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Application of interval-valued triangular fuzzy numbers and their functional to the healthcare problems

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Abstract. Aim. *In healthcare field there exist different types of uncertainty due to medical error generated by human and technologies. In general the crisp value generate loss of precision and inaccuracy about result and therefore the available data is not sufficient to assessed clinical process up to desired degree of accuracy. Therefore fuzzy set theory play as an important and advance role in accuracy of results in healthcare related problems.* **Methods.** *Here for more accuracy of result, we use functional fuzzy numbers in this paper. This study uses a new fuzzy fault tree analysis for patient safety risk modelling in healthcare. In this paper we will use level (λ, ρ) interval-valued triangular fuzzy number, their functional, t-norm operation and centre of gravity defuzzification method to evaluate fuzzy failure probability and estimate reliability of system. The effectiveness of these methods is illustrated by an example related to healthcare problems and then we analyse the result obtained with the other existing techniques. Tanaka et al.'s approach has been used to give the rank of basic events of the considered problems. Also, we use functional of fuzzy numbers to analyse the change in fuzzy failure probability.* **Results.** *The paper examines the application of the failure tree, t-norm and functional fuzzy numbers in the context of interval-valued triangular fuzzy numbers. The research examined two types of healthcare-specific problems and the corresponding defuzzification techniques for the purpose of reliability analysis using the existing methods. The authors concluded that t-norm is not associated with significant accumulation and identified how a functional fuzzy number affects reliability. Similarly, using the V index method, the least critical events were found for each system.*

Keywords: *healthcare, fuzzy sets, t-norm, interval-valued triangular fuzzy numbers and their functional, fault tree analysis, defuzzification.*

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1. Introduction

In classical or traditional set theory, an element either belonging to the set or not, that is the answer of any element belongs to the set become yes or no rather than more or less. Fuzzy set theory [25] provide means of uncertainty, that is it explained about degree of belonging of any element in a set,

Since healthcare safety related problems are major concern for healthcare institutions around the world, so the healthcare institutions have to point out the main reasons of different kinds of medical errors and to find out the ways for reducing their frequency. In healthcare, more proactive risk analysis techniques should be applied for better and safe medication processes [3].Fault tree analysis (FTA) has been extensively used technique in health problems [1-5, 18-19].Hyman and Johnson [10] presents a FTA of the patient harm-related clinical alarms failures. Park and Lee [18] constructed a FTA of hand washing process. Chen [6] proposed a new and faster method to analyze fuzzy system reliability using fuzzy number arithmetic operations.

Lee has developed chromosomes image [11-12] by using fuzzy logic in blood leukocyte, Butnariu [4] developed a neuron model (Acoustico- vestibulary nerve) as a fuzzy automation describe with this help. Similarly Rocha has been used fuzzy logic in nervous system [20-21]. A classical approach was developed by Forden and Bezdek to diagnosis of renovascular-hypertension [7]. Adlassning and kolarz used fuzzy logic in a consultative system for rheumatology known as Cadiag-2 [2].

A fuzzy iterative diagnostic expert called "SPINX" which is used to design to deal with diagnosis [8], it consist a dialogue system where patient data is entered and response to request for additional information and a decision system, here fault tree searches match patient to diagnosis, similarly Lesmo, Saitta and Torasso have developed a system to learning production rules for medical decision making [13]. The automatic learning system based on fuzzy set theory and works on linguistic variables. One another example of fuzzy system in Ophthalmological fuzzy consultative system developed by Oguntade [17], he asses patient status pre and post therapy. It is clear that there is a good deal of fuzzy logic with medical system and fuzzy inference model have been suggested and tested successfully.

2. Fuzzy Sets

Fuzzy sets were introduced independently by Lotfi A. Jadeh [24] and Dieter Klaua in 1965 as an extension of classical notion of a set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent conditions that is an element either belong to set or not, fuzzy set theory give the membership of elements in a set belong to the unit interval [0, 1]. Fuzzy set is a generalization of classical theory.

A fuzzy set is defined by a membership function from the universal set to the interval [0, 1], as given below;

$$\mu_A(x): X \to [0,1]. \tag{1}$$

Here $\mu_A(x)$ gives the degree to x belonging in the set A. A fuzzy set A can be expressed as follows:

$$\tilde{A} = \{(x, \mu_A(x)) : x \in X\}. \tag{2}$$

Fuzziness can be found in many areas of daily life such as in engineering, medicine, manufacturing and others.

3. Fuzzification and interval-valued triangular fuzzy number

Fuzzification is the process of transformation crisp value to the fuzzy value with the help of fuzzy membership functions. There are a variety of fuzzy membership functions exist for performing fuzzification including triangular, trapezoidal, Cauchy and Gaussian, etc. [16]. For analysing safety and healthcare related problems, inter-valued triangular fuzzy membership functions or more simply inter-valued triangular fuzzy number (IVTFNs) are often utilized to provide more precise descriptions and to obtain more accurate solutions. Mathematically, an interval-valued fuzzy set $\tilde{A}(i-v\ fuzzy\ set)$ on R is derived by $\tilde{A} \equiv \{(x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^u}(x)]) / x \in R\}$; $0 \le \mu_{\tilde{A}^L}(x) \le \mu_{\tilde{A}^u}(x) \le 1 \forall x \in R$, It is denoted by $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)] x \in R$ or $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$.

The i-v triangular fuzzy set $\tilde{A}$ indicates that, when the membership grade of x belongs to the interval $[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]$ the largest grade is $\mu_{\tilde{A}^U}(x)$ and the smallest grade is $\mu_{\tilde{A}^L}(x)$. where

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \dfrac{\lambda(x-a)}{b-a} & a \le x \le b \\ \dfrac{\lambda(c-x)}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases} \tag{3}$$

Therefore, $\tilde{A}^U = (a, b, c, \rho)$, $a < b < c$ similarly,

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \dfrac{\rho(x-a)}{b-a} & a \le x \le b \\ \dfrac{\rho(c-x)}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases} \tag{4}$$

Therefore $\tilde{A}^U = (a, b, c, \rho)$, $a < b < c$. Consider the case in which $0 < \lambda \le \rho \le 1$. From (3) and (4) we obtain $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$, $[(a, b, c; \lambda), (a, b, c; \rho)]$, which is called the level $(\lambda, \rho) i-v$ triangular fuzzy number Fig. 1.

