Original article
https://doi.org/10.21683/1729-2646-2022-22-1-13-19

# Optimizing the timeframe of scheduled repairs using valuation techniques

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**Abstract. Aim.** *The paper examines technical systems (machinery and equipment), whose condition deteriorates in the course of operation, yet can be improved through repairs (overhaul). The items are subject to random failures. After another failure, an item can be repaired or disposed of. A new or repaired item is to be assigned the date of the next scheduled repairs. Regarding a failed item, the decision is to be taken as to unscheduled repairs or disposal. We are solving the problem of optimization of such repair policy. At the same time, it proves to be important to take into consideration the effect of repairs, first, on the choice of appropriate indicators of item condition that define its primary operational characteristics, and second, on a sufficiently adequate description of the dynamics of items' performance indicators.* **Methods.** *Assigning the timeframe of scheduled repairs normally involves the construction of economic and mathematical optimization models that are the subject matter of a vast number of publications. They use various optimality criteria, i.e., probability of no failure over a given period of time, average repair costs per service life or per unit of time, etc. However, criteria of this kind do not take into account the performance dynamics of degrading items and do not fully meet the business interests of the item owners. The criterion of maximum expected total discounted benefits is more adequate in such cases. It is adopted in the theory of investment projects efficiency estimation and the cost estimation theory and is, ultimately, focused on maximizing a company's value. The model's formulas associate the item's benefit stream with its primary characteristics (hazard of failure, operating costs, performance), which, in turn, depend on the item's condition. The condition of non-repairable items is usually characterized by their age (operating time). Yet the characteristics of repairable items change significantly after repairs, and, in recent years, their dynamics have been described by various models using Kijima's virtual age indicator (a similar indicator of effective age has long been used in the valuation of buildings, machinery and equipment). That allows associating the characteristics of items in the first and subsequent inter-repair cycles. However, analysis shows that this indicator does not allow taking into consideration the incurable physical deterioration of repaired items. The paper suggests a different approach to describing the condition of such objects that does not have the above shortcoming.* **Conclusions.** *The author constructed and analysed an economic and mathematical model for repair policy optimisation that is focused on maximizing the market value of the company that owns the item. It is suggested describing the condition of an item with two indicators, i.e., the age at the beginning of the current inter-repair cycle and time of operation within the current cycle. It proves to be possible to simplify the dependence of an item's characteristics on its condition by using the general idea of Kijima models, but more adequately taking into consideration the incurable physical deterioration of such item. The author conducted experimental calculations that show a reduction of the duration of planned repairs as machinery ages at the beginning of an inter-repair cycle. Some well-known repair policies were critically evaluated.*

**Keywords:** *service life of a technical system, repair policy, optimization criterion, degradation, Kijima models, cost estimation, revenue approach.*

**For citation:** *Smolyak S.A. Optimizing the timeframe of scheduled repairs using valuation techniques. Dependability 2022;1: 13-19. https://doi.org/10.21683/1729-2646-2022-22-1-13-19*

**Received on:** *14.10.2021* / **Upon revision:** *10.02.2022* / **For printing:** *18.03.2022.*

## Basic definitions. Problem definition

The paper examines mass-produced and marketed technical systems (TS, i.e., machines and equipment) that are operated by companies, can be repaired and are subject to failures. We will call their state at the moment of release *new*. All TS that are identical in the new state are combined into a single *brand*. The paper examines the process of operation of single-brand TS. TS is useful for *market players* and therefore, according to valuation standards [1], has a certain *market value* (MV) that depends on the state of the TS. In the course of operation, the state of TS deteriorates due to physical wear (degradation). Possible failure of TS causes losses for the company. In case of failure, a TS is to be either disposed of, or submitted to emergency repairs. The efficiency of TS operation can be improved by assigning scheduled repairs (overhauls), as well as modifying its service life (i.e., the time of disposal). The MV of the disposed TS is called *salvage value* and is usually defined as the value of the elements (components, parts, scrap metal) suitable for further use less the cost of dismantling the TS and transporting its elements. Usually, this value is not high and we (for the purpose of simplification) will deem it equal to zero. The work performed by the TS is also useful for the market players and has a certain market value.

The *benefits* from the use of TS within a certain period are defined as the MV of the deliverables less the costs incurred within such period. Accordingly, the benefits of TS operation are equal to the MV of the performed work less the operating costs (that, among other things, include the cost of maintenance and scheduled repairs), while the benefits of TS disposal are zero.

