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Simulation of railway marshalling yards using the methods of the queueing theory

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Abstract. Aim. *The paper primarily aims to simulate the operation of railway transportation systems using the queueing theory with the case study of marshalling yards. The goals also include the development of the methods and tools of mathematical simulation and queueing theory.* **Methods.** *One of the pressing matters of modern science is the development of methods of mathematical simulation of transportation systems for the purpose of analysing the efficiency, stability and dependability of their operation while taking into account random factors. Research has shown that the use of the most mature class of such models, the single-phase Markovian queueing systems, does not enable an adequate description of transportation facilities and systems, particularly in railway transportation. For that reason, this paper suggests more complex mathematical models in the form of queueing networks, i.e., multiple interconnected queueing systems, where arrivals are serviced. The graph of a queueing network does not have to be connected and circuit-free (a tree), which allows simulating transportation systems with random structures that are specified in table form as a so-called "routing matrix". We suggest using the BMAP model for the purpose of describing incoming traffic flows. The Branch Markovian Arrival Process is a Poisson process with batch arrivals. It allows combining several different arrivals into a single structure, which, in turn, significantly increases the simulation adequacy. The complex structure of the designed model does not allow studying it analytically. Therefore, based on the mathematical description, a simulation model was developed and implemented in the form of software.* **Results.** *The developed models and algorithms were evaluated using the case study of the largest Russian marshalling yard. A computational experiment was performed and produced substantial recommendations. Another important result of the research is that significant progress was made in the development of a single method of mathematical and computer simulation of transportation hubs based on the queueing theory. That is the strategic goal of the conducted research that aims to improve the accuracy and adequacy of simulation compared to the known methods, as well as should allow extending the capabilities and applicability of the model-based approach.* **Conclusions.** *The proposed model-based approach proved to be a rather efficient tool that allows studying the operation of railway marshalling yards under various parameters of arrivals and different capacity of the yards. It is unlikely to completely replace the conventional methods of researching the operation of railway stations based on detailed descriptions. However, the study shows that it is quite usable as a primary analysis tool that does not require significant efforts and detailed statistics.*

Keywords: *transportation, marshalling yard, mathematical simulation, queueing theory, queueing network, simulation, computational experiment, dependability.*

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Introduction

In recent years, the application of the queueing theory [1] for simulating transportation systems of various levels became an important and relevant area of research, as it allows assessing the efficiency, stability and operational dependability of transportation while taking into account random factors. At the same time, the conventional toolkit of single-phase Markovian queueing systems (QSs) proved to be unsatisfactory, as: a) with a few exceptions, in transportation systems, service is carried out in several stages (phases); b) incoming vehicles cannot be considered as individual arrivals, as they may have a complex structure and differ in terms of capacity. For instance, such are the railway trains arriving to a freight or marshalling station (MS). The number of cars in a train may differ; while their types are also typically different [2]. Thus, a more complex simulation approach is required that uses non-Markovian and/or multi-phase QSs, as well as queueing networks (QNs). At the same time, since such mathematical objects are difficult to analyse, algorithmic and software tools need to be developed that would enable computer simulation.

Source overview

The applicability of two-phase QSs for railway station simulation was mentioned as early as in [2], yet at that time no systematic studies were carried out in this area. The reason probably consisted in the insufficiently mature mathematics. In the 21-st century, the situation changed. Thus, the Irkutsk School lead by Academy Member I.V. Bychkov and RAS Prof. A.L. Kazakov, to which the authors belong, for more than 10 years has been developing [3] an area of research associated with the application of multi-phase QSs as a model for processing the arrivals to transportation nodes. The nature of the latter may vary greatly from a metropolitan transport hub [4] to a railway freight station [3]. Similar research is also conducted abroad [5-7]. In [5], the queueing theory (QT) is used for identifying the capacity of railway lines, in [6], it is used for the mathematical description of the operation of stations and infrastructure facilities, in [7], it helps simulate railway node operations. However, the QT is much more often applied to the information and telecommunications technologies.

