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DEPENDABILITY INDICES OF MEAN TIME TYPE

The paper discusses mathematical definitions of such dependability indices as mean time to failure, mean time between failures and mean time before failure introduced in basic standards and theoretical literature related to dependability. The paper also describes methods for practical calculation of indices for design analysis of system dependability. It is demonstrated that the definition and use of such parameter as mean time before failure are incorrect.

Keywords: *dependability, mean time to failure, mean time between failures, mean time before failure, Markov models.*

The dependability index is a technical parameter that quantitatively determines one or several properties making up an object's dependability. The dependability index quantitatively describes to what degree certain properties specifying dependability are inherent in the given object or given group of objects. Dependability indices can have dimensions or can be dimensionless. Objects studied within the dependability theory can be divided into two large classes – recoverable and non-recoverable types. Dependability indices can also be divided into two classes – dependability indices of recoverable and non-recoverable objects.

Issues of the selection of basic dependability indices for various objects are sufficiently elaborated and regulated in GOST 27.003-90.

The important concept presented in many formulations of dependability parameters is *operation time.* Operation time is duration or volume of object work, i.e. operating time can be measured not only in time units but also in units of output, traveled distance and so forth. For example, in one of the foreign standards related to calculation of dependability of naval technical equipment, frequency parameters have dimension of 1/mile.

The purpose of the paper consists in the following:

• analysis of mathematical definitions of such indices as mean time to failure, mean time between failures and mean time before failure [1-8];

• identification of a mistake made in the mathematical definition of the index **mean time before failure** (this definition is offered in the generic GOST 27.002-89 and other literature);

• discussion of practical methods for calculation of mean time before failure index at the design analysis stage;

• comparison of all three indices among themselves.

Let us introduce mathematical definitions for such indices as mean time to failure, mean time between failures and mean time before failure.

1. Mean time to failure (T_1)

An object's mean time to failure is defined as expectation of random mean time to the first failure

$$T_1 = M[\xi_1] = \int_0^\infty \tau f(\tau)dx = \int_0^\infty \tau dQ(\tau) = \int_0^\infty P(\tau)d\tau, \quad (1)$$

where ξ_1 is an object's random mean time to the first failure, $P(t) = P(0,t) = Prob(\xi_1 \geq t) = 1 - F_1(t)$ is the probability of mean time to failure in the interval (0, t),

F (t), f (t) is the function and density of random distribution ξ_1.

Expression (1) is a mathematical definition as well as a real computing formula for obtaining a parameter value.

This parameter can characterize both recoverable and non-recoverable objects.

2. Mean time between failures ($T_{between}$)

Mean time between failures (as a random variable) is defined by a volume of object work (operating time) from the *k*-th up to (*k*+1)-th failure, where k = 1,2, … It should be noted that, first, here the definition says about object **operation**, i.e. recovery time after the *k*-th failure is not taken into account (therefore, it is possible to say "after the *k*-th recovery"), and second, mean time before the first failure is not taken into account in the given parameter.

Process of recoverable object operation on a stationary section represents a sequence of alternating casual intervals of work (ψ_k) and downtime (fig. 1). Downtimes take place after an object's failure when it undergoes recovery work. Recovery of operability can be complete (replacement by a similar new object) or partial (for example, repair of a faulty part only).

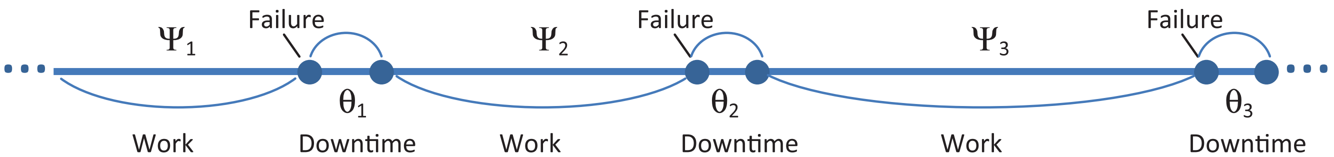

Fig. 1. Stationary section of recoverable object functioning

In general, in case of incomplete recovery, random times ψ_k of work after the *k*-1-th recovery and up to *the* k-th failure have a different distribution with densities $f_k(t)$. If T_k is mean time from the moment of termination of the *k*-th recovery up to (*k*+1)-th failure, then $T_{between}$ can be expressed as follows