Repairs improve the state of TS by eliminating some of the effects of physical wear. That is a case of *curable* deterioration. However, other effects accumulate and, eventually, may cause TS failure. Such deterioration is called *incurable* [1, 2]. The service life of TS is divided by repairs into inter-repair cycles (IRC). The *assigned duration* of IRC is the period, at the end of which the TS that has not failed earlier within this cycle is to be disposed of or repaired.

A *repair policy* is understood as the rule for assigning IRC durations and the rule for choosing a solution regarding a TS at the end of an IRC (that failed or reached the end of a designated period). We are solving the problem associated with the development of an optimal repair policy for single-brand TS. For that purpose, an optimality criterion is to be defined and the variation of TS characteristics in the course of operation is to be described. At the same time, almost until the end of the paper, we will assume the absence of inflation.

## Optimality criterion

The numerous works on the dependability theory used various optimality criteria, e.g., the life-average number or cost of repairs, total discounted costs [3] or equivalent annuity [4, 5], ratio of life-average costs to the average service life [6]. However, as it is correctly noted in [7], the optimality criteria were normally chosen without proper substantiation, out of real business context.

If we consider the acquisition and use of TS as an investment project, the optimal repair policy is to comply with its best version, the one that provides the highest expected net present value (NPV) [8; 9]. A similar criterion is also used in property valuation. Here, the *market value* (MV) is considered the primary type of value. This concept is defined and commented in the valuation standards [1], and we will not repeat that here. Let us just note that the MV of a valuation item at a certain date (valuation date) reflects both the price of the item in the transaction made on the valuation date between independent and economically rational market players under certain conditions (specified in the valuation standards) and the contribution of the item into the MV of the company that owns it. Three approaches are used for determining an item's MV.

In the comparative (market) approach, an item's MV is estimated based on the prices of the deals concluded on the valuation date with similar items.

In the cost-based approach, an item's MV is estimated based on the costs required for its creation. This approach is primarily used for evaluating buildings and structures, but its applicability is limited by evaluation standards and, in general, appears to be controversial, especially as regards machinery and equipment [10, 11].

The income-based approach is based on the principle of expected benefits that is mentioned, but not detailed in [1]. We will use the following definition [1, 2].

**The MV of the assessed item at the date of valuation is equal to the expected amount of discounted benefits from its use within the projection period and the item's MV at the end of the period, if the item is used most efficiently, and not less than the above sum if otherwise. The end of the projection period can be chosen arbitrarily, as the item's MV does not depend on it.**

A number of important comments should be made on this definition.

1. The term "expected", in the context of probabilistic uncertainty, is understood as the expectation (in [1], "weighted by probability"). In the following formulas, it is denoted as **E**.

2. Adding the item's MV at the end of the period can also be interpreted as benefits from the (virtual) sale of the item at the current MV. According to this interpretation, the most efficient use of an item may also include its sale at the MV at some point in time.

3. According to [1], the benefits are to be discounted at the after- or pre-tax rate, depending on whether the income tax was included in the cost. We assume the second option and discount the benefits at the pre-tax rate $r$.

As it can be seen, the item's MV reflects the maximum amount of the expected total discounted benefits (ETDB) that corresponds to its most efficient use. In this context, the ETDB from the use of TS is to be the criterion of optimal repair policy, which contributes to the growth of the company value. It is difficult to directly apply the principle of expected benefits to the optimization of the repair policy, as the MV of the work performed by the TS (denoted as *B*) is usually unknown. The following considerations help solve the problem. Let us consider a TS at the beginning of its use (moment in time 0). Its MV *K* is a known value that reflects the costs of acquisition, delivery and installation. Let P be a certain repair policy. With this policy, at time *t*, TS performance *Q*(t) and the rate of operating costs *C*(t) will be random functions of time. The moments $s_1, s_2, \ldots$ of repairs (and, in general, the cost of such repairs $R_1, R_2, \ldots$) will also be random.

The expected amount of discounted benefits of policy P, in this case, will be

$$B_\Sigma(\Pi) = \mathrm{E}\left\{\int_0^\infty \left[BQ(t) - C(t)\right]e^{-rt}dt + \sum_i R_i e^{-rs_i}\right\}.$$

However, due to the principle of expected benefits, this value is not greater than the TS market value *K* and is identical to *K* in case of an optimal policy. Out of that easily follows that

$$B \geq \frac{K + \mathrm{E}\left\{\int_0^\infty C(t)e^{-rt}dt + \sum_i R_i e^{-rs_i}\right\}}{\mathrm{E}\left\{\int_0^\infty Q(t)e^{-rt}dt\right\}},$$

with equality when the most efficient policy P is used.

