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RELIABILITY ANALYSIS OF NETWORKS CONSISTING OF IDENTICAL ELEMENTS

The paper offers analytical models, which allows obtaining expressions for determining the reliability indices of networks consisting of identical elements. For networks consisting of recoverable elements, the mean time between failures and the mean time to repair in steady-state operation are considered as reliability indices. In assessing the reliability of networks of non-repairable elements, such an index is the mean time to failure.

Keywords: *reliability indices of networks, Markov processes, mean time to failure, mean time to repair.*

Introduction

Many modern engineering systems have a network structure. For example, telecommunications, energy and transportation systems have such a structure. In the design of similar systems, a lot of attention is paid to providing the high level of reliability.

The reliability of a network depends on several factors: the reliability of components, the topology and control algorithms used. This paper proposes a method of estimating the reliability indices of a network according to its topology.

In the analysis of networks reliability, determining the values of parameters describing the behavior of network states is of considerable interest. For networks consisting of recoverable elements, the mean time between failures and the mean time to repair in steady-state operation are considered as reliability indices. In assessing the reliability of networks of non-repairable elements, such an index is the mean time to failure.

These reliability indices of networks can be determined if the process of changing network states is represented in the form of Markov process, which states are described by the number of failed elements, and network states.

The simplest criterion of network operability is its connectivity. The network is connected if between any pair of nodes, there is at least one path.

Network connectivity will be used in considered examples as criterion of network operability, but the obtained expressions are valid when using other criteria of network operability.

Model of network reliability consisting of identical recoverable elements

Networks refer to the class of systems with a complex structure. A characteristic feature of such systems is that under a certain number of failed elements the system can be either in upstate or in failed state. For example, considering the network shown in Fig.1 we cannot say for sure whether it would be connected, if you remove two or three edges.

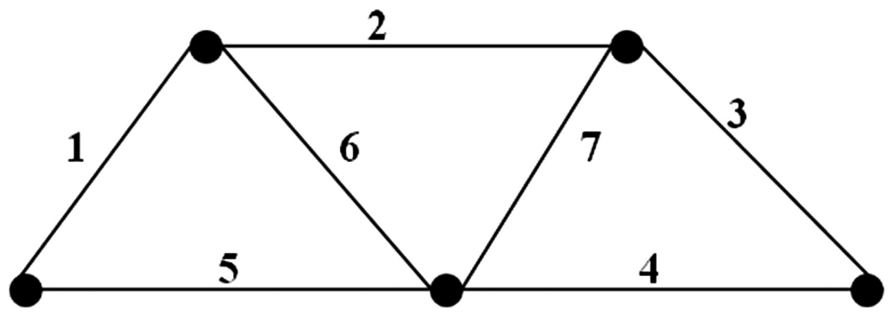

Fig. 1. Example of a network

We shall use the number of sections of power **i** as the main parameter that will allow us to determine the network reliability indices under consideration. We will denote this parameter as **Yi.**

Let us determine the values of **Yi** for the network shown in Fig. 1. Since the network under consideration is doubly connected, the removal of one edge cannot break its connectivity, therefore, Y_1**= 0.** We shall consider all the possible states of the network after removing 2 and 3 edges respectively in order to determine the values of $\mathbf{Y_2}$ and $\mathbf{Y_3}$.

As a result of the analysis of the network shown in Fig. 1, it is possible to determine that in this network there are two sections of power 2 and 14 sections of power 3, therefore, $\mathbf{Y_2}$=2, $\mathbf{Y_3}$=14. Any combination of **i** edges for **i** > 3 will be a section.

Knowing the values of Yi, we can determine the values of the probability that with **i** edges failed, the network will turn out to be disconnected. We denote this index as $\boldsymbol{Z_i}$.

The value of $\mathbf{Z_i}$ is equal to the ratio of $\mathbf{Y_i}$ to the total number of possible combinations of **i** elements out of *n*, where *n* is the number of edges in the network:

$$Z_i = \frac{Y_i}{\binom{n}{i}}. \tag{1}$$

Let us consider a Markov process, describing the change of network states in case of failure and restoration of its edges. Each state of the process is described by a particular combination of failed edges and the network status (connected and disconnected).

We denote the connected states and disconnected state of the network when ***i*** edges failed as ***(i')*** and ***(i")*** respectively.

We shall denote a set of connected states of the network as $\boldsymbol{E_+}$, and a set of disconnected states of the network as $\boldsymbol{E_-}$.