$$T_{между} = \lim_{k\to\infty} M[T_k] = \lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} T_i, \quad (2)$$

where each of mean times of an object from the moment of termination of the *k*-1-th recovery up to the *k*-th failure is determined as follows

$$T_k = M[\psi_k] = \int_0^\infty t f_k(t)dt. \quad (3)$$

Expressions (2), (3) are just a mathematical definition, real calculations of an index as per them are not carried out (it is practically impossible). Estimated expressions are based on the calculated values of stationary availability (K_A) and mean time to recovery (τ_{recov}) (if it can be determined), or stationary values of availability and parameter of failure rate (ω). Parameters K_A and ω are in essence computable by many methods, for example, by formulas using theorems for the probability of the sum, product of events and the theorem of total probability, and also using logical and probabilistic, Markov and asymptotic techniques. Let us introduce these expressions:

From the known expression for stationary availability $K_A = \frac{T_{BETWEEN}}{T_{BETWEEN} + \tau_{RECOV}}$, by using calculated values of K_A and τ_{recov} we obtain the following:

$$T_{BETWEEN} = \frac{K_A \cdot \tau_{RECOV}}{1 - K_A}; \tag{4}$$

From the known expressions for stationary availability and parameter of a failure rate $\omega = \frac{1}{T_{BETWEEN} + \tau_{RECOV}}$, by using calculated values of K_A and ω we obtain the value of $T_{between}$:

$$T_{BETWEEN} = \frac{K_A}{\omega}. \tag{5}$$

Formula (5) together with approximated (for $T_{between} >> \tau_{recov}$) expression (6) for the probability of non-failure operation P (t) of recoverable systems generalizes a significant part of results in asymptotic theory.

$$P(t) \approx \exp^{-\frac{t}{T_{BETWEEN}}}. \tag{6}$$

It should be noted that $T_{BETWEEN} \leq T_1$, as by definitions for reliability parameters P (t) and T_1 the system starts to function from the state of operability.

This parameter applies only to recoverable objects.

3. Mean time before failure (T_{before})

Normative (for example, GOST 27.002-89, GOST 27.003-90) and reference literature [1] introduces the definition of one more parameter related to the averaging of operating time, namely mean time before failure which is regulated as one of the basic parameters of dependability for recoverable objects,

$$T_{before}(t_t) = \frac{t_t}{M\{n(t_t)\}}, \tag{7}$$

i.e., the relation of an object's total operating time (t_t) to expectation of the failure number $n(t_t)$ for this operating time.

This parameter was brought in the normative literature in the beginning of 80th not as a parameter supplementing the list of the basic parameters of dependability and describing any feature of object application which is not reflected by other parameters, but instead of mean time between failures. Let us present the results of the critical analysis of parameter T_{before}.

a) Parameter T_{before} is a function of operating time, therefore it should be normalized and calculated for the time, i.e. it would be more correct to designate it as $T_{before}(t_t)$ rather than T_{before}.

b) Let us compare mean time before failure with other operating times appearing in definitions of indices. Mean time to failure (T_1) describes only operating time to the first failure. As all parameters (in normative documentation) are determined for completely operable initial state of an object, then mean time to the first failure will be maximal for distributions with the non-decreasing function of a failure rate. Mean time between failures ($T_{between}$) does not cover operating times to the first failure, but averages (on infinity) operating times after the first failure. For distributions with the non-decreasing function of a failure rate, mean time between failures will be the smallest one. Note that all reserved structures have the increasing function of a failure rate. Mean time before failure ($T_{before}(t_t)$ includes generally both mean time to failure and mean time between failures. Thus, the given explanations should result in the following relations between the considered parameters $T_1 \geq T_{to}(t_t) \geq T_{between}$. But it is not fulfilled and that will be shown by examples.

