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ASSESSMENT OF RELIABILITY PERFORMANCE UNDER THE ASSUMPTION OF INCOMPLETE RECOVERY

The paper describes modern techniques of parametrical estimation of the failure rate leading function and the assessed failure rate under the assumption of incomplete recovery using Kijima models. Models are compared among themselves and with the models assuming full or minimal recovery of a component (homogeneous and nonhomogeneous Poisson processes respectively). For estimating model parameters, the method of maximum likelihood function is used. For estimating the failure rate leading function given in implicit form, the most popular method of modeling is the Monte-Carlo method. The paper also offers an example of application of the developed technique for calculation of reliability characteristics of components, which are part of the regular equipment of NPP power units.

Keywords: *incomplete recovery, Kijima models, rate leading function, assessed failure rate.*

Introduction

The analysis of components reliability and systems of operating NPP is an urgent problem of the present stage in development of nuclear power. The probabilistic-statistical techniques for estimation of reliability characteristics are applied to carry out such researches. The most common for calculation of reliability of recovered systems are the models assuming full or minimal recovery. The functioning of modern technical systems, as a rule, is more complex process, for which incomplete or partial recovery is typical. Various types of recovery can be classified as follows:

1) full recovery; the component is restored up to the condition, which it had at the initial moment of time ("as new", in foreign literature the term "as good as new " is used). The given type of recovery can be treated as replacement of the failed component with a new one.

2) minimal recovery of a component "as it was before failure" ("as bad as old"). The component is recovered up to the condition, which it had at the moment of failure.

3) incomplete recovery of a component "is worse than new but better than it was before failure" ("better than old but worse than new"). After recovery, the component passes in some intermediate condition between two above described categories.

4) specific case "is worse than before failure" ("worse than old"). After recovery, the object is in a worse condition than before failure.

Accordingly, homogeneous process of recovery is used for modeling of full recovery, nonhomogeneous Poisson process is applied to modeling of minimal recovery. Such processes are well studied and widely used in practice. However, presently there are more demands for models, allowing us to consider incomplete recovery of a component, including a model of the generalized process of recovery. The study [1] is among the basic works on this subject. Kijima models are an object of the present research.

1. Generalized process of recovery, Kijima models

Let v_{i-1} is the virtual age of a component for the time of *(i-1)*-th recovery. *Xi* is the *i*-th time between failures. Then *Xi* has following function of distribution [2-3]:

$$F_i(x) = \frac{F(x+v_{i-1}) - F(v_{i-1})}{1-F(v_{i-1})}; \tag{1}$$

hence, the probability of non-failure operation can be written down in the following form:

$$P_i(x) = 1 - F_i(x) = \frac{P(x+v_{i-1})}{P(v_{i-1})}; \tag{2}$$

where *F (x)* is the function of distribution of time to the first failure (for a new component). The real age of the component can be written down in the following form:

$$S_n = \sum_{i=1}^{n} x_i, S_0 = 0;$$

where *Xi* is the *i*-th time to failure.

The model “Kijima I” assumes that the *n*-th recovery influences only the damages suffered by a component between *(n-1)*-th and *n*-th failures reducing the added age of the component from *Xi* up to *qXi*. The virtual age of a component after the *n*-th recovery can be written down as follows:

$$v_n = v_{n-1} + qX_n = q\sum_{i=1}^{n} X_i = qS_n; v_0 = 0; \tag{3}$$

where $q \geq 0$ is some parameter of the model describing the degree of the *n*-th recovery. Here and in what follows, it is supposed that *q=const*, though, generally, the value of *q* can differ for each recovery.

The model “Kijima II” assumes that the *n*-th recovery influences the total damages suffered by a component during the whole time of its functioning:

$$v_n = q(v_{n-1} + X_n) = q(q^{n-1}X_1 + q^{n-2}X_2 + \ldots + X_n). \tag{4}$$

Let us consider some particular values of parameter *q*:

The case *q=0* describes full recovery. Random processes (3-4) degenerate into homogeneous processes of recovery.

The case *q=1* describes the minimal recovery. Random processes (3-4) degenerate into heterogeneous Poisson processes.

The case *0 <q <1* describes incomplete recovery "worse than new but better than it was before failure" that is a subject of this paper study.

Finally, the specific case *q> 1*, i.e. the recovery "is worse than it was before failure", is not considered in this study.