There are a large number of schools of thought and groups of researchers involved in this area of research. Let us mention three of them, who, in the authors' opinion, occupy leading positions in all of the ex-Soviet countries: the Moscow school lead by Prof. V.V. Rykov (see, for example, [8-10]), the Tomsk school lead by Prof. A.A. Nazarov (see, for example, [11-13]) and the Belarusian school lead by Prof. A.N. Dudin (see, for example, [14-16]). Naturally, the list can be continued, yet this paper does not aim and cannot make a comprehensive review of the research findings in the field of QT application for the purpose of simulating information systems and technologies.

Going back to the activities of the Irkutsk school, we should note that, as the simulation results show, the common features inherent to all transportation systems in this case prevail over the various differences and allow examining them together. Although, of course, any simulation approach requires an adaptation to the object of research, which allows taking into account the structure and directions of the internal traffic flows.

The incoming traffic flows [17] deserve a separate discussion. As a model that allows capturing their complex and heterogeneous structure, we propose using the *BMAP* (Branch Markovian Arrival Process) that enables an integration of a number of different arrivals [14]. Essentially, this is a generalized case of the Poisson stream with grouped arrivals. This model was first suggested by the Italian mathematician D. Lucantoni back in 1991 [18], yet until now it has only been used for information system simulation [16].

Based on three-phase QSs, the authors, along with the above-mentioned transport hubs (in Moscow and Ekaterinburg) and freight stations [3], constructed mathematical models of MS operations [19-21]. The results were tested with specific transportation facilities both in Russia and abroad, and attracted the interest of transportation experts. However, the proposed approach has also shown a weakness that is due to the fact that the apparatus of a multi-phase QS is only able to describe linearly structured systems. They do not support the organization of loop motion of arrivals, which, in particular, is typical for some MSs.

Methods

For the purpose of solving the above problems and extending the capabilities of the simulation approach, we propose using a new class of objects, the queueing networks [21, 22]. A QN is understood as a set of interconnected QSs within which arrivals circulate (are serviced) [22, 23]. Unlike in the multi-phase QSs, a QN graph does not have to be connected and circuit-free (a tree), which allows a much more flexible simulation of structurally complex transportation systems that are defined by a "routing matrix". Unfortunately, in this case, the functional extension of the simulation apparatus has a downside. The object of research becomes fundamentally more complex. If with the multi-phase QSs, analytical results can sometimes be obtained, for QNs, only a numerical study is available using single toss-based simulation methods [24] (the Monte Carlo methods).

This paper is a follow-up to [21]. Based on the previously proposed concept of railway transportation system simulation, the authors suggest a method of QN-based MS simulation, thus developing upon the previously obtained results in the QS-based simulation of the above transportation facilities [19]. In particular, we have developed the appropriate numerical algorithms that are implemented as software that allows researching the properties and evaluating the parameters of multi-phase systems and queueing networks by means of specially organized computer experiments based on methods of statistical simulation. For the purpose of testing the proposed approach, a model station is examined. A computational experiment was carried out, conclusions were made regarding the specificity of the station's operation.

Mathematical model

As it is known, a marshalling system is a complex structure. It is designed for mass breaking-up of freight trains into individual groups of cars, their handling and accumulation with subsequent making of new trains out of them. The MSs perform standard operations and consist of similar elements. Let us identify the most important of them that will be taken into account in the mathematical model. Standard MS perform the following actions: acceptance of trains into the receiving yard (RY) and uncoupling of the locomotive; breaking-up of train on the hump; accumulation of cars in the marshalling yard (MY) in accordance with the train make-up plan; delivery of a train into the departure yard (DY), coupling of the locomotive and departure of the made train from the system. In each yard and at the hump there are service facilities of different capacities. The incoming train flow includes transit, local and other categories of trains, whose parameters may vary greatly. The trains arrive from several directions (two or more). An incoming train should be regarded as a group of arrivals, as cars are serviced independently from each other and occupy certain positions on the tracks of the yards. Therefore, the total incoming train flow consists of at least four sub-flows, each of which is a group of arrivals. Passenger trains usually bypass MYs and are not taken into account.

A significant part of the incoming train flow is transit trains that travel across the territory of Russia. This group is significantly affected by random factors due to the very long travel distances, therefore the traffic management system cannot effectively schedule all categories of trains on an individual railway line. As a consequence, trains significantly deviate from the schedule [25]. Therefore, it can be assumed that the arrival of trains is a random value.

