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A method of accounting for censored items as part of fatigue testing of composite materials

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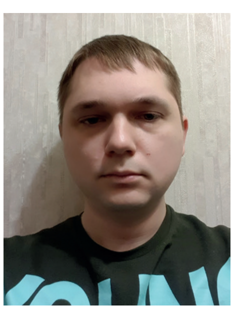

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Abstract. *The* **Aim** *of the paper is to create a method for including information on censorship for the purpose of adjusting the estimates of the S-N diagram of composite materials. Censored items are such items that did not fail by the end of testing, for which a certain operation time has been registered. It must be noted that, currently, researchers often ignore the operation time data of the items that did not fail by the end of testing, which does not appear to be justified in terms of cost saving and reliability of statistical conclusions. Censorship information is very important in terms of assessing durability. It only needs the right tool to use it. The proposed* **Method** *consists in bootstrapping-based simulation, a method from the group of computer-intensive methods. In the process of the method's development, the previously used approaches (for instance, as regards metals) were considered. In the examined example of the method's application, the data was taken from literary sources.* **Results.** *The paper shows an example of fatigue testing conducted by the authors that produced a large number of censored items. The results obtained using the method were compared with real data. It is shown that the quality of statistical estimation improves with the use of the method. The paper sets forth certain observations regarding the mechanical testing tool for quality control. The source of data dispersion associated with fatigue testing is discussed.* **Conclusions.** *The application of the method will help include the information on censored items in the estimation of the S-N diagram. For scientists who are involved in the experimental research of the fatigue resistance of composite materials, the suggested method might prove to be quite useful. It takes into consideration the characteristic features of the strength analysis of composite materials (large properties dispersion and absence of unlimited fatigue range). The method will allow taking onto account an important, but not always so far used information on the items that operated a certain number of cycles, but did not fail.*

Keywords: *polymer composite, properties dispersion, fatigue, censored samples, bootstrapping.*

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1. Introduction

The research of the mechanical properties of polymer composite materials (PCM) shows that they have serious advantages over the conventional structural metals and alloys that are mainly due to high specific characteristics of static strength. A polymer composite is a multiphase material, in which the reinforcement fillers are integrated with a polymer matrix, which causes synergistic mechanical properties that cannot be achieved by any of the individual components. At the same time, thanks to the scientific and technical progress, the cost of manufacture of various composite materials are on a continuous decline, while the quality is improving. Thus, this type of materials finds new applications. Due to the growing use of composite materials [1], currently, priority is given to quality requirements. Some advanced testing methods are successfully applied not only for technical state diagnostics but also for quality control [2, 3].

In [4, 5], the experience of application of advanced methods of nondestructive (NDI) and destructive inspection is described. Among other things, the items made of composite materials were submitted to Through Transmission Ultrasonic NDI (TTU) control that generated C-scans for quantitative estimation of flat damaged areas using image analysis software. Additional methods of inspection, e.g., microscopy and thermal imaging are also used in the research of damage resistance. Such extended capabilities may provide additional information on censored items [4]. In [6], the authors noted certain causes affecting the quality and reliability of the estimated values that do not allow obtaining stable and reliable physico-mechanical values. In [6], the focus is on the deformations of the manufactured items and type of their physico-mechanical failure.

Along with modern physical methods (see, e.g., [5]), of unchanging relevance in the context of quality analysis of composite materials is the research of the mechanical characteristics of entities, such as strength in static testing (tension, compression, shearing), stiffness (elasticity modulus). In [7], the shearing strength of carbon-reinforced composite materials was tested for the purpose of outlier analysis of the shearing strength values of items made of composite materials. Fig. 1 shows box-and-whisker diagrams [7, 8] for several batches of items. A significant dispersion can be observed even within a single batch. The aim of the research [7] was to develop a method of outlier removal. In this particular example, the indicator τ = 83.9 MPA was assumed to be an outlier, which meant that this value was subsequently removed and was not used for further analysis.

