# Estimation of the failure flow of a set of passenger car doors

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**Abstract.** *An estimation of the failure flows is a prerequisite for the operation of industrial products. It is based on statistical data about failures that occur within technical items in the process of their operation. In the technical product documentation, this indicator shall be featured in the "Dependability parameter estimation" section. The dependability analysis of rolling stock is still affected by the difficulty of defining the methodology for evaluating this parameter at various system levels. For the purpose of analysing a multicomponent system, a reliability block diagram should be developed, and the possible replacement (redundant) elements should be taken into consideration. Multicomponent systems are often represented through various block diagrams, where, among others, the "m-out-of-n" structure may be used referring to a system with a parallel arrangement of elements that is operable when at least m elements operate. An example of such system is a set of passenger car doors. The manufacturers and customers may have different approaches to calculating technical system dependability. First, the required dependability indicator for the entire train is defined that, in turn, defines the dependability requirements for a car. At the same time, the dependability indicator for a car is determined by the respective values of its components (subsystems, units and parts). However, the nature of the relationship between a car and its components is not always taken into account. At the same time, car manufacturers can and should define in the regulatory documentation (and later supervise in operation) the dependability indicators for a set of doors (components of a car in our case) as a single system. However, the failure criteria of a set of doors are not always defined. This paper examines the method of calculating the failure flow for a set of passenger car doors based on operational data and the failure flow of a single door.* **Aim.** *To propose a method for calculating the failure flow of a set of 6 car doors by analysing the possible reliability block diagrams with subsequent transition to transition and state graphs.* **Conclusions.** *A number of block diagrams were developed for the purpose of dependability calculation of sets of passenger car doors based on the system failure criterion. The failure flow of a set of car doors was calculated according to the developed block diagrams. It is concluded that the Markovian method of calculating the failure flow is of higher priority than the logic-and-probability approach, since it takes into account the recovery factor. A Markovian method was proposed for calculating the failure flow and recovery time of a set of car doors for the "3-out-of-4" reliability block diagram.*

**Keywords:** *dependability, failure flow, reliability block diagram, set of doors, Markovian processes.*

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## Introduction

One of today's pressing problems consists in ensuring high dependability of railway transportation. The safety of passenger transportation is directly associated with the estimation of the dependability indicators of rolling stock components. The suppliers of passenger car components are to estimate the dependability indicators over the period of time agreed with the customer (car-building plant). The manufacturers, in turn, report to transportation companies with estimated dependability indicators of the products, i.e., the cars. Hence the difference in the approaches to the quantitative estimation:

1) the car manufacturers calculate the dependability indicators of a car, while the suppliers calculate those of the components and elements;

2) the method of estimating the dependability indicators for cars differs from that for rolling stock components due to the different levels of operation and design. That is expressed in the fact that the estimation of the mean operation time of a car could be done using the exponential distribution law, while that of steps, for instance, depending on the version, using both exponential and normal, as well as the Weibull distribution [1]. The exponential distribution is typical for complex items consisting of many elements with different operation time distributions [2]. Since a set of car doors consists of mechanical (manual) and automatic (electromechanical) side and gangway doors, the exponential distribution law should be used when estimating the dependability indicators. The assumption of the Markovian nature of transitions within a complex system is due to the fact that if each of a systems' elements has an approximately exponential law of fault-free operation distribution, then the behaviour of the entire system can be described with a Markovian process [3]. An automatic door kit includes a programmable control unit designed for opening/closing doors, as well for processing information generated by inductive displacement sensors and obstruction-in-the-opening sensors. Additionally, the failure flow of doors is assumed to be constant in the course of normal operation in accordance with the results of the criterion's application according to GOST R IEC 60605-6-2007. Consequently, it should be assumed that a door's time to failure is exponentially distributed, and it was decided to use Markovian analysis for calculating the failure flow of a set of car doors.

The passenger car manufacturers take into consideration the dependability indicators for entire sets of car doors (4 to 6, depending on the model) according to the following rule: the door's failure flow parameter is multiplied by the number of such doors in the car, which, in terms of structural dependability, indicates an elementary serial structure, where the failure of one door in most cases equals the failure of the car, which is a questionable assumption.