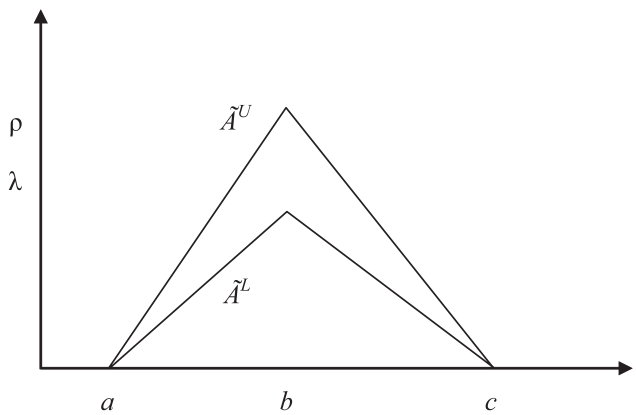

Fig. 1. i-v triangular fuzzy number

4. t-norm

Zadeh [24] suggest that the intersection of fuzzy set is minimum operator and algebraic product. The minimum product and bounded difference operator belong to so called triangular norm or t-norm. A t-norm is a binary function, t : [0, 1]×[0, 1]→[0, 1] which satisfies the axioms of (i) Commutativity, (ii) Associativity (iii) Monotonicity and (iv) Boundary condition. In literature there are various t-norms operation such as $t(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$, $p < a < b < c < d < r$.

$$0 < \lambda \le \rho \le 1 \text{ and}$$

$$t(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = \begin{cases} \mu_{\tilde{A}}(x) & \textit{where } \mu_{\tilde{B}}(x) = 1 \\ \mu_{\tilde{B}}(x) & \textit{where } \mu_{\tilde{A}}(x) = 1 \cdot \\ 0, & \textit{otherwise} \end{cases}$$

Here the last t-norm operation was applied by Lin et al. [14] for triangular fuzzy numbers and we use this operation to find the failure probability of system. t–norm operation give smaller fuzzy accumulation which is the best advantage in fuzzy arithmetic operation.

5. Functional of fuzzy numbers

It is defined as a function of function of x that is, membership value have a membership degree also. It is defined as follow

$$\mu_{\tilde{A}}(x) : X \to [0,1],\ \mu\left(\mu_{\tilde{A}}(x)\right) : \mu_{\tilde{A}}(x) \to [0,1],$$

$$\tilde{A} = \left\{\left(\left(x, \mu_{\tilde{A}}(x)\right), \mu\left(\mu_{\tilde{A}}(x)\right)\right) : x \in X\right\}. \quad (5)$$

Here a interval-valued triangular functional fuzzy number shown in Fig. 2 here the lower interval-valued triangular fuzzy number membership is (1,2,3:0.5) and upper membership value is (1,2,3:1.0) the z coordinate can be calculated as $\mu\left(\mu_A(x)\right) = \dfrac{1}{x^2+1}$;

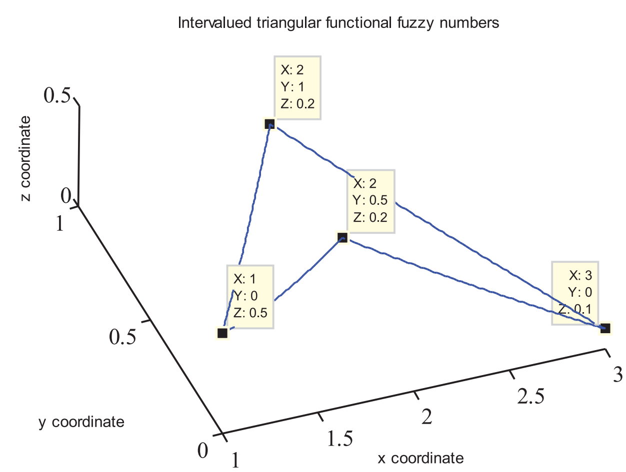

Fig. 2.Interval-valued Triangular Functional fuzzy numbers

Table 1. Gate Operation

Approach	Gate	Operation	Equation
Traditional FTA	OR	Conjunction	$P_{\text{OR}} = 1 - [(1-q_1)\times(1-q_2)\times\ldots\times(1-q_n)]$
	AND	Intersection	$P_{\text{AND}} = q_1 \times q_2 \times \ldots. \times q_n$
Traditional FFTA	OR	Conjunction	$P_{\text{OR}} = \tilde{1}\Theta[(\tilde{1}\Theta\tilde{q}_1)\otimes(\tilde{1}\Theta\tilde{q}_2)\otimes\ldots.\otimes(\tilde{1}\Theta\tilde{q}_n)]$
	AND	Intersection	$P_{\text{AND}} = \tilde{q}_1 \otimes \tilde{q}_2 \otimes \ldots\ldots \otimes \tilde{q}_n$
t-norm FFTA	OR	Conjunction	$P_{\text{OR}} = \tilde{1}\Theta_t[(\tilde{1}\Theta_t\tilde{q}_1)\otimes_t(\tilde{1}\Theta_t\tilde{q}_2)\otimes_t\ldots.\otimes_t(\tilde{1}\Theta_t\tilde{q}_n)]$
	AND	Intersection	$P_{\text{AND}} = q_1 \otimes_t q_2 \otimes_t \ldots. \otimes_t q_n$

Table 2. Fuzzy operation of two interval-valued Triangular fuzzy numbers (TFN's)

Operation	Triangular fuzzy interval-valued numbers
Multiplication	$\begin{pmatrix} a_1, b_1, c_1 : \rho \\ a_1, b_1, c_1 : \lambda \end{pmatrix} \times \begin{pmatrix} a_2, b_2, c_2 : \rho \\ a_2, b_2, c_2 : \lambda \end{pmatrix} = \begin{pmatrix} a_1a_2, b_1b_2, c_1c_2 : \rho \\ a_1a_2, b_1b_2, c_1c_2 : \lambda \end{pmatrix}$
Complement	$1 - \begin{pmatrix} a, b, c : \rho \\ a, b, c : \lambda \end{pmatrix} = \begin{pmatrix} 1-c, 1-b, 1-a : \rho \\ 1-c, 1-b, 1-a : \lambda \end{pmatrix}$

