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An analysis of estimate bias of steady-state availability for various test plans

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Abstract. *Any process of technical product development may involve dependability testing. If in the course of operation, the recovery of an entity after a failure is the norm, then test plans of types NRect, NRecR, NNoRect и NNoRecR are normally used, where N is the number of tested same-type entities; t is the testing time of each of the N entities; R is the number of failures; Rec (NoRec) is the characteristic of the plan that indicates that the entity's operability after each failure within the testing time is recovered (not recovered). Normally, NRect and NRecR indicate that, in the process of testing, failures are recovered immediately. In order not to confuse plans NRect, NRecR, NNoRect and NNoRecR with test plans with long recovery times, let us denote the latter as NRec!t, NRec!R, NNoRec!t and NNoRec!R respectively. Let us simplify the problem description and require, for test plans of types NRec!t, NRec!R, NNoRec!t and NNoRec!R, the fulfilment of condition D = R, where D is the number of recoveries, i.e. after the conclusion of testing, at the moment of time t, the recovery of entities continues until the last of R failed entities is recovered. We will denote such test plans NRec!t(D=R), NRec!R(D=R), NNoRec!t(D=R) and NNoRec!R(D=R). As the dependability model, an exponential distribution is adopted. Steady-state availability is normally defined as the composite dependability indicator of recoverable entities. Finding efficient estimates is one of the primary goals of the dependability theory. Since the 1960s, Russian scientific literature has featured next to no research dedicated to the properties of steady-state availability estimates. The best known work in the steady-state availability estimates for a NRecR test plan is in the book: Beletsky B.R. [Dependability theory of radio engineering systems (mathematical foundations). Study guide for colleges]. Moscow: Sovetskoye radio; 1978. This paper makes up for this deficiency. In order to identify the efficient steady-state availability estimate out of infinite many, first, an estimate efficiency comparison criterion is to be constructed. The paper* **Aims** *to construct a simple criterion of steady-state availability estimation for test plans with long recovery times and identify the efficient estimate out of the available ones using the constructed criterion.* **Methods of research.** *The efficient estimate was found using integral numerical characteristics of the accuracy of estimate, i.e., the sum square of the displacement of the expected realization of an estimate from the considered parameters of the distribution laws.* **Conclusions.** *The authors constructed simple criteria of efficiency of steady-state availability estimation for test plans with long recovery time (case of N≥1). Estimate $G_3=(1+VR/S(R1))^{-1}$ is bias-efficient out of those available for test plans of types NRec!t(D=R) and NNoRec!t(D=R). Conventional estimate $G_1=(1+V/S)^{-1}$ is bias-efficient out of those available for test plans of types NRec!R(D=R) and NNoRec!R(D=R).*

Keywords: *estimation, efficient estimate, efficiency criterion, availability coefficient, steady-state availability.*

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Introduction

Any process of technical product development may involve dependability testing. If in the course of operation, the recovery of an entity after a failure is the norm, then test plans of types *NRect*, *NRecR, NNoRect* и *NNoRecR* are normally used, where *N* is the number of tested same-type entities; *t* is the testing time of each of the *N* entities; *R* is the number of failures; *Rec* (*NoRec*) is the characteristic of the plan that indicates that the entity's operability after each failure within the testing time is recovered (not recovered) [1–3]. Normally, *NRect* and *NRecR* indicate that, in the process of testing, failures are recovered immediately. In order not to confuse plans *NRect*, *NRecR, NNoRect* and *NNoRecR* with test plans with long recovery times, let us denote the latter as *NRec!t*, *NRec!R, NNoRec!t* and *NNoRec!R* respectively.

Let us simplify the problem description and require, for test plans of types *NRec!t*, *NRec!R, NNoRec!t* and *NNoRec!R*, the fulfilment of condition $D = R$, where D is the number of recoveries, i.e., after the conclusion of testing, at the moment of time t, the recovery of entities continues until the last of R failed entities is recovered. We will denote such test plans *NRec!t(D=R)*, *NRec!R(D=R), NNoRec!t(D=R)* and *NNoRec!R(D=R)*. As the dependability model, an exponential distribution is adopted.