For the purpose of correctly defining the operating procedure of doors as part of a car subject to the hierarchy of connections, the structural approach was considered. A dependability block diagram is a graphical representation of the operational state of a system. It shows the logical connections between the operating components required


Fig. 1. The models of passenger cars examined in the context of reliability block diagrams are as follows: a) model 61-4447 (couchette car); b) model 61-4462 (compartment car); c) model 61-4460 (dining car); d) model 61-4445 (staff car)

for the system to perform as expected [4]. However, the reliability block diagram-based simulation methods are designed for non-recoverable systems, where the failure order is irrelevant. In the case of recoverable facilities and systems, where the failure order is to be taken into consideration, such methods as the Markovian analysis [5-8] are more appropriate. In the course of the study, the operation of an entire system was represented using the structural approach that enabled – taking into account the restoration of the system – the transition to the state and transition graph [4].

## Object of research

Exterior doors are designed for all types of passenger cars with the design speed of up to 200 km/h (Fig. 1). The doors ensure comfort and safety: gangway doors allow moving from car to car; side doors allow entering and exiting a car, protect against sudden changes of pressure and temperature, prevent dust and moisture from entering a car's vestibule, provide noise and heat insulation of a car vestibule under all modes of train operation.

Fig. 2 shows the location of different types of doors of a complete set.

## Reliability block diagram of a set of car doors

A reliability block diagram of a set of car doors can be built in a number of ways. In terms of model analysis, it is assumed that in the examined cases the system is under complete control, while the switches are perfectly dependable. The simplest representation of a set of car doors is a non-redundant system (Fig. 3), where the failure of any element causes the failure of the entire system. Then, the system's probability of no-failure is calculated using a well-known formula:

$$P_{c1} = \prod_{i=1}^{N} P_i, \tag{1}$$

where $P_i$ is the probability of no-failure of the element; $N$ is the number of elements in the system.


Fig. 3. Linear reliability block diagram of a set of passenger car doors, where Ei is a door with electromechanical operation; Bi is a gangway door

The second possible representation of the reliability block diagram of a set of car doors is segregated constant redundancy with integer multiplicity. The various methods of redundancy and their respective dependability benefits are discussed in detail in [9]. The methods of assessing the impact of maintenance on the efficiency of redundancy are described in [10]. In general, the probability of no-failure of this diagram is calculated using the formula:

$$P_p = 1 - \prod_{i=1}^{N} (1 - P_i). \tag{2}$$

For a set of car doors, let us consider composite diagrams:

• manual door backed-up by another manual door (Fig. 4a);

• automatic side doors backed-up by manual doors (Fig. 4b).

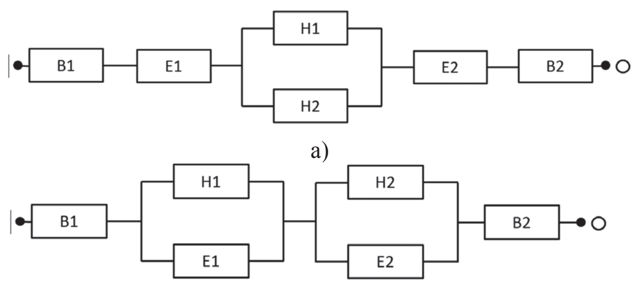



Fig. 4. Redundant reliability block diagrams of a set of passenger car doors, where Ei is a side door with electromechanical operation, Bi is a gangway door, Hi is a door with mechanical operation

Thus, we obtain two variants of a mixed system with element redundancy of individual units.

The probability of no-failure with a back-up manual door (Fig. 4a) is calculated using formulas (1) and (2):

