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Method of estimating the size of an SPTA with a safety stock

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Abstract. Aim. *To modify the classical method [1, 4] that causes incorrect estimation of the required size of SPTA in cases when the replacement rate of failed parts is comparable to the SPTA replenishment rate. The modification is based on the model of SPTA target level replenishment. The model considers two situations: with and without the capability to correct requests in case of required increase of the size of replenishment. The paper also aims to compare the conventional and adjusted solution and to develop recommendations for the practical application of the method of SPTA target level replenishment.* **Methods.** *Markovian models [2, 3, 5] are used for describing the system. The flows of events are simple. The final probabilities were obtained using the Kolmogorov equation. The Kolmogorov system of equations has a stationary solution. Classical methods of the probability theory and mathematical theory of dependability [6] were used.* **Conclusions.** *The paper improves upon the known method of estimating the required size of the SPTA with a safety stock. The paper theoretically substantiates the dependence of the rate of backward transitions on the graph state index. It is shown that in situations when the application is not adjusted, the rates of backward transitions from states in which the SPTA safety stock has been reached and exceeded should gradually increase as the stock continues to decrease. The multiplier will have a power-law dependence on the transition rate index. It was theoretically and experimentally proven that the classical method causes SPTA overestimation. Constraint (3) was theoretically derived, under which the problem is solved sufficiently simply using the classical methods. It was shown that if constraint (3) is not observed, mathematically, the value of the backward transition rate becomes uncertain. In this case, correct problem definition results in graphs with a linearly increasing number of states, thus, by default, the problem falls into the category of labour-intensive. If the limits are not observed, a simplifying assumption is made, under which a stationary solution of the problem has been obtained. It is shown that, under that assumption, the solution of the problem is conservative. It was shown that, if the application is adjusted, the rate of backward transition from the same states should gradually decrease as the stock diminishes. The multiplier will have a hyperbolic dependence on the transition rate index. This dependence results in a conservative solution of the problem of replenishment of SPTA with application adjustment. The paper defines the ratio that regulates the degree of conservatism. It is theoretically and experimentally proven that in such case the classical method causes SPTA underestimation. A stationary solution of the problem of SPTA replenishment with application adjustment has been obtained. In both cases of application adjustment reporting, a criterion has been formulated for SPTA replenishment to a specified level. A comparative analysis of the methods was carried out.*

Keywords: *Markovian analysis, graph, graph state, probability of transition, failure rate, replenishment rate, SPTA, safety stock, application adjustment.*

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Introduction

The sufficiency indicator (SI) is the primary quantitative characteristic of SPTA stock. The proposed methods for estimating the size of SPTA usually assume that SI is defined in the performance specifications (PS) for the development of a product or SPTA kit.

An estimate of a stock type is understood as the identification of the required size of SPTA, under which the required SI value is attained. The stock estimation of an SPTA kit consists of the estimates of each individual stock type.

There are several strategies of SPTA replenishment:

– scheduled replenishment (SR). In case of SR, the SPTA replenishment period T_R is defined, after which the stock is replenished to the initial level. If the stock is exhausted before that moment and another failure occurs, the system goes into an inoperable state until the next replenishment;

– scheduled replenishment with emergency deliveries (RED). For the case of RED, the period T_R is also defined. At the same time, upon the SPTA exhaustion and after another failure, an early (emergency) delivery of spare parts (SP) is organized for the purpose of replacing the failed module and replenishing the stock to the initial level;

– continuous replenishment (CR). In case of CR, a stock replenishment request is generated and submitted for execution after each failure and use of each SP. SPTA replenishment may be interpreted as repairs of a failed SP, that after a 100% recovery is added to the SPTA;

– replenishment to the specified level (RL). In case of RL, a stock replenishment application is generated after the stock has reduced to the specified level, including zero, even before the next failure occurs. An application is always generated when the system is operational. That is the difference between RL and RED. This strategy is examined and modified in this paper.

The required size of SPTA is normally estimated using Markovian analysis according to GOST R 51901.15.

