Original article
https://doi.org/10.21683/1729-2646-2022-22-1-38-43

Correlations between states and events in the simulation of dependability using Markov processes

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Abstract. *The paper examines the correlations between states and events that are used in the construction of process diagrams that describe the dependability of items. Based on the constructed state and event diagram, input data is generated and the mathematical method is selected that is implemented in accordance with the problem at hand. The distinctive features and advantages of the matrix method are presented.* **Aim.** *To improve the simulation methods by clarifying the correlation between states and events and using matrix methods of calculation.* **Methods.** *The examined causal relationships between states and events allowed establishing correlations between them, i.e., an event can be the cause of a state change, then a state change is a consequence; a state can be the cause of an event, then an event is a consequence of a state. Under this approach, an event can cause a state change, while at the same time an event is a consequence of a state. The situation with states is similar. A state can be the cause of an event, while at the same time a state is the consequence of an event. It is also noted that a single state may cause a number of events, while an event can also cause a number of states. Examples of such correlations are given. It is noted that the duration of a state can be constant, random or zero. The examined correlations between states and events enable a substantiated construction of a diagram of states and transitions. A substantiated construction of a diagram of states and transitions results from a conceptual model, in which all states and events are given a physical and technical interpretation that transforms into a formal state-transition diagram. A special attention is given to the matrix methods that have a number of advantages, i.e., compactness and simplicity of converting the input characteristics into output characteristics, availability of standard software, use of verification procedures, feasibility of implementation using standard computer-based tools. The input data is also generated in matrix form. The paper indicates the characteristics of a state-transition diagram that can be calculated from the input data. Note is made of the use of methods based on semi-Markov processes. The author points out that, while using matrix methods, cycles should be generated. A relevant matter associated with the large number of states and the consequent problem of aggregation of states is touched upon. Two approaches to the aggregation of states are set forth that allow keeping the system's output characteristics unchanged.* **Results.** *A proposal is formulated for the construction of a dependability model involving a number of stages, i.e., definition of the goal of simulation with the indication of the used dependability indicators, description of the conceptual model, construction of a substantiated state-transition diagram, selection of the mathematical method, calculations, discussing the findings, conclusions and suggestions based on the performed simulation.* **Discussion and conclusions.** *A dependability model should take into consideration the causal relationships between states and events that are established based on the physical, as well as the engineering and technical features of the item. Taking these relationships into account, a state diagram is generated that enables initial data compilation. The matrix method is efficient and has a number of useful features. The above considerations are methodological in their nature. They can be helpful for generating dependability models of technical systems and studying the dependability theory in educational institutions.*

Keywords: *item dependability, state transition diagram, matrix methods for Markov process simulation.*

For citation: *Zelentsov B.P. Correlations between states and events in the simulation of dependability using Markov processes. Dependability 2022;1: 38-43. https://doi.org/10.21683/1729-2646-2022-22-1-38-43*

Received on: *26.11.2021* / **Revised on:** *21.01.2022* / **For printing:** *18.03.2022.*

Introduction

One of the problems of complex systems research consists in constructing such models of actual systems that are suitable for theoretical and experimental study of their properties. At the same time, item dependability models are to adequately represent actual processes in existing systems. Mathematical simulation is the most common and promising method for studying complex systems that allows conducting research at the design stage, solving analysis and synthesis problems, predicting the quality and efficiency of system operation, substantiating the required or optimal structure when designing new and improving existing systems and correctly interpreting statistical data.

Normally, dependability models are constructed on the basis of a discrete set of states, transitions between which occur in continuous time. Such processes are graphically represented as a state-transition diagram. The paper examines the causal correlations between states and events (transitions between states) that form a diagram. Further use of the diagram is associated with the selection of a mathematical method and calculations in accordance with the defined dependability process simulation objective.

The considerations presented in the paper are methodological in their nature and reflect the author's individual opinion as regards technical system dependability simulation.

Source overview

State standard [1] establishes guidelines for the application of Markov methods for simulating the dependability of systems with discrete states in continuous time. Markov methods can be used for dependability simulation of various technical systems. When applying Markov analysis, a state-transition diagram is used, which is a graphical representation of the conceptual model and simulates the behaviour of the system over time. The rules for constructing state and transition diagrams are described and examples of applying these rules are given. Accordingly, the state space analysis is used. State space analysis is used in the study of the dependability of various system architectures, i.e., redundant systems, systems with complex maintenance strategies, etc. It is stressed that the key problem solved by Markov analysis is the correct construction of the state space diagram. Additionally, a homogeneous Markov process is completely characterized by a transfer rate matrix.