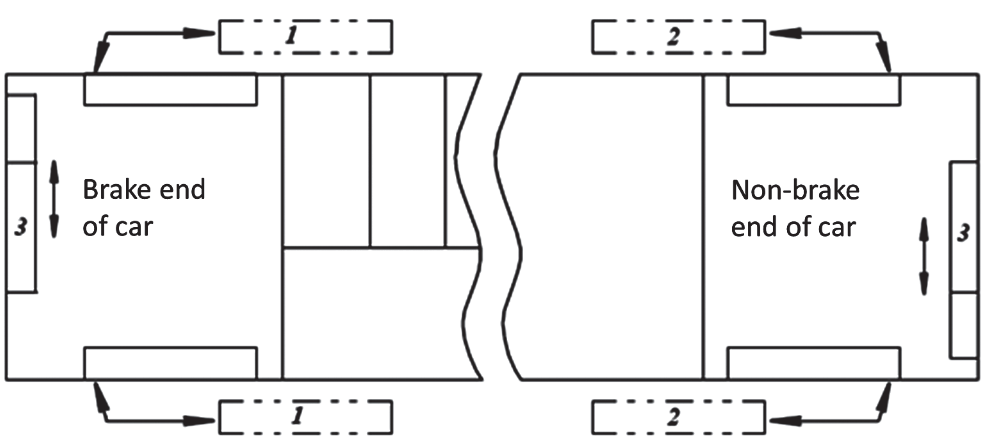



Fig. 2. Passenger car doors: 1 – single side door with electromechanical operation; 2 – single side door with mechanical operation; 3 – single gangway door with electromechanical operation.

$$P_{c2} = P_{B1} \cdot P_{E1} \cdot P_{B2} \cdot P_{E2} \cdot \left[1-\left(1-P_{H1}\right)\cdot\left(1-P_{H2}\right)\right].$$

The probability of no-failure with manual doors backing-up electromechanical side doors (Fig. 4b) is calculated as follows:

$$P_{c3} = P_{B1} \cdot P_{B2} \cdot \left[1-\left(1-P_{E1}\right)\cdot\left(1-P_{H1}\right)\right]\cdot\left[1-\left(1-P_{E2}\right)\cdot\left(1-P_{H2}\right)\right].$$

Another possible approach to structural evaluation involves representing a set of car doors as a "*m*-out-of-*n*" structure [4]. A system of this type can be considered a variant of a system with a parallel arrangement of elements that is operable when at least $m$ elements out of $n$ ($m < n$) are operable.

Let us describe three variants:

- a "3-out-of-4" system that is considered operable when doors H1, E1 and E2 are operable (Fig 5a);