The assumptions of the Markovian analysis regarding the probability of transition can be defined as follows:

– state transitions are statistically independent events;

– failure rate λ and recovery rate μ are constant;

– the probabilities of transition from one state to another within a small period of time Δt (Δt is little) are defined by the values $\lambda\Delta t$ and/or $\mu\Delta t$;

– the conditions and mode of operation of all same-type modules are assumed to be identical.

Let us examine the following mathematical model that allows calculating the required SPTA size under condition that a certain critical level m of safety stock is reached [1, 4]. The replacement process with no repair of failed components corresponds to the random death and birth process that is described by the following state graph (Fig. 1). The system state graph has $k + 2$ states.

Let us introduce the following designations:

n is the number of same-type elements in the system;

k is the planned SPTA size;

m is the level of safety stock, that, when reached, causes the next SPTA replenishment to the planned value k ($1 \le m \le k$);

λ is the failure rate of one element;

$\mu = 1/T_D$ is the rate of scheduled SPTA replenishment;

T_D is the average time of SP delivery from the moment the replenishment application is generated.

On the graph, the numbers 0, 1, 2, ..., k show the operable states corresponding to the expended share of the SPTA. State $k + 1$ indicates that the SPTA has been exhausted. The arrows represent transitions from one state to another. Above the arrows are the corresponding transition rates.

Case 1. The application is not adjusted

First, let us assume that the delivery application is not adjusted (see Fig. 1) if further failures occur prior to the delivery of the SP. Let us identify the transition rates for cases when the SP replenishment application has been sent and additional failures have occurred.

Obviously, $\gamma_0=1$. The time of transition from state $k–m+1$ to state 1 will be less than T_D, as the SP replenishment application was previously submitted in state $k–m$. The proportion of the average transition time from state $k–m+1$ to state 1 relative to the total average time of consecutive transition from state $k–m$ to state $k–m+1$, then to state 1 is $\frac{1/\mu}{1/\mu+1/n\lambda}$. Thus, the average time of transition from state $k–m+1$to state 1 will be equal to $\frac{(1/\mu)^2}{1/\mu+1/n\lambda}$ and its rate, respectively, equal to $(1+\rho)\mu$, where $\rho = \frac{\mu}{n\lambda}$. I.e., the multiplier $\gamma_1=1+\rho$ (see Fig. 1).

Let us summarize. Let us examine a transition from state $k–m+j$ to state j. The transition is possible if the random time η of SPTA replenishment application completion turned out to be longer than the $\xi_1 + \ldots + \xi_j = \sigma_j$ total operation time. The average transition time (in terms of mathematical expectation) will be defined by the following integral:

$$E\left[(\eta-\sigma_j) I\{\eta>\sigma_j\}\right] = \int_0^\infty f_{\sigma_j}(x) \int_x^\infty (y-x) f_\eta(y)\,dy\,dx, \quad (1)$$