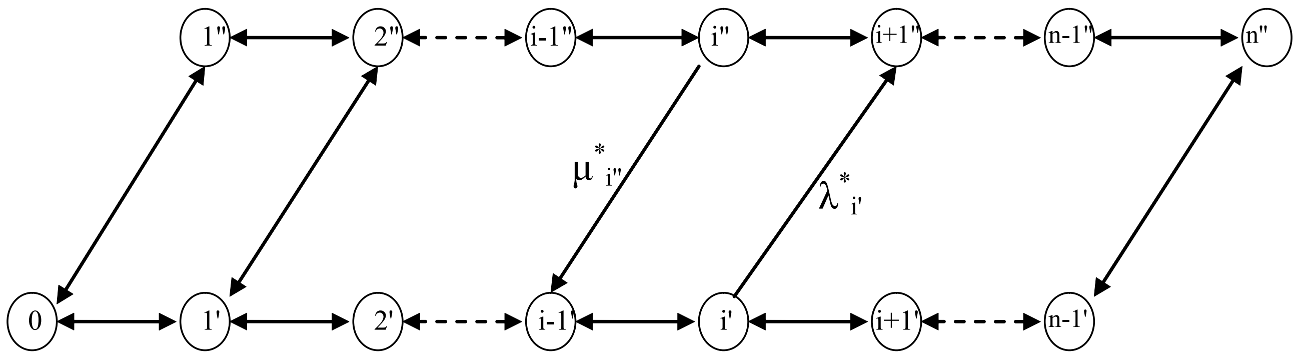

Fig. 2. Markov process of network states changes

In [1] it is shown that the mean stay time of a Markov process on the set of states $\boldsymbol{E}_+$ to the first transition into one of the states of the set $\boldsymbol{E}\_$ can be determined from expression (2).

$$T_{E_+} = \frac{\sum_{i=0}^{n} P_{i'}}{\sum_{i=0}^{n} P_{i'} \lambda^*_{i'}} \tag{2}$$

The mean stay time of the process on the set of disconnected states $\boldsymbol{E}\_$ to the first transition into one of the states $\boldsymbol{E}_+$ can be determined from expression (3).

$$T_{E\_} = \frac{\sum_{i=0}^{n} P_{i''}}{\sum_{i=0}^{n} P_{i''} \mu^*_{i''}}, \tag{3}$$

where $P_{i'}$, $P_{i''}$ are the probabilities of states, and
$\lambda^*_{i'}$ is the transition rate from the state (i') to the state $(i+1)''$;
$\mu^*_{i''}$ is the transition rate from the state (i'') to the state $(i-1)'$.

The total failure rate of edges in the state $\boldsymbol{i}$ is equal to $\boldsymbol{(n\text{-}i)\ \lambda}$, the total recovery rate is equal to $\boldsymbol{i\mu}$**;** therefore:

$$\lambda^*_{i'} = (n-i)\lambda Z^*_i \tag{4}$$

$$\mu^*_{i''} = i\mu Z^{**}_i, \tag{5}$$

where Z^*_i is the probability that at the failure of one edge the network will transit from a connected state (i') into a disconnected state $(i+1)''$;
Z^{**}_i is the probability that at the recovery of one edge the network will transit from a disconnected state (i'') into a connected state $(i-1)'$.

The relationship of values $\mathbf{Z}^*_{\mathbf{i}}$, $\mathbf{Z}^{**}_{\mathbf{i}}$ and $\mathbf{Z_i}$ was established in the study [2].
The values Z^*_i and Z^{**}_i can be determined from expressions (6) and (7) respectively.

$$Z_i^* = \frac{Z_{i+1} - Z_i}{1 - Z_i} \tag{6}$$

$$Z_i^{**} = \frac{Z_{i-1}}{Z_i} \tag{7}$$

And by substituting the values $\lambda^*_{i'}$ and $\mu^*_{i''}$ from expressions (4) and (5) into expressions (2) and (3) respectively, we will obtain the following:

$$T_{E_+} = \frac{\sum_{i=0}^{n} P_{i'}}{\lambda \sum_{i=0}^{n} P_{i'} (n-i) Z_i^*} \tag{8}$$

$$T_{E_-} = \frac{\sum_{i=0}^{n} P_{i''}}{\mu \sum_{i=0}^{n} i P_{i''} (1 - Z_i^{**})}. \tag{9}$$

The values of $P_{i'}$, $P_{i''}$ may be determined as follows.