The mathematical model of an MS is constructed in two stages. At the first stage, the incoming arrivals are described. For that purpose, a *BMAP* model is used that allows aggregating several different arrival flows into a single structure. At the second stage, the processing of arrivals in the system is described. In order to take into account the complex hierarchical structure of the system, it is suggested to use a QN.

The ***BMAP*** **arrival** (Batch Markovian Arrival Process) differs from the simple batch arrival in that: a) the rate of batch arrivals λ_v depends on the state number of the Markov control chain v_t with continuous time and finite space-state $\{0,1,\ldots,W\}$; b) the time of Markov chain v_t being in state v has an exponential distribution with parameter λ_v; c) after the time of the chain's being in state v has elapsed, it, with a given probability $p_k(v,v')$, will change into a different state v'; a batch of size $k \geq 0$ is generated in the process; d) transition probabilities $p_k(v,v')$ comply with the normalization requirement $\sum_{k=0}^{\infty}\sum_{v=0}^{W} p_k(v,v') = 1$. The transition rates of the Markov chain are conveniently stored in matrices

$$(D_0)_{v,v} = -\lambda_v,\ v = \overline{0,W};\ (D_0)_{v,v'} = \lambda_v p_0(v,v'),\ v,v' = \overline{0,W};\ (D_k)_{v,v'} = \lambda_v p_k(v,v'),\ v,v' = \overline{0,W},\ k \geq 1. \quad (1)$$

A **QN** is the sum of the finite number S of QSs (hereinafter referred to as nodes), in which arrivals are transferred from one node to another in accordance with the routing matrix P [22, 23]. Let us accept that arrivals are received into a QN from an external source. If it is accepted as an additional node with index 0, the route of an arrival is defined by the stochastic matrix $P = \|P_{ij}\|$ of size $(S+1) \times (S+1)$. Its elements are P_{ij}, the probabilities of the arrival moving from node i to node j $(i, j = \overline{1,S})$, P_{0j} and P_{j0}, are, respectively, the probability of an arrival into the j-th node from the source and the probability of an arrival leaving the network after being serviced in j-th node $(j = \overline{1,S})$. Obviously, $\sum_{j=0}^{S} P_{ij} = 1\ (i = \overline{0,S})$, $P_{00} = 0$ [22, 23].

Marshalling yard simulation

Let us examine a model marshalling station (MMS). Its characteristics correspond to the Ekaterinburg-Sortirovochny (E-S) station of the Sverdlovsk Railway, the largest MS in Russia. E-S is a two-system MS with a serial yard arrangement that handles trains from five lines: 1) Tagil, 2) Kungur, 3) Kazan, from the stations, 4) Ekaterinburg-Tovarny and 5) Ekaterinburg-Passazhirsky. The down system serves lines 4) and 5), the up system serves lines 1), 2), 3). The two systems are almost identical in terms of the performed operations. The up system is currently undergoing an upgrade and we are unaware of its performance parameters. Therefore, the MMS uses the up system numbers (see Fig. 1): the RY includes 11 specialized tracks for receiving freight trains with a total capacity of 716 conventional cars (conv. cars) serviced by two humping engines that move trains to the hump along two tracks with a total capacity of about 100 conv. cars; the hump has a large capacity and can handle up to 5500 conv. cars per day; the MY has 35 tracks with the total capacity of 2535 conv. cars, train formation is ensured by three engines that move trains to the DY; it has 15 tracks with the total capacity of 980 conv. cars, trains depart from it along three lines. The tracks in the yards differ in length, the longest being able to accommodate over 80 conv. cars. E-S accepts a car flow that includes three categories of trains: a) transit with rehandling; b) transit without rehandling; c) local. The a) and c) trains arrive to the receiving yard and undergo the