There are no methods for calculation of T_{before} (t_t) in any literature known to us (and referred to partially in the end of the paper). It means that there is no precisely formulated method for obtaining parameter values and examples of its calculation for reserved recoverable structures. It is related to the fact that operating time for specified calendar time of object operation is a random variable.

c) Practically always when parameter T_{before} is normalized or calculated, one keeps in mind $T_{between,}$ i.e. mean time between failures, and sometimes T_1, i.e. mean time to failure.

d) For $t_t \to \infty$, $T_{before}(t_t) \to T_{between}$, for this reason (probably even not understanding) one says and writes T_{before}, though an asymptotic (stationary) value equal to $T_{between}$ is defined.

e) For $t_t \to 0$, parameter $T_{before}(t_t)$ grows, to infinity as well.

f) If should be, mean time before failure index ($T_{before}(t_t)$) should be present in normative literature only as a special parameter for objects, functioning strictly over certain operating time (after which the object is removed from operation even if it does not finish certain task performance). It should be removed from the basic parameters, and the mean time between failures index should be included into the list of the basic dependability parameters.

Once again it is necessary to emphasize that practically always mean time between failures is specified in requirements and estimated at the design analysis, but it is named as mean time before failure! **You never specify any operating time where it is necessary to define mean time before failure.**

In the foreign normative and technical literature there is no parameter similar to $T_{before}(t_t)$. In standards and literature of USA, England, Germany there are such parameters as MTTF – mean time to failure which is defined in the same way as T_1 – mean operating time to failure (for the parameters of reliability determined up to the first failure, i.e. with absorption in states corresponding to failure, operating time if it is expressed in terms of time, coincides with mean time to failure), and parameter MTBF – mean time between failures. Parameter MTBF is equal to the sum of mean operating time between failures and mean time of recovery MTBF = $T_{between} + \tau_{recov}$. In our literature, in design materials, requirements MTBF is often specified, but $T_{between}$ is meant and calculated, and that is incorrect!

Let us consider the examples of calculation of mean time before failure.

Example 1

Let us consider a redundant unit with recovery, whose Markov model of dependability is presented below

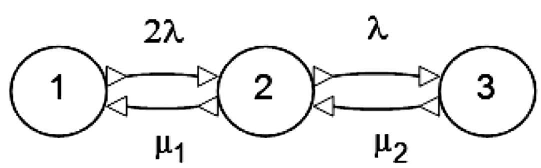

Fig. 2. Markov model for dependability of a redundant unit with recovery

State 1 – both units are in upstate; state 2 – one of the units fails, and the other one is in upstate; state 3 – both units fail. States 1 and 2 correspond to operability of a redundant structure, state 3 – failure of a redundant structure. Failure rate of each unit is equal to λ, repair rate of any unit in state 2 is equal to μ_1, and repair rate of any unit in state 3 is equal to μ_2.

The method of calculation of $T_{before}(t_t)$ will consist in the following. First, we shall obtain an analytical expression for the mean number of failures $N_1(t)$ using Markov income processes (t is the calendar time of structure operation, which consists of non-failure operating time t_t and time of being in nonserviceable state 3). Let us head μ_2 for infinity, then recovery from a structure failure state will be instant and time of being in state 3 will be equal to zero. Therefore, all modeling time *t* will appear to be equal to non-failure operating time t_t. Further we shall carry out calculations as to formula (7) dividing *t* by $N_1(t)$.

The basic ratios for Markov income processes are briefly stated in the appendix of this paper. In the most detailed way, the methodology of Markov income processes is given in [9,10].

The system of differential equations for the mean number of failures has the following form:

$$\begin{aligned} \dot{N}_1(t) &= -2\lambda N_1(t) + 2\lambda N_2(t) \\ \dot{N}_2(t) &= \mu_1 N_1(t) - (\lambda+\mu_1)N_2(t) + \lambda N_3(t) + \lambda \\ \dot{N}_3(t) &= \mu_2 N_2(t) - \mu_2 N_3(t). \end{aligned} \tag{8}$$

It should be reminded (see Appendix) that N_i (t) is the mean number of failures (determined by a choice of the gain matrix with unit at the point of transition from state 2 into state 3 (in W matrix, element w_{23}=1, others w_{ij} = 0)) for the initial state i.