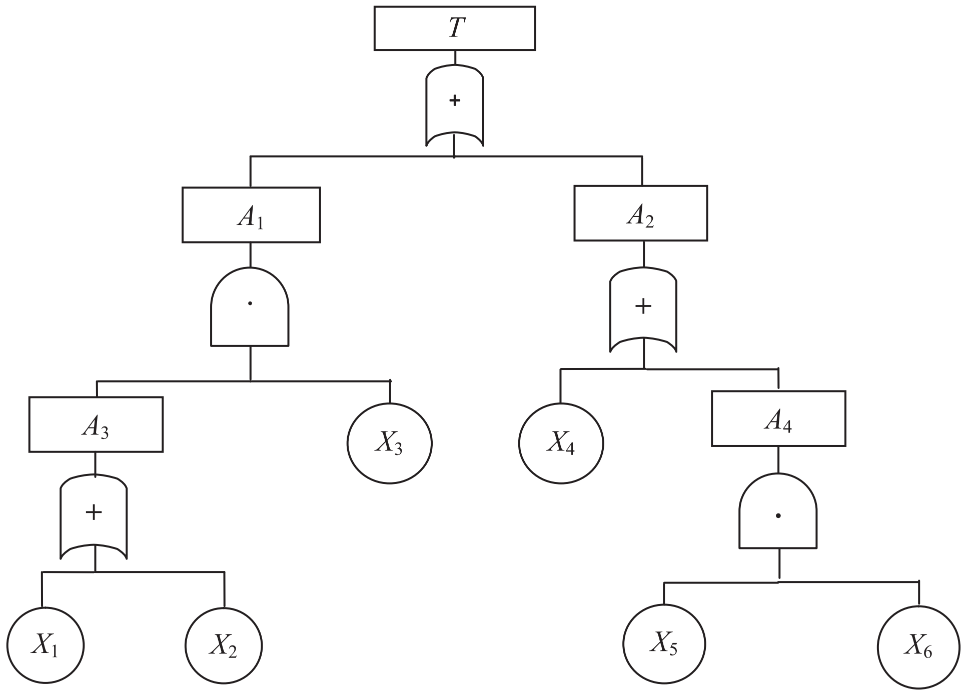

Fig. 3. Fault tree diagram

6. Fault tree analysis (FTA) and fuzzy probability

Fault-tree analysis (FTA) is a tree analysis in which undesired state of a system is analyzed using Boolean logic to combine a series of lower level basic events. Furthermore, if the failure probabilities of system components are known then the probability of the top event can be calculated. In fault tree diagram there are two gates are used one is "AND" and another is "OR". "AND" (conjunction) means the failure probability is depend on that entire event, and the event associated with OR gate means they are work independently to failure next event. The fuzzy failure probability can be calculated by following arithmetic operation on fuzzy numbers, here when we take probability in fuzzy sense the FTA is called FFTA (fuzzy fault tree analysis). In FTA operation we take crisp failure probability and in FFTA we use fuzzy failure probability.

The fault tree Fig. 3 associated with the basic events name as X_1, X_2, X_3, X_4, X_5 and X_6. The Top event is T and T can be expressed as the following equation;

$$T = A_1 \cup A_2,$$

$$T = (A_3 \cap X_3) \cup (X_4 \cup A_4),$$

$$T = \left[\left\{ \left(X_1 \cup X_2 \right) \cap X_3 \right\} \cup \left\{ X_4 \cup \left(X_5 \cap X_6 \right) \right\} \right]. \quad (6)$$

7. Reliability

Reliability is defined to be the probability that a component or system will perform a required function for a given period of time, when used under stated operating conditions. It is the probability of a non failure over time. Reliability is a property of consistency, measurement, test or life of any system or accuracy of experiment. It's an estimation about how much the error is smaller and the value is around the true value .A high reliable system produce similar result under various conditions i.e. highly reliable score are accurate.

7.1. Computation of Reliability of a system

In analyzing a complex system, a particular failure law may be applied to the entire system. However, an alternative approach is to determine an appropriate reliability or reliability model for each component of the system and by applying the rules of probability according to the configurations of the components within the system. Compute a system reliability there are following configurations;

7.1.1. Serial (Series) Configuration

In series configuration all components must run for the system to function. Under this concept, if either of two serially related components fails, the system will fail. The series relationship is represented by the block diagram in Fig. 4.

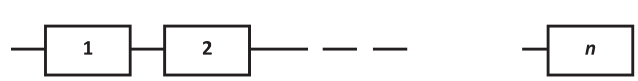

Fig. 4.serial configuration

The failure probability of series system may be determined by 1 minus the probability that no component fail (i.e. the probability that the system operate). The component failure probabilities in the following way;

$E1$: the event that component 1 fail.
E_2: the event that component 2 fail.
Then let failure probability of component 1 is $P(E_1)=q_1$ and component 2 is $P(E_2)=q_2$, then non failure probability of components are $P(E_1^C)=1–q_1$ and $P(E_2^C)=1–q_2$.then non failure probability of system is equal to $(1–q_1)\cdot(1–q_2)$.

Therefore failure probability is $P_s=1– (1–q_1)\cdot(1–q_2)$.

Assuming that the two components are independent, Generalising to n mutually independent components in series;

$$P_S = 1-(1-q_1)\cdot(1-q_2)\cdot(1-q_3)\cdot(1-q_4)\cdot\ldots\cdot(1-q_n). \quad (7)$$

Now to determine reliability we take the complement of failure probability of system.

7.1.2. Parallel Configuration

Two or more components are in parallel or redundant configuration if all the units must fail for the system to fail, i.e. if one or more units operate the system continuous to operate. Parallel units are represented by block diagram in Fig. 5.

The failure probability for n parallel and independent components is found by the probability that all components fail, i.e.

$$P_S = q_1 \times q_2 \times q_3 \times \ldots \times q_n. \quad (8)$$

Now to determine reliability we take the complement of failure probability of system.