The fatigue strength characteristics stand apart. Despite the fact that composite materials are more and more frequently used in aircraft, spacecraft and other products exposed to repeated loads, there is still an opinion that the static strength of composite materials is a sufficient indicator of strength. That is partly due to the fact that, in reality, the slope of the S-N diagram *m* of items made of composite materials is indeed very high as compared with the similar value for metals. If for metals, the value $m = 5 \ldots 9$ (indicator of the slope of the S-N diagram) is typical, in composite materials an $m = 15$ can often be observed. Such a high value of *m* primarily implies a significant scatter of the fatigue properties, not surprisingly. For composite materials, a large dispersion is the case for static characteristics as well (see Fig. 1), and, as noted above, fatigue always increases dispersion [9]. Two important factors increase the dispersion of the fatigue characteristics of composite materials. First, even during static testing, the strength of composite materials demonstrates a large dispersion as compared with metals. Second, fatigue-related dispersion is always higher than during static testing.

This research involved compression tests on small-size items. Fig. 2a shows the initial item, 2b shows the item that failed as the result of fatigue testing. The items were made of carbon fiber and binding substance based on epoxy and phenolformaldehyde resins normally used in the manufacture of aircraft and spacecraft products and components. At the first stage, the items were submitted to static compression testing. Table 1 shows the summary of the static $\sigma_c^{\text{в}}$ compression testing. The variation coefficient is quite high: V = 0.25, which indicates low stability of properties.

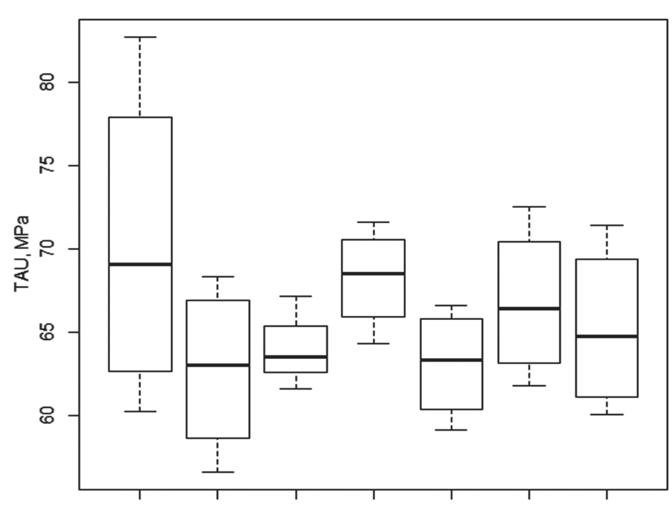
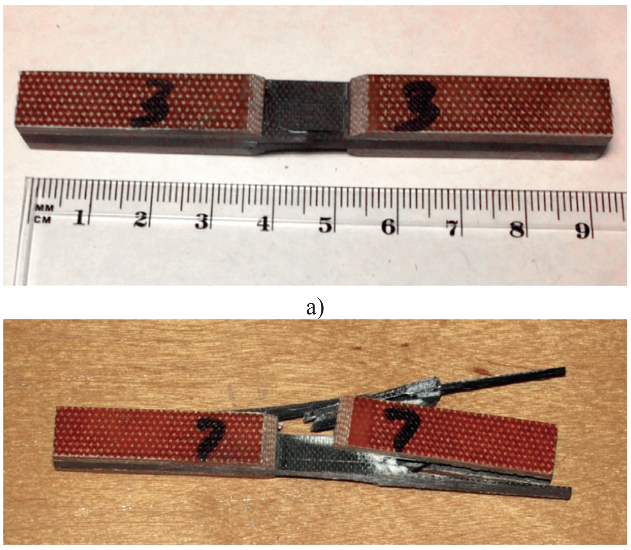


Fig. 1. Box-and-whisker diagrams for several batches of composite materials [7]

Table 1. Report on compression tests (static) $\sigma_c^в$

Mean $\sigma_c^в$	701.3	MPA
MSD	175.0	MPA
Variation V	0.25	



a)

b)

Fig. 2. Small-size item made of carbon fiber for compression testing. a) before testing; b) after fatigue failure

The item after fatigue failure, is shown in Figure 2b. It demonstrates a destruction with disrupted connections between fibers. The fiberboard pieces also show partial destruction. As such failure of the tested items is quite common, this result in terms of fatigue (specifically, item no. 7, figure 2b) was included in the general table.

Fig. 3 shows preliminary fatigue testing data:

The results shown in Fig. 3 demonstrate not only a large dispersion, but also an extremely high rate of censorship, i.e., the presence of a large number of items that did not fail by the end of testing. This figure can, very conditionally, be called an S-N diagram. It is more of agglomerated data that remotely indicate the presence of a dependence between the number of cycles and the range of stress. The high dispersion may be associated with: a) small size of the items that does not ensure the averaging of the properties with respect to the width; b) deformation of fibers; c) uneven compression at the stage of fabrication.