Steady-state availability is normally defined as the composite dependability indicator of recoverable entities. Steady-state availability is defined as the probability that an entity will be in the up state at the given moment in time sufficiently remote from the start of the tests[1].

The formula for steady-state availability (C_a) used in practice is as [1–3]

$$C_a = T/(T+H) = 1/(1+H/T),$$

where $T = 1/\lambda$ is the mean time to failure of an entity, where λ is the failure rate of an entity, $H=1/h$ is the mean time to recovery (replacement) of an entity, where h is the renewal (replacement) rate of an entity. As the availability coefficient estimate, the following general formula is used

$$G = \check{T} / (\check{T} + \hat{H}) = 1 / (1 + \hat{H} / \check{T}),$$

where $\check{T}$ is the estimate of the mean time to failure based on the results of entities' testing, $\hat{H}$ is the estimate of the mean time to restoration based on the results of entities' testing. Out of the form of estimate G follows that there are infinite many estimates of steady-state availabilities C_a. For instance, for test plans of types *NRect* and *NRecR,* the estimate[2] $\check{T} = Nt / (R + 1)$, $R \geq 1$ and $\check{T} = Nt / R$, $R \geq 1$ [1–4] respectively can be chosen as $\check{T}$. As the formula for $\hat{H}$, $\hat{H} = V/D = R$ is normally chosen, where $R \geq 1$, V is the total recovery time. Finding efficient estimates is one of the primary goals of the dependability theory. Since the 1960s, Russian scientific literature has featured next to no research dedicated to the properties of steady-state availability estimates. The best-known work in the steady-state availability estimates for a *NRecR* test plan is in [3]. This paper makes up for this deficiency.

In order to identify the efficient steady-state availability C_a out of infinite many, first, an estimate efficiency comparison criterion is to be constructed.

The Aim of the paper

The paper aims to construct a simple criterion of steady-state availability estimation for test plans with long recovery times ($N \geq 1$) and identify the efficient estimate out of the available ones using the constructed criterion.

Methods of research

The efficient estimate was found using integral numerical characteristics of the accuracy of estimate, i.e. the sum square of the displacement of the expected realization of an estimate from the considered parameters of the distribution laws [4].

Constructing the efficient criterion of steady-state availability estimate for test plan of type *NRec!t(D=R)*

Let us examine the test plan of type *NRec!t(D=R)*. Such tests are primarily intended for steady-state availability estimation [1–3] (determinative tests). Let there be a certain number of entities undergoing tests and, in the process of testing, the time to failure and restoration time of an entity are random values and follow the exponential probability distribution [1–3]. In the course of testing, an entity may be in either of two states: up and down. In case of failure, an entity is recovered. Its operation time is restored in full (through replacement or repairs), which, in the process of testing, allows considering the parameters of the distribution law invariant.

Thus, an *NRec!t(D=R)* test can be represented as two sets of tests conducted according to classic plans *NRect* (recovery can be considered immediate) and *(N=R)Rec(D=R)* (we assume that failures occur immediately), i.e., a plan with limited testing time and a plan with a limited number of recoveries (is a random value) respectively.

In the process of *NRect* testing, a Poisson failure flow with the rate of $N\lambda$ can be observed [1–3]. Without violating the integrity of reasoning, we will denote the rate $N\lambda$ of the combined failure flow as λ, i.e., λ is equivalent to $N\lambda$, which should not cause confusion, then the set of independent tests is represented as the testing of one entity $N = 1$ characterized by the combined failure flow.

Let us introduce the following designations:

S, total operation time of entities;

V, total recovery time.

[1] GOST 27.002-2015. Dependability in technics. Terms and definitions. Moscow: Standartinform; 2016. (in Russ.)

[2] GOST R 50779.26-2007. Statistical methods. Point estimates, confidence intervals, prediction intervals and tolerance intervals for exponential distribution. Moscow: Standartinform; 2008. (in Russ.)

