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THE ISSUE OF ACCURACY OF INFORMATION ASSIGNMENT FOR OPTIMIZATION OF PREVENTIVE REPLACEMENTS

The paper offers a method for researching the stability and sensitivity of models of preventive replacement optimization that allows for justification of the accuracy of initial economic information assignment.

Keywords: *model, optimization, replacement, stability, sensibility, accuracy.*

1. State of the art

Currently, optimization calculations for parameters of preventive replacements (PR) of technical devices (TD) are usually carried out under the assumption of strict reliability and unambiguity of raw information and, consequently, strong unambiguity of the solutions obtained. When solving practical optimization problems of PR, it is inevitable to face greater or lesser uncertainty of source information that presents itself in unreliable knowledge of numerical values of initial indicators or their probabilistic description. Initial information in optimization problems of PR can be divided into four types:

1. deterministic;
2. probabilistically certain, when functions and parameters of the distribution of random variables are known;
3. probabilistically uncertain, when the distribution functions of random variables are not known.

Deterministic initial information includes information about the cost of PR, the average value of which is unambiguously defined by regulations. Information about the cost of emergency recovery can be considered as probabilistically certain, because it cannot be unambiguously determined due to the dependence from the number of random factors (device sudden failure, qualification of maintenance personnel, etc.). Depending on the completeness of source data, information about the damage due to TD failures because of the random and sometimes not sufficiently certain nature can be attributed to the probabilistically certain or probabilistically uncertain information.

Particular difficulties arise in practice when selecting a reliability function (RF) because of the small amount of statistic data about TD failures. The distribution function can be determined by existing methods of mathematical statistics when the number of failures is more than one hundred. In this case, the information will be probabilistically certain, and otherwise it will be probabilistically uncertain, because several possible distribution functions can be obtained.

The uncertainty of initial information leads to methodological and practical difficulties of PR optimization. In this case, the dimension of the problem increases significantly, since there is a large number of possible combinations of information about the reliability function and cost parameters. This leads to the ambiguity of the optimization problem solution, because each interval of PR under various combinations of initial information is conditionally optimal.

Therefore, the problem of investigating the influence of the accuracy in determining the initial information about the reliability function (probability distribution function) and cost parameters while optimizing the PR remains relevant.

In the paper [1], the author for the first time introduced the concept of economic solutions' stability and variation sensitivity of the initial data functions of operational cost per unit in relation to the optimization model of preventive replacements. The research of stability and sensitivity of PR optimization models for mean time between failures and group replacements known in the reliability theory has been carried out [2]. For a specified accuracy of calculations, the areas of equally economic intervals have been defined and the region of admissible deviation of models' parameters in the area of economic sustainability assessed. The paper [3] as a continuation of this work in order to substantiate the accuracy of assignments of initial economic data presents similar research of operational cost per unit function of PR optimization models known in the reliability theory with minimal repair at failure for the Weibull distribution function [2].

Later due to the urgency of this issue, a number of publications on this problem have been offered. Thus, the paper [4] considers the case where the type of time-to-failure distribution law has been set and its parameters evaluated. It examines only the effect of deviations of the distribution parameters to choose the cycle of preventive maintenance. In this case, the maximum value of availability factor is used as a criterion, and the average duration of preventive and emergency maintenance works are considered as deterministic. It should be noted that the above example of determining the period of preventive maintenance with an exponential distribution law is incorrect. This law describes failures of ageing-resistant systems for which preventive maintenance is not appropriate [2].

Similarly, the paper [5] shows the results of researches as regards the effect of deviations of the shape parameter estimated values in Weibull distribution (in this case, the scale parameter is accepted as deterministic) on the optimal values of replacement ages and on the optimal values of operating costs using the strategy of PR according to mean time between failures. Here, the distribution function is considered specified, and information on the cost of preventive and emergency replacements is accepted as deterministic.

The paper [6] attempted to justify requirements for the accuracy of reliability factor estimates to meet the challenges of extending the lifetime of radio electronic systems life. The authors refer only to publication [1], although fully utilize the results of previous studies [3] by replacing the designation parameters in formulas. While the paper [3] gives the assessment of allowable accuracy of initial economic data based on the obtained results of stability and sensitivity operating costs, publication [6] actually repeats the study of stability of the known model for PR with minimal repair at failure for the Weibull distribution function, but there is no any solution of the stated problem in [6].

Thus, from the above analysis of publications, it is evident that the research issues and justification of the accuracy requirements for the initial information in the optimization of preventive replacements have not been completed so far. In previous publication [7], the author presented the results of the research of economic stability optimization models of PR under probabilistically uncertain information about the reliability function (RF as a function of time). In this case, for the estimated value of the variation coefficient, a family of distribution functions is specified and the equivalence of the solutions obtained is shown in relation to determination of the range of optimal values of PR periodicity.