Conclusion: special methods are required in order to account for the censorship while constructing S-N diagrams of items made of composite materials.

2. Accounting for information on censorship (interruptions)

Fatigue testing of composite materials of various types, as well as the behaviour of materials under cyclic loads is the focus of attention of many researchers [10 – 14]. As it was noted above and based on common sense, it can be concluded that the information on the interruptions (censorship) should be somehow included into statistical conclusions. That would improve the statistical data on fatigue. Indeed, we know that a specific item (that did not fail by the end of the test cycle) has endured a certain (large!) number of cycles without breaking. It probably did not break before

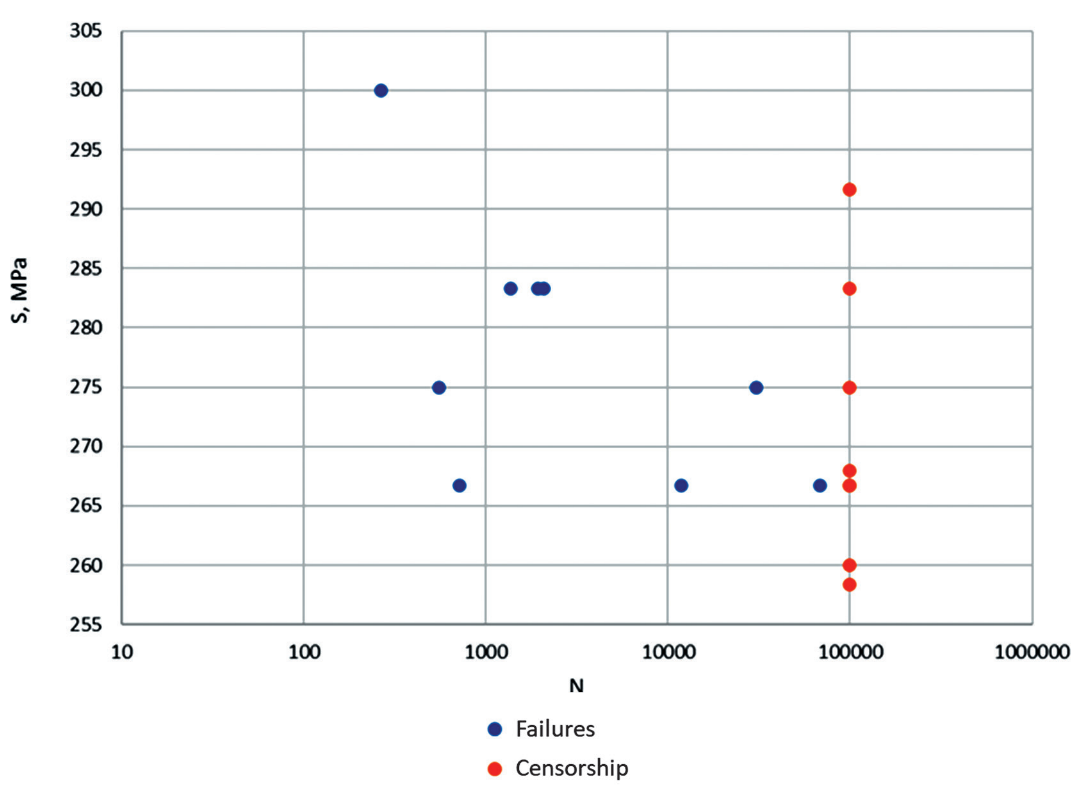


Fig. 3. Results of fatigue testing involving compression of small-size samples made of carbon fiber

the target number of cycles due to 1) a combination of individual positive strength characteristics of the item; or 2) unreasonably low selected stress amplitude σ_a. The normally recommended stress range for the first fatigue test is $\sigma_a = 2^{-1}$ (0.75… 0.85) $\sigma_c^{в}$, where $\sigma_c^{в}$ is the average value of the limiting compression stress evaluated based on 6 or 5 items. It can be stated that in the process of evaluation of $\sigma_c^{в}$ also an error is quite possible (see Table 1).