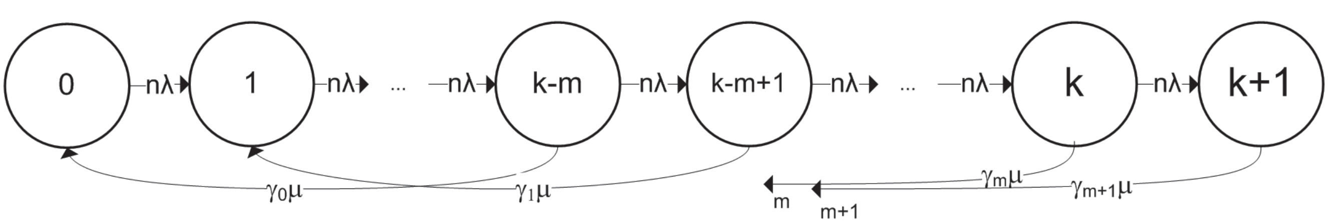

Fig. 1. Transition graph of the death and birth process

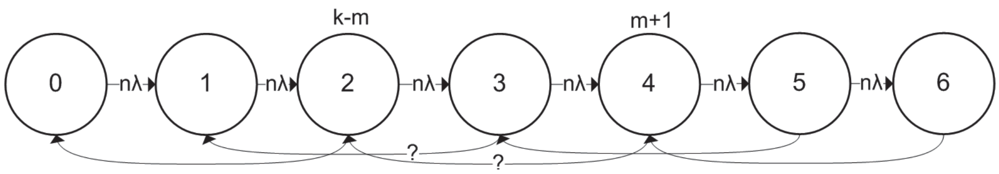

Fig. 2. Transition graph with uncertain transition rate

where $E[\,]$ is the expectation operator, $I\{A\}=\begin{cases}1,\text{if } A \text{ is true}\\0,\text{if } A \text{ is false}\end{cases}$ is the indicator function, $f_{\sigma_j}(x)$ is the distribution density of the total operation time, $f_\eta(y)$ is the distribution density of application execution time. As $\xi_1,\ldots,\xi_j$ are independent, identically distributed random values that follow an exponential distribution with the rate of $n\lambda$, then the total operation time will have gamma distribution with the rate of $n\lambda$ and shape variable j. By substituting the exponential probability density function with the rate of j into (1) as $f_\eta(y)$, after some simple transformations, the following result is obtained:

$$E\left[\left(\eta-\sigma_j\right)I\left\{\eta>\sigma_j\right\}\right]=\frac{1}{\mu}\left(\frac{n\lambda}{n\lambda+\mu}\right)^j.$$

Therefore, coefficients γ_j (see Fig. 1) will be equal to:

$$\gamma_j=(1+\rho)^j,\text{where } \rho=\mu/n\lambda. \quad (2)$$

The rate of transition from state $k–m+j$ into state j will be $\gamma_j\mu$.

The process of death and birth can be described with a Kolmogorov system of equations. A number of situations is possible involving various numbers of incoming and outgoing transitions.

First, let us introduce a supplementary condition:

$$k \geq 2m+1. \quad (3)$$

This restriction is to prevent uncertainty in the transition rates (see Fig. 2). Input data: k=5, m=3. On the one hand, the rate of transition from state 3 to state 1 should be $\gamma_1\mu$ if the system has transitioned from state 2 to state 3. On the other hand, if the system has transitioned from state 5 into state 3 and the SPTA was replenished as a result, the rate should be $\gamma_0\mu$. The similar uncertainty characterizes the rate of transition from state 4 to state 2. It is either $\gamma_2\mu$ or $\gamma_0\mu$. The constraint (3) arises from the solution of inequality $k–m\geq m+1$, a condition under which no uncertainty arises (see Fig. 2).

In total, there may be five cases.

1. $k\geq 2m+3$,

$$\begin{cases}P_0'(t)=-n\lambda P_0(t)+\gamma_0\mu P_{k-m}(t);\\P_i'(t)=n\lambda P_{i-1}(t)-n\lambda P_i(t)+\gamma_i\mu P_{k-m+i}(t),1\leq i\leq m+1;\\P_i'(t)=n\lambda P_{i-1}(t)-n\lambda P_i(t),m+2\leq i\leq k-m-1;\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_{i-k+m}\mu\right)P_i(t),k-m\leq i\leq k;\\P_{k+1}'(t)=n\lambda P_k(t)-\gamma_{m+1}\mu P_{k+1}(t).\end{cases}$$

2. k=2m+2,

$$\begin{cases}P_0'(t)=-n\lambda P_0(t)+\gamma_0\mu P_{k-m}(t);\\P_i'(t)=n\lambda P_{i-1}(t)-n\lambda P_i(t)+\gamma_i\mu P_{k-m+i}(t),1\leq i\leq m+1;\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_{i-k+m}\mu\right)P_i(t),m+2\leq i\leq k;\\P_{k+1}'(t)=n\lambda P_k(t)-\gamma_{m+1}\mu P_{k+1}(t).\end{cases}$$

3. k=2m+1,

$$\begin{cases}P_0'(t)=-n\lambda P_0(t)+\gamma_0\mu P_{k-m}(t);\\P_i'(t)=n\lambda P_{i-1}(t)-n\lambda P_i(t)+\gamma_i\mu P_{k-m+i}(t),1\leq i\leq m;\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_0\mu\right)P_i(t)+\gamma_i\mu P_{k-m+i}(t),i=m+1\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_{i-k+m}\mu\right)P_i(t),m+2\leq i\leq k;\\P_{k+1}'(t)=n\lambda P_k(t)-\gamma_{m+1}\mu P_{k+1}(t).\end{cases}$$