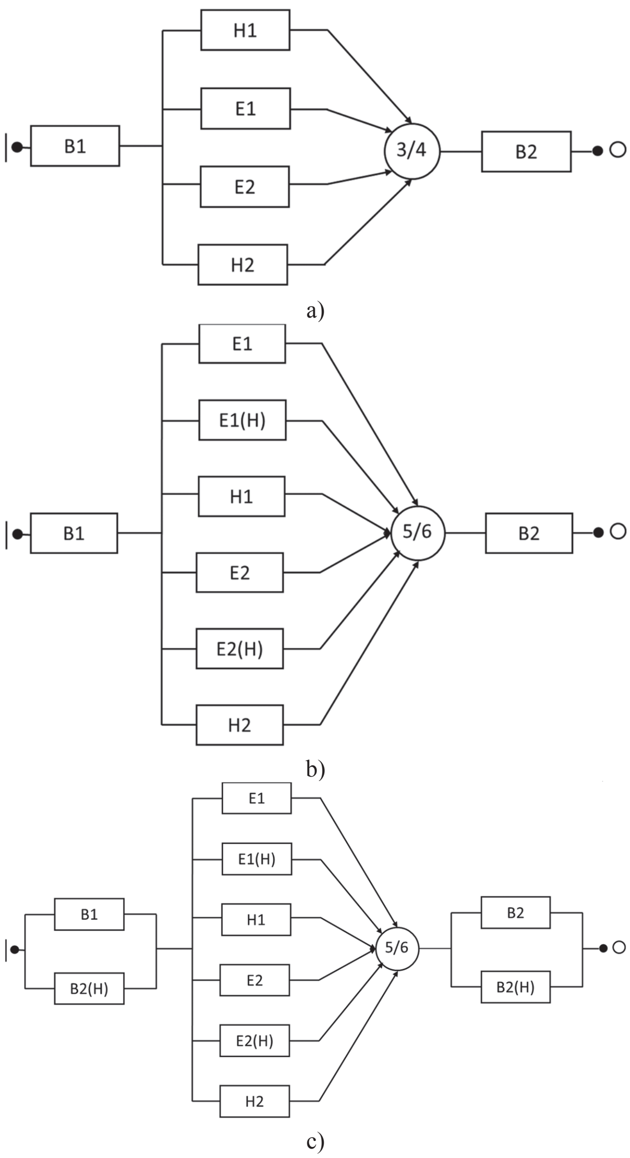



Fig. 5. "*M*-out-of-*n*" block diagrams for evaluating the dependability of a set of passenger car doors, where Ei is a door with electromechanical operation, Ei(H) is a door with electromechanical operation in manual mode, Bi is a gangway door, Bi(H) is a gangway door with electromechanical operation in manual mode, Hi is a door with mechanical operation

- a "5-out-of-6" system that implies the option of manual operation for electromechanical doors Ei(H) (Fig. 5b);
- a "5-out-of-6" system that takes into account the possible manual operation of electromechanical side and gangway doors, EI(H) and Bi(H) (Fig. 5b).

Both of the above structures are mixed. Sequentially connected or element-redundant gangway doors are added to the "*m*-out-of-*n*" structure.

Then, provided that all elements of the "*m*-out-of-*n*" are equally dependable, the probability of no-failure of the structure shown in Fig. 5a will be as follows:

$$P_{c4} = P_{B1} \cdot P_{B2} \cdot \left(4 \cdot P_m^3 - 3 \cdot P_m^4\right),$$

where $P_m$ is an element of the "*m*-out-of-*n*" structure.

The probability of no-failure for the structure shown in Fig. 5b under the same conditions is as follows:

$$P_{c5} = P_{B1} \cdot P_{B2} \cdot \left(6 \cdot P_m^5 - 5 \cdot P_m^6\right).$$

Finally, the probability of no-failure of the structure shown in Fig. 5c that implies both automatic and manual operation of a gangway door will be as follows:

$$P_{c6} = \left[1-\left(1-P_{B1}\right)\cdot\left(1-P_{B1(H)}\right)\right]\cdot$$
$$\cdot\left[1-\left(1-P_{B2}\right)\cdot\left(1-P_{B2(H)}\right)\right]\cdot\left(6 \cdot P_m^5 - 5 \cdot P_m^6\right).$$

Failure flow calculation for the block diagrams of car door sets

Let us estimate the failure flow for the structures shown in Fig. 4b, 5a-c. The mean time to system failure can be represented as follows:

$$T = \int_0^\infty P(t)dt = \int_0^\infty \left[1-\left(1-e^{-\lambda t}\right)^n\right]dt,$$

where $P(t)$ is the probability of no-failure.

Let us substitute variables

$$1-e^{-\lambda t} = x \Rightarrow t = \frac{1}{\lambda}\ln\frac{1}{1-x} \Rightarrow dt = \frac{1}{\lambda\left(1-x\right)}dx.$$

Then,

$$T = \frac{1}{\lambda}\int_0^\infty \frac{1-x^n}{1-x}dx = \frac{1}{\lambda}\int_0^\infty \frac{-\left(x-1\right)\left(x^{n-1}+x^{n-2}+\ldots+x+1\right)}{1-x}dx =$$
$$= \frac{1}{\lambda}\left(\left.\frac{x^n}{n}\right|_0^1 + \left.\frac{x^{n-1}}{n-1}\right|_0^1 + \ldots + \left.\frac{x^2}{2}\right|_0^1 + \left.x\right|_0^1\right) = \frac{1}{\lambda}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right).$$