Since all edges are identical in terms of reliability, then all states with ***i*** failed edges are equiprobable. The probability that ***i*** edges will be in failure state in the network is equal

$$P_i = \frac{(\lambda / \mu)^i \binom{n}{i}}{(1 + \lambda / \mu)^n} \tag{10}$$

from the definition of Z_i we obtain the following:

$$P_{i'} = (1 - Z_i) P_i \tag{11}$$

$$P_{i''} = Z_i P_i. \tag{12}$$

The probability of the network stay in connected state at an arbitrary time point is the following:

$$R = \sum_{i=0}^{n} P_{i'}. \tag{13}$$

The value of R may also be determined from the expression:

$$R = \frac{T_{E_+}}{T_{E_+} + T_{E_-}}. \tag{14}$$

Table 1. The values of $Y_i, Z_i, Z_i^*, Z_i^{**}$ for the network shown in Fig. 1

i		Yi	Zi	Z*i	Z**i
0	1	0	0	0	0
1	7	0	0	0,095238	0
2	21	2	0,095238	0,336842	0
3	35	14	0,4	1	0,238095
4	35	35	1	0	0,4
5	21	21	1	0	1
6	7	7	1	0	1
7	1	1	1	0	1

By substituting the values of $Y_i, Z_i, Z_i^*, Z_i^{**}$ from Table 1 in expression (8), (9), (14) for λ = 0,1 (1/h), and μ = 1 (1/h), we obtain the following values of reliability indices under consideration.

$T_{E_+} = 23{,}769$ h.

$T_{E_-} = 0{,}469$ h.

$R = 0{,}980645$

Rough estimates of reliability indices

Analyzing the limits of the expressions obtained for $\lambda / \mu \to 0$, [3], it is possible to obtain rough estimates of reliability indices.

$$T_{E_+} = \frac{1}{\lambda(\lambda/\mu)^{s-1} * S * Y_s} \tag{15}$$

$$T_{E_-} = \frac{1}{s * \mu} \tag{16}$$

For the network shown in Fig. 1 s=2, Y_s=2

By substituting these values into expression 14,15,16 for λ = 0,1 (1/h) and μ = 1 (1/h), we will obtain the following:

$T_{E_+} = 25$ h.

$T_{E_-} = 0{,}5$ h.

$R = 0{,}980392$

Comparison with known results

In order to verify the obtained expressions, we shall define the values of reliability indices under consideration for the system presented in Fig. 3, for which analytical estimates are known.

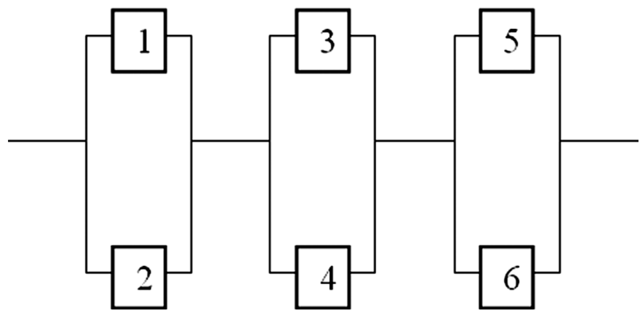

Fig. 3. A serial-parallel system

The following expressions can be found in [4].

For *s* identical elements connected in parallel with the parameters λ, μ.

$$t_{E_+} = \frac{1}{s*\mu}\left[\left(1+\frac{\mu}{\lambda}\right)^S - 1\right]$$

$$t_{E_-} = \frac{1}{s*\mu}$$

$$r = \frac{t_{E_+}}{t_{E_+} + t_{E_-}}$$

For *k* identical serially connected recoverable subsystems.

$$T_{E_+} = \frac{1}{k*(1/t_{E_+})}$$

$$R = r^k$$

$$T_{E_-} = T_{E_+}\frac{1-R}{R}$$

By substituting s = 2, k = 3, λ = 0,1 (1/h), μ = 1 (1/h) in the above expressions, we will obtain the following:

$T_{E_+} = 20{,}000$ hours

$R = 0{,}975411$

$T_{E_-} = 0{,}504$ hours

To determine these reliability indices, using expressions (8), (9) and (14) it is necessary to define the values of Yi and Zi.