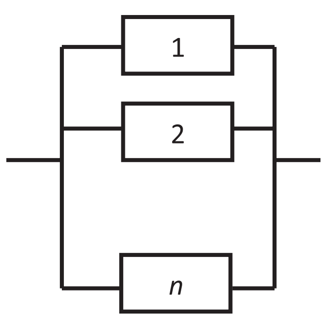

Fig. 5.Components in Parallel

8. V Index

Let $\tilde{q}_T$ denote the fuzzy failure probability of the system top event, which depends on its components whose fuzzy failure probabilities ($q_{i's}$) are interval-valued TFN's then the fuzzy Failure probability of system top event is given by the equations;

$$\tilde{q}_T = \tilde{q}_T(\tilde{q}_1, \tilde{q}_2 \ldots\ldots \tilde{q}_i \ldots\tilde{q}_n) = \\ = \left[\left(\tilde{q}^L_{T_1}, \tilde{q}^L_{T_2}, \tilde{q}^L_{T_3} : \lambda\right), \left(\tilde{q}^L_{T_1}, \tilde{q}^L_{T_2}, \tilde{q}^L_{T_3} : \rho\right)\right], \quad (9)$$

where $\tilde{q}^L_T = \left[\left(\tilde{q}^L_{T_1}, \tilde{q}^L_{T_2}, \tilde{q}^L_{T_3} : \lambda\right), \left(\tilde{q}^L_{T_1}, \tilde{q}^L_{T_2}, \tilde{q}^L_{T_3} : \rho\right)\right]$ is a λ, ρ i-v TFNs.

Let $\tilde{q}_{T_i}$ be the fuzzy failure probability of system top event after preventing system i[th] component failure (i.e. $\tilde{q}_i = \tilde{0}$) then the value of $\tilde{q}_{T_i}$ is given by the equation.

$$\tilde{q}_T = \tilde{q}_T(\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_i, \ldots, \tilde{q}_n) = \\ = \left[\left(\tilde{q}_{Ti_1}, \tilde{q}_{Ti_2}, \tilde{q}_{Ti_3} : \lambda\right), \left(\tilde{q}_{Ti_1}, \tilde{q}_{Ti_2}, \tilde{q}_{Ti_3} : \rho\right)\right], \quad (10)$$

where $\tilde{q}_T = \left[\left(\tilde{q}_{Ti_1}, \tilde{q}_{Ti_2}, \tilde{q}_{Ti_3} : \lambda\right), \left(\tilde{q}_{Ti_1}, \tilde{q}_{Ti_2}, \tilde{q}_{Ti_3} : \rho\right)\right]$ is λ, ρ i-v TFN.

Then the index V given by H. Tanaka [23], measure the difference between E_1 and $\tilde{q}_{E_1}$ is defined as;

$$V\left(\tilde{q}_T, \tilde{q}_{T_i}\right) = \left(\tilde{q}_{T_i} - \tilde{q}_{T_{i_1}}\right) + \left(\tilde{q}_{T_i} - \tilde{q}_{T_{i_2}}\right) + \left(\tilde{q}_{T_i} - \tilde{q}_{T_{i_3}}\right) > 0. \quad (11)$$

$V\left(\tilde{q}_T, \tilde{q}_{T_i}\right)$ Indicates the extent of improvement in eliminating the failure of the i[th] component. If $V\left(\tilde{q}_T, \tilde{q}_{T_i}\right) > V\left(\tilde{q}_T, \tilde{q}_{T_j}\right)$ then preventing failure of i[th] component is more effective than the preventing failure of j[th] component of the system.

9. Definitions

9.1. Definition 1

Let $\tilde{A} = \left[\left(a_1, b_1, c_1 : \lambda\right), \left(a_1, b_1, c_1 : \rho\right)\right]$ and $\tilde{B} = \left[\left(a_2, b_2, c_2 : \lambda\right), \left(a_2, b_2, c_2 : \rho\right)\right]$ are two i-v triangular fuzzy numbers then the failure probability $P(\tilde{A} \cup \tilde{B})$ for $\tilde{A} > 0$ and $\tilde{B} > 0$ can be defined using OR operator [9, 25] as

$$P(\tilde{A} \cup \tilde{B}) = 1\Theta\left[\left(1\Theta P(\tilde{A})\right) \otimes \left(1\Theta P(\tilde{B})\right)\right]. \quad (12)$$

9.2. Definition 2

Let $\tilde{A} = \left[\left(a_1, b_1, c_1 : \lambda\right), \left(a_1, b_1, c_1 : \rho\right)\right]$ and $\tilde{B} = \left[\left(a_2, b_2, c_2 : \lambda\right), \left(a_2, b_2, c_2 : \rho\right)\right]$ be two i-v triangular fuzzy numbers then the failure probability $P(\tilde{A} \cap \tilde{B})$ for $\tilde{A} > 0$ and $\tilde{B} > 0$ can be defined using AND operator [9, 22] as;

$$P(\tilde{A} \cap \tilde{B}) = P(\tilde{A}) \otimes P(\tilde{B}). \quad (13)$$

10. Methodology

The following steps are performed for developing proposed FFTA.

Step 1. Construct a fault tree of any healthcare related problem associated with intermediate and basic events.

Step 2. Transform bottom events failure possibilities in the form of interval-valued triangular fuzzy numbers with the help of expert knowledge and discussion.

Step 3. Calculate top event fuzzy failure probability using simple arithmetic operation and t-norm based fuzzy arithmetic operations on interval-valued triangular fuzzy numbers using operations given in Table 2, and expressions given in Table 3 (third row) for 'OR' and 'AND' gates which are based on eq. (11) and (13), respectively.

Step 4. With the help of functional of fuzzy numbers we associate the membership function with its membership degree in the form of a continuous function and analyse the change in fuzzy failure probability of top event.

Step 5. We Compute system top event fuzzy reliability, which is equal to one minus the fuzzy failure probability of the top event.

Step 6. Find the most and least influential bottom events using the definition of index V given in eq. (11), calculate $V\left(\tilde{q}_T, \tilde{q}_{T_i}\right)\forall i$ by eliminating the ith bottom event in the fault-tree diagram, and find the most and least influential events by finding max $V\left(\tilde{q}_T, \tilde{q}_{T_i}\right)$ and min $V\left(\tilde{q}_T, \tilde{q}_{T_i}\right)$ values, respectively for the whole system.

Step 7. Defuzzified top event failure probability and can be easily computed with the help of COG defuzzification method and proposed COG method for functional of fuzzy number and then analyze the results and give suggestions based on it for improving the performance of system.

