Model of item monitoring system under unreliable supervision

Boris P. Zelentsov, *Siberian State University of Telecommunications and Information Sciences, Russian Federation, Novosibirsk*
zelentsovb@mail.ru

Boris P. Zelentsov

Abstract. Aim. *The conducted research aims to develop an analytical model of item dependability for situations of technical state monitoring with constant inspection frequency and subject to inspection errors and failures of various types. The primary purpose of the model is the calculation and prediction of dependability indicators that depend on specified conditions.* **Methods.** *The model is based on the Markovian process theory. Models of two types are used, i.e. the continuous-time discrete process model and semi-Markovian model. The mathematical operations involved in the model implementation were performed in matrix form. An items' operation is presented in the form of recurrent cycles separated from each other by the recovery state. A continuous-time model allows obtaining state probabilities within the periods between inspections, mean active state times and state probabilities at the end of a period. The probabilities of entering states at the end of a period are a priori for the semi-Markovian model, using which the mean numbers of active states within one cycle were obtained.* **Results.** *The mean up and down time within a cycle were calculated using mean state frequency and mean time of active state. Based on those parameters, formulas were obtained for calculating the availability and non-availability coefficients. Out of the above model follows that the dependability indicators depend on the frequencies of explicit and hidden failures, inspection frequency and inspection errors. The paper sets forth the calculation data for the mean cycle duration and non-availability coefficient under various failure rates and various probabilities of inspection errors. It is shown that the mean cycle duration significantly depends on the probability of inspection errors of the I kind and practically does not depend on the probability of inspection errors of the II kind. However, the non-availability coefficient practically does not depend on the probability of inspection errors of the I kind, yet there is a strong dependence on the probability of inspection errors of the II kind.* **Conclusions.** *The presented model allows calculating and predicting dependability indicators taking into consideration explicit and hidden failures, as well as the monitoring system parameters. While designing new and improving the maintenance procedures of existing systems, the effect of various factors on the dependability level should to be taken into consideration.*

Keywords: *item technical condition monitoring, explicit and hidden failures, inspection frequency, inspection errors.*

For citation: *Zelentsov B.P. Model of item monitoring system in presence of inspection errors. Dependability 2020;4: 3-12. https://doi.org/10.21683/1729-2646-2020-20-4-3-12*

Received on: *29.09.2020* / **Revised on:** *08.10.2020* / **For printing:** *18.12.2020*

Introduction

As it is known, an item's dependability depends not only on the kinds and parameters of failures, but also on the maintenance system, one of the components of which is the technical state monitoring that consist in supervising the item in order to obtain information on its technical state and operational characteristics. For this reason, research of dependability of items in view of technical state monitoring remains relevant.

In the course of operation, an item may be in various states, if used as intended, be submitted to various types of maintenance, including technical state inspections. From this perspective, an item's operation can be represented as three phases: intended use (operation), technical state inspection and recovery.

When an item is used as intended, its technical state is monitored, i.e. the item is observed in order to obtain information on its technical state and operational characteristics. The monitoring is implemented in the form of technical state inspection operations that are performed continuously or at intervals. Two technical states associated with failures are considered, i.e. the failure is detected or not detected at the moment of occurrence. In this context, explicit and hidden failures are distinguished. A monitoring system is intended for recurrent inspections, performance supervision, parameter measurements and – based on the above – identifying the presence of hidden failures.

Source overview

Technical state monitoring is used in a number of technical fields with respective specificity. Thus, in the energy industry, one of the major problems is relay protection of power systems, where it is required to supervise such events as false operation of the monitoring system, functional failure, internal and external faults [22, 23]. An analytical functional model of relay protection of power systems that takes into consideration three kinds of failures and a performance monitoring system are examined in [19]. The model allowed obtaining failure rates, predicting the system availability and identifying the required frequency of relay protection system technical state inspection.

Telecommunication network equipment is classified as long-term use systems, within which various sections of the network are submitted to continuous and recurrent monitoring, which allows establishing the required network availability taking into consideration equipment redundancy and recovery characteristics [2].

Monitoring simulation enables the research, design and improvement of technical systems. For that purpose, the discrete Markovian continuous and discrete-time process theory is sued. Both in Russia, and abroad various research activities are conducted in this area.

Many researchers examine transitions between states in continuous time. Such transitions are described with a system of differential equations. This approach was used in [13, 15, 17] while generating dependability models of complex systems. In [11], based on a system of differential equations, a number of technical systems were described.

Self-monitored systems have been studied by many authors. Thus, in [16], function accuracy control in restorable systems was examined. The model is based on the continuous-time Markovian process theory. The state transitions are described using a system of differential equations.

The research widely uses models based on semi-Markovian processes [12, 18]. In [10], the theory and application examples of semi-Markovian processes are provided. Embedded Markov chains were used to examine the characteristics of nonstationary processes, for instance, temporal characteristics in queueing systems.

In [13], the authors examine the effect of the completeness, depth and reliability of inspections on the simulation of the dependability of redundant systems. Models of standard dependability structures were developed. The simulation results allow making substantiated requirements for the monitoring system characteristics.