It is obvious that $Y_1 = 0$ and $Z_1 = 0.0$. Upon failure of two elements, three combinations lead to disconnection (downstate) of the system. These combinations are (1,2), (3,4), (5,6). The total number of possible combinations is 15, therefore, $Y_2 = 3$ and $Z_2 = 3/15$. To determine Y_3 and Z_3, we shall consider all possible combinations out of three elements (see Table 2). From the results presented in Table 1, it follows that $Y_3 = 12$, and $Z_3 = 12/20$. In case of failure of four or more elements, the system will be disconnected, therefore, $Z_4 = Z_5 = Z_6 = 1$

Table 2. States of the system shown in Fig. 3 for failure of three elements

1	123	-	11	234	-
2	124	-	12	235	+
3	125	-	13	236	+
4	126	-	14	245	+
5	134	-	15	246	+
6	135	+	16	256	-
7	136	+	17	345	-
8	145	+	18	346	-
9	146	+	19	356	-
10	156	-	20	456	-

The results of other calculations are presented in Table 3.

Table 3. The results of characteristics calculation of the system presented in Fig. 3

i		Pi	Yi	Zi	Z*i	Z**i	Qi	Ri
0	1	0,5644739301	0	0	0	0	0,0000000000	0,5644739301
1	6	0,3386843580	0	0	0,2	0	0,0000000000	0,3386843580
2	15	0,0846710895	3	0,2	0,5	0	0,0169342179	0,0677368716
3	20	0,0112894786	12	0,6	1	0,33333	0,0067736872	0,0045157914
4	15	0,0008467109	15	1	0	0,6	0,0008467109	0,0000000000
5	6	0,0000338684	6	1	0	1	0,0000338684	0,0000000000
6	1	0,0000005645	1	1	0	1	0,0000005645	0,0000000000

By substituting the data from Table 3 in expression (26), (27), (37), we will obtain the following:
$T_{E_+} = 20{,}000$ hours
$R = 0{,}975411$
$T_{E_-} = 0{,}504$ hour

An example of reliability analysis of a network consisting of identical recoverable elements

Let us denote the number of nodes as **m** and the number of edges as **n**. It should be noted that, for each pair of values of **n, m**, there is a topology that provides the highest level of reliability. Algorithms proposed by G.T. Artamonov in [5] were used to determine these topologies.

The network with the parameters m = 20, n = 24.

For achieving the maximal reliability, it is necessary to find the network with a minimum diameter and with the number of nodes equal to 2*(n – m) having the degree of 3, and evenly distribute the nodes of degree 2 over the edges of this network. As a result, the network presented in Fig. 4 will be obtained. The values of Y_i of the network can be determined by exhaustive search of all possible states. The results are presented in Tables 4a and 4b.

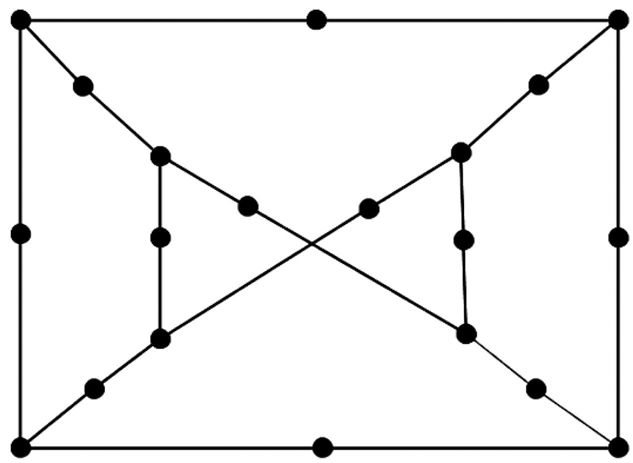

Fig.4 The network with parameters m = 20, n = 24

Table 4a. The values of Y_i and Z_i for the network shown in Fig. 4

i	1	2	3	4	5	i ≧ 6
Y_i	0	12	328	4082	29960	
Z_i	0	0,043478	0,162055	0,384252	0,704875	1

Table 4b. The values of T_{E+}, T_{E-}, R for the network shown in Fig. 4

λ	T_{E+}	T_{E-}	R
0,1	2,13	0,3334	0,864576
0,01	390,86	0,4845	0,998762
0,001	41415,64	0,4986	0,999988

The use of other criteria of operability and models of network reliability

Up to this point, we have used connection of all nodes across the network as a criterion of its operability. However, it is appropriate to ask a question: can the loss of connection with one of the nodes be considered as a failure? If we consider the loss of connection with h nodes to be permissible, then it is possible to use the following criterion of network operability: the network is considered operable if the number of connected nodes ≧ **m – h**. This criterion of network operability was analyzed in the study [6].