4. $m+2\leq k\leq 2m$,

$$\begin{cases}P_0'(t)=-n\lambda P_0(t)+\gamma_0\mu P_{k-m}(t);\\P_i'(t)=n\lambda P_{i-1}(t)-n\lambda P_i(t)+\gamma_0\mu P_{k-m+i}(t),1\leq i\leq k-m-1;\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_0\mu\right)P_i(t)+\\+\gamma_{(k-2m-1+i)\vee 0}\mu P_{k-m+i}(t),k-m\leq i\leq m+1;\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_{i-m-1}\mu\right)P_i(t),m+2\leq i\leq k;\\P_{k+1}'(t)=n\lambda P_k(t)-\gamma_{k-m}\mu P_{k+1}(t).\end{cases}$$

5. k=m+1,

$$\begin{cases}P_0'(t)=-n\lambda P_0(t)+\gamma_0\mu P_{k-m}(t);\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_0\mu\right)P_i(t)+\gamma_0\mu P_{k-m+i}(t),1\leq i\leq m;\\P_i'(t)=n\lambda P_{i-1}(t)-\left(n\lambda+\gamma_0\mu\right)P_i(t)+\gamma_1\mu P_{k-m+i}(t),i=m+1;\\P_{k+1}'(t)=n\lambda P_k(t)-\gamma_1\mu P_{k+1}(t).\end{cases}$$

Let us search for a stationary solution. For that purpose, let us assume the derivatives on the left sides of the equations equal 0 and apply the normalization condition: $\sum_0^{k+1}P_i=1$.

1. $k\geq 2m+3$,

$$\begin{cases}P_0=\rho P_{k-m};\\P_{i-1}=P_i-\gamma_i\rho P_{k-m+i},1\leq i\leq m+1;\\P_{i-1}=P_i,m+2\leq i\leq k-m-1;\\P_{i-1}=\left(1+\gamma_{i-k+m}\rho\right)P_i,k-m\leq i\leq k;\\P_k=\gamma_{m+1}\rho P_{k+1}.\end{cases}$$

2. k=2m+2,

$$\begin{cases} P_0 = \rho P_{k-m}; \\ P_{i-1} = P_i - \gamma_i \rho P_{k-m+i}, 1 \le i \le m+1; \\ P_{i-1} = \left(1+\gamma_{i-k+m}\rho\right)P_i, m+2 \le i \le k; \\ P_k = \gamma_{m+1}\rho P_{k+1}. \end{cases}$$

3. k=2m+1,

$$\begin{cases} P_0 = \rho P_{k-m}; \\ P_{i-1} = P_i - \gamma_i \rho P_{k-m+i},, 1 \le i \le m; \\ P_{i-1} = \left(1+\rho\right)P_i - \gamma_i \rho P_{k-m+i}, i = m+1; \\ P_{i-1} = \left(1+\gamma_{i-k+m}\rho\right)P_i, m+2 \le i \le k; \\ P_k = \gamma_{m+1}\rho P_{k+1}. \end{cases}$$

4. m+2≤k≤2m,

$$\begin{cases} P_0 = \rho P_{k-m}; \\ P_{i-1} = P_i - \rho P_{k-m+i}, 1 \le i \le k-m-1; \\ P_{i-1} = \left(1+\rho\right)P_i - \gamma_{(k-2m-1+i)\vee 0}\rho P_{k-m+i}, k-m \le i \le m+1; \\ P_{i-1} = \left(1+\gamma_{i-m-1}\rho\right)P_i, m+2 \le i \le k; \\ P_k = \gamma_{k-m}\rho P_{k+1}. \end{cases}$$