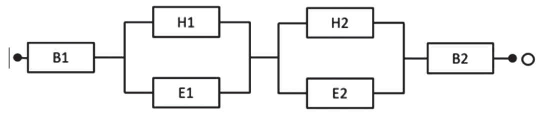
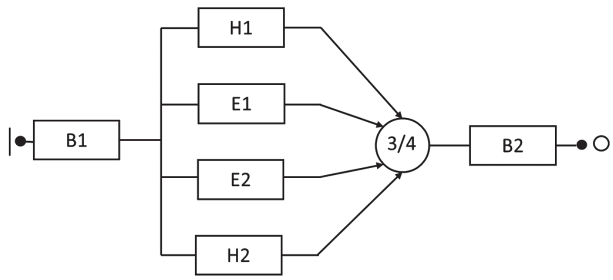
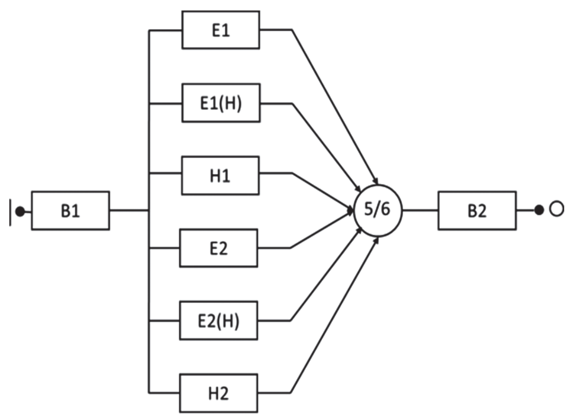
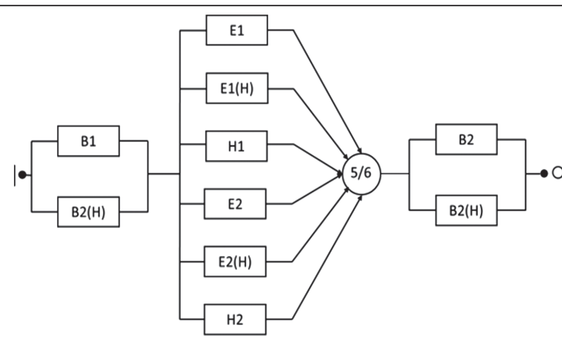
Thus, for a "*m*-out-of-*n*" system we obtain:

$$T = \frac{1}{\lambda}\left(\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}+\ldots+\frac{1}{m}\right). \quad (3)$$

The initial data for calculation are the values of failure flow $\lambda = 1.667\times10^{-6}$ 1/km and recovery flow $\mu = 0.041$ 1/km of one door.

By substituting the numerical values into (3), we obtain the following as summarized in Table 1.

**Table 1. Results of failure flow calculation for the various block diagrams of a set of car doors**

| Type of diagram | Graphical representation | Failure flow parameter, $\lambda$, km$^{-1}$ |
|---|---|---|
| Automatic side doors backed-up by manual doors |   | $\lambda_c = 5.5566\times10^{-6}$ |
| An “*m*-out-of-*n*” system with sequentially connected gangway doors |   | $\lambda_c = 6.1917\times10^{-6}$ |
| A “5-out-of-6” system that implies the option of manual operation for electromechanical doors |   | $\lambda_c = 7.88\times10^{-6}$ |
| A “5-out-of-6” system that takes into account the possible manual operation of electromechanical side and gangway doors |   | $\lambda_c = 6.7694\times10^{-6}$ |

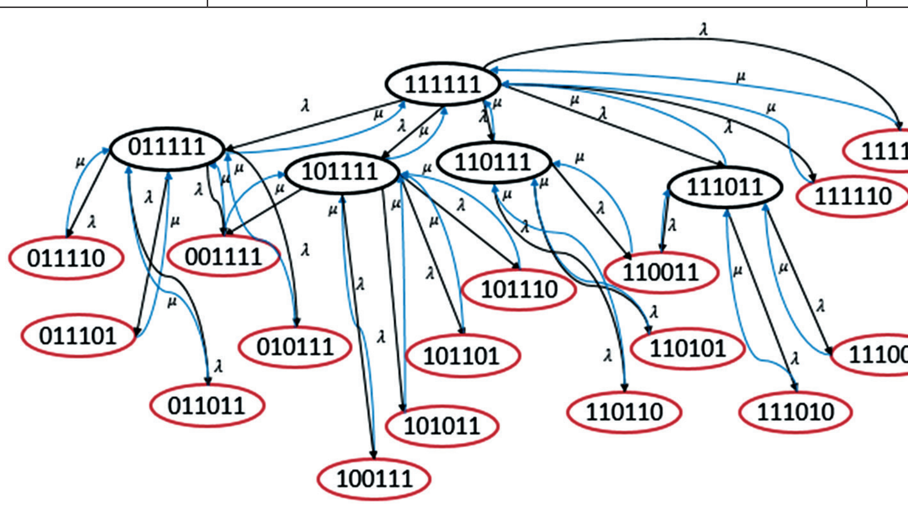



Fig. 6. State graph for the “3-out-of-4” structure with sequentially connected gangway doors

We obtained a series of failure flow values that depend on which of the reliability block diagrams can be considered. A team of engineers that was involved with this type of doors at various life cycle stages has adopted the “3-out-of-4” diagram (Fig. 5a).