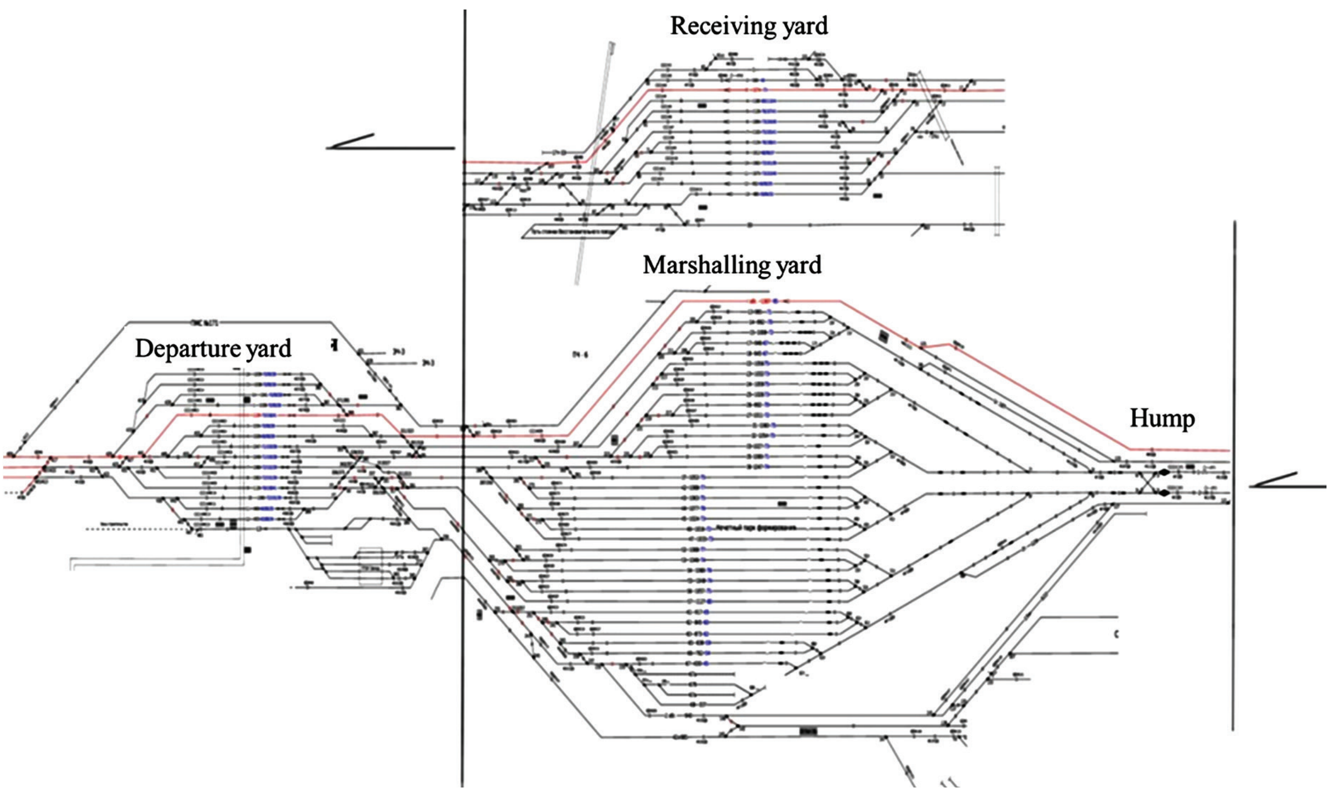

Fig. 1. Down system of the E-S marshalling station

rest of the service. The b) trains are serviced in the RY or DY, then leave the system.

While preparing paper [25], we collected statistical data on trains operating on the Sverdlovsk Railway, in particular, freight trains: actual and scheduled times of arrival at station, number of cars in trains and their numbers. They might be partially obsolete, but we did not manage to obtain more recent data, therefore further station simulation will be based on the available information. They show that in a half of the cases, schedule violations by freight trains exceed 30 minutes and more than two hours in 28% of cases. Consequently, the time between the arrival of trains to the station can be taken as a random value. The number of cars in the transit and most local freight trains follows a binomial distribution $B(80, 0.9)$. Their average number is 70, the maximum number is 80 for all categories.