By solving the system of equations (8), we shall obtain a non-homogeneous differential equation of the third order

$$\dddot{N}_1(t) + (3\lambda+\mu_1+\mu_2)\ddot{N}_1(t) + (2\lambda^2+2\lambda\mu_2+\mu_1\mu_2)\dot{N}_1(t) = 2\lambda^2\mu_2, \tag{9}$$

whose common solution has the following form

$$N_1(t) = C_1 + C_2 e^{x_1 t} + C_3 e^{x_2 t} + \frac{2\lambda^2\mu_2 t}{2\lambda^2+2\lambda\mu_2+\mu_1\mu_2}, \tag{10}$$

where the first three summands with constants C_i and roots of the characteristic equation x_0=0, and x_1, x_2 represent the general solution of the homogeneous differential equation corresponding to the system of equations (8), and the fourth summand is the partial solution of heterogeneous equation (9).

By studying solution (10), constants C_i and roots of the characteristic equation x_j for $\mu_2 \to \infty$ we shall obtain the finite solution

$$N_1(t) \xrightarrow[\mu_2\to\infty]{} \frac{2\lambda^2 t}{(2\lambda+\mu_1)} + \frac{2\lambda^2}{(2\lambda+\mu_1)^2} e^{-(2\lambda+\mu_1)t} - \frac{2\lambda^2}{(2\lambda+\mu_1)^2}. \tag{11}$$

Now we shall give expressions (they are obtained by solving algebraic equations of typical Markov process) for calculation of parameters T_1 and $T_{between}$ using the model presented in fig. 2:

$$\begin{aligned} &T_1 = \frac{3\lambda+\mu_1}{2\lambda^2}; \quad K_Г = \frac{2\lambda\mu_2+\mu_1\mu_2}{2\lambda^2+2\lambda\mu_2+\mu_1\mu_2}; \\ &\omega = \frac{2\lambda^2\mu_2}{2\lambda^2+2\lambda\mu_2+\mu_1\mu_2}; \quad T_{между} = \frac{2\lambda+\mu_1}{2\lambda^2}. \end{aligned} \tag{12}$$

Let us make some numerical calculations for T_{before} (t_t). Let = λ = 0,001 $hour^{-1}$ (it corresponds to mean time to failure T_{to}=1000 hour), μ_1 = 0,1 $hour^{-1}$ (it corresponds to mean time to repair τ_r =10 hour.). Values (in hours) of $T_{before}(t_t)$ for different t_t, mean time between failures $T_{between}$ and mean time to failure T_1 are presented in table 1.

Table 1. Results of calculation of parameters T_{before} (t_t), $T_{between}$ and T_1

	t_t = 0,1	t_t = 1	t_t = 10	t_t = 100	t_t = 2000
$T_{before}(t_t)$	14285714,3	1030927,8	136680,8	56543,2	51252,2
$T_{between}$	51000				
T_1	51500				

Example 2

Let us consider one recoverable element with exponentially distributed time to failure (with λ rate) and recovery time (with μ rate). Now we shall give formulas for calculation of dependability basic parameters.

Probability of non-failure operation	$P(t) = e^{-\lambda t}$
Mean time to failure	$T_1: = T_{to} = 1/\lambda$
Stationary availability	$K_a = \mu/(\lambda+\mu)$
Stationary failure rate parameter	$\omega = \mu\cdot\lambda/(\lambda+\mu)$
Mean time between failures	$T = T_{between} = 1/\lambda$
Mean number of failures	$N_1(t) = [\mu\cdot\lambda\cdot t/(\lambda+\mu)] + [\lambda^2/(\lambda+\mu)^2]\cdot[1 - e^{-(\lambda+\mu)\cdot t}]$
Mean time before failure	$T_{before}(t_t) = 1/\lambda$, as for $\mu \to \infty$ $N_1(t) \to \lambda t$

For the given example (one element) $T_1 = T_{before}(t_t) = T_{between}$.

Example 3

Let us consider a redundant unit similar to the unit described in example 1, but in this case we shall assume impossibility of recovery during functioning. Namely, the system operates before fails, and then recovers into completely operable state. The model is represented in fig. 3.

