this relationship should be considered as the Laplace-Stieltjes transform $\tilde{G}(\nu)=\int_0^{\infty}e^{-\nu x}G(x)dx=Me^{-\nu\theta}$ of the function $G^{*(k)}(x)$, since $\overline{F}_{\chi_t}(x)=e^{-\nu x}$ for the exponential distribution of the random variable χ_t, then we have

$$P(t)=\sum_{k=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^k}{k!}\int_0^{\infty}e^{-\nu x}dG^{*(k)}(x)=\sum_{k=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^k}{k!})\left(\tilde{G}(\nu)\right)^k=$$
$$=\sum_{k=0}^{\infty}e^{-\lambda t}\frac{\left(\lambda t\tilde{G}(\nu)\right)^k}{k!}=e^{-\lambda t(1-Me^{-\nu\theta})}. \tag{4}$$

Similarly to (2), let us calculate the distribution function of the cumulative load for the time *t* using relation (3), assuming that the process of a product's operation is the process of impact loads. Under general assumptions, the distribution function of cumulative load for the time *t* is defined as follows

$$P(t)=\sum_{k=0}^{\infty}F'*F^{*(k)}(t)\int_0^{\infty}(G^{*(k)}(x)-G^{*(k+1)}(x))dF_{\chi}(x),$$

where $F_t(x)$ is the distribution function of the inverse residual time τ_t.

To further simplify the obtained relation for $P(t)$, we shall rewrite the integral appearing in this equation as

$$\int_0^{\infty}(G^{*(k)}(x)-G^{*(k+1)}(x))dF_{\chi}(x)=\int_0^{\infty}\left(P\left(\sum_{i=1}^{k}\theta_i\leq x\right)-P\left(\sum_{i=1}^{k+1}\theta_i\leq x\right)\right)dF_{\chi}(x)=$$
$$=\int_0^{\infty}\overline{F}_{\chi}(x)d\left\{P\left(\sum_{i=1}^{k}\theta_i\leq x\right)-P\left(\sum_{i=1}^{k+1}\theta_i\leq x\right)\right\}.$$

Given that the quantity $\chi_t=\chi$ is exponentially distributed with the parameter ν, i.e. $F_{\chi}(x)=1-e^{-\nu x}$ and calculating the Laplace-Stieltjes transform of the function $P\left(\sum_{i=1}^{k}\theta_i\leq x\right)-P\left(\sum_{i=1}^{k+1}\theta_i\leq x\right)$, then the integral under consideration can be rewritten again

$$\int_0^\infty (G^{*(k)}(x) - G^{*(k+1)}(x))dF_\chi(x) =$$

$$= \int_0^\infty e^{-\nu x} d\left\{ P\left(\sum_{i=1}^{k} \theta_i \le x\right) - P\left(\sum_{i=1}^{k+1} \theta_i \le x\right)\right\} = Me^{-\nu k\theta} - Me^{-\nu(k+1)\theta} = Me^{-\nu k\theta}(1 - Me^{-\nu\theta}).$$

Since the process of impacts is a Poisson process, which has the property of no-aftereffect, then in view of this remarkable property the random variable τ_t has the same distribution as the variable τ. Taking into account the convolution functions $F(t)$, we rewrite the probability as follows

$$P(t) = \sum_{k=0}^{t} F^{*(k+1)}(t) Me^{-\nu k\theta}(1 - Me^{-\nu\theta}).$$

Performing the Laplace-Stieltjes transform of the function $F(t)$, we shall rewrite

$$\tilde{P}(s) = \frac{(1 - Me^{-\nu\theta})\tilde{F}(s)}{1 - \tilde{F}(s)Me^{-\nu\theta}}.$$

Taking into consideration that $\tilde{F}(s) = \int_0^\infty e^{-st} dF(t) = \frac{\lambda}{\lambda + s}$ for the exponential law of random variable distribution τ, we can rewrite $\tilde{P}(s)$ as

$$\tilde{P}(s) = \frac{1 - Me^{-\nu\theta})\frac{\lambda}{\lambda + s}}{1 - \frac{\lambda}{\lambda + s} Me^{-\nu\theta}} = \frac{\lambda(1 - Me^{-\nu\theta})}{s + \lambda(1 - Me^{-\nu\theta})}.$$

Hence, we obtain the final expression for the distribution function of the cumulative load for the time *t*, which we write, using the theorem of residues

$$Q(t) = 1 - e^{-\lambda t(1 - Me^{-\nu\theta})} = 1 - e^{-\lambda t} e^{\lambda t Me^{-\nu\theta}}. \tag{5}$$

If we assume that the function $G(x) = 1 - e^{-\mu x}$, then $Q(t) = 1 - e^{-\lambda t \frac{\nu}{\mu + \nu}}$ and $P(t) = e^{-\lambda t \frac{\nu}{\mu + \nu}}$.