In the MY, the hump is considered to be the primary facility that defines the system's performance. Its capacity defines the size of the car stream the station can handle. We were unable to obtain the statistics of the incoming train traffic, so we shall calculate the number of the trains arriving for breaking-up in such a way as to make the hump loading 70% of the maximum possible. This value corresponds to the average planned loading for such facilities. Then, the system is to accept 3850 cars or 55 trains per day. We shall take the number of trains without breaking-up to be one quarter of the number of those broken-up, i.e., 14 per day. We deduce that the MMS accepts 69 freight trains from two lines per day, i.e., 35 trains from line 4) and 34 from line 5). In the model, we divide the trains depending on the handling procedure at the station, i.e., with breaking-up (trains A) and with no breaking-up (trains B), and the line. Thus, the incoming car flow will consist of four sub-flow groups. We will use a *BMAP* flow model for the purpose of mathematical description of such car flow. It will include 81 4 × 4 D_k matrices, $k=\overline{0,80}$. Their elements are calculated using formulas (1), where $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda$ 69 / 24 = 2.875, p_0 = 28 / 69 = 0.41, p_2 = 27 / 69 = 0.39, $p_1 = p_3$ = 7 / 69 = 0.1, $p_k(v,v') = p_{v'} f(k)$, $v,v' = \overline{0,3}$, $k=\overline{0,80}$, $f(k)$ is the probability of the arrival of a group of cars of size k that follows the law $B(80, 0.9)$.

The MMS model in the form of a QN is as follows. The system has four service nodes: 1) in the RY, the two shunting engines are the channels, the yard tracks are the queue, then Node 1 is a QS with two channels and a queue with 716 positions; 2) at the hump, one hump track and the shunting device are one channel, the second track is the queue, then Node 2 is a QS with one channel and a

Table 1. Parameters of the channel operation in the MMY subsystems

	Node 1	Node 2	Node 3	Node 4
F	$N(20, 2)$	$N(20, 3)$	$N(40, 5)$	$N(40, 3)$
X	$B(80, 0.9)$	$B(80, 0.9)$	$B(80, 0.9)$	$B(80, 0.9)$

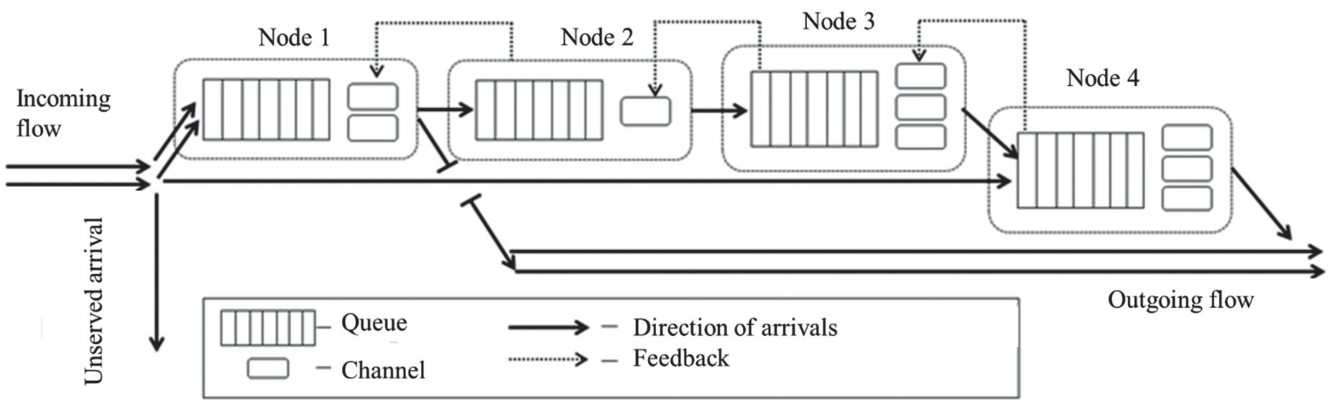

Fig. 2. QN diagram

queue with 100 positions; 3) in the MY, the three engines are the channels, the yard tracks are the queue, then Node 3 is a three-channel QS with a queue with 2535 positions; 4) in the DY, the three departure tracks are the channels (three lines), the yard tracks are the queue, then Node 4 is a three-channel QS with 980 positions. The service time in the node channels is taken as a random normally distributed value. The choice is due to the fact that the station staff strive to bring the duration of technical operations closer to the standard values. Deviations are due to the effect of random factors. An example would be a train that has cars with only two or three lines (destination stations). Then, it can be broken-up on the hump in 12 minutes instead of the standard 20. The MMS operation parameters are presented in Table 1.