11. Defuzzification

Defuzzification is the process of converting the fuzzy value to crisp value. There are various methods to transform a number into a fuzzy set and then defuzzified [25]. The simplest method to defuzzification is chose to set the highest membership function, another a common and useful defuzzification technique is centre of gravity. For the interval valued triangular fuzzy number we will take the mean of the COG of upper and lower triangular fuzzy numbers. In this paper we also use COG method to Interval-valued triangular functional fuzzy numbers.

Table 3. t-norm operation in i-v triangular fuzzy numbers

Operation	Fuzzy expression t-norm definition
(1) Addition	$\tilde{A} \oplus_{T_w} \tilde{B}$ = (b_1+b_2-max(b_1-a_1, b_2-a_2), b_1+b_2, , c_1+c_2+max(c_1-b_1, c_2-b_2))
(2) Multiplication	$\tilde{A} \otimes_{T_w} \tilde{B}$ = (b_1b_2-max((b_1-a_1)c_2, (b_2-a_2)c_1), b_1b_2, c_1c_2+max((c_1-b_1)a_2, (c_2-b_2)a_1)
(3) Subtraction	$\tilde{A}\Theta_{T_w} \tilde{B}$ = (b_1-b_2-max(b_1-a_1, c_2-b_2), b_1-b_2, b_1-b_2+max(c_1-b_1, b_2-a_2))
(4) Compliment	$1\Theta_{T_w} \tilde{B}$ = (1-c_2, , 1-b_2, 1-a_2)

11.1. COG Method

The centre of gravity defuzzification method can be expressed as

$$x^* = \frac{\int x \cdot \mu_A(x)dx}{\int \mu_A(x)dx} \quad (14)$$

From Fig. 1 we evaluate COG of both of the membership function upper and lower as follow;

$$x^{U*} = \frac{\int_a^b x\cdot\rho\left(\frac{x-a}{b-a}\right)dx + \int_b^c x\cdot\rho\left(\frac{c-x}{c-b}\right)dx}{\int_a^b \rho\left(\frac{x-a}{b-a}\right)dx + \int_b^c \rho\left(\frac{c-x}{c-b}\right)dx},$$

$$x^{U*} = \frac{a+b+c}{3}; \quad (15)$$

$$x^{L*} = \frac{\int_a^b x\cdot\lambda\left(\frac{x-a}{b-a}\right)dx + \int_b^c x\cdot\lambda\left(\frac{c-x}{c-b}\right)dx}{\int_a^b \lambda\left(\frac{x-a}{b-a}\right)dx + \int_b^c \lambda\left(\frac{c-x}{c-b}\right)dx},$$

$$x^{L*} = \frac{a+b+c}{3}. \quad (16)$$

Then average of both of the value we can get;

$$x^* = \frac{1}{2}\left[x^{U^*} + x^{L^*}\right], \quad (17)$$

$$x^* = \frac{a+b+c}{3}. \quad (18)$$

11.2. Proposed COG Method

In proposed methods the membership function possesses a variable degree of membership. In this method the centre of gravity define as;

$$x^* = \frac{\int x\cdot\mu_A(x)\cdot\mu(\mu_A(x))dx}{\int \mu_A(x)\cdot\mu(\mu_A(x))dx} \quad (19)$$

Let $\mu(\mu_A(x)) = \frac{1}{x^2+1}$, and $0 < \frac{1}{x^2+1} \le 1$, for any real value of x then

We evaluate the defuzzified value of each lower and upper trapezoidal fuzzy numbers and then we take the mean value of both defuzzification values.

$$x^{U*} = \frac{\int_a^b x\cdot\rho\left(\frac{x-a}{b-a}\right)\cdot\frac{1}{x^2+1}dx + \int_b^c x\cdot\rho\left(\frac{c-x}{c-b}\right)\cdot\frac{1}{x^2+1}dx}{\int_a^b \rho\left(\frac{x-a}{b-a}\right)\cdot\frac{1}{x^2+1}dx + \int_b^c \rho\left(\frac{c-x}{c-b}\right)\cdot\frac{1}{x^2+1}dx}, \quad (20)$$

$$x^{L*} = \frac{\int_a^b x\cdot\lambda\left(\frac{x-a}{b-a}\right)\cdot\frac{1}{x^2+1}dx + \int_b^c x\cdot\lambda\left(\frac{c-x}{c-b}\right)\cdot\frac{1}{x^2+1}dx}{\int_a^b \lambda\left(\frac{x-a}{b-a}\right)\cdot\frac{1}{x^2+1}dx + \int_b^c \lambda\left(\frac{c-x}{c-b}\right)\cdot\frac{1}{x^2+1}dx}, \quad (21)$$

$$x^{*}=\frac{1}{2}\left[x^{U^{*}}+x^{L^{*}}\right], \tag{22}$$

$$x^{U^{*}}=x^{L^{*}}=\frac{\left[\begin{array}{l}\frac{1}{b-a}\left\{b-a-\frac{a}{2}\ln\left(\frac{b^{2}+1}{a^{2}+1}\right)-\tan^{-1}\left(\frac{b-a}{1+ab}\right)\right\}- \\ -\frac{1}{c-b}\left\{c-b-\frac{c}{2}\ln\left(\frac{c^{2}+1}{b^{2}+1}\right)-\tan^{-1}\left(\frac{c-b}{1+bc}\right)\right\}\end{array}\right]}{\left[\begin{array}{l}\frac{1}{b-a}\left\{\frac{1}{2}\ln\left(\frac{b^{2}+1}{a^{2}+1}\right)-a\tan^{-1}\left(\frac{b-a}{1+ab}\right)\right\}- \\ -\frac{1}{c-b}\left\{\frac{1}{2}\ln\left(\frac{c^{2}+1}{b^{2}+1}\right)-c\tan^{-1}\left(\frac{c-b}{1+bc}\right)\right\}\end{array}\right]}, \tag{23}$$

$$x^{*}=\frac{1}{2}\left[x^{U^{*}}+x^{L^{*}}\right]=x^{U^{*}}\ \text{или}\ x^{L^{*}},\ \text{т.к.}\ x^{U^{*}}=x^{L^{*}}. \tag{24}$$

12. Example

12.1. Fault Tree of medication Pump

The FTA Diagram of medication pump failure [15] is shown in Fig.6. In this example of Fault tree contain four combination of failures which lead to top event, i.e. top event is associated with 'OR' gate, that means all are each independently associated. Similarly the pump and the alarm work together associated with AND gate.