5. k=m+1,

$$\begin{cases} P_0 = \rho P_1; \\ P_{i-1} = \left(1+\rho\right)P_i - \rho P_{i+1}, 1 \le i \le m; \\ P_m = \left(1+\rho\right)P_{m+1} - \gamma_1 \rho P_{m+2}, i = m+1; \\ P_{m+1} = \gamma_1 \rho P_{m+2}. \end{cases}$$

Next, for the first case, let us express the obtained recursions and the final solution. For convenience, let us denote $\rho = \theta/n = \mu/n\lambda$.

k≥2m+3. The solution is found "from top to bottom".

$$P_k = \gamma_{m+1}\rho P_{k+1};$$

$$P_{i-1} = \gamma_{m+1}\rho P_{k+1}\prod_{j=m-k+i}^{m}\left(1+\gamma_j\rho\right), k-m \le i \le k;$$

$$P_{i-1} = P_i, m+2 \le i \le k-m-1.$$

Therefore,

$$P_{m+1} = P_{m+2} = \ldots = P_{k-m-1} = \gamma_{m+1}\rho P_{k+1}\prod_{j=0}^{m}\left(1+\gamma_j\rho\right) = A_m P_{k+1},$$

where $A_m = \gamma_{m+1}\rho\prod_{j=0}^{m}\left(1+\gamma_j\rho\right)$.

General formula

$$P_i = \left(A_m - \gamma_{m+1}\rho\prod_{j=i+1}^{m}\left(1+\gamma_j\rho\right)\right)P_{k+1}, 0 \le i \le m-1.$$

Additionally,

$$P_m = \left(A_m - \gamma_{m+1}\rho\right)P_{k+1}.$$

After applying the normalization condition, we will obtain the following result

$$\left(k-m\right)A_m + 1 = \frac{1}{P_{k+1}}.$$

The second and third situations cause similar conclusions.

Thus, under restriction (3), the probability of SPTA failure will be defined by the expression:

$$P_{k+1} = \frac{1}{1+\left(k-m\right)A_m}, \text{ where } A_m = \gamma_{m+1}\rho\prod_{j=0}^{m}\left(1+\gamma_j\rho\right). \quad (4)$$

Now, let us drop the condition (3), i.e., let us examine cases 4 and 5 and implement the accurate model of SP supply using a Markovian graph (Fig. 3). The graph, due to the awkwardness that increases as k grows, is represented for the special case k=3, m=2.

First, let us consider case 4. In order to avoid uncertainty in the rate of recurrent transitions, additional states have been introduced into SPTA replenishment: (2.0), (2.1), (3.0), (3.1), (3.2), …. The first number is the number of SP taken from the SPTA, the second one is the number of failures that occurred since the submission of the replen-

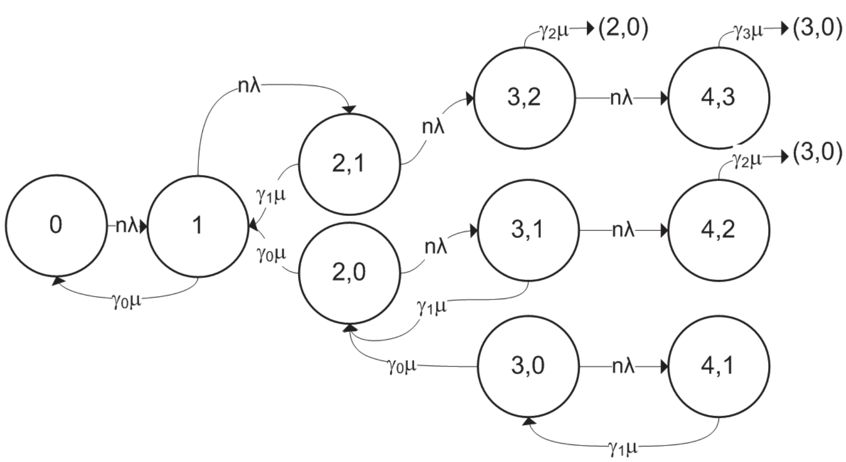

Fig. 3. System transition graph in case of k≤2m

ishment application. For example, (2.0) means that there have been two failures (and 2 replacements, respectively) and there are no unfilled applications, and it is time to send a new SPTA *k–m*=1 replenishment application. (3.1) means that there have been three failures (3 replacements, respectively), there is an unsatisfied application submitted in a situation where two SPs from the SPTA have been used and it is time to send a new SP *k–m*=1 replenishment application. (3.2) means that there have been three failures (3 replacements, respectively), there is an unsatisfied application submitted in a situation where another SP from the SPTA has been used and it is time to send a new SP *k–m*=1 replenishment application.