The obtained results can be used in this case, but it is only necessary to make appropriate changes to the algorithm determining the values Yi. Table 5 shows the values $\mathbf{T_{E+}}$ and ***R***, for different values of h in the network shown on figure 4.

Table 5. The values of T_{E+} and *R* for different values of *h* in the network shown in Fig. 4 (λ=0,01; μ=1)

h	T_{E+}	R
1	5780,49	0,999943591
2	9843,50	0,999967012
3	31455,53	0,999990157

You can loosen the restriction on reliability equality of edges. Each edge can be presented as the serial connection of a certain number of elements with equal reliability. At that, dummy nodes are introduced into a network structure. The values of Z_i. are determined after such changes. If one of the elements constituting an edge has failed, then this edge is removed from the network. Network connection is checked without dummy nodes.

It should be noted that the obtained results can be used when nodes have failed and edges are completely dependable. Node failure can be simulated by removing all the edges outgoing from that node.

This method of reliability analysis of networks can be used even in those cases when only those states are considered as operable ones, in which the quantity of certain network parameters, such as the transmission time or capacity, will satisfy the specified values.

For large-scale networks, the statistical estimation of values of Z_i can be determined using the Monte Carlo method. First, a random combination out of **i** failed elements is generated, and then the network operability is checked. The ratio of the number of tests, in which the network would be in downstate, to the total number of tests will be a statistical estimate of Z_i.

However, please note that in order to achieve the high accuracy of statistical evaluation of Z_i, the number of trials must be large enough (10^5-10^6).

Reliability model of network consisting of the identical non-recoverable elements

We assume that the network nodes are absolutely dependable, and the edges are identical in terms of reliability, and they failed independently of each other and have an exponential distribution of operation time to failure.

Let us consider a Markov chain, which describes the change of the network states in moments of failure of its elements.

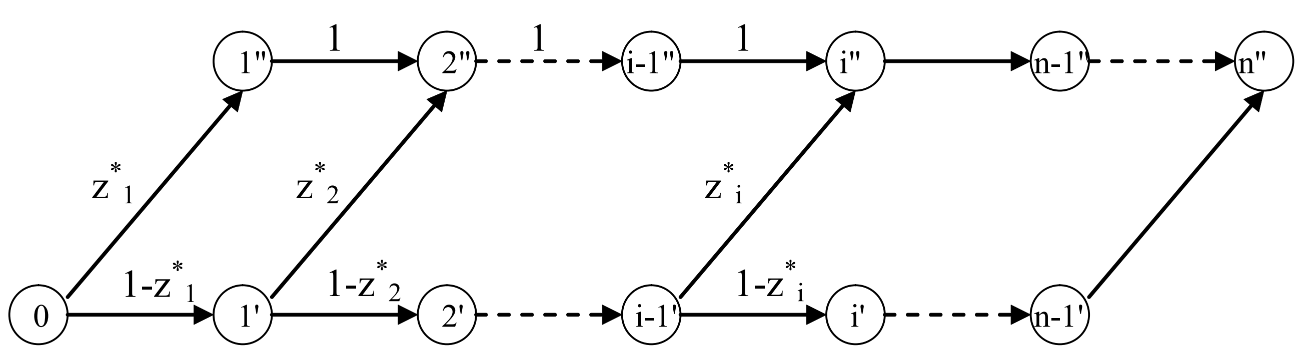

Fig. 5. The diagram of network state changes

If with ***i-1*** of failed elements the network was operable, then in case of failure of the *i*-th element the network transits into downstate with the probability $\mathbf{Z^*_i}$, or with the probability $\mathbf{1- Z^*_i}$ it will remain in upstate.

If with ***i-1*** of failed elements the network was in downstate, then in case of failure of the *i*-th element, the network will remain in downstate with the probability equal to one.

Transition diagram of the process under consideration is shown in Fig. 5. The states ***i'*** corresponds to the network upstates, and the states ***i"*** correspond to the network downstates when ***i*** elements failed.