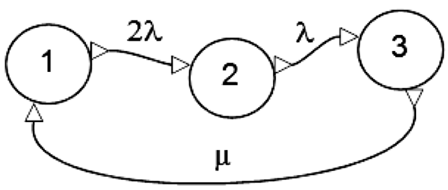

Fig. 3. Markov model of dependability for the redundancy unit with recovery from failure state

Let us give expressions for calculation of parameters T_1 and $T_{between}$ according to the model in fig. 3:

$$T_{до} = \frac{3}{2\lambda}; \quad K_г = \frac{3\mu}{2\lambda + 3\mu}; \quad \omega = \frac{2\lambda\mu}{2\lambda + 3\mu}; \quad T_{между} = \frac{3}{2\lambda}, \qquad (13)$$

i.e. $T_{between}$ coincides with T_{to}, as it should be, since all mean times between failures and mean times to the first failure are identical (the system after recovery always begins operating from state 1).

To define $T_{before}(t_t)$ we shall make the same, as in example 1. The system of equations of Markov income processes for definition of the failure number has the following form:

$$\begin{aligned} \dot{N}_1 &= -2\lambda \cdot N_1 + 2\lambda \cdot N_2 \\ \dot{N}_2 &= -\lambda \cdot N_2 + \lambda \cdot N_3 + \lambda. \\ \dot{N}_3 &= \mu \cdot N_1 - \mu \cdot N_3. \end{aligned} \tag{14}$$

Let us introduce the final result without intermediate calculations, since we shall obtain it by another way as well. By solving the equations and heading μ for infinity, we shall obtain the following:

$$N_1(t) \xrightarrow[\mu\to\infty]{} \frac{2}{9}e^{-3\lambda \cdot t} + \frac{2}{3}\lambda t - \frac{2}{9}. \tag{15}$$

Now we will show one more method of calculation. Expression (15) can be obtained by another technique if to apply results of renewal theory. There is a model known as *simple renewal process*. The essence of this process consists in the fact that a new element that started to function at the moment t=0, and if failed during the moment t_1, then it is instantly replaced by a new one, which having worked during the time t_2 and failed, is also instantly replaced by the following new one, etc. Density function of time for elements of non-failure operation is identical and equal to f (t). There is a known expression for renewal function – mean number of recoveries over the time t – (for simple renewal process) in the form of Laplace transform [11]. This expression has the following form:

$$N(s) = \frac{f(s)}{s \cdot (1 - f(s))}, \tag{16}$$

where N (s) is the Laplace transform from renewal function; f (s), s is the Laplace transform from density function and a variable of transformation, accordingly.

It should be noted that, first, the entire time *t* is total operating time since recovery time is equal to zero, and, second, the number of recoveries is equal to the number of failures since the moment t=0 is not considered as recovery, and the moment of last failure (during the moment *t*) is also the moment of recovery because of instant recovery.

Let us find density of distribution for example 3. As this is usual redundancy without recovery from operable state with one failure, we may not solve a system of differential equations for the model shown in fig. 3 with “absorbing screen” and directly write the formula for the function and density of distribution F (t), f (t):

$$\begin{aligned} F(t) &= (1 - e^{-\lambda \cdot t})^2 \\ f(t) &= F'(t) = 2\lambda \cdot (1 - e^{-\lambda \cdot t}) \cdot e^{-\lambda \cdot t} \end{aligned}. \tag{17}$$

The Laplace transform from f (t) has the following form:

$$f(s) = \frac{2\lambda}{s + \lambda} - \frac{2\lambda}{s + 2\lambda}. \tag{18}$$

By substituting in (16) and making an inverse Laplace transform, we obtain the following

$$N(s) = \frac{2\lambda^2}{s^2 \cdot (s + 3\lambda)} \Rightarrow \frac{2}{9} \cdot e^{-3\lambda \cdot t} + \frac{2}{3} \cdot \lambda t - \frac{2}{9} = N(t), \tag{19}$$

and that coincides with (15) and indicates the correctness of the approach using Markov income process models.

Finally, for example 3 we have:

$$T_{на} = \frac{t}{\frac{2}{9} \cdot e^{-3\lambda \cdot t} + \frac{2}{3} \cdot \lambda t - \frac{2}{9}}. \tag{20}$$

It is obvious that for t → ∞, T_{before} → (3/2λ) = $T_{between}$ = T_{to}. And for t → 0, T_{before} → ∞ (differentiating the numerator and the denominator before it by L'Hospital rule).