Expectation of the average number of failures on an interval *(0; t]* is also known as "function of recovery", "the failure rate leading function" (in foreign literature as "Cumulative Intensity Function", for example, in [3]). In what follows, the term "the failure rate leading function" is used in the text of this paper. For a case of incomplete recovery, the given value determined by the equation [1, 3]:

$$H(t)=\int_0^t (g(\tau\,|\,0)+\int_0^\tau h(x)g(\tau-x\,|\,x)dx)d\tau, \tag{5}$$

where:

$$g(t\,|\,x)=\frac{f(t+qx)}{1-F(qx)};h(t)=\frac{dH(t)}{dt};f(t)=\frac{dF(t)}{dt}.$$

The solution of (5) is impossible analytically, and even the numerical solution is difficult [3]. In 1998 Kaminsky and Krivtsov offered the method for calculation of the failure rate leading function using a Monte-Carlo approach. Now this approach is widely applied to research of the generalized processes of recovery. For modeling based on the Monte-Carlo method, it is necessary to assume the form of distribution function of operating time to the first failure, parameters of distribution function, and the value of the parameter *q*. The estimation of parameters is possible in various ways.

2. Estimation of model parameters under the assumption of Weibull distribution

Kaminsky and Krivtsov in the work [3] suggest using the least square method:

$$\min_{\theta_1,\theta_2...,\theta_p,q}(\sum_{i=1}^{n}(H_{эмп}(t_i)-H_{MK}(t_i,\theta_1,\theta_2...,\theta_p,q))^2),$$

wher $H_{эмп}$ is an empirical failure rate leading function,
H_{MK} is estimation of the function using the Monte-Carlo method,
$(\theta_1,\theta_2...,\theta_p)$ is a vector of distribution parameters.

The estimation of the vector $(\theta_1,\theta_2...,\theta_p)$ and the model parameter q in a similar way entails heavy great computing expenses.

Let us consider in more detail the alternative approach consisting in use of a method of maximal likelihood [2, 4, 5]. We shall assume that the mean time to the first failure has a Weibull distribution. It is necessary to notice that the function of this distribution has various forms of writing. In this paper, we shall use the following kind of distribution simplifying a little bit the further calculations:

$$F(x) = 1 - \exp(-\lambda x^{\beta}). \tag{6}$$

Let us rewrite the probability of non-failure operation for the *i*-th time to failure (2) using (6):

$$P_i(x) = \frac{P(x + v_{i-1})}{P(v_{i-1})} = \frac{\exp(-\lambda(x + v_{i-1})^{\beta})}{\exp(-\lambda v_{i-1}{}^{\beta})} = \exp(\lambda(v_{i-1}{}^{\beta} - (x + v_{i-1})^{\beta}));$$

where $v_{i-1} = v_{i-1}(x_1, x_2, ... x_{i-1}, q)$ is the virtual age of a component after the *(i-1)*-th failure is equal to (3) or to (4) depending on a model used. Further, we shall find the density of distribution:

$$f_i(x) = -\frac{\partial P(x)}{\partial x} = \lambda\beta\,(x + v_{i-1})^{\beta-1} \exp(\lambda(v_{i-1}{}^{\beta} - (x + v_{i-1})^{\beta})).$$

The function of likelihood:

$$L(\lambda, \beta, q) = \prod_{i=1}^{n} f_i(x_i) = (\lambda\beta)^n \prod_{i=1}^{n} (x_i + v_{i-1})^{\beta-1} \exp(\lambda(v_{i-1}{}^{\beta} - (x_i + v_{i-1})^{\beta})).$$

Let us take the natural logarithm from both parts:

$$\ln L(\lambda, \beta, q) = n(\ln \lambda + \ln \beta) + \lambda \sum_{i=1}^{n} [v_{i-1}{}^{\beta} - (x_i + v_{i-1})^{\beta}] + (\beta - 1) \sum_{i=1}^{n} \ln(x_i + v_{i-1}). \tag{7}$$

Then, we can derive the received logarithmic function of likelihood (7) on each of arguments λ, β, q, to equate derivatives to zero and to solve the received system in relation to λ, β, q. Unfortunately, the analytical solution for such a system has not been found, therefore numerical methods are applied to the solution [2, 4, 5]. It is necessary to notice that modern software for mathematical modeling (Matlab, etc.) allow calculating effectively the maximum of the given function from many variables saving us from often bulky operations of deriving and explicit reference to a system of equations.