Finding an accurate analytical solution in the general case is a time-consuming task.

Therefore, we will solve its simplified version, assuming that in case of uncertainty the multiplier takes the minimal possible values, i.e., γ_0.

$$\gamma_i = \min\left(\gamma_0, \gamma_{i,j}\right) = \gamma_0 = 1. \quad (5)$$

That assumption leads to a conservative estimate of the required SPTA size, since in the simplified version the rate of recurrent transitions into "improved" states decreases, the response to applications slows down. As before, we will find stationary solutions using the same notations

$$\begin{cases} P_0 = \rho P_{k-m}; \\ P_{i-1} = P_i - \rho P_{k-m+i}, 1 \le i \le k-m-1; \\ P_{i-1} = (1+\rho) P_i - \gamma_{(k-2m-1+i)\vee 0} \rho P_{k-m+i}, k-m \le i \le m+1; \\ P_{i-1} = (1+\gamma_{i-m-1}\rho) P_i, m+2 \le i \le k; \\ P_k = \gamma_{k-m} \rho P_{k+1}. \end{cases}$$

The solution of system (6) in an explicit form is quite cumbersome. In order to solve the problem numerically, it is suggested using a replacement-based algorithm

$$P_i = a_i P_{k+1}, i = 0, 1, \ldots, k+1. \quad (7)$$

Coefficients a_i will be defined by a "backward" recursion:

$$\begin{cases} a_{k+1} = 1; \\ a_k = \gamma_{k-m} \rho; \\ a_{i-1} = (1+\gamma_{i-m-1}\rho) a_i, m+2 \le i \le k; \\ a_{i-1} = (1+\rho) a_i - \gamma_{(k-2m-1+i)\vee 0} \rho a_{k-m+i}, k-m \le i \le m+1; \\ a_{i-1} = a_i - \rho a_{k-m+i}, 1 \le i \le k-m-1. \end{cases}$$

In (6) and (8), for the purpose of reduction, symbol $\vee$ is used that means $a \vee b = \max(a,b)$. The probability of SPTA shortage will equal

$$P_{k+1} = \left(\sum_{i=0}^{k+1} a_i\right)^{-1}. \quad (9)$$

Let us consider the fifth case: $k=m+1$.

From the first two equations we obtain a recurrence equation:

$$P_{i-1} = \rho P_i, i = 1, \ldots, m+1.$$

By using the latter equation, we deduce:

$$P_i = \gamma_1 \rho^{m+2-i} P_{m+2}, i = 0, \ldots, m+1.$$

Out of the normalization condition, if $\rho \ne 1$, we obtain:

$$P_{k+1} = \frac{1-\rho}{1-\rho+\gamma_1\rho-\gamma_1\rho^{m+3}} = \frac{1-\rho}{1+\rho^2-(1+\rho)\rho^{m+3}}. \quad (10)$$

If $\rho = 1$, we obtain:

$$P_{k+1} = \frac{1}{1+2(m+2)} = \frac{1}{2m+5}. \quad (11)$$

Case 2. The application is adjusted

Now, let us assume that the delivery application is adjusted in a situation when further failures have occurred before the delivery of an SP kit (Fig. 4).