Now we shall calculate other indices of reliability, such as a failure rate of a product.

Failure rate of a product

Using (2), we obtain the relation for the failure rate $\Lambda(t)$ of a product, which is opposed to a sequence of impact loads. $\Lambda(t)$ can be written according to the definition of a failure rate as

$$\Lambda(t)=\frac{\sum_{k=0}^{\infty} f^{*(k+1)}(t)\int_0^{\infty}(G^{*(k)}(x)-G^{*(k+1)}(x))dF_{\chi}(x)}{\sum_{k=0}^{\infty}\int_0^{\infty} G^{*(k)}(x)dF_{\chi}(x)(F^{*(k)}(t)-F^{*(k+1)}(t))},$$

where the density of distribution $f^{*(k+1)}(t)$ is determined by successive integration of $f^{*(k+1)}(t)= =\int_0^t f^{*(k)}(t-x)f(x)dx$, where $f(t)=\dfrac{dF(t)}{dt}$.

In view of the fact that further analytical simplifications of the written relation of the failure rate $\Lambda(t)$ are not possible, then again, we assume that there is a process of impact loads, for which the quantities τ and $\chi_t=\chi$ are exponentially distributed random variables, then we have

$$f^{*(k+1)}(t)=\lambda\frac{(\lambda t)^k}{k!}e^{-\lambda t}.$$

Considering these assumptions and substituting the density of distribution $f^{*(k+1)}(t)$ in relation to the product failure rate $\Lambda(t)$, we have

$$\Lambda(t)=\frac{\sum_{k=0}^{\infty}\lambda\frac{(\lambda t)^k}{k!}e^{-\lambda t}\left(\tilde{G}(\nu)\right)^k(1-\tilde{G}(\nu))}{\sum_{k=0}^{\infty}\left(\tilde{G}(\nu)\right)^k e^{-\lambda t}\frac{(\lambda t)^k}{k!}}=\lambda(1-\tilde{G}(\nu)). \quad (6)$$

Equation (6) can be obtained directly from (5) and (4). Thus, we obtain a somewhat unexpected result: the failure rate of an “aging” product does not depend on time.

Since the trajectories of a product operation process is a stepwise function, then the time to failure is a discrete random variable, so to estimate the failure rate of a product, you can use a discrete analog ([2], p. 35). Using the above obtained, we have

$$\lambda_{k+1}=\frac{MG^{*(k+1)}(\chi_t)}{\sum_{i=k}^{\infty}MG^{*(i+1)}(\chi_t)}=\frac{\left(\tilde{G}(\nu)\right)^{k+1}(1-\tilde{G}(\nu))}{\sum_{i=k}^{\infty}\left(\tilde{G}(\nu)\right)^{i+1}(1-\tilde{G}(\nu))}=\frac{\left(\tilde{G}(\nu)\right)^{k+1}}{\sum_{i=k}^{\infty}\left(\tilde{G}(\nu)\right)^{i+1}}=1-\tilde{G}(\nu),$$

where λ_{k+1} is the probability that the product is in good order after the *k*-th impact, but it fails after the $k+1$-th impact. In case if $G(x)=1-e^{-\mu x}$, then $\lambda_{k+1}=\dfrac{\nu}{\nu+\mu}$, $\lambda_{k+1}\le 1$.

Thus, the assumptions about exponential distributions of rates of impact loads and magnitude of endurance allowed us to obtain explicit quantitative indices of a product’s reliability necessary for engineering practice, from the mathematical model of a product's reliability, operating under multiple effects of impact disturbance. Final relations of reliability indices are simple, which makes them attractive for practical use.

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