It follows that an optimal repair policy is to ensure minimal expected unit costs (EUC), i.e., the ratio of the expected discounted costs for the purchase, operation and repair of the TS to the expected discounted scope of work that it performs. For the deterministic case, a similar criterion was proposed in [1-3] and practically used in the development of depreciation rates of construction machines. However, since in our case the repair policy cannot be defined with a finite number of scalar parameters, it proves to be difficult to optimize it according to the EUC criterion. Further on, we will suggest a more convenient solution of this problem that is based on the same idea.

## Characterization of the state of TS under repair

Kijima has suggested [14] characterizing the state of repaired items by *virtual age* (VA) that increases synchronously with the chronological age, but after repairs, rapidly decreases proportionally to the item's VA before the repairs (model I) or the duration of the previous IRC (model II). If proportionality coefficient $\beta = 1$ or 0, the item's state after the repairs becomes either new, or same as before the repairs. We consider both of these cases unrealistic, and assume that $0 < \beta < 1$.

Meanwhile, the basic idea of describing the state of a TS with a single indicator was proposed much earlier. Thus, in [15], it was stated that some valuators use the *effective age* (EA) for appraising the value of buildings. This indicator reflects the age of a typically used similar building that is in the same state as the one being evaluated. Since the 1950s, the concept of EA has been used, first in the US, then in other countries for the purpose of valuating buildings, machinery and equipment. Initially, the valuators assessed the EV of items using expert methods. Later, more substantiated methods and tables were developed, of which neither Kijima, nor his followers were apparently aware. In this context, Kijima models can be considered an application of the EV concept in dependability. These models have been studied by many authors (e.g., in [16]) and used for solving practical problems.

However, the virtual age and similar indicators cannot adequately describe the state of a repaired TS. Indeed, otherwise, after the first repairs, when the virtual age of TS decreases, it will be in the same state as it was at some point in time within the first IRC. But then it is to further be used in the same way, i.e., work until failure or until the assigned time of the first repairs, etc. In this case, its service life will prove to be infinite, which is impossible for the TS exposed to *incurable* wear. At the same time, the state of regular TS can be adequately described by *two* indicators, i.e., age *s* at the beginning of the current IRC and the time of operation within this cycle *t* [17, 18]. Then, the dependence of TS characteristics on its state will have to be described by functions of two variables. It turns out that they can be simplified using Kijima's idea.

Let us take a certain TS operational characteristic (e.g., performance). We will denote its value for the TS in state *(s,t)* as *Z(s,t)* and assume that *z(t)=Z(0,t)*. As with the Kijima model I, we will assume that the characteristic of a TS that underwent the first repairs at age *s* becomes the same as that of a TS of a smaller age of $\beta s$: $Z(s,0) = Z(0,\beta s) = z(\beta s)$. But the incurable wear of the first TS is greater than that of the second one, therefore, further on, its characteristic will deteriorate faster, and the faster the greater the age difference. Generally speaking, in its regard, the time will as if "accelerate" by a certain rate $k(s) > 1$ times, i.e., after time *t*, it will be $z(\beta s + k(s)t)$. If the first TS, after repairs at age *s*, undergoes second repairs after time *s'*, then, in the next IRC, for it, time should "accelerate" by another *k(s')* times, i.e., by *k(s)k(s')* times compared to the second TS. It is logical to assume that the result of the second repairs will be the same as that of the second TS that underwent repairs at the same age $s + s'$, which means an "acceleration" of $k(s + s') > 1$ times. But then $k(s + s') = k(s)k(s')$, and that is only possible if $k(s) = \gamma^s$, where $\gamma > 1$ is the "degradation acceleration" coefficient. In such case, the characteristic of a TS that underwent repairs at age *s* will be $Z(s,0) = z(\beta s)$ after the repairs, while after more time *t* it will be $Z(s,t) = z(\beta s + t\gamma^s)$. Such model that is applicable to any characteristics of a TS can be called a modified Kijima model. It appears that it more adequately describes the dynamics of the characteristics of repaired TS. A similar model of geometrical process, in which "degradation acceleration" is associated with the *ordinal number of the IRC*, was proposed in [19] and subsequently examined by many authors.