Many estimates of $G(D = R, R, S = Nt, V)$ can be proposed. Comparing them efficiency-wise requires constructing an efficiency criterion of steady-state availability estimation. For that purpose, let use the experience of such construction presented in [4]. The estimate G is considered to be the most bias-efficient compared to other estimates if its expectation EG has the lowest bias from the true steady-state availability C_a that always depends on the parameters of the distribution laws λ, h. The bias (m) in most cases is defined as the square of deviation EG from the adopted availability values, namely:

$$m(G, C_a) = (EG - C_a)^2.$$

In principle, the form of estimates G of steady-state availability may have any functional form $G(N, S = Nt, V, D = R, R, \ldots)$, $N = 1$. In this paper, we should restrict ourselves to the simple form, namely ($S = Nt$):

$$G_1 = \frac{1}{1+\frac{V}{S}\cdot\frac{R}{D=R}} \text{ (conventional estimate)},$$

$$G_2 = \frac{1}{1+\frac{V}{S}\cdot\frac{R+1}{D=R}},\ G_3 = \frac{1}{1+\frac{V}{S}\cdot\frac{R}{R+1}}.$$

Then, assuming that $EG(V, R)$ is finite, $EG(V, R) = E_R(E_V(G|R))$ [6].

For each of the R failed entities, the density of the probability function for a sum of independent identically distributed random variables of recovery time τ_{vij} with distribution density $he^{-ht} - V$ has the form of a special Erlang distribution $F\left(D=R, R=r, H=10^j, T=10^i\right) = E\left(G-\frac{T}{T+H}\right)^2$ [1, 2, 5]. Conditional expectation E_V of estimate G of steady-state availability has the form:

$$E_V(G\,|\,R, D=R, h) = \int_0^\infty G\frac{h(hV)^{(D-1)}e^{-hV}}{(D-1)!}dV.$$

The failure flow is a Poisson flow with distribution function $L(r) = \sum_{k=0}^{r} e^{-\Delta}\frac{\Delta^k}{k!}$, $(\Delta = \lambda t)$. The expectation EG is calculated according to formula [1–3]:

$$EG = E_R\left(E_V G\right) = \sum_{r=0}^{\infty}\left(E_V G\right)\cdot e^{-\Delta}\frac{\Delta^r}{r!}.$$

Bias $m(G)$, like C_a, also depends on the parameters of the chosen distribution laws (T, H). In order to construct the efficiency criterion of steady-state availability estimation, we should summarize the displacement per all parameters of the selected distribution laws (T, H) and test plan ($R=r$, t, $N=1$):

$$A(G) = \int_{10^3}^{10^5}\int_{10^1}^{10^4}\int_{10^4}^{10^7} m(G)dHdTdt. \quad (1)$$

Let us note that parameter N of the $NRec!t(D = R)$ test plan is not critical and, without violating the integrity of reasoning, is taken equal to one $N = 1$. If we do not restrict the summation, the value of the constructed functional $A(G)$ for most estimates will always be infinite. Therefore, the limits of summation (expressed in hours) are restricted by reasonable intervals of the values of parameters (T, H, t, V) and $R = 10$.

Among the estimates with minimal total bias $A(G)$, the one with the minimal total deviation is to be considered efficient. For that purpose, the functional is constructed:

$$B(G) = \int_{10^3}^{10^5}\int_{10^1}^{10^4}\int_{10^4}^{10^7} y(G)dHdTdt, \quad (2)$$

where $y(G) = E_R(E_V((G - K_г)^2|R))$ [4, 6],

$$E_V\left(\left(G-K_г\right)^2|\,R, D=R, h\right) = \int_0^\infty \left(G-K_г\right)^2\frac{h(hV)^{(D-1)}e^{-hV}}{(D-1)!}dV.$$