In this case, the transition rate multipliers will also be non-constant, as additional time will be required for the replenishment per the previous application. Let us suppose that it takes an average time of τ to consolidate an SPTA kit in a warehouse, while the delivery to the destination takes π. Then, the transition from state *k–m* into state 0 will on average take $m\tau + \pi$. The transition from state *k–m*+1 to state 1 will on average take $(m+1)\tau + \pi$. In general, the transition from state *k–m+j* into state *j* will on average take $(m+j)\tau + \pi$. Additionally, $j = 0, 1, \ldots, m+1$. As before, $\gamma_0 = 1$. Then,

$$\gamma_j = \frac{m\tau+\pi}{(m+j)\tau+\pi}. \quad (12)$$

If the times τ and π are unknown, we can use the estimate

$$\frac{m\tau+\pi}{(m+j)\tau+\pi} \ge \frac{m}{m+j}.$$

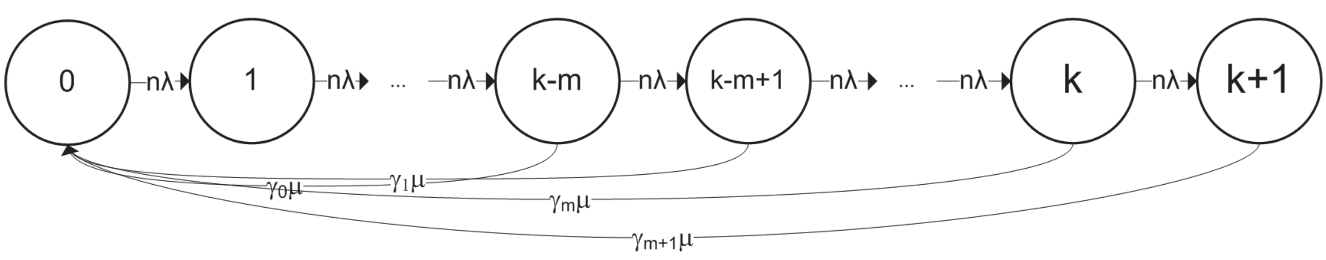

Fig. 4. System transition graph with adjustment of SPTA application

If we assume that

$$\gamma_j = \frac{m}{m+j}, \tag{13}$$

then the recurrent transition rates will be underestimated and the required SPTA size will be overestimated, i.e., conservative. The degree of conservatism will be defined by formula π/τ. If (π/τ)>1, the delivery time is longer than the time of SPTA kit consolidation, the result will be strongly conservative, and an adequate result should be obtained using the classical method of computation. If (π/τ)<1, the delivery time is shorter than the time of single SPTA kit consolidation, the result will be mildly conservative, the error caused by the application of (13) instead of (12) will be small. In any case, if π/τ is known, (12) is preferable.

The following system of differential equations corresponds to the graph shown in Fig. 4.

$$\begin{cases} P_0'(t) = -n\lambda P_0(t) + \mu\sum_{i=0}^{m+1}\gamma_i P_{k-m+i}(t); \\ P_i'(t) = n\lambda P_{i-1}(t) - n\lambda P_i(t), 1 \le i \le k-m-1; \\ P_i'(t) = n\lambda P_{i-1}(t) - (n\lambda + \gamma_{i-k+m}\mu)P_i(t), k-m \le i \le k; \\ P_{k+1}'(t) = n\lambda P_k(t) - \gamma_{m+1}\mu P_{k+1}(t). \end{cases} \tag{14}$$

As we did before, let us seek a stationary solution "from top to bottom".

$$P_k = \gamma_{m+1}\rho P_{k+1};$$

$$P_{i-1} = \gamma_{m+1}\rho P_{k+1}\prod_{j=m-k+i}^{m}(1+\gamma_j\rho), k-m \le i \le k;$$

$$P_{i-1} = P_i, 1 \le i \le k-m-1.$$

Therefore,

$$P_1 = P_2 = \ldots = P_{k-m-1} = \gamma_{m+1}\rho P_{k+1}\prod_{j=0}^{m}(1+\gamma_j\rho) = A_m P_{k+1}.$$

The probability of the original state is defined by the sum

$$P_0 = \rho\sum_{i=0}^{m+1}\gamma_i P_{k-m+i} = \gamma_{m+1}\rho P_{k+1}\left[1 + \gamma_m\rho + \sum_{i=0}^{m-1}\gamma_i\rho\prod_{j=i+1}^{m}(1+\gamma_j\rho)\right].$$