## Optimization model

Let us first find out how the value of a TS varies within a single IRC. We will characterize each IRC not by its ordinal number, as it is usually done, but by the age of the TS at the beginning of the cycle. Let us introduce the following designations: $M_s$ is the IRC, at the beginning of which the TS has the age of $s$, $T_s$ is its designated duration, $B$ is the MV of the work performed by an operable TS within a small unit of time, $R$ is the cost of TS repairs (we deem it to be identical for scheduled and emergency repairs), $L$ is the company's losses caused by TS failure, $Q(s,t)$ is the TS performance in state $(s,t)$, $C(s,t)$ is the rate of its operational costs, $\lambda(s,t)$ is the hazard of TS failure, $\Lambda(s,t)=\int_0^t \lambda(s,x)\,dx$ is the mean number of failures over time $t$ within cycle $M_s$.

We will assume that function $Q(s,t)$ is non-increasing, while functions $\lambda(s,t)$ and $C(s,t)$ are non-decreasing with respect to their arguments, while at least one of them grows indefinitely if $s\to\infty$ and $t\to\infty$. The time of disposal and repair is considered negligible.

Let us denote the cost of a TS in state $(s,0)$ as $f(s) = V(s,0)$. Let us evaluate function $f(s)$ from above. In order to do that, let us note that the TS in state $(s,t)$ brings benefits with the rate of $BQ(s,t) - C(s,t)$. In particular, a TS at the beginning of cycle $M_s$ brings benefits with the rate of $B_0 = BQ(s,0) - C(s,0)$ and has the hazard of failure $\lambda(s,0)$. Then, those characteristics deteriorate until the TS enters the next IRC or is disposed of. It can be seen that the cost of the TS is not greater than the MV $W(s)$ of a virtual item that always provides benefits with the rate $B_0$, whose failures occur with a constant rate $\lambda(s,0)$ and do not cause losses. But such item, within the short time $dt$, fails with the probability $\lambda(s,0)dt$, therefore, requiring expected repair costs $\lambda(s,0)Rdt$ and providing expected benefits $[B_0 - \lambda(s,0)R]dt$. That is why the ETDB from its use over an infinite service life is equal to $[B_0 - \lambda(s,0)R]/r$. If this value is positive, it is identical to the MV of the virtual item $W(s)$, otherwise using such item is inefficient and it has $W(s) = 0$. Noting that the TS at the beginning of the cycle $M_s$ has a MV $f(s)$ that does not exceed $W(s)$, we obtain: $f(s) \le W(s) = \max\{[BQ(s,0)-C(s,0)-\lambda(s,0)R]/r;\ 0\}$. But if $s\to\infty$, at least one of the functions $C(s,0)$ and $\lambda(s,0)$ increases indefinitely, therefore if $s$ is sufficiently large $f(s) = 0$.

Let $g(x)$ be the cost of a TS with the age of $x$, that needs to be disposed of or repaired. It corresponds to the total benefits from the best possible further use of such TS. But disposing of the TS provides zero benefits, while repairs require costs $R$ and put the TS at the beginning of the next cycle, i.e., state $(x,0)$, where it will have a MV $f(x)$. Therefore,

$$g(x)=\max\left[f(x)-R;0\right]. \quad (4)$$

Cycles $M_s$, in which $f(s) > 0 = g(s + T_s)$, will be called *terminal*. In them, using a TS for its intended purpose is efficient, but at the end of the cycle it should be disposed of.

In this situation, the repair policy consists in assigning for each cycle $M_s$ a duration $T_s$ and specifying, which of them are terminal.

Let us assume that for cycle $M_s$ duration $T$ was assigned. Let us take a TS at the beginning of this cycle and find the expected sum $G(s,T)$ of discounted (at the beginning of the cycle) benefits from its use in cycle $M_s$ (including the cost of the TS at the end of the cycle).

Note that the duration of cycle $M_s$ is random. With probability $e^{-\Lambda(s,T)}$, it is equal to $T$, while with probability $\lambda(s,x)e^{-\Lambda(s,x)}dx$, it lies within the interval $(x, x+dx)$ if $x < T$.

In the first case, there will be no loss from failure, and at the end of the cycle, the TS will have an age of $s + T$ and PC $g(s + T)$. In the second case, the TS fails having operated for time $x$, i.e., at the age of $s + x$. Its MV will be $g(s + x)$ and there will be failure-related losses L.

The benefits of using the TS for its intended purpose at time $x$ after the start of the cycle, i.e., in state $(s,x)$, for the small period $dx$ are $[BQ(s,x) – C(s,x)]dx$. However, the TS will provide them only if it does not fail during time $x$ of its operation within the cycle, i.e., with probability $e^{-\Lambda(s,x)}$.