The channel performance parameters (see Table 1) are defined based on the standard MY operation process taking into account the specificity of E-S. The service time in Node 1 is taken as the time of humping from the departure park and return of the engine. The service time interval is [12.0; 28.0] minutes, the average time is 20 minutes. The hump capacity (Node 2) is 5500 conv. cars per day or 4 conv. cars per minute. Let the service time interval be [11.0; 29.0] minutes. In Node 3 (MY), the service time is taken as the time of engine coupling to the train, completion of train formation and its relocation to RY, as well as the return of the engine to the MY. The service time interval is [25.0; 55.0] minutes, the average time is 40 minutes. In the DY (Node 4) we take into account the time of engine coupling to the train, short brake testing and the train's departure from the system. Let the average service time be 40 minutes and the interval be [31.0; 49.0] minutes.

Trains A and B arrive into the system. The former enter Node 1 and proceed along the entire system. The latter are

Table 2. Results of experiment 1

	Received	Lost	T_{syst} (m)	P_G	P_R
Groups of arrivals	2431.80	0	161.76	0	0
Arrivals	175199.20	0			
	K	l	t_{ph} (m)	t_{lock} (m)	P_{lock}
Node 1	1.00	41.46	35.68	2897.00	0.05
Node 2	0.74	24.77	32.61	0.00	0.00
Node 3	1.43	0.00	42.43	0.00	0.00
Node 4	1.59	26.55	51.05	-	-

Table 3. Results of experiment 2

	Received	Lost	T_{syst} (m)	P_G	P_R
Groups of arrivals	2903.50	0	166.13	0	0
Arrivals	209049.50	0			
	K	l	t_{ph} (m)	t_{lock} (m)	P_{lock}
Node 1	1.23	61.10	39.20	4681.25	0.08
Node 2	0.83	32.65	34.48	0.00	0.00
Node 3	1.60	0.00	42.44	0.00	0.00
Node 4	1.79	26.64	50.02	-	-

Table 4. Results of experiment 3

	Received	Lost	T_{syst} (m)	P_G	P_R
Groups of arrivals	3453.50	1.00	178.23	0.0003	0.0003
Arrivals	248605.50	72.82			
	K	l	t_{ph} (m)	t_{lock} (m)	P_{lock}
Node 1	1.50	111.15	48.60	7216.50	0.12
Node 2	0.90	42.58	36.96	0.00	0.00
Node 3	1.74	0.00	42.45	0.00	0.00
Node 4	1.97	29.85	50.23	-	-

received in Node 1 or Node 4, are serviced and leave the system. We use a routing matrix to account for different train service procedures. Let us assume that trains B may be admitted to Node 1 or Node 4 with an equal probability of $p_b = 0.5$. Then, trains from the total incoming flow arrive at Node 4 with the probability of $p_{0;4} = (2 \cdot 7 / 69)p_b = 0.1$, and at Node 1 with the probability of $p_{0;1} = 1 - p_{0;4} = 0.9$. After servicing in Node 1, trains B leave the system with the probability of $p_{1;0} = 7 / 62 = 0.11$. Trains A go to Node 2 with the probability of $p_{1;2} = 1 - p_{1;0} = 0.89$, then proceed across Nodes 3 and 4. In QT terms, the MMY model will be of the form of QN with routing matrix P formed by the above probabilities and the following nodes: 0 is the source of the incoming flow; 1 is $BMAP / G^B/2/716$; 2 is $* / G^B / 1 / 100$; 3 is $* / G^B / 3 / 2535$; 4 is $* / G^B / 3 / 980$, where B is the binomial distribution, G is the random law of service time distribution. The diagram of the above QN is shown in Fig. 2.

Let us study the obtained QN numerically using the simulation model [24]. The sought characteristics, the performance indicators, are [1, 22, 23]: P_R and P_G are the probabilities of handling an application and a group of applications, T_{syst} is the application's average system time, l is the average queue length, K is the average number of busy channels, t_{ph} is an application's average time in the node, t_{lock} is the total blocking time of all of a node's channels, P_{lock} is the probability of blocking of a single channel of a node.