Appendix A

Laboratory No.5 in the Institute of Management of the Russian Academy of Science in the middle of 80th developed the method and algorithms of parameters' calculation for modeling multilevel systems based on Markov income processes [9, 10]. The method of Markov income processes (MIP method) somewhat generalizes the classical method of Markov processes with continuous time and discrete space of states. Its application allows us to calculate a number of parameters, which computation cannot be carried out directly in usual (classical) Markov processes.

Now we shall give the basic relations for calculation of dependability parameters and technical efficiency. The expectation of income H (t) satisfies to the system of differential equations:

$$\frac{dH(t)}{dt} = \Lambda H(t) + R, \tag{A.1}$$

where H (t) is the column vector of income expectation; $H^T(t) = (H_1(t), H_2(t), \ldots, H_n(t))$ is the transposed column vector; n is the number of system states; $H_i(t)$ is the expectation of income of the system during the operation time *t* if during the moment t=0 the system was in state *i*; Λ is the matrix of transfer rate from the *i*-th state into the *j*-th state.

R is the column vector of absolute terms:

$$R_i = w_i + \sum_{j, j \neq i} \lambda_{ij} w_{ij}; \tag{A.2}$$

w_{ij} is the income received in the system at transition from the *i*-th state into the j-th state;
w_i is the income per unit of time if the system is in state *i*.

Stationary values of parameters (if they exist) are obtained from the system of equations

$$\Lambda \cdot H + R = 0. \tag{A.3}$$

The possibility of calculating the large spectrum of dependability parameters and efficiency (practically any parameter, which is met in normative and technical documents) is achieved by a special choice of income matrix W = (w_{ij}). We shall not list here all types of income matrixes for all parameters; it can be found in [9].

As an example, we shall note only some parameters:

1. Probability of system failure Q over the time t

For calculation Q (t) it is necessary to take a matrix of incomes W whose elements of columns, corresponding to system failure state are equal to unit value, and all other elements are equal to zero. And failure states should be made absorbing as the given parameter characterizes system behavior to its first failure. Calculations are carried out as to formula (A1).

2. Mean time to failure Tm

Matrix W should have diagonal elements w_{ii}, corresponding to operable states, equal to unit, and other elements are equal to zero. Failure states should be made absorbing. As this parameter is determined by (0, ∞), calculation is carried out as to formula (A3).

The first two parameters are also calculated in common Markov processes with continuous time, and the following parameter (N(t)) can be calculated directly without additional integration only with the use of «income» model.

3. Failure rate ω(t) *and mean failure number N (t)*

Failure rate ω(t) is a differential parameter in relation to integrated parameter N(t) (to mean failure number for (0, t)). That is ω(t) = dN (t)/dt and it is determined during the moment *t*. Calculation of ω(t) is carried out in two stages. First, N(t) is determined as to formula (A1). For this purpose we should put in W matrix elements w_{ij} equal to unit (i≠j), if *i* is operable state and *j* is failure state. Other elements are equal to zero. Then, the obtained vector N(t) is substituted in (A1) and a derivative is defined, which in this case is equal to failure rate.

$$\frac{dH(t)}{dt} = \frac{dN(t)}{dt} = \omega(t). \tag{A.4}$$

In MIP model, calculation of the following dependability and efficiency parameters is also developed (including multilevel models of a system):

- mean time of recovery;
- system failure rate at the moment *t*;
- dispersion (and all other moments) of time of non-failure operation;
- probability of being at the i-th level of functioning at the moment of time *t* (in two-level model (operability/failure) this parameter is availability/unavailability (downtime));
- probability to find a system at the *i*-th level of functioning at the moment of time *t* and not to step down below this level in time τ from the moment *t* (in two-level model, this parameter refers to operational availability);
- mean number of transitions from the *i*-th onto the *j*-the level of operability (in two-level model, this parameter refers to mean number of failures /recoveries);
- mean total time of a system being at the *i*-th level of functioning on the interval (0, t);
- expectation of efficiency (level of functioning) at the moment of time *t*;
- expectation of integral efficiency of functioning on the interval (0, t);
- value of functioning efficiency averaged on the interval (0, t);
- factor of efficiency retention on the interval of functioning (0, t);
- parameters averaged on the interval of time (0, t).

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