Standard [1] further notes the advantages of Markov analysis, i.e., the ability to simulate various maintenance strategies. The assumption of constant recovery rate is to be substantiated, if the mean recovery time is not negligible compared to the corresponding mean time to failure. It is also noted that the use of Markov analysis requires special precautions associated with the increasing number of system states. In case of a large number of states and transitions, the probability of errors and distortions grows. Additionally, the computational methods also become more complex and may require the use of special software. For practical reasons, it is allowed excluding states with very low probabilities from the model of system operation.

There are numerous publications in the Russian and foreign literature dedicated to the study of the properties of Markov processes in discrete and continuous time, as well as their application to simulating probabilistic systems of various purpose, e.g., [2 – 4]. Of particular note is the widespread use of the Markov process theory involving state transition diagrams employed for solving dependability-related problems. Thus, in [5], using continuous-time Markov processes, functional models of recoverable and non-recoverable systems were developed, methods for calculating dependability indicators (availability coefficient, mean time between failures, etc.) were given for various conditions associated with equipment specificity. In particular, [5] sets forth dependability models of systems tested at random periods. In [6], methods are examined for calculating dependability based on Markov processes taking into account the completeness of testing. The above works examine Markov models, in which the future state of a system does not depend on the evolution of states up to the current one.

When Markov methods are used, mathematical models of dependability clearly show the process of state transition of the item (element, system). This process reflects the actual processes within technical systems. First of all, let us define the term "state of item".

The technical state of an item (technical state, state of item, state) is a set of the item's properties that are subject to change during its manufacture, operation, transportation and storage that are characterized by documented parameter values and/or qualitative characteristics [7]. Out of the above definition follows that a state is characterized by the time elapsed from the beginning of the state to its end, while the beginning and the end of the state are events. It should be noted that [1] uses the term "state transition", which is synonymous with the term "event". The term "event" (more precisely, "random event") is a basic term of probability theory. Further, we will use both terms that have the same meaning.

In [8], recovery is considered as a process and an event that consists in an item's transition from a non-operable to an operable state. Out of this definition, as well as the definition of recovery rate, follows that what is meant here is an event associated with the completion of recovery. This understanding involves that recovery is associated with two events, i.e., the beginning of recovery and the completion of recovery. Mathematical models often use the "recovery rate" parameter that characterises the completion of recovery, provided that the "beginning of recovery" event has taken place. Note that a "beginning of recovery" event may occur under various conditions, i.e., immediately after a failure, with a delay due to limited recovery capabilities, operation of the item in nonoperable state after a hidden failure, etc.

Correlation between the terms «state» and «event»

For a common understanding of the process of state transition, let us note the causal relationships between the terms “state” and “event”. Mathematical models generally assume that events (state transitions) occur instantaneously.

An event can cause a state transition, whereas the changed state is a consequence of the event. Examples:

– failure causes the operable state to change into inoperable state, i.e., the inoperable state is a consequence of the failure;

– completion of recovery causes transition from recovery to operable state, i.e., the operable state is a consequence of the completion of recovery.

– failure detection can cause the start of recovery or blocking of an item, i.e., the start of recovery or blocking of the item are consequences of failure detection.

A state can be the cause of an event. In this case, an event is a consequence of a state. Examples:

– using an item for its intended purpose is the cause of the failure, i.e., the failure is a consequence of the item being used;

– repair (restoration) of an item is the cause of the “start of operation” or “start of storage” events, i.e., “start of operation” or “start of storage” are the consequences of repair (restoration);

– an operation of incorrect technical condition inspection can cause such events as a type I inspection error and type II inspection error, i.e., type I and II inspection errors are the consequences of an incorrect inspection operation.

Thus, an event can be the cause of a state change, while at the same time being a consequence of a state. The situation is similar with a state. It may cause an event, while at the same time being a consequence of an event.

It should be noted that the same event is normally the end of one state and the beginning of another. Therefore, events may have different names depending on what state they are assigned to when the model is developed.

In the examples given in standard [1], the beginning of recovery coincides with the item’s failure. In actual systems, different situations may take place. Transition into the recovery state may occur with the following events:

– recovery after waiting in queue (restricted recovery);

– detection of a hidden failure during item diagnostics;

– inspection error causes false recovery of operable item.

Cases may be noted, whereas a failure does not cause recovery at the moment of failure:

– a hidden failure occurs;

– an explicit failure occurs and the item is queued for recovery (restricted recovery);

– upon verification of the technical state, no failure was detected.

It should be kept in mind that, within a single state, several events may occur, e.g., when an item is used for its intended purpose, hidden failures, explicit failures, pre-failures, damage may occur. An event can also cause a number of states: a technical state inspection operation may be valid or may cause type I and II inspection errors.