After applying the normalization condition, we will obtain the following result

$$\gamma_{m+1}\rho\sum_{i=1}^{m}\prod_{j=i}^{m}(1+\gamma_j\rho) + (k-m)A_m + \gamma_{m+1}\rho + 1 = \frac{1}{P_{k+1}},$$

from which the probability of SPTA shortage will be defined by the formula:

$$P_{k+1} = \frac{1}{(k-m)A_m + \gamma_{m+1}\rho + 1 + B_m}, \tag{15}$$

where $A_m = \gamma_{m+1}\rho\prod_{j=0}^{m}(1+\gamma_j\rho)$, $B_m = \gamma_{m+1}\rho\sum_{i=1}^{m}\prod_{j=i}^{m}(1+\gamma_j\rho)$.

As before, we select k so that probability (15) is less than a certain number ε. Finally, we obtain the following criteria for the assessment of the required stock:

$$k \ge \frac{\frac{1-\varepsilon}{\varepsilon} - \gamma_{m+1}\rho - B_m}{A_m} + m, \tag{16}$$

where $A_m = \gamma_{m+1}\rho\prod_{j=0}^{m}(1+\gamma_j\rho)$.

Comparative analysis of the obtained results

Let us compare the estimates of the required SPTA size obtained using the classical method with constant rates of recurrent states and the new method taking into account the adjustments γ_i, $i = 1, 2, \ldots m+1$.

Table 1. Estimation of the required size of SPTA. ρ = 1. The application is not adjusted

Method	ε		
	0.1	**0.05**	**0.01**
Classical	4	6	26
New	3	3	6

Table 2. Estimation of the required size of SPTA. ρ = 1. The request is adjusted

Method	ε		
	0.1	**0.05**	**0.01**
Classical	3	5	25
New	10	20	100

Table 3. Estimation of the required size of SPTA ρ = 2. The application is not adjusted

Method	ε		
	0.1	**0.05**	**0.01**
Classical	2	3	7
New	2	2	3

Table 4. Estimation of the required size of SPTA ρ = 2. The request is adjusted

Method	ε		
	0.1	**0.05**	**0.01**
Classical	2	2	7
New	3	6	26

Table 5. Estimation of the required size of SPTA ρ = 5. The application is not adjusted

Method	ε		
	0.1	**0.05**	**0.01**
Classical	2	2	2
New	2	2	2

Table 6. Estimation of the required size of SPTA ρ = 5. The request is adjusted

Method	ε		
	0.1	**0.05**	**0.01**
Classical	1	1	2
New	2	2	4

Tables 1 to 6 show the calculated required SPTA size under various values of $\rho = \frac{\mu}{n\lambda}$ and probabilities of SPTA shortage ε. m=1 is taken as the safety stock.

Having analysed the calculation results, we can note that, first, previous conclusions on the overestimation or underestimation of the size of SPTA proved to be correct. Second, the amendments proposed in the paper should be taken into account if $\rho \leq 5$, i.e., when the value of SPTA replenishment rate μ is comparable to the total failure rate $n\lambda$. In this situation, a tangible economic effect can actually be obtained if the application is not adjusted. In the same case, if the application is adjusted, a conservative, yet correct, estimate of the required size of SPTA can be obtained.

On the other hand, it should be noted that the paper examined and investigated the solution of a stationary problem, that will probably significantly differ from the solution of the original non-stationary problem in cases of relatively short calculation time. The authors will dedicate further research activities to the computational solution of a non-stationary problem.

Conclusion

The paper improved the method of estimating the required size of an SPTA with a safety stock. The application results of the adjusted and classical estimates were analysed. It was revealed that the most pronounced effect from the application of the adjusted estimate will be observed if the failure rates are comparable to the SPTA replenishment rates.

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The authors' contribution

The authors have modified and expanded the method of evaluating SPTA with safety stock. A comparative analysis of the classical and modified methods was carried out.

Valery A. Chepurko developed a method of assessing the size of SPTA in situations when the SPTA replenishment request is not adjusted.

Alexey N. Chernyaev developed a method of assessing the size of SPTA in situations when the SPTA replenishment request is adjusted and conducted a comparative test.

Conflict of interests

The authors declare the absence of a conflict of interests.