Direct calculation of functionals $A(G)$ and $B(G)$ (formulas (1) and (2)) is quite complicated, as it requires significant computational powers. Therefore, formulas (1) and (2) should be simplified and given a more practical form according to ($R = r$)

$$A_1 = \sum_{k=3}^{5}\sum_{j=1}^{4}\sum_{i=4}^{7}(C)^2, \quad (3)$$

$$\text{where } C = \left[\sum_{r=1}^{10} E_V G\left(D=r, R=r, H=10^j, t=10^k\right)\cdot e^{-\frac{t=10^k}{T=10^i}}\frac{\left(-t=10^k\right)^r}{\left(T=10^i\right)^r r!}\right] - \frac{10^i}{10^i+10^j},$$

$$B_1 = \sum_{k=3}^{5}\sum_{j=1}^{4}\sum_{i=4}^{7}\sum_{r=1}^{10} e^{-\frac{t=10^k}{T=10^i}}\frac{\left(-t=10^k\right)^r}{\left(T=10^i\right)^r r!}E_V F, \quad (4)$$

$$\text{where } F = \left[G\left(D=r, R=r, t=10^k\right) - \frac{10^i}{10^i+10^j}\right]^2.$$

The results of substitution into formulas (3) and (4) of the suggested estimates of steady-state availability for the $NRec!t(D = R)$ test plan are shown in Table 1.

Table 1. Results of substitution into formulas (1) and (2) of the suggested steady-state availability estimates for test plan of type *NRec!t(D = R)*

	G_1	G_2	G_3
$A1$	34.057	34.263	33.906
1 – 1000	114	170	93

Out of Table 1 follows that, for the test plan of type $NRec!t(D=R)$, estimate $G_3 = (1 + VR / S(R + 1))^{-1}$ is bias-efficient out of those available.

Example. Two items of complex equipment were submitted to trial operation for a period of three months (2190 h). As the dependability indicator, the specifications indicated the steady-state availability $C_a = 0.92$. In the course of trial operation, a failure was detected. Due to complicated logistics, the equipment spent 500 h under repairs.

Let us examine two solutions:

1) Conventional estimate

Based on the results of trail operation, the estimate of steady-state availability was

$G_1 = (1 + V / S)^{-1} = (1 + 500 / (2190 + 2190))^{-1} = 0.897$, which does not comply with the specifications.

2) Using the efficient estimate of steady-state availability $G_3 = (1 + VR / S(d + 1))^{-1}$.

Based on the results of trial operation, the estimate of steady-state availability was

$G_3 = (1+VR/S(R1))_{-1} = (1500/2(21902190))-1 = 0.946$, which does not comply with the specifications.

Constructing the efficient criterion of steady-state availability estimate for test plan of type *NNoRec!t(D=R)*

Let us examine the test plan of type *NNoRec!t(D=R)*. An *NNoRec!t(D=R)* test can be represented as two sets of tests conducted according to classic *NNoRect* plans (no recovery) and *(N = R)NoRec(D = R)* i.e., respectively, according to the binomial plan and a plan with a limited number of recoveries (a random value).

Let us examine the test plan of type *(N = R)NoRec(D = R)*. As in the previous section, for each of the *R* tested (or failed, in case of the *NNoRect*) entities, the density of the probability function for a sum of independent identically distributed random variables of recovery time τ_{vij} with distribution density $he^{-ht} – V$ has the form of a special Erlang distribution [1, 2, 5]. Then, the conditional expectation E_V of estimate *G* of steady-state availability is calculated according to formula:

$$E_V(G \mid R, D=R, h) = \int_0^{\infty} G \frac{h(hV)^{(D-1)} e^{-hV}}{(D-1)!} dV.$$

Let random value *R* have binomial distribution $p_N(R = r)$ [7, f. 1.4.55] with parameters *N* and $p,\ 0 \le p \le 1$, i.e., $R = r$ equal to the number of successes in a series of *N* independent experiments with the probability of success $p = 1 – e^{-\lambda t}$ assumes integer value 0, 1, 2, …, *N* with probabilities $p_N(r) = C_N^r p^r (1-p)^{N-r}$.