Thus, states and events are temporally associated with causal relationships. The new state of an item (an element or an entire system) is a consequence of a certain event, while any event is a consequence of the preceding state. Each state corresponds to two events, the beginning of the state and its completion.

The duration of a state may be of three types: constant (fixed, regular, deterministic), random or zero. If the state duration is zero, the beginning and completion of the state coincide. Such state can be called both a state, and an event.

An example of a constant-duration state is diagnostics of an item with a constant diagnostic time. An example of a random-duration state is the random time of item recovery (repair). An example of a zero-duration state: item diagnostics operation is performed within a time that is significantly shorter than the duration of other states, therefore, in models, the duration of the diagnostic operation is assumed to be zero.

When building dependability models, the system features are taken into account that cannot be covered in a single paper. However, the following factors can be noted:

– presence of hidden failures, explicit failures, pre-failures;

– application of a technical state monitoring system;

– use of maintenance system;

– maintenance with periodic or continuous inspection;

– possibility of type I and II inspection errors

and much more.

Hence, the state transition $s_i \rightarrow s_j$ is an event that is a consequence of state s_i and the cause of state s_j, i.e., state s_j is a consequence of this event. It should be noted that, in [1], state transitions (events) are often given with not due explanation. The expressions “state transition”, “transition from one state to another” and “return from one state to another” are used.

In order to define a substantiated diagram, a complete description of the states and transitions (events) should be done. For each state, the following is to be specified:

1) name;

2) transition into a state as a consequence of an event;

3) termination of a state as a consequence of another event.

For each event (transition), the following should be specified:

1) name;

2) the state that causes the event;

3) the state that is a consequence of the event.

Mathematical methods based on Markov chains and processes use various types of states associated with diagrams, i.e., neighbouring states, reachable states, communicating states, isolated state, absorbing state, non-essential and essential state, recurrent and non-recurrent state. The specificity of diagrams is expressed in the use of different types of sets and subsets of states, i.e., associated set, isolated and non-isolated set, transitive subset, subset of essential and non-essential states, ergodic set. These terms are extensively covered in academic and research literature.

When formulating the specific features of state-transition diagrams, "terminological perfection" is to be ensured. It comes down to the non-ambiguity of the terms, consistency within themselves and with state standards [9].

Matrix methods for Markov process simulation

Standard [1] notes that a homogeneous continuous-time Markov process is fully characterized by a rate matrix that is used for constructing and solving a matrix differential equation that allows finding the probabilities of states or events as function of time. It also refers to a method based on algebraic equations for calculating the limit probabilities of states. Hybrid models are mentioned, i.e., fault tree analysis, dependability structure diagram, Petri nets.

The matrix method is one of the most efficient mathematical methods for simulating Markov processes. The initial data of the matrix method for a continuous-time process are in the rate matrix. The use of constant rates for the time of occurrence of events or the duration of states is to be substantiated.

There are numerous Russian and foreign publications associated with studying the properties of discrete-time and continuous-time Markov processes and their application for simulating various probabilistic systems, e.g., for dependability-related purposes. In [10], matrix methods are presented for simulating discrete-time and continuous-time Markov processes that allow calculating probabilistic, temporal and frequency characteristics of states and subsets of states associated with the specificity of the examined system. Those characteristics are easily converted into dependability indicators, such as probability of no failure, mean time to failure, failure rate, availability and unavailability coefficients, etc.

The interest in the matrix methods is due to their advantages, i.e., compactness and simplicity of converting the input characteristics into output characteristics, availability of standard software, feasibility of implementation using state-of-the-art computer-based tools. It should be noted that matrix methods are classified as numerical analytical methods, i.e., applicable for both numerical calculations, and analytical studies.

Let us briefly represent the matrix method for a continuous-time process described in [10]. A transfer rate matrix is compiled based on the state transition diagram. The rate matrix can be used to calculate the following:

– state probabilities as functions of time under any initial state and specified initial distribution by defining and solving a matrix differential equation;

– limiting state probabilities for an ergodic process according to two analytical formulas, i.e., using matrix inversion and determinants;

– probabilities of being in a subset of states;

– mean time spent in a subset of states by inverting the rate matrix;

– variance of the time spent by the system in a subset of states using an inverted rate matrix and the matrix made on the basis of the initial distribution of state probabilities.

Depending on the chosen dependability model, the process of state transition may be either discrete-time, or continuous-time. In dependability, mathematical models are most often continuous-time. Similar procedures and characteristics for the discrete-time process are described in [10].