The problem of censored items also exists in the context of processing of fatigue testing results of metals. In [15 – 17], the problem of censored welded metal items is examined. The authors believe that their items have an unlimited fatigue endurance σ_∞. That complicated the problem even further. They try to estimate the fatigue endurance σ_∞ based on the information on the samples that did not fail before the target number of cycles. In order to solve the problem, they use the maximum likelihood method (MLM) (the method of estimation of population parameters, e.g., regression coefficients) for the purpose of estimating the values with the highest probability of obtaining the observed data.

There is a basic linear relationship between lg σ_a and lg N in the form of Basquin's equation:

$$\lg N = \lg A - m \cdot \lg \sigma_a, \qquad (1)$$

where m is the indicator of fatigue (slope), while lg A is the point of intersection. Here, σ_a is the stress range, normally [MPA], while N is the number of cycles to failure (or another failure criterion in case of composite materials testing).

The most detailed research of the fatigue durability of composite materials used in aircraft can be found in [4]. Following the conclusions of an earlier work [17], the authors [4] use the Weibull distribution not only for the purpose of analyzing dispersion, but also for complementing information on the items broken in the process of cyclic loading with information on 1) static testing; 2) censored items. The concept of Load Enhancement (consisting in increasing the maximum allowed loading by taking into account the information on censored items) is widely used in [4]. Fig. 4 shows the S-N diagram from [4]. The information in Fig. 4 will be further used in the appraisal of the developing method.

Earlier there were no research aimed at directly estimating σ_∞ in composite materials (e.g., through ladder method) [10]. Fortunately, the times have changed; new testing methods appeared, and researchers started delving into gigacycle fatigue, i.e. $N > 10^8$ cycles [18]. In order to speed up the testing, special ultrasound machines have been developed [19]. This novel approach to testing allowed (although not without certain doubts) shedding light on whether σ_∞ exists in composite materials.

Fig. 5 shows the test results [19] of carbon fiber reinforced composite materials (CFRP) that are widely used in the aerospace industry. The tests were conducted with the asymmetry of R = 0.2 and (please note) frequency of 965 Hz. For that testing method, a special cooling system was developed. Fig. 5 shows that the curve $\sigma_a - N$ goes down even when the number of loading cycles is above 10^6. In Fig. 5, a plateau may be seen between 10^7 and 10^8. When the number of cycles is above 10^8, the fatigue strength continues to decline. The authors conclude that the S-N diagram $\sigma_a - N$ does not have an unlimited fatigue endurance σ_∞ similar to the one that can be observed in metals.

3. Method

The method was developed for improving the quality of linear regression of the S-N diagram (1) through special accounting for censorship. The assumptions underlying the developed method: 1) there is no unlimited fatigue endurance for composite materials; 2) equation (1) is correct if lg(σ_a)

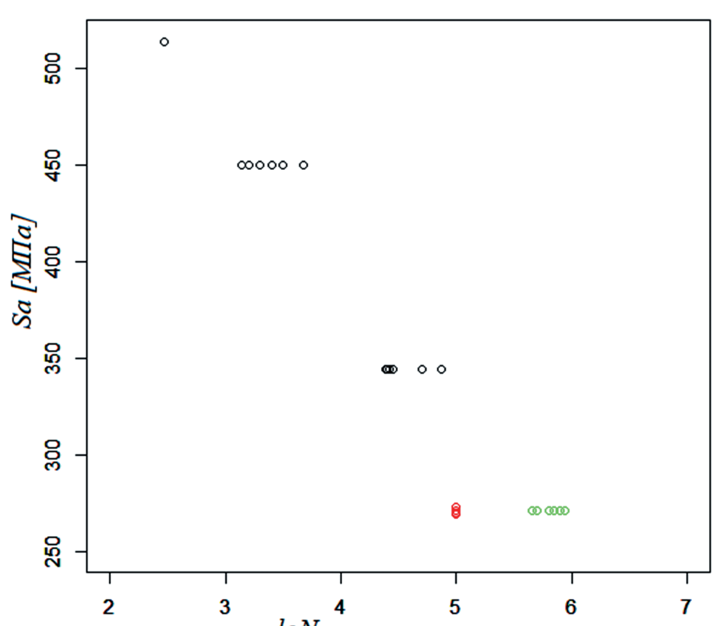


Fig. 4. S-N diagram of the AS4-PW composite material (according to [4]) (failures are marked in black and green, the red dots mark the conditionally censored samples; details below)