Due to the fact that the logic-and-probability approach, although convenient for calculation, does not take into account the restoration of doors, the Markovian calculation of the failure flow was considered.

## Calculation of the failure flow parameters of a set of car doors based on Markovian random processes

Let us estimate the dependability indicators of a set of car doors for the “3-out-of-4” structure with sequentially connected gangway doors using Markovian analysis. After determining the system state using the known *transition probability densities* $\lambda$ *and* $\mu$ of the doors, let us construct a state graph for this structure (Fig. 6).

Next, let us construct a system of Kolmogorov differential equations, where the number of equations in the system will be equal to the number of states:

$$\begin{cases}
\frac{dP_{111111}}{dt} = -6\lambda P_{111111}(t) + \mu\left[P_{011111}(t) + P_{101111}(t) + P_{110111}(t) + P_{111011}(t) + P_{111101}(t) + P_{111110}(t)\right]; \\
\frac{dP_{011111}}{dt} = -(5\lambda+\mu)P_{011111} + \mu\left[P_{011110}(t) + P_{011101}(t) + P_{001111}(t) + P_{010111}(t) + P_{011011}(t)\right] + \lambda P_{111111}(t); \\
\frac{dP_{101111}}{dt} = -(5\lambda+\mu)P_{101111} + \mu\left[P_{001111}(t) + P_{100111}(t) + P_{101011}(t) + P_{101101}(t) + P_{101110}(t)\right] + \lambda P_{111111}(t); \\
\frac{dP_{110111}}{dt} = -(5\lambda+\mu)P_{110111} + \mu\left[P_{010111}(t) + P_{100111}(t) + P_{110110}(t) + P_{110101}(t) + P_{110011}(t)\right] + \lambda P_{111111}(t); \\
\frac{dP_{111011}}{dt} = -(5\lambda+\mu)P_{111011} + \mu\left[P_{011011}(t) + P_{101011}(t) + P_{110011}(t) + P_{111010}(t) + P_{111001}(t)\right] + \lambda P_{111111}(t); \\
\frac{dP_{111101}}{dt} = -\mu P_{111101}(t) + \lambda P_{111111}(t); \\
\frac{dP_{111110}}{dt} = -\mu P_{111110}(t) + \lambda P_{111111}(t); \\
\frac{dP_{011110}}{dt} = -\mu P_{011110}(t) + \lambda P_{011111}(t); \\
\frac{dP_{011101}}{dt} = -\mu P_{011101}(t) + \lambda P_{011111}(t); \\
\frac{dP_{001111}}{dt} = -2\mu P_{001111}(t) + \lambda\left[P_{011111}(t) + P_{101111}(t)\right]; \\
\frac{dP_{010111}}{dt} = -2\mu P_{010111}(t) + \lambda\left[P_{011111}(t) + P_{110111}(t)\right]; \\
\frac{dP_{011011}}{dt} = -2\mu P_{011011}(t) + \lambda\left[P_{011111}(t) + P_{111011}(t)\right]; \\
\frac{dP_{100111}}{dt} = -2\mu P_{100111}(t) + \lambda\left[P_{101111}(t) + P_{110111}(t)\right]; \\
\frac{dP_{101011}}{dt} = -2\mu P_{101011}(t) + \lambda\left[P_{101111}(t) + P_{111011}(t)\right]; \\
\frac{dP_{101101}}{dt} = -\mu P_{101101}(t) + \lambda P_{101111}(t); \\
\frac{dP_{101110}}{dt} = -\mu P_{101110}(t) + \lambda P_{101111}(t); \\
\frac{dP_{110011}}{dt} = -2\mu P_{110011}(t) + \lambda\left[P_{110111}(t) + P_{111011}(t)\right]; \\
\frac{dP_{110101}}{dt} = -\mu P_{110101}(t) + \lambda P_{110111}(t); \\
\frac{dP_{110110}}{dt} = -\mu P_{110110}(t) + \lambda P_{110111}(t); \\
\frac{dP_{111001}}{dt} = -\mu P_{111001}(t) + \lambda P_{111011}(t); \\
\frac{dP_{111010}}{dt} = -\mu P_{111010}(t) + \lambda P_{111011}(t).
\end{cases}$$

Using MATLAB, let us evaluate the system’s probability of no-failure. We will obtain $P$ = 0.9917. This probability value is calculated for a period equal to the service life of the doors.