3. Application of Monte-Carlo method for estimation of characteristics of recovery process

3.1. Estimation of failure rate leading function

Assuming that the form of a distribution function is known and having estimations of parameters, we can model random processes (3-4). For estimation of the failure rate leading function at point *T*, it is necessary to use a sequence of random variables – mean times to failure. N_j, i.e. the number of failures which have occurred till time *T*, is the simulated value of recovery function, and further, simulation is repeated a necessary number of times *S*. The final estimation of function $H_{MK}(T)$ is calculated as an average from the simulated values [5]:

$$H_{ì\ Ê}(T)=\frac{1}{S}\sum_{i=1}^{S}N_{i}. \tag{8}$$

Let us apply the method of inverse functions [6, p. 371-373] for modeling a mean time between failures. The distribution function of mean time between failures is determined as (1). We shall designate $F_i(x)=U$ and write down the obtained expression:

$$U=\frac{F(x+v_{i-1})-F(v_{i-1})}{1-F(v_{i-1})};$$

from which we shall find:

$$x=F^{-1}((1-F(v_{i-1}))U+F(v_{i-1}))-v_{i-1}. \tag{9}$$

An inverse Weibull distribution function in the form of (6) is equal to the following:

$$F^{-1}(y)=[-\frac{\ln(1-y)}{\lambda}]^{\frac{1}{\beta}}. \tag{10}$$

Having substituted (6) and (10) in (9), we shall obtain the final formula for modeling the *i*-th mean time between failures:

$$x_i=[(v_{i-1})^{\beta}-\frac{\ln U}{\lambda}]^{\frac{1}{\beta}}-v_{i-1}.$$

To carry out simulation, it is necessary to use *U~U [0; 1]*.

3.2. Estimation of failure rate

GOST [7] distinguishes an instantaneous parameter of failure rate (IPFR), as a limit ratio of the number of failures on an interval to the length of that interval tending to zero:

$$h(t)=\lim_{\Delta t\to 0}\frac{E(N(t+\Delta t)-N(t))}{\Delta t}=\frac{dH(t)}{dt};$$

and also the average parameter of failure rate (APFR) is the average value of IPFR on a finite interval of time:

$$\overline{h}(t_1;t_2)=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}h(t)dt;$$

In this study, we are interested in estimation of APFR, which can be obtained by a Monte-Carlo method similarly to the failure rate leading function (8):

$$h_{MK}(T) = \frac{1}{S}\sum_{i=1}^{S}\frac{n_i}{\Delta_i}; \tag{11}$$

where n_i is the number of failures on an interval Δ_i, and T is the middle of an interval Δ_i.

3.3. Estimation of an error of calculations

As is known, when using a Monte-Carlo method, it is possible to obtain an error estimation, which is not guaranteed, but only insured with some degree of confidence. According to [8, p.234] and the central limit theorem, we obtain the upper boundary of an error of integral calculations using a Monte-Carlo method (8) with confidence factor β:

$$\delta \le t_\beta \sqrt{\frac{D(H_{MK})}{S}};$$

where t_β is the value of the argument of Laplace function $\Phi(t)$, for which $\Phi(t) = \beta/2$ [6, p. 365]. t_β = 3 for $\beta = 0,997$ (three sigma rule).

The unknown dispersion $D(H_{MK})$ can be replaced by its unbiased estimate D:

$$D = H_{MK}^2 - (H_{MK})^2 = \frac{S}{S-1}\left(\sum_{i=1}^{S} N_i^2 - \left(\sum_{i=1}^{S} N_i\right)^2\right).$$

From which the required formula for an error of calculation the failure rate leading function by using a Monte-Carlo method takes the following form at some point:

$$\delta \le t_\beta \sqrt{\frac{\sum_{i=1}^{S} N_i^2 - \left(\sum_{i=1}^{S} N_i\right)^2}{S-1}}; \tag{12}$$

It should be noted that $\delta \sim \sqrt{1/S}$, where S is the number of iterations of modeling the value $H_{MK}(T)$ at point T. The choice of the value of order 10^6 as S allows us to achieve the accuracy acceptable for the majority of calculations and is not a problem for modern computing means.

4. An example of calculation

Let us calculate parameters of the presented model using real data. For this purpose, we shall take advantage of the information on failures of accumulation and processing data devices, operating in the structure of the NPP regular equipment. For the chosen device during operation $T \approx 4,5*104$ h, 112 failures have been registered. A Weibull distribution has been selected as distribution of mean time to the first failure. The used models, the corresponding values of logarithmic function of likelihood (7) and estimations of parameters are presented in Table 1:

Table 1. Estimations of model parameters

Model	lnL	λ	β	q
Full recovery	-253.1316	0.3588	0.8563	-
Minimal recovery	-242.0335	0.0038	1.7130	-
Kijima I	-242.0336	0.0038	1.7131	0.9998
Kijima II	-241.8521	0.0017	1.8863	0.9930