Now, taking into account the amount of possible benefits and their probabilities, we find:

$$\begin{aligned}G(s,T)&=e^{-rT}\cdot e^{-\Lambda(s,T)}g(s+T)+\\&+\int_0^T e^{-rx}\cdot\left[g(s+x)-L\right]\lambda(s,x)e^{-\Lambda(s,x)}dx+\\&+\int_0^T e^{-rx}\cdot e^{-\Lambda(s,x)}\left[BQ(s,x)-C(s,x)\right]dx=\\&=e^{-N(s,T)}g(s+T)+\int_0^T e^{-N(s,x)}H(s,x)\,dx,\end{aligned} \quad (5)$$

where

$$\begin{aligned}N(s,x)&=rx+\Lambda(s,x);\\H(s,x)&=BQ(s,x)-C(s,x)+\lambda(s,x)\left[g(s+x)-L\right].\end{aligned} \quad (6)$$

Let us note that the cost of TS $f(s)$ at the beginning of cycle $M_s$ is equal to the maximum value of $G(s,T)$, out of which and by virtue of (5) we obtain:

$$\begin{aligned}f(s)&=\max_T G(s,T)=\\&=\max_T\left\{e^{-N(s,T)}g(s+T)+\int_0^T e^{-N(s,x)}H(s,x)\,dx\right\}.\end{aligned} \quad (7)$$

The optimal $T_s$ will be the value of $T$, under which $G(s,T)$ is maximal. But, perhaps, such $T$ are more than one, or $T = \infty$. Let us consider both of the options.

1. Let us assume that the maximum $G(s,T)$ is reached both if $T = T'$, and if $T = T'' > T'$. But a TS operating within cycle $M_s$, before it reaches state $(s,T'')$, must first be in state $(s,T')$, where it will be decided upon its repairs or disposal. Therefore, if used rationally, it simply will not "live until" state $(s,T'')$. That means that $T_s$ must be the smallest of the values of $T$ that maximize $Q(T)$.

2. The case of $T = \infty$ is impossible, as $G(s,T)$ decreases if $T$ are large. Indeed, as it was shown above, if $s$ are sufficiently large, $f(s) = 0$. Out of that and (4) follows that if $T$ is sufficiently large, $g(s + T) = 0$. But then by virtue of (5) and (6) $G(s,T)=\int_0^T e^{-N(s,x)}H(s,x)dx$ and $G'_T(s,T)=e^{-N(s,T)}\left[BQ(s,T)-C(s,T)-\lambda(s,T)L\right]$. Since at least one of the functions $C(s,t)$ and $\lambda(s,t)$ increases without limit if $t\to\infty$, $G'_T(s,T)$ becomes negative if $T$ is sufficiently large, while $G(s,T)$ will decrease, which was to be proven.

Let us note that function $G(s,T)$ is continuous, but not monotone in terms of $T$, and therefore can have several local maxima, out of which only one will be chosen as $T_s$, i.e. the global one. However, if $s$ changes, some of the local maxima may simply disappear, while the global maximum may "jump" from one local maximum to another. As a result, the dependence of $T_s$ on $s$ may be discontinuous, and, in the points of discontinuity, the maximum value of $G(s,T)$ will be achieved at two points at once. A similar situations may arise if $s$ is fixed if the initial data, e.g., loss value $L$ or dependencies $\lambda(s,t)$ and $C(s,t)$, are modified.

## Algorithm of model solution

To solve the problem, let us substitute (4) into formula (7) and represent it as follows:

$$f(s)=\mathrm{G}\left(f(s)\right), \tag{8}$$

where **G** is an operator that translates the function of one variable $\varphi(s)$ into another function $\phi(s)$ as follows:

$$\phi(s)=G\left(\varphi(s)\right)\triangleq\max_{T\geq 0}\Big\{\max\left[\varphi(s+T)-R;0\right]e^{-N(s,T)}+ +\int_0^T e^{-N(s,x)}\begin{bmatrix}B-C(s,x)+\lambda(s,x)\cdot \\ \cdot\max\left[\varphi(s+x)-R;0\right]-\lambda(s,x)L\end{bmatrix}dx\Big\}. \tag{9}$$

It is easy to see that if function $\varphi(s)$ is continuous, non-negative and confined for $s \geq 0$, then function $\phi = \mathbf{G}(\varphi)$ will be identical. It is also easy to see that operator **G** is monotone: if $\varphi_1(s) \geq \varphi_2(s)$, then $\phi_1(s) \geq \phi_2(s)$, too. That allows solving equation (8) using the iterative method. For example, for the first approximation we can take $f_1(s) = 0$, and the subsequent ones we can find using formula $f_{n+1} = \mathbf{G}(f_n)$. Then, sequence $\{f_n(s)\}$ will be monotone and bounded, and, therefore, will have a limit (the "fixed point" of operator G), the required $f(s)$ equal to zero for sufficiently large $s$. In case of a numerical solution, the values of $f(s)$ were identified at the points of a uniformly spaced small-stepped grid, while the integrals were calculated using the Simpson formula.