Let us use a simpler method of calculating the reliability indicators. For this purpose, let us use a Laplace transformation for a system of differential equations [8]:

$$\begin{cases}
\frac{dP_{111111}}{dt} = -6\lambda P_{111111}(t) + \mu\begin{bmatrix} P_{011111}(t) + P_{101111}(t) + \\ + P_{110111}(t) + P_{111011}(t) \end{bmatrix}; \\
\frac{dP_{011111}}{dt} = -(5\lambda+\mu)P_{011111}(t) + \lambda P_{111111}(t); \\
\frac{dP_{101111}}{dt} = -(5\lambda+\mu)P_{101111}(t) + \lambda P_{111111}(t); \\
\frac{dP_{110111}}{dt} = -(5\lambda+\mu)P_{110111}(t) + \lambda P_{111111}(t); \\
\frac{dP_{111011}}{dt} = -(5\lambda+\mu)P_{111011}(t) + \lambda P_{111111}(t).
\end{cases}$$

We use Laplace transformations [8, 11] in order to obtain an expression for the mean time to failure, out of which we deduce the numerical value of the failure flow. For $P(t)$, we will obtain the following representation:

$$P(z) = \int_0^\infty P(t)e^{-zt}dt.$$

Then, we obtain the following system of equations:

$$\begin{cases}
-1 = -6\lambda T_{111111} + \mu\left[T_{011111} + T_{101111} + T_{110111} + T_{111011}\right]; \\
0 = -(5\lambda+\mu)T_{011111} + \lambda T_{111111}; \\
0 = -(5\lambda+\mu)T_{101111} + \lambda T_{111111}; \\
0 = -(5\lambda+\mu)T_{110111} + \lambda T_{111111}; \\
0 = -(5\lambda+\mu)T_{111011} + \lambda T_{111111}.
\end{cases}$$

By solving the system, we obtain:

$\lambda_{syst}$= 3.334×10$^{-6}$ 1/km if $\lambda$ = 1.667×10$^{-6}$ 1/km and $\mu$ = 0.041 1/km.

Thus, using Markovian analysis and Laplace transformation, the rated value of the failure flow parameter of a set of car doors was calculated based on the data on the failure and recovery flow parameters of a single door.

## Conclusion

The paper elaborated upon a number of block diagrams for calculating the reliability indicators of sets of passenger car doors. Unlike the logic-and-probability approach to defining the failure flow parameter, the Markovian method is of higher priority, since it takes into account the recovery factor. Thus, the latter value of the system failure flow parameter is applicable to the “3-out-of-4” reliability block diagram under consideration for the purpose of monitoring the dependability indicators of sets of car doors in the course of operational testing.

The approaches examined in the paper enable the rolling stock component consumers and the manufacturer to agree on the methods of dependability calculation of complex systems and to avoid conflicting methods of reliability indicator calculation when moving from an element to a system. The described method is currently used for calculating the rated value of the mean time to failure of a set of car doors installed in long-distance trains.

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## The authors' contribution

**Belousova M.V.** Calculated the parameters of the failure flow for block diagrams of a set of car doors, developed state graphs for the 3-out-of-4 set structure, evaluated the dependability indicators for a set based on Markovian processes.

**Bulatov V.V.** Described the object of research, generally evaluated the applicability of the considered mathematical methods to the given rolling stock systems and developed reliability block diagrams for a set of passenger car doors.

**Smirnov N.V.** Together with Belousova M.V. formulated the mathematical problem, verified the correctness of all arguments and calculations, proposed an interpretation of the obtained results.

## Conflict of interests

The author declares the absence of a conflict of interests.