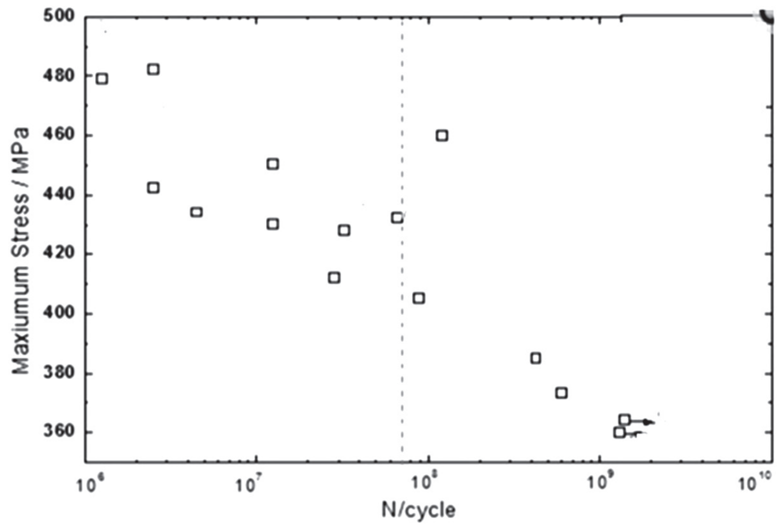


Fig. 5. Gigacycle fatigue of a composite material carbon fiber / epoxy resin [19]

is adopted as an independent variable (factor) and lg (N) as the function (dependent variable, normal distribution). As a reminder, historically, the S-N diagram is always mapped alternatively: the horizontal axis for lg(N) and vertical axis for lg(σ_a). Similarly, in this paper, the S-N diagrams are mapped in this way on graphs. During the estimation of regression coefficients, correctness is reestablished, i.e.: $y \rightarrow$ lg (N) and x $\rightarrow$ lg (σ_a).

Let us assume that there are p pairs of values σ_a, N, that are recorded failures during fatigue tests. We also have q censored tests at the lower level of σ_a*, for which only the number of tested items and their times to censoring are known.

As the limit number of cycles to completion of testing, the conventional $N_b = 10^5$ (target number of cycles of fatigue tests) was preliminary adopted. This type of data limitation is called single right censoring. This type of censorship takes place when a subject is removed from the research before an event occurred or when the research concludes before an event occurred. Unlike in operational dependability tests in service, when the times to censorship differ [20], in fatigue tests, all the censored elements usually have the same life, namely the target number of test cycles N_b.

In order to recover the lost data on the potential durability of non-failed q items in accordance with the developed method, it was proposed to create the bootstrap samples [20, 21]. Each bootstrap sample consists of p elements selected in a special random order. Based on the original sample, it appears to be possible estimating the regression equation of the form (1) using the least square method. The bootstrap sample consists of the same number of pairs (lgσ_a, lg$N$) and from the same pairs, but with a different probability of occurrence. The j-th bootstrap sample is generated through a random selection with replacement. Table 2 shows an example of selection of six possible combinations from 10 pairs in the original sample for bootstrap samples per the method's rules, according to which the random selection is done with replacement [21]:

The indices in Table 2 are related to the information on lg σ_a and lg$N$. For each j-th sample, a regression equation is derived. In this example, we have 6 equations with unique coefficients lg A and m in equation (1). Using those 6 equations, we can obtain 6 extrapolated values of N_i, i = 1… 6 for σ_a*, and all of them will be different. Believing those values to be relevant, at the next stage we include them in the adjusted estimate of the S-N diagram and obtain improved statistical parameters of the S-N diagram.

The method is further explained using an example.

The data for this example are taken from [4]. In Fig. 4, some points are shown in black (failures), p = 13, while some are shown in red (censored, q = 6). The green points in the figure show the "as though unknown failures" if the operation time $N > 10^5$. They were conditionally deemed unknown, as for the purpose of method appraisal it was initially deemed that testing was conducted for up to 10^5 cycles. Using the failure data and the least square method, the S-N diagram equation was estimated based on 13 points (Fig. 4, black points):

Table 2. An example of selection of six random indices for bootstrap samples based on the original sample.

j	1	2	3	4	5	6	7	8	9	10
1	6	7	7	8	4	7	6	3	9	1
2	4	2	4	4	2	8	3	5	9	4
3	1	8	7	3	5	3	10	2	2	10
4	6	1	2	7	9	2	1	10	9	2
5	7	10	8	10	6	2	8	2	5	1
6	3	6	3	7	3	5	1	3	9	6

$$\lg N = 39.49 - 16.94 \lg \sigma_a \quad (2)$$

After a bootstrap simulation, we obtained 6 additional breaking points N_i , $i = 1 \ldots 6N_i$, $i = 1 \ldots 6$ if $\sigma_a^* = 112$ MPA.