In the course of the solution, for each cycle $M_s$, its assigned duration $T_s$ is also defined as the least $T$ that enables the maximum of (5). Naturally, the durations $T_s$ of different IRC will be different, which was revealed back when the deterministic situation was considered, e.g., in [17]. Further, all the terminal cycles can be identified (they will have $f(s) > 0 = g(s + T_s)$ along with the maximum service life of TS $T_{max}$ (it corresponds to the least $s$, under which $f(s) = 0$). Knowing $f(s)$, the cost of TS in any other states $(s,t)$ can be calculated as well. The corresponding formulas are derived in the same way as formulas (5) and (7), but we do not need them.

In the above procedure, the MV of the work performed by a serviceable TS per unit of time $B$ was considered known, although the owners of machinery and equipment are not usually aware of it, and valuators almost never estimate the cost of work (except, probably, that of construction, installation and repair activities). To solve this problem, let us note that the above procedure can be performed for different values of $B$, and all the costs of $f(s)$ will be non-decreasing functions of $B$. That is also true for the cost of the TS at the beginning of its use $f(0)$. But this cost is known and is equal to $K$. Therefore, the desired value $B$ must be the root of the equation $f(0) = K$. This method of estimating the cost of work strictly corresponds to the cost-based approach to valuation, although, in this form, it has not yet been used by valuators.

The model assumed there was no inflation. However, the model is also applicable under conditions of inflation, if, according to valuation standards [1], cost indicators are measured in prices at the valuation date, and the real (rather than nominal) pre-tax discount rate is used. Such procedure can also be substantiated by the method described in [12, section 4.3].

## Experimental calculations

According to model (7), experimental calculations were performed with the following input data:

- TS performance at the beginning of operation is adopted as 1: - $Q(0,0) = 1$, rate of operating costs $C_0 = C(0,0) = 40$, market value $K = 100$;
- repair costs $R = 25$;
- in the first IRC, as the TS ages, its performance deteriorates exponentially at a rate of $\alpha = 0.02$ 1/year, the rate of operating costs increases linearly at the rate of $i = 0.03$, while the failures have a Rayleigh distribution with the parameter $\omega$ (mean time to failure is equal to $\omega\sqrt{\pi/2}$).
- in other IRCs, the TS characteristics were described by a modified Kijima model:

$$Q(s,t)=e^{-\alpha\left(\beta s+t\gamma^{s}\right)};\ C(s,t)=C_0\left[1+i\left(\beta s+t\gamma^{s}\right)\right];$$
$$\lambda(s,t)=\left(\beta s+t\gamma^{s}\right)/\omega^2,$$

where $\beta = 0.4$, $\gamma = 1.2$.

Failure-related losses $L$ and the failure distribution parameter $\omega$ varied. The MV of work performed by the TS per a unit of time, $B$, was determined from the condition $f(0) = K$.

We examined the effect of parameters $L$ and $\omega$ on the assigned times of preventive repairs and the maximum service life of the TS $T_{max}$. Let us set forth only some of the findings (in their entirety, they would take up too much space).

Value $L$ ranged from 100 to 1000 (from one to ten times the MV of the TS cost). Its effect, if ω = 4 and 8 years, is shown in Fig. 1 to 3. Figure 1 shows the dependence of the assigned time of the first repairs ($T_0$, years) on $L$. Disturbances on the graph occur when $L$ is small. They correspond to the above situations, whereas the maximum value of $Q$(s,T) is reached at two points at once. The dependence of the maximum service life of the TS ($T_{max}$, years) on $L$ is shown in Fig. 2.