Computational experiment. Tables 2-4 below show the results of scenario-based simulation of the operation of the above QN (see Fig. 2) under various parameters of the incoming car flow. Each table shows the average results of 10 program starts. The simulation time is five weeks for all experiments, the minimal time required for the simulation model to calculate the stationary QN characteristics. The primary indicator of the fact the QN is successfully handling the load is $P_R = 0$, which, for large industrial and transportation systems means the absence of the risk of accidents. Thus, in particular, this indicates that the station handles all trains and the operation is fault-free.

Experiment 1. Table 2 shows simulation results for the case of car flow λ = 4 per hour.

*E*xperiment *2*. Table 3 shows the simulation results for the case of total flow of 5880 cars or almost 84 trains per day, of which 17 do not require handling (λ = 3.5).

*E*xperiment *3*. Table 4 shows simulation results for the case of arrival rate λ = 4 per hour.

Discussion of the simulation results

Out of the results of Experiment 1, it can be seen that the average system dwell time of an arrival (car) is 2.7 hours. The probability of failure is zero, i.e., the car flow is accepted to the station continuously, which is an important indicator of the dependability of railway transportation. Thus, the MMY successfully handles the car flow; a dependable and reliable station operation is ensured.

Experiment 2. Node 1 is the busiest as the average time of channel blocking is 2.2 hours a day, which is not critical, yet it affects the station's operation. In such situation, the traffic controllers arrange freight trains at adjacent railway stations, where they await the release of tracks in the receiving yard. The marshalling station operates normally and handles the load, however, due to longer blocking of Node 1 channels, the goods delivery time increases, while the stability of the transportation system is not fully ensured.

Experiment 3. The results of the simulation show that the channels of Node 1 are on average blocked for 3.4 hours a day, which is critical for the system, since the queue of Node 1 overflows and the probability of failure becomes non-zero, dependable and reliable operation of MMY is not ensured.

Based on the results of all experiments, the following general conclusion can be made. The system operates normally at maximum rate of incoming train flow of λ = 3.5 trains per hour. The limit value of a station's operation is the rate of incoming train flow of λ = 4 trains per hour. The system's bottleneck is the insufficient capacity of the receiving yard (Node 1). In order to relieve the load, all trains should be redirected to Node 4 (DY) without handling. Further improvement of the system's performance requires increased hump capacity. However, that will require its reconstruction involving substantial material costs.

Conclusion

Summing up the results of the research, we should note that, as the authors hope, it is only the first major step on a long way that is to culminate in the creation of a single methodology for mathematical and computer simulation of transportation hubs using QN. That will enable improved accuracy and adequacy of the simulation aimed at assessing the efficiency, stability and operational dependability of transportation systems, as well as enhancing the model-based approach. Another important problem to be solved as part of the transportation systems simulation is that the software products are alienable from the developers. The most natural solution would be to use intellectualization tools in the creation of an intelligent system for managing the development of the region's transportation and logistics infrastructure [26] led by academy member I.V. Bychkov.

In conclusion, let us note that it is unlikely that the proposed model-based approach will completely replace the conventional methods of studying the operation of railway stations based on their detailed description. However, as we have identified, it can quite be used as a primary analysis tool that does not require significant efforts and detailed statistics.

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References

1. Gnedenko B.V., Kovalenko B.V. [Introduction into the queueing theory]. Moscow: LKI; 2007. (in Russ.)

2. Akulinichev V.M., Kudriavtsev V.A., Koreshkov A.N. [Mathematical methods in the operation of railways]. Moscow: Transport; 1981.

3. Kazakov A.L., Maslov A.M. [Construction of a model of an uneven traffic flow. Case study of a railway freight station]. *Modern Technologies. Systems Analysis. Modeling* 2009;3:27-32. (in Russ.)

4. Zhuravskaya M.A., Kazakov A.L., Zharkov M.L. et al. Simulating the operation of transport hub of a metropolis as a three-phase queueing system. *Transport of the Urals* 2015;3:17-22. (in Russ.)