The purpose of the present paper is to study stability and sensitivity optimization models of preventive replacements to justify the accuracy of the initial economic information assignment.

2. Justification of the accuracy of the initial economic information assignment

When optimizing the PR periodicity of technical devices, the average values of the initial economic data are considered as a guiding line. In practice, the cost of emergency repair taking into account the damage owing to repair downtime of devices has significant deviations from the average value. In this case, the parameters of mathematical models cannot be defined unambiguously. Under these conditions, when solving the problem, its own optimal values of PR periodicity correspond to each combination of parameter values.

Based on the results obtained previously in [1], the problem of justifying the accuracy by which the initial data should be determined should be solved in two stages. First, it is necessary to carry out a study of economic stability of the operational cost per unit function and for a given accuracy of calculations to determine the acceptable deviations of PR periodicity in the area of optimal values.

Second, it is necessary to investigate the sensitivity of the operational cost per unit to alteration of its parameters and using an acceptable deviation range of PR periodicity to justify the determination accuracy of the initial economic data.

We shall show the solution of this problem using the example of preventive replacements strategy with minimal repair at failure when the operational cost per unit (if operating time between failures has the Weibull distribution) is determined according to [2]

$$C(\tau) = [B+A(k_b\, \tau T^{-1})^b]\, \tau^{-1}], \quad (1)$$

where *B* is the cost of preventive replacement; *A* is the cost of emergency repair in case of failure, with the damage taken into account; *b* is the shape parameter of the Weibull distribution; $k_b = G(1- b^{-1})$. Here *G* is the gamma function, and *T* is mean time between failures (MTBF); ф is the periodicity of PR.

Dividing equation (1) by *A/T*, we will obtain the value of the relative cost per unit in the dimensionless form

$$y = C(\tau)A^{-1} = \eta x^{-1}+k_b^b x^{b-1}, \quad (2)$$

where $з = B/A$ is a cost factor, $x = ф /T$ is the relative periodicity of preventive replacements in fractions of MTBF.

The optimal value x_0 at minimum relative operating costs per unit is determined from the condition $dx/dy = 0$ by the following formulas:

$$X_0=k_b^{-1}(\eta(b-1)^{-1})^{1/b}; \quad (3)$$

$$y_0= bk_b^b x_0^{b-1}. \quad (4)$$

Transforming (2) through (3) and (4), we shall obtain an equation of the following form:

$$y_x = (b-1 +x_x^b)(bx_x)^{-1}, \quad (5)$$

where $y_x= y/y_0$; $x_x= x/x_0$ are relative deviations of operating costs per unit and periodicity of preventive replacements from their optimal values respectively.

Equation (5) has a generalized character, does not depend on the parameters η, *x* and k_b of initial mathematical model (2) and allows to explore the economic sustainability of the operating costs per unit. Assuming the value y_x = 1+δ, it is possible to determine the permissible relative deviations of optimal periodicity of preventive replacements corresponding to the accepted accuracy of calculations of relative operating costs per unit.

The diagrams of dependence y_x on x_x for different values of *b* are presented in Fig. 1, which shows the following. With the growth of the shape parameter of the Weibull distribution *b* (decreasing coefficient of variation), economic sustainability of the function of operating costs per unit decreases. If we specify, for example, д = 0,05 (see a dashed line in Fig. 1), it is possible to obtain the following admissible deviations of preventive replacements: for *b* =2 from the lower $\underline{x_x}$ = 0.73 to the upper $\overline{x_x}$ = 1.37, for *b* = 4 from the lower $\underline{x_x}$ = 0.82 to the upper $\overline{x_x}$ = 1.19. Then the optimal τ_0, the permissible lower $\underline{\tau_0}$ and the upper $\overline{\tau_0}$ values of the periodicity of preventive replacements are defined as follows

$$\tau_0 = x_0 T,\ \underline{\tau_0} = \underline{x_x}\tau_0,\ \overline{\tau_0} = \overline{x_x}\tau_0 .$$