In [21], based on analytical methods of monitoring, failure and damage detection and diagnostics models were developed for complex systems. Specific research activities associated with the frequency of preventive maintenance are set forth in [15].

The monitoring system is strongly associated with the matters of operational tests that are a reliable source of information on the initial dependability characteristics [3, 14, 20]. Those characteristics are used in the construction of various models that reflect actual processes in technical systems. Rational organization of operational tests affects the reliability of obtained information and cost of the monitoring system.

Conceptual model

In the course of operation, an item may be in two states: up and down. An item enters the down state as a hidden failure occurs and is not detected at the moment of its onset. Such failures are detected during technical state inspections as part of check operations.

Thus, in terms of technical state monitoring, an item's failures are divided into two types: hidden and explicit.

An item operates and is occasionally submitted to inspections. Between inspections, an item may fail, resulting in its transition from the up into the down state. The item is used as intended both in the up, and the down state. If an up item is inspected, then upon such inspection it is returned into operation. If a down item is inspected, it is submitted to recovery, upon which it is returned into operation.

An item is observed in operation in order to obtain information on its technical state in the following cases:

1) the item enters the down state as a hidden failure occurs and is detected by the next inspection;

2) upon the onset of an explicit failure, it is detected by the continuous monitoring system, upon which the item is submitted to recovery.

Explicit failures are detected by the continuous monitoring system at the moment of their onset, while hidden failures are detected during scheduled inspections. Therefore, the duration of the period between inspections may be:

1) specified, if no explicit failure occurred;

2) below specified, if an explicit failure occurred.

In this model, the following conditions and assumptions are adopted:

1) during an item's operation, hidden and/or explicit failures may occur;

2) hidden and explicit failures occur at constant rates, i.e. failures occur at random moments in time, while the time to failure is distributed exponentially;

3) item state is supervised at fixed periods, at the same time, with each period starting with the beginning of operation upon recovery or next inspection;

4) is a failure is detected, the item is submitted to recovery, after which operation starts in the up state;

5) in the course of scheduled inspection, inspection errors of the I and II kind are possible;

6) the duration of inspections and recovery are assumed to be negligibly small.

The last assumption is for the purpose of simplifying the model. Such assumption allows estimating the effect of various factors in the "pure form". For instance, in accordance with the established norms, the availability coefficient is the probability of up state disregarding planned periods of no intended use of the item. If necessary, the model allows taking into consideration the final active time of supervision and recovery.

The Aim of the paper is to make a model of item dependability with constant periods between inspections taking into consideration the above conditions and limitations.

Methods

The models set forth in this paper are based on the Markovian process theory. Models of two types are used, i.e. the continuous-time discrete process model and semi-Markovian model.

Using the continuous-time Markovian model, the state probabilities are found. The input is the rate of transition between states λ_{ij} represented in the form of a transition rate matrix $\Lambda = \left\| \lambda_{ij} \right\|$ over a certain set of states. Out of matrix Λ, the image of the state probabilities in matrix form is found:

$$P(s) = (sE - \Lambda)^{-1} \qquad (1)$$

where s is the Laplace variable; E is the identity matrix.

Using inverse Laplace transformation, the state probability matrix $P(t) \div P(s)$ is found, where matrix $P(t) = \left\| p_{ij}(t) \right\|$, element $p_{ij}(t)$ of such matrix is the probability that in the moment in time t the process is in the j-th state, provided that the i-th state is the initial one. If the initial probability distribution $p(0)$ is known, the state probabilities can be represented as a string [4, 5]:

$$p(t)=p(0)\times P(t). \qquad (2)$$

Note. Finding the state probabilities does not require composing and solving a system of differential equations. State probabilities are found using standard computer operations.

Further in this model, the relative frequency method is used that is based on the semi-Markovian process theory [4, 6]. The input parameters are the probabilities of passage. Probability of passage q_{ij} is the probability of transition from the i-th state into the j-th state provided that the i-th state is exited.

Let U be a certain set of nonexistent states. In the passage probability matrix, over set U, Q_{UU}, transitions are shown only between the states of set U. Out of matrix Q_{UU}, the relative frequency matrix N_U on set U is found:

$$N_U = \left\| n_U(i, j) \right\| = (E - Q_{UU})^{-1}, \qquad (3)$$

where $n_U(i,j)$ is the average number of entries into the j-th state before leaving the set U provided that the i-th state is the initial one when entering set U. Elements of the relative frequency matrix are referred to as the relative state frequencies.

If the initial probability distribution $q(0)$ is known, the relative state frequencies can be represented as string

$$n_U = \left\| n_U(j) \right\| = q(0) \cdot N_U. \qquad (4)$$

Out of the relative state frequencies and continuous-time state probabilities, the item' dependability indicators are found. As part of the examined model, the following will be calculated:

– mean duration of the up and down states;

– mean recovery frequency;

– availability coefficient and non-availability coefficient.

The operations in the course of model development can be performed manually or in a computer mathematics system.