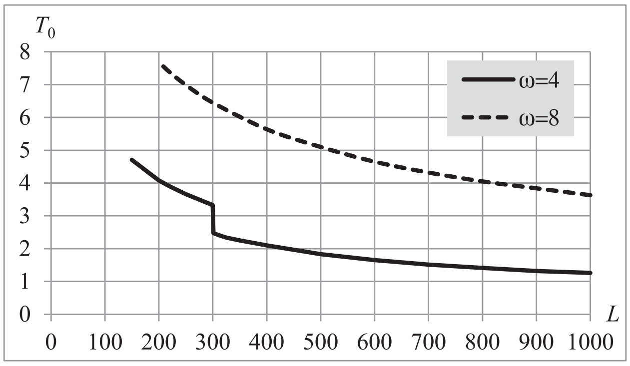


Fig. 1. Dependences of the assigned time of the first repairs ($T_0$, years) on $L$ if ω = 4 and 8 years

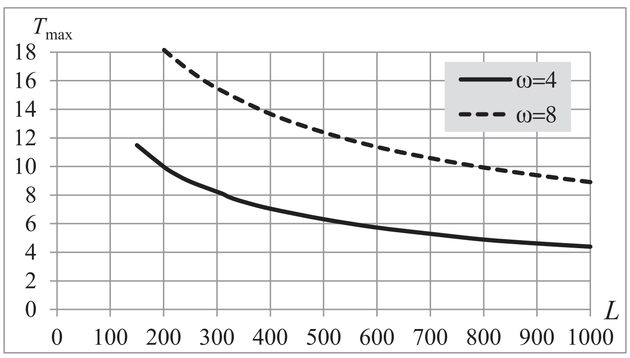


Fig. 2. Dependences of the maximum TS service life ($T_{max}$, years) on $L$ if ω = 4 and 8 years

Figures 3 and 4 show the dependences of the cycle duration ($T_s$, years) on the age of the TS at the beginning of the cycle ($s$, years) for various combinations of $L$ and ω. It can be seen that the optimal policy is significantly different from the common one, whereas the time of scheduled repairs is assigned identical or according to the serial number of the repairs regardless of the failure-related damage. Note that TS of a "sufficient" age is to be assigned very short times of scheduled repairs, which is technically inconvenient and provides a small economic effect. Therefore, such TS should not be assigned a time of the next preventive repairs at all. They should be disposed of only upon the next failure.

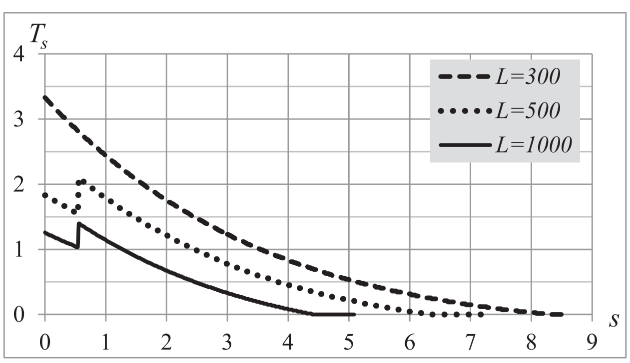


Fig. 3. Dependences of the assigned time of preventive maintenance ($T_s$, years) on the age of the TS at the beginning of the cycle ($s$, years) if ω = 4 years and varied $L$

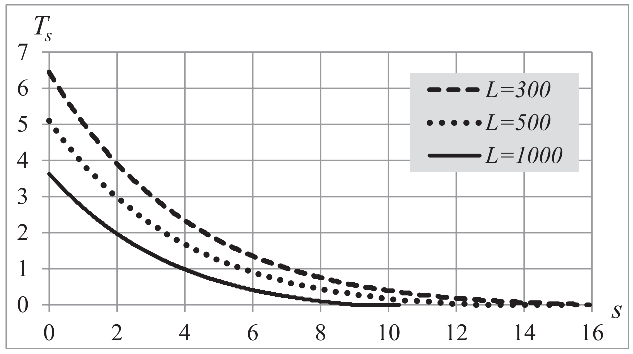


Fig. 4. Dependences of the assigned time of preventive maintenance ($T_s$, years) on the age of the TS at the beginning of the cycle ($s$, years) if ω = 8 years and varied $L$

## Conclusion

The cost criteria used in the dependability theory for optimising TS repair policy do not fully meet the business interests of companies. An economically substantiated solution of such problems is ensured by methods and criteria used in the valuation theory. They allow estimating the cost of work (services) performed by the TS, and, in particular cases, result in the criterion of the minimum expected discounted unit costs that can be rarely found in dependability-related literature.

In it, changes in the performance characteristics of TS after repairs are described by Kijima's virtual (effective) age models. Half a century before Kijima, a similar indicator was proposed for the valuation of assets and is still practically used by appraisers today. However, we show the inadequacy of describing the condition of repairable TS by any one such indicator. It appears to be more appropriate to characterize their condition by two indicators, i.e., the operating time at the beginning of the IRC and in the course of such cycle.