Marx and Slonim [15] considered the failure probability of basic events 0.001(column 3 of Table 4) However, this

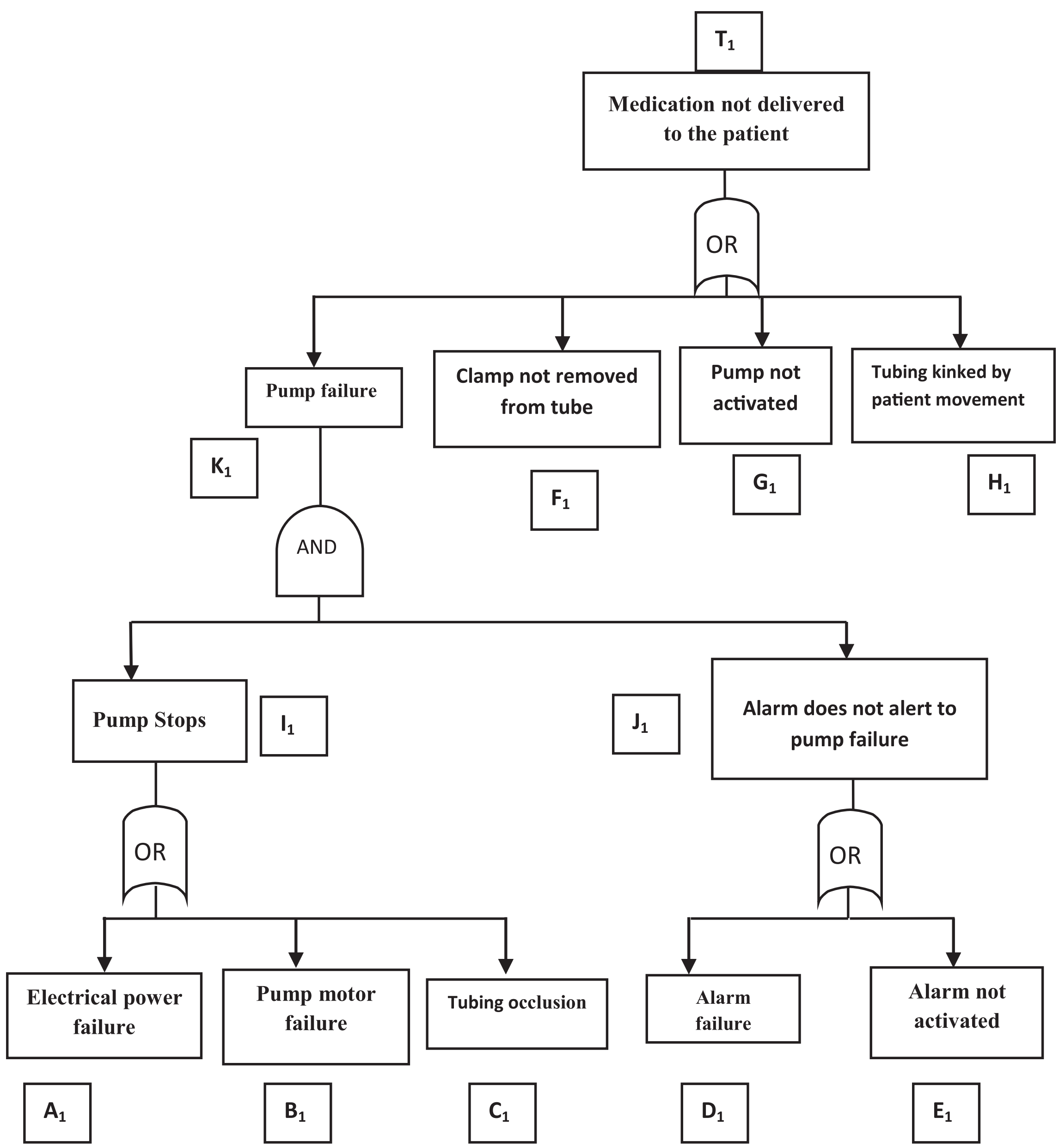

Fig. 6. Fault tree of medication pump

could not be possible for real system, and so we transform these values as different interval-valued TFNs (Triangular fuzzy numbers) as given in Table 4 (column 4).

Mathematical expression of event is given by;

$$T = K_1 \cup F_1 \cup G_1 \cup H_1 = (I_1 \cap J_1) \cup F_1 \cup G_1 \cup H_1 = \\ = ((A_1 \cup B_1 \cup C_1) \cap (D_1 \cup E_1)) \cup F_1 \cup G_1 \cup H_1. \quad (25)$$

By definition we can establish mathematical formula of this expression is given as:

$$q_{T_i} = 1 - \left[(1 - q_{K_1}) \times (1 - q_{F_1}) \times (1 - q_{G_1}) \times (1 - q_{H_1}) \right] = \\ = 1 - \left[(1 - q_{I_1} \times q_{J_1}) \times (1 - q_{F_1}) \times (1 - q_{G_1}) \times (1 - q_{H_1}) \right] = \\ = 1 - \left[(1 - (1 - (1 - q_{A_1}) \times (1 - q_{B_1}) \times (1 - q_{C_1})) \times \right. \\ \left. \times (1 - (1 - q_{D_1}) \times (1 - q_{E_1}))) \times (1 - q_{F_1}) \times (1 - q_{G_1}) \times (1 - q_{H_1}) \right]. \quad (26)$$

Now for calculating the output result for the crisp value, we use traditional method. For operate the fuzzy numbers, we use traditional fuzzy fault tree operation and in proposed method, we use the t-norm operation on interval-valued triangular fuzzy numbers. The operation between 'OR' gate and 'AND' gate in various method shown in Table 2 and Table 3.

13. Fuzzy Failure Probability and Reliability by various method

13.1. Max-Min Method

Huang et al. [9] max min method is applicable when the failure probability is extremely small. In this method we use maximum failure possibility among those events which are associated with 'OR' operation and minimum for which are associated with 'AND' operation. The crisp probability is in Table 4 (column 3)

$$P_{OSS}(I_1) = \max(P_{OSS}(A_1),\ P_{OSS}(B_1), P_{OSS}(C_1)) = \\ = \max(0.001, 0.001, 0.001) = 0.001,$$

$$P_{OSS}(J_1) = \max(P_{OSS}(D_1),\ P_{OSS}(E_1)) = \\ = \max(0.001, 0.001) = 0.001,$$

$$P_{OSS}(K_1) = \min(P_{OSS}(I_1),\ P_{OSS}(J_1)) = \\ = \min(0.001, 0.001) = 0.001.$$

Then, the top event failure possibility of top event "medication not delivered to the patient" is 0.001 and the reliability of "Medication delivered to the patient" is 0.999.