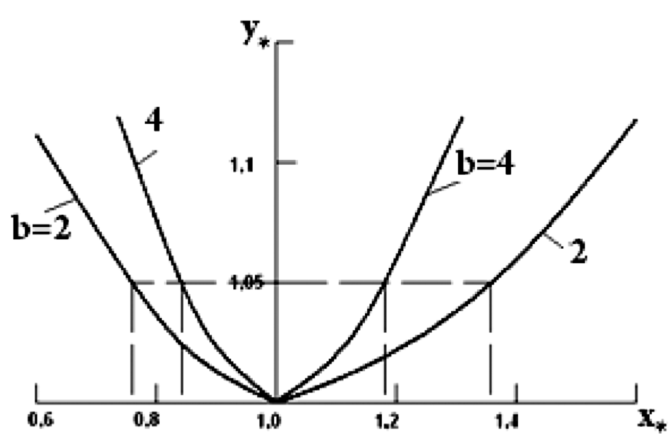

Fig.1. Results of the economic stability study

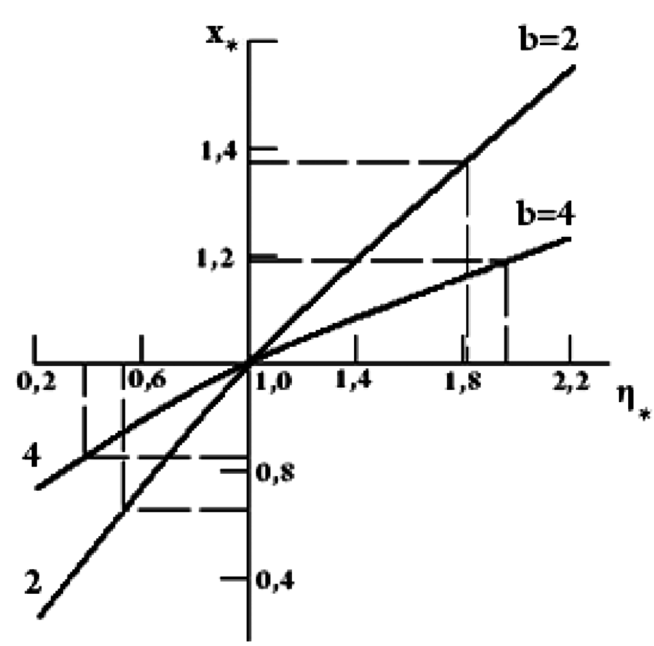

Fig. 2. Results of the sensitivity study

Determination of the sensitivity function of operating costs per unit for change of the cost coefficient з is carried out using equation (3). Sensitivity analysis does not require knowledge of the numerical values of the cost coefficient, as it is executed in relative units $\eta_x = \eta/\eta_o$, where η_o is the basic value of the cost coefficient corresponding to the optimum periodicity of preventive replacements.

The results of sensitivity analysis are presented in Fig. 2, which shows that with the increase of the shape parameter *b* the sensitivity to changes of η_x decreases, and initial economic data can be determined with lower accuracy.

For example, in the area of economic stability function of operating costs per unit at д = 0,05 (see a dashed line in Fig. 2) the following relative deviations of cost coefficient: for *b* = 2 – from 0.52 to 1.84, and for *b* = 4 – from 0.46 to 2.0 of its baseline value.

Conclusion

As a result of sensitivity analysis, it has been found that in the area of economic stability of the function of operating costs per unit the initial economic data can be determined with lower accuracy. For example, if the calculation accuracy makes up 5% of operating costs per unit in case of the replacements strategy with minimal repairs at failures, permissible relative deviations of the coefficient cost in t case of the Weibull distribution with a shape factor equal to 2 from 0.52 to 1.84, and with a shape factor equal to 4 from 0.46 to 2.0 of its basic value. In considered cases, these values can be taken as acceptable accuracy of determining initial economic data.

References

1. **Volodarsky V.A.** Optimizing the periodicity of preventive replacements under uncertainty initial information // Dependability and quality control. – 1984. – # 8. – pp. 39-44.

2. **Barlow R., Proschan F.** Mathematical theory of reliability. – Moscow: Soviet Radio, 1969. – 488 p.

3. **Volodarsky V.A.** Justification of initial economic data accuracy for optimizing the periodicity of preventive replacements // Dependability and quality control. – 1986. – # 4. – pp. 36-39

4. **Golikov V.F.** About the influence of determining the accuracy of dependability characteristics to choose the period of preventive maintenance // News of the USSR Academy of Sciences. Engineering Cybernetics. 1986. – # 1. – pp. 66-69.

5. **Bayhelt F., Franken P.** Dependability and Maintenance. Mathematical approach. – Moscow: Soviet Radio, 1988. – p. 392.

6. **Lanetsky B.N., Kobzev V.V.** Justification of requirements for the accuracy of estimates of radio electronics' reliability factor in exploited surface-to-air missile systems for solving the problem of assigned service life extension (of resources) // Information processing systems – 2006. – Issue 4 (53). – pp.110-117.

7. **Volodarsky V.A.** Studies of economic stability optimization models for preventive replacements / / Reliability. – 2012. – № 1 (40). – S. 36-43.