For the purpose of simulating processes that describe the dependability of systems, semi-Markov process-based methods can be used. The difference between a semi-Markov process and a discrete-time and continuous-time Markov process is that transitions are considered not at discrete moments of time and not in continuous time, but at moments of exiting states (or moments of state transition).

The process of state transition in a semi-Markov process is defined by the so-called probabilities of passing. A passing probability matrix can be defined based on a transition probability matrix for a discrete-time process and based on a rate matrix for a continuous-time process. The probabilities of passing a semi-Markov process can also be calculated for cases with a constant or random duration with an unknown distribution.

Probabilities of passing do not contain information on the duration of states. If such characteristics are required for simulating the system, those are defined together with the probabilities of passing as initial data. Such input data may include, e.g., the mean times in states after entering.

This approach involves that an event is a dependent event if it is caused by a certain state. In this case, events should be characterized with conditional probabilities. In this context, the events reflected in dependability models may be deterministic or random. A deterministic event is the only event that is a consequence of a state. Its conditional probability is 1. Random events include those whose conditional probabilities are below 1.

An example of a random event is a failure of an item that has been used for its intended purpose for some time with no failure during that period, i.e., opposite random events. An example of a deterministic event is commencement of the use of an operable item after recovery.

Based on the semi-Markov process, the following can be calculated:

– expected number of times in the states of a subset called mean relative state rates and represented in matrix form;

– mean time in a subset of states based on the mean relative state rates.

Computation and verification should be implemented using computer mathematics. In particular, ready-made formulas are used to calculate states as functions of time, limit probabilities of states, mean time and variance of time spent in a subset of states and other characteristics. The use of computer mathematics allows reducing the relevance of many states.

Computational procedures based on computer mathematics can be performed both numerically and analytically.

Relevant problems solved using the matrix method

Let us briefly mention two problems solved using the matrix method, i.e., aggregation of states and cyclic system operation.

In standard [1], it is noted that if the number of states is large, difficulties associated with possible errors and distortions may arise. At the same time, it is allowed to exclude from the system operation model the very-low-probability states. Research literature notes that reducing the number of states by discarding unlikely ones may cause significant errors in the system's output characteristics. Therefore, such approaches are to include error calculation, i.e., with no error calculating such approach is irrelevant.

When using the matrix methods, the difficulties associated with the large number of states are quite easily addresses by means of verification procedures for both initial data generation, and calculation. Verification procedures allow quickly finding data input and computation errors; therefore, verification procedures help improve the efficiency of mathematical methods.

In [10], two approaches are described to the problem of aggregation of states, i.e., aggregation using truncation of matrix characteristics and frequency-based. These approaches use the matrix method and allow keeping the system's output characteristics unchanged.

The operation of long-term use systems is associated with repeating cycles. That fact was one of the reasons that determined the development of the model of cyclic system operation [10]. The model of cyclic operation describes transitions between subsets of states by means of manipulations with matrices. This approach allowed calculating system characteristics in transient and stationary modes. Formulas are given for calculating the mean times in subsets of states in the transient and stationary modes, as well as for limit probabilities of subsets.

Results

Out of the above reasoning follows that dependability simulation should be carried out in the following stages.

1. Definition of the simulation objective specifying the employed dependability indicators.

2. Presentation of the conceptual model that contains the initial representation of the item. It sets out the physical and operational features of the facility and provides an engineering description of the processes in terms of dependability.

3. Construction of the state transition diagram based on the conceptual model.

4. Selection of the mathematical method. There should be a clear understanding of the source data and the output characteristics obtained using the method.

5. Calculations.

6. Findings, suggestions and conclusions based on the conducted simulation.

Conclusions

1. A proposal was defined for dependability model preparation that contains the goal, conceptual model, state transition diagram, mathematical method, calculations and conclusions.

2. When constructing state-transition diagrams, the causality relationships between states and events should be taken into consideration. The establishment of these relationships is based on the physical and engineering features of the examined systems.

3. One of the efficient methods of Markov process simulation is the matrix method that has a number of useful features, i.e., compactness and simplicity of transformations, availability of standard software.

4. The above matrix method allows constructing analytical and algorithmic models of equipment operation as part of various technical systems.

5. The matrix method provides verification procedures for every stage of the simulation for the purpose of eliminating errors and distortions in the input data generation and computational procedure implementation.

6. Matrix manipulations should be performed using such modern software tools as Mathcad and Matlab.

The above materials can be used as guidelines for efficient construction of dependability models of technical systems and in studying the dependability theory in educational institutions.

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The author's contribution

The author examined the possible causal relationships between states and events and provided examples of a significant dependence of the future on the past in dependability process simulation.

Conflict of interests

The author declares the absence of a conflict of interests.