Point estimations of values of the failure rate leading function FRLF (8) and APFR (11) are presented in Figures 1-6 respectively. In the given figures, the following designations are accepted:

$H_{эмп}(t)$ and $h_{эмп}(t))$ are empirical FRLF and APFR;
$H_{полн}(t)$ and $h_{полн}(t))$ are estimations of FRLF and APFR under the assumption of full recovery.;
$H_{мин}(t)$ and $h_{мин}(t)$ are estimations of FRLF and APFR under the assumption of minimal recovery;
$H_1(t)$ and $h_1(t)$ are estimations of FRLF and APFR for Kijima I model;
$H_2(t)$ and $h_2(t)$ are estimations FRLF and APFR for Kijima II model.

It should be noted that if the choice of *S* (the number of iterations for modeling of function value) is equal to the size of order 10^6, the error (12) of function estimation using a Monte-Carlo method makes up, as a rule, no more than 1% with the level of confidence probability 0,997.

According to the values of the function of likelihood presented in Table 1, the best model (a minimal value of function of likelihood) for the specific processed data set is the Kijima II model. Based on a small difference in values of function of likelihood (within the limits of 0,5 %), it is possible to conclude that, for the processed data, the difference between Kijima models and the model of heterogeneous Poisson process under the assumption of the minimal recovery is insignificant and does not allow us to specify

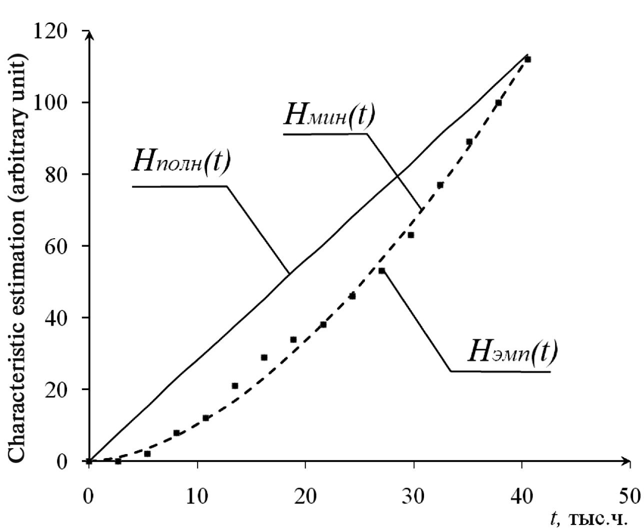

Fig. 1. Estimations of the failure rate leading function for models of full and minimal recovery