5. Weik N., Niebel N. Capacity analysis of railway lines in Germany – a rigorous discussion of the queueing based approach. *Journal of Rail Transport Planning & Management* 2016;6(2):99-115.

6. Dorda M., Teichmann D. Modelling of freight trains classification using queueing system subject to breakdowns. *Mathematical Problems in Engineering* 2013;2013:11.

7. Nießen N. Waiting and loss probabilities for route nodes. In: Proceedings of the 5th International Seminar on Railway Operations Modelling and Analysis. Copenhagen (Denmark); 2013.

8. Bronshtein O.I., Raykin A.A., Rykov V.V. [On a queueing system with losses]. *Izvestiia Akademii nauk SSSR: Tekhnicheskaia kibernetika* 1964;4:39-47. (in Russ.)

9. Kitaev M.Yu., Rykov V.V. Controlled queueing system. N.Y.: CC Press; 1995.

10. Rykov V.V. Controllable queueing systems from the very beginning up to nowadays. *Materials of Information Technologies and Mathematical Modelling named after A.F. Terpugov* 2017;1:25-26.

11. Grachev V.V., Moiseev A.N., Nazarov A.A. et al. Multistage queueing model of the distributed data processing system. *Proceedings of TUSUR University* 2012;2-2(26):248-251. (in Russ.)

12. Shklennik M., Moiseeva S., Moiseev A. Optimization of two-level discount values using Queueing tandem model with feedback. *Communications in Computer and Information Science* 2018;912:321-332.

13. Galileyskaya A.A., Lisovskaya E.Yu., Moiseeva S.P. et al. On sequential data processing model that implements the backup storage. *Modern Information Technologies and IT-Education* 2019;3:579-587. (in Russ.)

14. Dudin A.N., Klimenok V.I. [Queueing systems with correlated flows]. Minsk: BSU; 2000. (in Russ.)

15. Kim C., Dudin A., Dudina O. et al. Tandem queueing system with infinite and finite intermediate buffers and generalized phase-type service time distribution. *European Journal of Operational Research* 2014;23:170-179.

16. Vishnevskii V.M., Dudin A.N. Queueing systems with correlated arrival flows and their applications to modeling telecommunication networks. *Automation and remote control* 2017;8:3-59. (in Russ.)

17. Gasnikov A.V., editor. [Introduction into the mathematical modelling of traffic flows]. Moscow: MIPT; 2010. (in Russ.)

18. Lucantoni D.M. New results on single server queue with a batch Markovian arrival process. *Commun. Statist. Stochastic Models* 1991;7:1-46.

19. Kazakov A.L., Pavidis M., Zharkov M.L. Multiphase systems of mass service in switchyard modeling. *Herald of the Ural State University of Railway Transport* 2018;2:4-14.

20. Kazakov A.L., Pavidis M. On one approach to model operation of marshalling stations. *Transport of the Urals* 2019;1(60):29-35. (in Russ.)

21. Zharkov M.L., Pavidis M.M. Modelling of railway stations based on queueing networks. *Aktualnye Problemy Nauki Pribaikalia* 2020;3:79-84. (in Russ.)

22. Walrand J. An introduction to queueing networks. Mir; 1993.

23. Ivnitsky V.A. Queueing network theory. Moscow: FIZMATLIT; 2004. (in Russ.)

24. Kelton D.W, Law A.M. Simulation modelling and analysis. Saint Petersburg: Piter; 2004.

25. Zharkov M.L., Parsyurova P.A., Kazakov A.L. Modeling operation of railway stations and rail network sections based on studying train schedule deviations.

Proceedings of Irkutsk State Technical University 2014;6(89):23-31. (in Russ.)

26. Bychkov I.V., Kazakov A.L., Lempert A.A. et al. [An intelligent system for managing the transportation and logistics infrastructure of a region]. *Control Sciences* 2014;1:27-35. (in Russ.)

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The authors' contribution

Zharkov M.L. stated the problem and developed the model-based approach. **Pavidis M.M.** made a source review, performed model identification and computational experiment, drew conclusions based on the simulation results.

Conflict of interests

The authors declare the absence of a conflict of interests.