Then, expectation $EG(V, R) = E_R(E_V(G|R))$ has the form of

$$EG(V,R) = \sum_{r=0}^{N} p_N(r) E_V(G \mid R = r).$$

Similarly (see the previous section), the following expectation is constructed:

$$EG\left((G-K_г)^2\right) = \sum_{r=0}^{N} p_N(r) E_V\left((G-K_г)^2 \mid R=r\right).$$

In order to construct the efficiency criterion of steady-state availability estimation, we should summarize the displacement per all parameters of the selected distribution laws *(T, H)* and test plan ($R = r$, *t*, $N \le 10$):

$$A(G) = \sum_{N=1}^{10} \int_{1000}^{10^5} \int_{1}^{10^4} \int_{10^4}^{10^7} m(G)\, dH dT dt, \qquad (5)$$

$$B(G) = \sum_{N=1}^{10} \int_{1000}^{10^5} \int_{1}^{10^4} \int_{10^4}^{10^7} y(G)\, dH dT dt. \qquad (6)$$

Direct calculation of functionals *A(G)* and *B(G)* (formulas (5) and (6)) is quite complicated, as it requires significant computational powers. Therefore, formulas (5) and (6) transform as follows:

$$A_1 = \sum_{N=1}^{10} \sum_{k=3}^{5} \sum_{j=1}^{4} \sum_{i=4}^{7} (C)^2, \qquad (7)$$

where

$$C = \begin{bmatrix} \sum_{r=1}^{N} E_V G\left(D=r, R=r, N, H=10^j, t=10^k\right) \\ \cdot C_N^r p^r (1-p)^{N-r} \end{bmatrix} - \frac{10^i}{10^i + 10^j},$$

$$B_1 = \sum_{N=1}^{10} \sum_{k=3}^{5} \sum_{j=1}^{4} \sum_{i=4}^{7} \sum_{r=1}^{N} C_N^r p^r (1-p)^{N-r} E_V F, \qquad (8)$$

where $F = \left[G\left(D=r, R=r, N, t=10^k\right) - \frac{10^i}{10^i + 10^j} \right]^2$.

Let us note that total operation time *S* is calculated based on average $S = R \cdot t / 2 + (N – R) \cdot t$.

The results of substitution into formulas (7) and (8) of the suggested estimates of steady-state availability for the test plan of type *NNoRec!t(D=R)* are shown in Table 2.

Table 2. Results of substitution into formulas (1) and (8) of the suggested steady-state availability estimates for test plan of type *NNoRec!t(D = R)*

	G_1	G_2	G_3
A1	271	272	270
B1·100	240	272	239

Out of Table 1 follows that, for the *NNoRec!t(D = R)* test plan, estimate $G_3 = (1 + VR / S(R + 1))^{-1}$ is bias efficient out of those available.

Constructing the efficient criterion of steady-state availability estimate for test plan of type *NRec!R(D=R)* and *NNoRec!R(D=R)*

An *NNoRec!R(D=R)* test can be represented as two sets of tests conducted according to classic *NNoRecR* (no recovery) and *(N = R)NoRec(D = R)* plans, i.e., respectively, according to the binomial plan with a limited number of recoveries that, in this case, is not a random value. A similar *NRec!R(D = R)* test can be represented as two sets of tests conducted according to classic *NRecR* (with recovery) and *(N = R)Rec(D = R)* plans.

For any of R failed entities, the duration of recovery τ_{vij} and operation τ_{ij}, $i = 1, 2, \dots, R$; $j = 1, 2, \dots, N$ do not depend on each other and each one has its own distribution density he^{-ht} and $\lambda e^{-\lambda t}$ respectively. The dependence of the occurrence of failures and, consequently, recoveries does not affect the duration of recovery τ_{vij} and operation τ_{ij}. Consequently, the S, V are independent too. For the purpose of constructing expectation EG, the distribution function of the sums of S, V and number of failures R must be known. For each of the $R = r$ $(D = d = r)$ entities submitted to recovery, the density of the probability function for the sum of independent identically distributed random variables of recovery time τ_{vij} with distribution density $he^{-ht} - V$ has the form of a special Erlang distribution $\frac{h(hV)^{(d-1)}e^{-hV}}{(d-1)!}$ [1, 2, 5]. For each of the $R = r$ failed entities, the density of the probability function for the sum of independent identically distributed random times to failure with distribution density $\lambda e^{-\lambda t} - S$ also has the form of a special Erlang distribution $\frac{\lambda(\lambda S)^{(r-1)}e^{-\lambda S}}{(r-1)!}$. The same is true for the set R of failed entities [1, 2, 5].