4. Analysis of the results

In Fig. 6, the normal probability graph (the "qqplot" function [22]) shows two distributions of the random durability values lg N for the items tested at σ_a*=112 MPA. One distribution (shown in blue) was obtained for real values obtained in [4]. Let us reveal a secret. In reality, the experiment was conducted up to $N_{b=}10^6$ cycles, and the cluster of points marked in red in Fig. 4 was adopted in the censored form for the purpose of method appraisal. Marked in red are those obtained using the above method, i.e. the bootstrapping distribution. As we can see, the distributions match well. The measures of distributions shown in Table 3 also demonstrate good concordance.

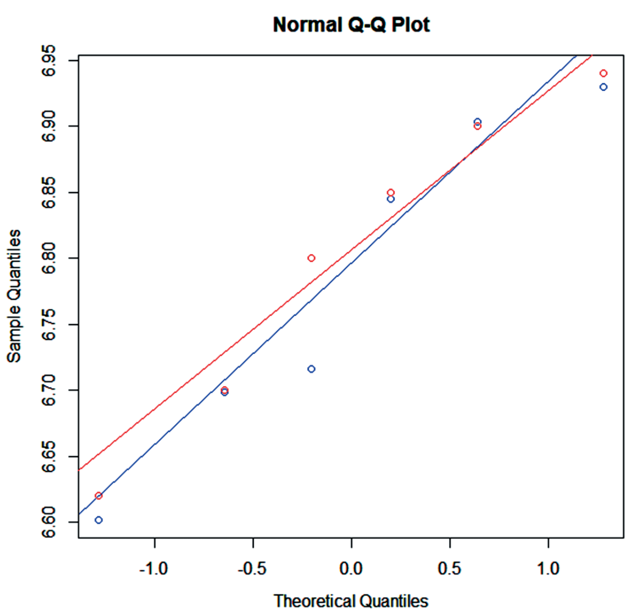


Fig. 6. Distribution of random value lg(N). Marked in blue is the natural distribution; marked in red is the one simulated for those tested under the range of $\sigma_a = 112$ [MPA]

Table 3. The parameters of experimental and model samples for random value lg N (σ_a) for those tested under the rate of σ_a = 112 [MPA]

	Experimental sample	Simulated sample
mean	6.78244	6.801667
RMS	0.129617	0.122052

Upon the acquisition of 6 additional points, the S-N diagram's equation was updated:

$$\lg N = 39.23 - 15.82 \cdot \lg \sigma_a \quad (3)$$

The quality of the regression equation can be characterized by the coefficient of determination R^2:

$$R^2 = D((y))\hat{}/ D(y) = 1 - D(e) / D(y) \quad (4)$$

where $D(y)$ is the total sum of squares; $D(e)$ is the dispersion of the model predictions.

Based on (4), the coefficient of determination for original curve (2) is: $R^2 = 0.909$, while for the adjusted curve (3) it is: $R^2 = 0.941$, therefore, it can be stated that an improvement was made.

5. Conclusions

The paper discussed the causes and methods of dispersion analysis of strength tests of composite materials, both under statistic, and fatigue loading. Based on the recent years' research, both in the area of metal, and composite materials fatigue, a method of regression equation improvement was developed. Using the developed model, the method takes into consideration the censorship results. The simulation ~~is~~ was based on statistical bootstrapping. Using the example of actual testing quoted in literature, the applicability of the proposed method was shown.

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The authors' contribution

Gadolina I.V. conducted the experiment, ensured the repeatability of the fatigue tests.

Maydanov I.S. conducted the research literary sources, defined the problem.

Smelov S.A. prepared tests specimens, supervised the quality of their manufacture, verified the geometrical parameters.

Suslova Yu.V. processed the results, prepared the article.

Conflict of interests

The authors declare the absence of a conflict of interests.