We shall introduce the notation $Q_{i'}$ as the probability of upstate, and $Q_{i''}$ as the probability of downstate of the network when ***i*** elements failed. From the definition of Z_i we have the following:

$$Q_{i'} = 1 - Z_i \tag{17}$$

$$Q_{i''} = Z_i \tag{18}$$

In accordance with the diagram of network state changes shown in Fig. 5 we can write the following:

$$Q_{i'} = Q_{(i-1)'}(1 - Z^*_i) \tag{19}$$

whence

$$Z^*_i = \frac{Q_{(i-1)'} - Q_{(i)'}}{Q_{(i-1)'}}. \tag{20}$$

By substituting the values of $Q_{i'}$ and $Q_{i''}$ from (17) and (18) in (20), we will obtain

$$Z^*_i = \frac{Z_i - Z_{i-1}}{1 - Z_{i-1}} \tag{21}$$

Also, from expression (19) it follows

$$Q_{i'} = \prod_{j=1}^{i} 1 - Z^*_j = 1 - Z_i. \tag{22}$$

To determine the mean time to failure, we shall consider the continuous Markov process that describes the behavior of a system over time. A number of failed elements and the state of the network [7] specify states of the process. State transition diagram of the process is shown in Fig.6.

Let us denote the failure rate of elements by λ. Suppose that at some point in time there are *i* failed elements, and this network is operable. For an infinitely small time interval Δt, any of the following events can occur:

- The network will remain in upstate. The probability of this event is equal to **(1- (n-i)λ Δt);**
- One more element will fail, and the network transits into downstate. The probability of this event is equal to **(1-(n-i)λ** $\boldsymbol{Z^*_{i+1}}$ **Δt);**

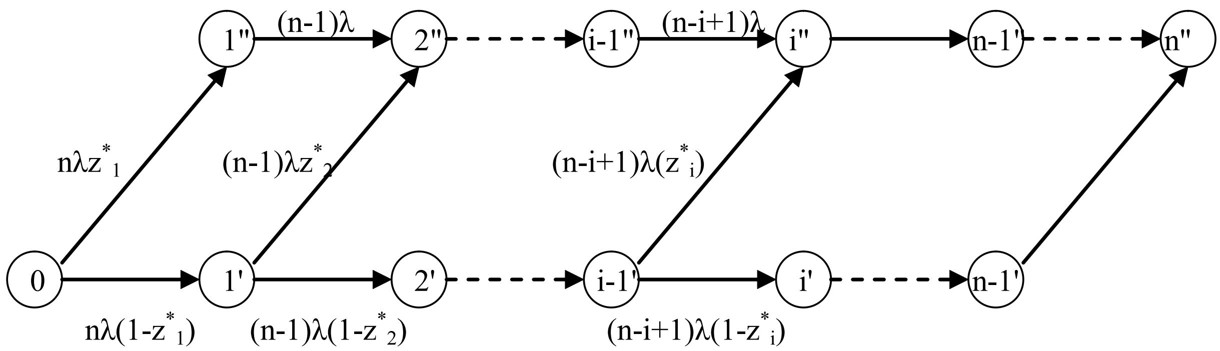

Fig. 6. Markov process of network state changes

- One more element will fail, but the network will remain in upstate. The probability of this event is equal to **(1-(n-i)λ *(1-Z*$_{i+1}$)* Δt).**

In order to simplify the analytical calculations, we shall replace a set of downstates by one absorbing state (see Fig. 7).