Then, the expectation of estimate G of steady-state availability is calculated according to formula

$$E(G, D = d = r, R = r, \lambda, h) = \\ = \int_0^\infty\int_0^\infty G \frac{h(hV)^{(d-1)}e^{-hV}}{(d-1)!}\frac{\lambda(\lambda S)^{(r-1)}e^{-\lambda S}}{(r-1)!}dVdS.$$

Similarly,

$$E\left((G-K_r)^2, D = d = r, R = r, \lambda, h\right) = \\ = \int_0^\infty\int_0^\infty (G-K_r)^2 \frac{h(hV)^{(d-1)}e^{-hV}}{(d-1)!}\frac{\lambda(\lambda S)^{(r-1)}e^{-\lambda S}}{(r-1)!}dVdS.$$

In order to construct the efficiency criterion of steady-state availability estimation, we should summarize the displacement per all parameters of the selected distribution laws *(T, H)* and (*R = 10*) test plan:

$$A(G) = \sum_{r=1}^{10}\int_1^{10^4}\int_{10^4}^{10^7} m(G)dTdH, \tag{9}$$

$$B(G) = \sum_{r=1}^{10}\int_1^{10^4}\int_{10^4}^{10^7} E(G-K_z)^2 dHdT. \tag{10}$$

Direct calculation of functionals *A(G)* and *B(G)* (formulas (9) and (10)) is quite complicated, as it requires significant computational powers. Therefore, formulas (9) and (10) should be simplified and given a more practical form respectively:

$$A_1 = 10^4 \sum_{j=1}^{4}\sum_{i=4}^{7}\sum_{r=1}^{10} C\left(D = R, R = r, H = 10^j, T = 10^i\right), \tag{11}$$

where

$$C\left(D = R, R = r, H = 10^j, T = 10^i\right) = \left(EG - \frac{T}{T+H}\right)^2,$$

$$B_1 = 10^2 \sum_{j=1}^{4}\sum_{i=4}^{7}\sum_{r=1}^{10} F\left(D = R, R = r, H = 10^j, T = 10^i\right), \tag{12}$$

where

$$F\left(D = R, R = r, H = 10^j, T = 10^i\right) = E\left(G - \frac{T}{T+H}\right)^2.$$

A normalizing factor is introduced into formulas (11) and (12), simplifying the form of the result. The results of substitution of the suggested steady-state availability estimates into formulas (11) and (12) for test plans of type *NRec!R(D=R)* and *NNoRec!R(D=R)* are shown in Table 3.

Table 3. Results of substitution of the suggested steady-state availability estimates into formulas (11) and (12) for test plan of type *NRec!R(D=R)* and *NNoRec!R(D=R)*

	G_1	G_2	G_3
*A*1	202	1199	325
1	49	66	42

Out of Table 3 follows that estimate $G_1 = (1 + V / S)^{-1}$ is the bias-efficient out of those available ones.

Conclusions

1) The authors constructed simple criteria of efficiency of steady-state availability estimation for test plans with long recovery time (case of $N \geq 1$).

2) Estimate $G_3 = (1 + VR / S(R+1))^{-1}$ is the bias-efficient out of those available for test plans of types *NRec!t(D = R)* and *NNoRec!t(D = R)*.

3) Conventional estimate $G_1 = (1 + V/S)^{-1}$ is bias-efficient out of those available for test plans of types *NRec!R(D = R)* and *NNoRec!R(D = R)*.

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The authors' contribution

The authors constructed simple criteria of efficiency of steady-state availability estimation for test plans with long recovery time (case of $N \geq 1$). Efficient estimates were obtained out of those considered.

Dmitry M. Rudkovsky constructed the efficient criterion of steady-state availability estimate for test plans of type *NRec!R(D = R)* and *NNoRec!R(D = R)*.

Viktor S. Mikhailov constructed the efficient criterion of steady-state availability estimate for test plans of type *NRec!t(D = R)* and *NNoRec!t(D = R)*.

Conflict of interests

The authors declare the absence of a conflict of interests.