13.2. Tanaka et al. Method

Using Tanaka et al. method and Table 4 (column 4), the fuzzy failure probability of top event is $\begin{pmatrix} 0.0015511, 0.00297288494329, 0.004705098 : 0.8 \\ 0.0015511, 0.00297288494325, 0.004705098 : 1.0 \end{pmatrix}$ and fuzzy reliability is

$$\begin{pmatrix} 0.995294902, 0.99702711505671, 0.9984489 : 0.8 \\ 0.995294902, 0.99702711505671, 0.9984489 : 1.0 \end{pmatrix}.$$

Table 4. Transformation crisp value into triangular fuzzy numbers

Basic Event	Failure Probability	Crisp possibility	TFNs representation
A_1	$\tilde{q}_{A_1}$	0.001	$\begin{pmatrix} 0.0006, 0.0010, 0.0015 : 0.8 \\ 0.0006, 0.0010, 0.0015 : 1.0 \end{pmatrix}$
B_1	$\tilde{q}_{B_1}$	0.001	$\begin{pmatrix} 0.0006, 0.0010, 0.0015 : 0.8 \\ 0.0006, 0.0010, 0.0015 : 1.0 \end{pmatrix}$
C_1	$\tilde{q}_{C_1}$	0.001	$\begin{pmatrix} 0.00055, 0.0010, 0.0014 : 0.8 \\ 0.00055, 0.0010, 0.0014 : 1.0 \end{pmatrix}$
D_1	$\tilde{q}_{D_1}$	0.001	$\begin{pmatrix} 0.0006, 0.00095, 0.00145 : 0.8 \\ 0.0006, 0.00095, 0.00145 : 1.0 \end{pmatrix}$
E_1	$\tilde{q}_{E_1}$	0.001	$\begin{pmatrix} 0.0005, 0.0010, 0.0016 : 0.8 \\ 0.0005, 0.0010, 0.0016 : 1.0 \end{pmatrix}$
F_1	$\tilde{q}_{F_1}$	0.001	$\begin{pmatrix} 0.0005, 0.0010, 0.0016 : 0.8 \\ 0.0005, 0.0010, 0.0016 : 1.0 \end{pmatrix}$
G_1	$\tilde{q}_{G_1}$	0.001	$\begin{pmatrix} 0.00055, 0.00097, 0.0015 : 0.8 \\ 0.00055, 0.00097, 0.0015 : 1.0 \end{pmatrix}$
H_1	$\tilde{q}_{H_1}$	0.001	$\begin{pmatrix} 0.0005, 0.0010, 0.0016 : 0.8 \\ 0.0005, 0.0010, 0.0016 : 1.0 \end{pmatrix}$

13.3. t-norm proposed Method

Using the proposed t-norm operation and from **Table 3**, the fuzzy failure probability is

$$\begin{pmatrix} 0.00247417296853, 0.00297288494329, 0.00357225265339 : 0.8 \\ 0.00247417296853, 0.00297288494329, 0.00357225265339 : 1.0 \end{pmatrix},$$

and fuzzy reliability is

$$\begin{pmatrix} 0.99642774734661, 0.99702711505671, 0.99752582703147 : 0.8 \\ 0.99642774734661, 0.99702711505671, 0.99752582703147 : 1.0 \end{pmatrix}.$$

14. Defuzzification

14.1. Defuzzification by Tanaka et al. method

Defuzzifying by Tanaka et al. by traditional method using **eq.18** the failure probability of top event is 0.0814379427968 and reliability is 0.9185620572032.

14.2. Defuzzification of functional of interval-valued fuzzy probability of top event

Defuzzifying by proposed method the failure probability of top event using equation (23) and equation (24) is 0.0799309006739733 and reliability of top event is 0.920069099326026027.

The difference of both results is 0.00150704212282669 which is towards the left to failure probability obtained by traditional method.

15. Result

For obtaining the critical basic events of top event "Medication not delivered to the patient". We calculated the difference $V\left(\tilde{q}_{T_i}, \tilde{q}_{T_{1i}}\right)$ for each basic event using equation (11) and results are given in Table 7. Based on the value of index V in Table 7, it is analyzed that the most critical basic events are F_1 and H_1 whereas least critical basic events is C_1. The orders of all critical basic events are given below in decreasing manner;

$$(F_1, H_1) > G_1 > E_1 > D_1 > (A_1, B_1) > C_1. \quad (27)$$

The comparison between various methods is shown in **Table 5** and **Table 6.**

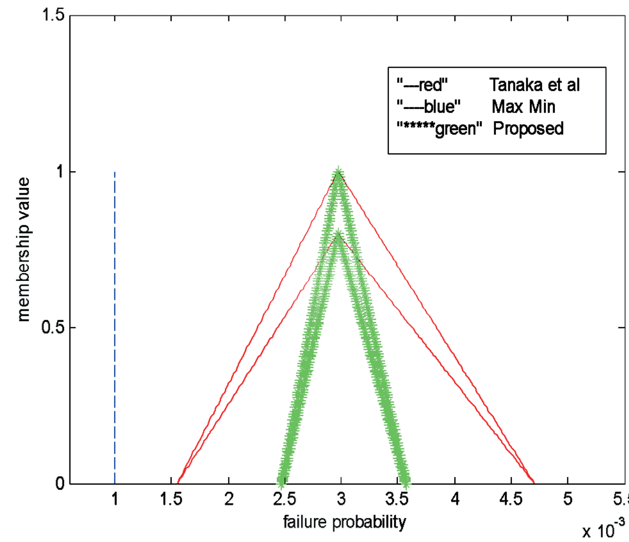

Fig. 7. Failure Probability of "Medication not delivered to Patient

Table 5. Comparison between Max-Min and Tanaka et al. method

Membership value	Max-Min Method	Tanaka et.al method			
		Left Points		Right Points	
0.1	0.001	0.00169327849433	0.00172882311791	0.00448857136791	0.00453187669433
0.2	0.001	0.00183545698866	0.00190654623582	0.00427204473582	0.00435865538866
0.3	0.001	0.00197763548299	0.00208426935373	0.00405551810373	0.00418543408299
0.4	0.001	0.00211981397732	0.00226199247165	0.00383899147164	0.00401221277732
0.5	0.001	0.00226199247165	0.00243971558956	0.00362246483956	0.00383899147164
0.6	0.001	0.00240417096597	0.00261743870747	0.00340593820747	0.00366577016597
0.7	0.001	0.00254634946030	0.00279516182538	0.00318941157538	0.00349254886030
0.8	0.001	0.00268852795463	0.00297288494329	0.00297288494329	0.00331932755463
0.9	0.001	0.00283070644896			0.00314610624896
1	0.001	0.00297288494329			0.00297288494329