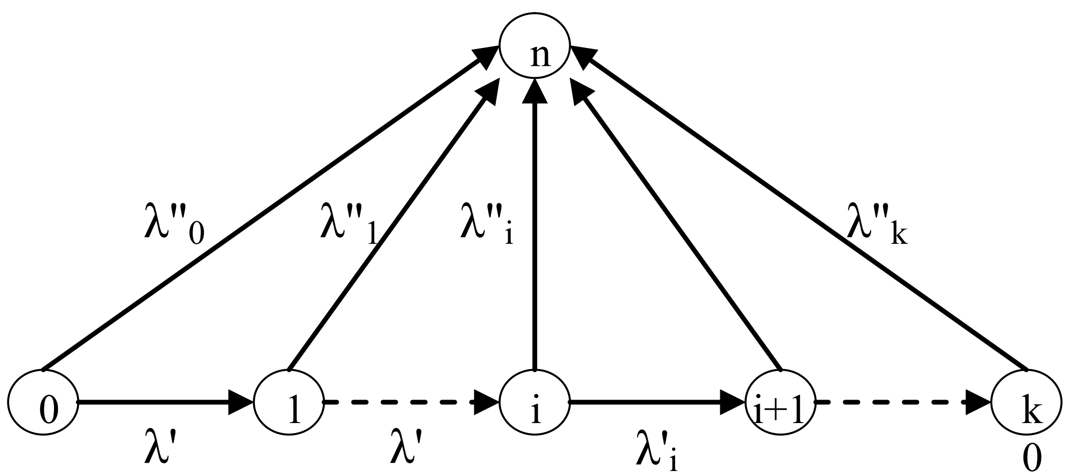

Fig. 7. Markov process of network state changes with absorbing state

The probabilities of process stay in different states at an arbitrary time point $\mathbf{P_i(t)}$ may be found by solving the following system of differential equations:

$$\frac{\partial P_0(t)}{\partial t} = -\lambda_0^{'} P_0(t) - \lambda_0^{"} P_0(t) = -\lambda_0 P_0(t)$$

$$\frac{\partial P_i(t)}{\partial t} = \lambda_{i-1}^{'} P_{i-1}(t) - \lambda_i^{'} P_i(t) - \lambda_i^{"} P_i(t) = \lambda_{i-1}^{'} P_{i-1}(t) - \lambda_i P_i(t) \qquad (23)$$

$$\frac{\partial P_n(t)}{\partial t} = \sum_{i=0}^{k} \lambda_i^{"} P_i(t),$$

where λ_i=(n-i)λ λ'_i=(1- Z^*_{i+1}) λ_i
λ''_i=Z^*_{i+1} λ_i
k is the maximum number of elements after the failure of which the network can be in upstate.
Let us solve the system of differential equations (23), using the Laplace transform with the initial conditions $P_0(0)$=1, $P_i(0)$=0 $\forall i \neq 0$.

We shall introduce the following notation $F(s)=\int\limits_0^\infty P(t)e^{-st}\partial t$

Then the system of differential equations (23) is reduced to the system of algebraic equations with regard to F (s).

$$sF_0(s)=1-\lambda_0 F_0(s)$$

$$sF_i(s)=\lambda'_{i-1}F_{i-1}(s)-\lambda_i F_i(s) \tag{24}$$

$$sF_n(s)=\sum_{i=0}^{k}\lambda''_i F_i(s)$$

Hence, we have the following:

$$F_0(s)=\frac{1}{s+\lambda_0}$$

$$F_i(s)=\frac{\lambda'_{i-1}}{s+\lambda_i}F_{i-1}(s) \tag{25}$$

$$F_n(s)=\frac{1}{s}\sum_{i=0}^{k}\lambda''_i F_i(s)\ .$$

This solution allows obtaining the expression for the mean time to failure:

$$T=\sum_{i=0}^{k}F_i(s)\Big|s=0 \tag{26}$$

Expanding expressions (25) we will obtain

$$F_0(0)=\frac{1}{\lambda_0}=\frac{1}{n\lambda} \tag{27}$$

$$F_i(0)=\frac{\lambda'_{i-1}}{\lambda_i}F_{i-1}(0)=\frac{\prod\limits_{j=1}^{i}(1-Z_j^*)}{(n-i)\lambda}\ . \tag{28}$$

Using (22), expression (28) can be simplified as follows:

$$F_i(0) = \frac{\prod_{j=1}^{i}(1-Z_j^*)}{(n-i)\lambda} = \frac{1-Z_i}{(n-i)\lambda}. \qquad (29)$$

Then from expression (26) it follows

$$T = \frac{1}{\lambda}\sum_{i=0}^{k}\frac{(1-Z_i)}{n-i}. \qquad (30)$$

The expression (30) allows determining the value of the mean time to failure by means of the known values of $\boldsymbol{Z_i}$ and λ.

Now we shall determine the value of T for the network shown in Fig. 1, at λ = 0,01 (1/h)

$$T = 100(\frac{1}{7}+\frac{1}{6}+(\frac{19}{21})\frac{1}{5}+(\frac{21}{35})\frac{1}{4}) = 100*\frac{269}{420} \approx 64,047(hour).$$

Comparison with known results

To verify the obtained expression, we should define the mean time to failure of the system, for which analytical estimates are known (see Fig. 3).

In [4], the following expressions can be found.