The above provisions allow constructing models for optimising a repair policy that meets the economic interests of market players. It is shown that, in each IRC, the time of the next scheduled repairs is to be assigned depending on the damage caused by a failure and the age of the TS at the beginning of the cycle, not on the serial number of repairs.

## References

1. International Valuation Standards (IVS) (2019). Effective 31 January 2020. International Valuation Standards Council.

2. Fedotova M.A., Koroliov I.V., Kovaliov A.P., et al. Fedotova M.A, editor. [Evaluation of machines and equipment:

Textbook. Second edition, updated and revised]. Moscow: INFRA-M; 2018. (in Russ.)

3. Aven T. Optimal replacement under a minimal repair strategy. *Advances in Applied Probability* 1983;15(1):198-211. DOI: 10.2307/1426990.

4. Bergman B. Optimal Replacement under a general failure model. Advances in Applied Probability 2;10(2):431-451.

5. Christer A.H., Waller M.W. Tax-Adjusted Replacement Models. *Journal of the Operational Research Society* 1987; 38(11):993-1006. DOI: 10.1057/jors.1987.170.

6. Jiang R. Performance evaluation of seven optimization models of age replacement policy. *Reliability Engineering & System Safety* 2018;180(C):302-311. DOI: 10.1016/j.ress.2018.07.030.

7. Van Horenbeek A., Pintelon L., Muchiri P. Maintenance optimization models and criteria. *International Journal of System Assurance Engineering and Management* 2010;1(3):189-200. DOI: 10.1007/s13198-011-0045-//.

8. [Guidelines for efficiency assessment of investment projects. Second edition. Approved by the Ministry of Economy of the RF, Ministry of Finance of the RF, Gosstroy of the RF, 21.06.1999, no. VK477]. Moscow: Ekonomika; 2000. (in Russ.)

9. Vilensky P.L., Livshits V.N., Smolyak S.A. [Efficiency assessment of investment projects: theory and practice. Study guide. Fifth edition]. Moscow: PoliPrintServis; 2015. (in Russ.)

10. Mikerin G.I., Smolyak S.A. [Efficiency assessment of investment projects and property valuation: convergence opportunities]. Moscow: CEMI RAS Publishing; 2010. (in Russ.)

11. Smolyak S.A. [Preventive repairs of machines in a Kijima-type model]. *Vestnik CEMI RAS* 2018;1(2). Available at: https://cemi.jes.su/s111111110000091-3-1/. DOI: 10.33276/S0000091-3-1. (in Russ.)

12. Smolyak S.A. [Valuation of machines and equipment (secrets of discounted cash flow)]. Moscow: Optsion; 2016. (in Russ.)

13. Livshits V.N., Smolyak S.A. [Service life of fixed assets in an optimal plan]. In: Proceedings of the First Conference for Optimal Economy Planning and Management. Section 1, Issue 1. Moscow: CEMI RAS; 1971. P. 352-357. (in Russ.)

14. Kijima M. Some results for repairable systems with general repair. *Journal of Applied Probability* 1989;26:89-102.

15. Welch R.B. Depreciation of buildings for assessment purposes. Chicago: International Association of Assessing Officers; 1943.

16. Chumakov I.A., Chepurko V.A., Antonov A.V. [On some properties of Kijima incomplete recovery models]. Dependability 2015;3(54):10-15.

17. Smolyak S.A. Overhaul policy optimization and equipment valuation concerning its reliability. *Journal of the New Economic Association* 2014;2(22):102-131. (in Russ.)

18. Smolyak S.A. Optimization of the Number and Frequency of Repairs. *Economics of Contemporary Russia* 2019;2:84-103. DOI: 10.33293/1609-1442-2019-2(85)-84-103. (In Russ.)

19. Lin Ye (Lam Yeh). Geometric processes and replacement problem. *Acta Mathematicae Applicatae Sinica* 1988,4:366-377. DOI: 10.1007/BF02007241.

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## The author's contribution

The author analysed the existing methods of accounting for the effects of repairs on the operational characteristics of technical systems, suggested an alternative method. A model was constructed for the purpose of optimizing the timeframe of scheduled repairs of technical systems that is based on the valuation theory and ensures maximized market value of companies that own such systems. Calculations using the model show that an optimal timeframe of scheduled repairs of a technical system significantly depends not only on the failure rate, but the system's age at the start of the inter-repair cycle and losses in production caused by its failures.

## Conflict of interests

The author declares the absence of a conflict of interests.