Table 6. Comparison between max-min and t-norm proposed method

Membership value	Max-Min Method	Proposed t-norm method			
		Left Points		Right Ponts	
0.1	0.001	0.00252404416601	0.00253651196538	0.00349733168963	0.00351231588238
0.2	0.001	0.00257391536348	0.00259885096222	0.00342241072586	0.00345237911137
0.3	0.001	0.00262378656096	0.00266118995906	0.00334748976210	0.00339244234036
0.4	0.001	0.00267365775843	0.00272352895591	0.00327256879834	0.00333250556935
0.5	0.001	0.00272352895591	0.00278586795275	0.00319764783458	0.00327256879834
0.6	0.001	0.00277340015339	0.00284820694960	0.00312272687081	0.00321263202733
0.7	0.001	0.00282327135086	0.00291054594644	0.00304780590705	0.00315269525632
0.8	0.001	0.00287314254834	0.00297288494329	0.00297288494329	0.00309275848531
0.9	0.001	0.00292301374581			0.00303282171430
1	0.001	0.00297288494329			0.00297288494329

Table 7. Ranking of basic event of Example 1 using failure difference

Eliminated event	$\tilde{q}_{T_{1i}}$	$V\left(\tilde{q}_{T_i},\tilde{q}_{T_{1i}}\right)$	Rank
$A_1(i=1)$	$\begin{pmatrix}0.0024722231738, 0.00297094556083, 0.00357031413827 : 0.8\\ 0.0024722231738, 0.00297094556083, 0.00357031413827 : 1.0\end{pmatrix}$	0.000005827692319999422	5
$B_1(i=2)$	$\begin{pmatrix}0.00247222317380.002970945560830.00357031413827 : 0.8\\ 0.00247222317380.002970945560830.00357031413827 : 1.0\end{pmatrix}$	0.000005827692319999422	5
$C_1(i=3)$	$\begin{pmatrix}0.00247222332310.00297094556083, 0.00357031413826 : 0.8\\ 0.00247222332310.00297094556083, 0.00357031413826 : 1.0\end{pmatrix}$	0.000005827635119999477	6
$D_1(i=4)$	$\begin{pmatrix}0.00247133568504, 0.002970004907872, 0.00356941849523 : 0.8\\ 0.00247133568504, 0.002970004907872, 0.00356941849523 : 1.0\end{pmatrix}$	0.000008551477067999993	4
$E_1(i=5)$	$\begin{pmatrix}0.002471186078590.00296989967328, 0.00356926886526 : 0.8\\ 0.002471186078590.00296989967328, 0.00356926886526 : 1.0\end{pmatrix}$	0.000008955948079999217	3
$F_1(i=6)$	$\begin{pmatrix}0.001475612719830.00197485980309, 0.00257452629985 : 0.8\\ 0.001475612719830.00197485980309, 0.00257452629985 : 1.0\end{pmatrix}$	0.00299431174244	1
$G_1(i=7)$	$\begin{pmatrix}0.001505632544530.00200482962803, 0.00260452717178 : 0.8\\ 0.001505632544530.00200482962803, 0.00260452717178 : 1.0\end{pmatrix}$	0.00290432122087	2
$H_1(i=8)$	$\begin{pmatrix}0.001475612719830.00197485980309, 0.00257452629985 : 0.8\\ 0.001475612719830.00197485980309, 0.00257452629985 : 1.0\end{pmatrix}$	0.00299431174244	1

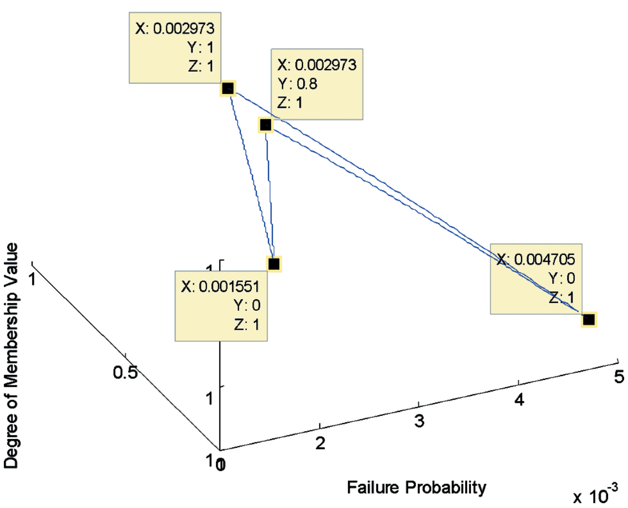

Fig. 8. Functional fuzzy failure Probability of Top Event

16. Conclusion

This whole study developed FTA, t-norm and functional fuzzy numbers for interval valued triangular fuzzy numbers. This study consist two types of healthcare related problems and their defuzzification methods to analyse the reliability from various existing techniques and therefore we conclude that t norm method gives small accumulation and how functional fuzzy number changes the reliability. Similarly from V index method we found the least critical events about any system and health experts can use these methods to reduce the failure probability of any clinical process.

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The authors' contribution

Kapil Naithani changed the fuzzy problem of a clinical system into a functional fuzzy set and applied the centre of gravity defuzzification methods in functional value and compare with other defuzzification techniques in fuzzy set to find fluctuation of reliability value among different methods.

Rajesh Dangwal gave an example of a clinical methods and a fault tree with the failure probability of basic events to find exact possibility of failure of events to increase the reliability of system.