The probability of failure-free operation of the system consisting of two identical elements connected in parallel

$$P_{(t)} = 1-(1-e^{-\lambda t})^2 = 2e^{-\lambda t} - e^{-2\lambda t}.$$

The probability of failure-free operation of the system consisting of three identical serially connected subsystems.

$$P_{\tilde{n}(t)} = (P_{(t)})^3$$

$$P_{\tilde{n}(t)} = (2e^{-\lambda t} - e^{-2\lambda t})^3 = 8e^{-3\lambda t} - 12e^{-4\lambda t} + 6e^{-5\lambda t} - e^{-6\lambda t}$$

The mean time to failure of the system is equal to

$$T = \int_0^\infty P_{\tilde{n}(t)}\partial t = \frac{1}{\lambda}(\frac{8}{3}-\frac{12}{4}+\frac{6}{5}-\frac{1}{6}) = \frac{1}{\lambda}(\frac{42}{60}) = 0,7\frac{1}{\lambda}.$$

To determine the mean time to failure by using expression (30), first, it is necessary to define the values of Z_i. For the system under consideration, they are as follows:

Z_1=0,0; Z_2=3/15; Z_3=12/20; Z_4 = Z_5 = Z_6=1.

By substituting these values into (30), we will obtain:

$$T = \frac{1}{\lambda}\left(\frac{1}{6} + \frac{1}{5} + \left(\frac{12}{15}\right)\frac{1}{4} + \left(\frac{8}{20}\right)\frac{1}{3}\right) = \frac{1}{\lambda}\left(\frac{42}{60}\right) = 0,7\frac{1}{\lambda}$$

This example also shows that the proposed method can be used to determine the mean time to failure of complex serial-parallel systems consisting of identical non-recoverable elements.

An example of reliability analysis of the network consisting of identical non-recoverable elements

Let us denote by **m** the number of nodes and by **n** the number of edges. Now we shall consider a network with parameters m = 20, n = 24 (see Fig. 4). The values of **Zi** of the network can be determined by exhaustive search of all possible states.

The proposed method can be used with other criteria of the network operability. For example, the network is considered operable if the number of connected nodes ≧ **m – h**. The obtained results can be used in this case, but it is only necessary to make appropriate changes to the algorithm determining the values of $\mathbf{Z_i}$. Table 6 shows the values of $\mathbf{Z_i}$, for different values of *h* for the network shown in Fig. 4. Determination of $\mathbf{Z_i}$ for k > 0 was carried out by using the Monte Carlo method. The number of tests was 10^6.

Table 6. The values of Z_i for different values of *h* for the network shown in Fig. 4

i / h	0	2	4	6	8	10
1	0,0000000	0,00000	0,00000	0,00000	0,00000	0,00000
2	0,0434783	0,00000	0,00000	0,00000	0,00000	0,00000
3	0,1620553	0,01581	0,00000	0,00000	0,00000	0,00000
4	0,3841521	0,07980	0,01498	0,00435	0,00296	0,00000
5	0,7048748	0,25247	0,08813	0,03329	0,01644	0,00000
6	1,0000000	0,56194	0,27850	0,13070	0,05976	0,00324
7	1,0000000	0,86648	0,56942	0,33252	0,17912	0,04445
8	1,0000000	1,00000	0,82642	0,58123	0,36764	0,16783
9	1,0000000	1,00000	0,96544	0,79849	0,57894	0,34029
10	1,0000000	1,00000	1,00000	0,93211	0,76173	0,52593
11	1,0000000	1,00000	1,00000	0,98807	0,89262	0,69336
12	1,0000000	1,00000	1,00000	1,00000	0,96470	0,82566
13	1,0000000	1,00000	1,00000	1,00000	0,99360	0,91743
14	1,0000000	1,00000	1,00000	1,00000	1,00000	0,97046
15	1,0000000	1,00000	1,00000	1,00000	1,00000	0,99403
α	0,214850615	0,29501	0,354027	0,411813	0,476755	0,571528

$$\alpha = \sum_{i=0}^{k} \frac{(1-Z_i)}{n-i}$$

$$T = \alpha * \lambda$$

Conclusion

The analytical expressions for determining the values of reliability indices of networks consisting of elements that have an exponential distribution of time to failure and recovery time have been obtained. At the same time, nodes and edges of the network can be considered as completely dependable. The accuracy of the obtained result is confirmed by calculating the reliability indices of a serial-parallel system for which analytical estimates are known.

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