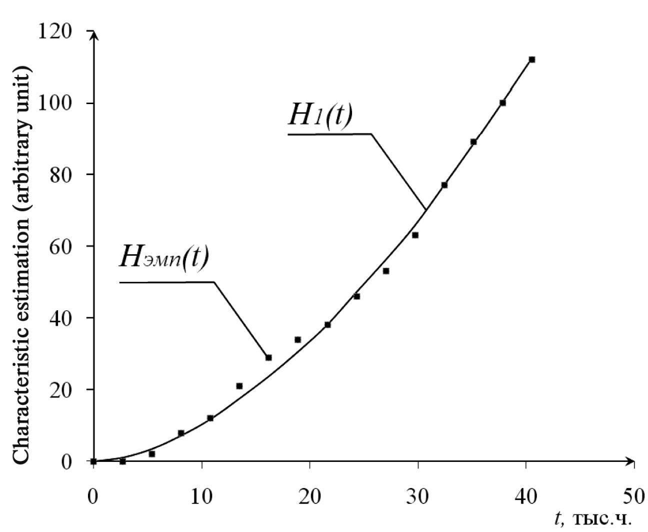

Fig. 2. Estimation of the failure rate leading function for Kijima I model

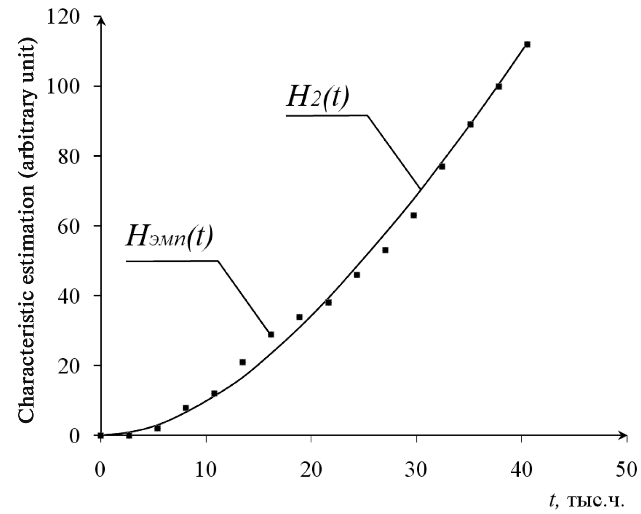

Fig. 3. Estimation of the failure rate leading function for Kijima II model

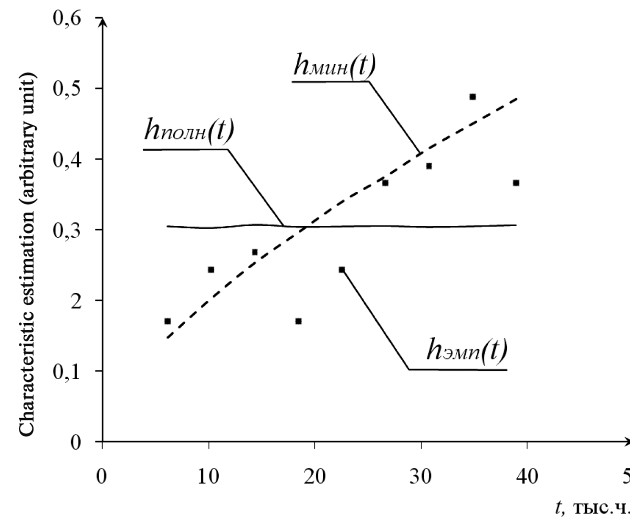

Fig. 4. Estimations of APFR for models of full and minimal recovery

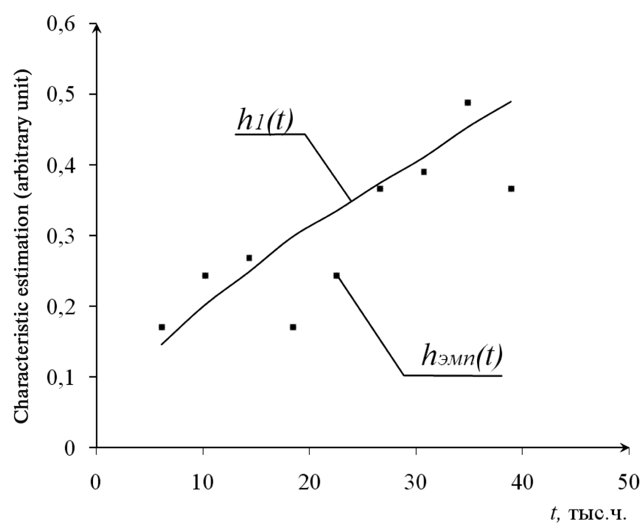

Fig. 5. Estimation of APFR for Kijima I model

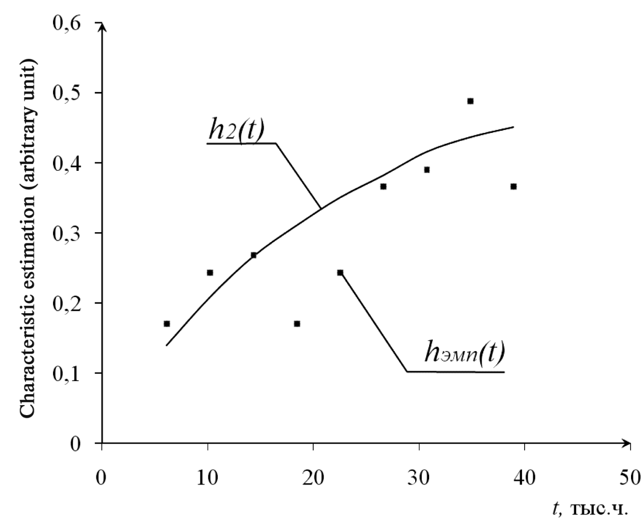

Fig. 6. Estimation of APFR for Kijima II model

with the high confidence the best model from investigated ones. The similar conclusion is also supported by the similar estimations of parameters of Weibull distribution presented in Table 1 and the parameter q describing a recovery degree.

Conclusion

Unlike homogeneous and nonhomogenous processes of recovery, a model of the generalized process of recovery, including Kijima models, allow us to take into account a case of incomplete recovery “worse than new but better than it was before failure”. This paper describes the method of estimation of the failure rate leading function and the mean assessed failure rate with use of models “Kijima I” and “Kijima II” showing conformity to real data. Calculations of model parameters according to accumulation and processing operation data of devices functioning within the regular equipment of NPP power units have been presented.

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