State probabilities within one period

Within one period, both hidden, and explicit failures may occur. An explicit failure may occur both in the up state, and the down state. It should be taken into consideration that an explicit failure may occur after a hidden failure, however, a hidden failure cannot occur after an explicit failure, as an explicit failure is detected by the monitoring system at the moment of its occurrence and the item is submitted to recovery. Both hidden, and explicit failures occur within a continuous-time period.

Let the initial state of the period be up. The diagram of continuous-time single-period states is shown in Fig. 1, where 1U is the up state; 2H is the down state with hidden failure: 3HE is the state with two types of failures; 4E is the state only with an explicit failure. State transitions occur as the result of a hidden and explicit failures at a random moment in time at the rates of λ_h and λ_e.

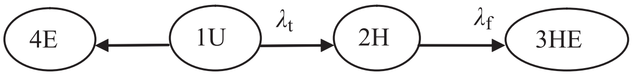

Fig. 1. Continuous-time single-period state diagram

The initial matrix of single-period transition rate:

$$\Lambda = \begin{pmatrix} -\lambda_h - \lambda_e & \lambda_h & 0 & \lambda_e \\ 0 & -\lambda_e & \lambda_e & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Having performed the required transformations (1) and (2), we obtained the state probabilities within one period with the initial state 1 that are the elements of the first matrix row $P(t)$:

$$p_{11}(t) = \exp(-\lambda \cdot t);\ p_{12}(t) = \exp(-\lambda_e \cdot t) - \exp(-\lambda \cdot t);$$

$$p_{13}(t) = \frac{\lambda_c}{\lambda} + \frac{\lambda_e}{\lambda} \cdot \exp(-\lambda \cdot t) - \exp(-\lambda_e \cdot t);$$

$$p_{14}(t) = \frac{\lambda_e}{\lambda} - \frac{\lambda_e}{\lambda} \cdot \exp(-\lambda \cdot t), \quad (5)$$

where $\lambda = \lambda_h + \lambda_e$ is the total failure rate.

Obviously, $p_{11}(t) + p_{12}(t) + p_{13}(t) + p_{14}(t) = 1$. The probability of an explicit failure equals the sum of the state probabilities 3 and 4:

$$p_e(t) = p_{13}(t) + p_{14}(t) = 1 - \exp(-\lambda_e \cdot t). \quad (6)$$

It should be taken into consideration that states 1 and 2 are registered by the monitoring system at the end of the interval between inspections, while states 3 and 4 are registered at the moment of explicit failure.

If state 2 is normal, then

$$p_{21}(t) = p_{24}(t) = 0;\ p_{22}(t) = \exp(-\lambda_e \cdot t);$$

$$p_{24}(t) = 1 - \exp(-\lambda_e \cdot t). \quad (7)$$

Let us introduce a parameter that we will name the reduced failure rate:

$\rho_h = \lambda_h \cdot T, \quad \rho_e = \lambda_e \cdot T, \quad \rho = \lambda \cdot T$. The reduced failure rate is the average number of failures within period T: ρ_h and ρ_e are the reduced rates of hidden and explicit failures, ρ is the reduced total rate of hidden and explicit failures. Using one parameter instead of two allows simplifying the formulas and calculations.

The state probabilities at the end of the period will be expressed in terms of reduced rates. If the initial state is 1:

$$p_{11}(T) = \exp(-\lambda \cdot T) = \exp(-\rho);$$

$$p_{12}(T) = \exp(-\lambda_e \cdot T) - \exp(-\lambda \cdot T) =$$
$$= \exp(-\rho_e) - \exp(-\rho) = \exp(-\rho_e) \cdot \left(1 - \exp(-\rho_h)\right);$$

$$p_{13}(T) = \frac{\lambda_h}{\lambda} + \frac{\lambda_e}{\lambda} \cdot \exp(-\lambda \cdot T) - \exp(-\lambda_e \cdot T) =$$
$$= \frac{\rho_h}{\rho} + \frac{\rho_e}{\rho} \cdot \exp(-\rho) - \exp(-\rho_e);$$

$$p_{14}(T) = \frac{\lambda_e}{\lambda} - \frac{\lambda_e}{\lambda} \cdot \exp(-\lambda \cdot T) =$$
$$= \frac{\rho_e}{\rho} + \frac{\rho_e}{\rho} \cdot \exp(-\rho) = \frac{\rho_e}{\rho} \cdot \left(1 - \exp(-\rho)\right). \quad (8)$$

If the initial state is 2:

$$p_{21}(T) = p_{24}(T) = 0;$$

$$p_{22}(T) = \exp(-\lambda_e \cdot T) = \exp(-\rho_e);$$

$$p_{23}(T) = 1 - \exp(-\lambda_e \cdot T) = 1 - \exp(-\rho_e). \quad (9)$$

A period my start with an up or down state and end with any state. Therefore, a period can be characterized by the initial or final state. The model under consideration may involve 6 types of periods shown in Table 1. Shown in Table 1 are: U and D are the up and down states of the item, HF and EF are hidden and explicit failures.

Table 1. Types of periods between consecutive inspections

Type of period	Initial state	Events within period	Final state	Period designation	Probability of period
1. Up period	U	–	U	UU	$p_{uu}=p_{11}(T)$
2. Period with HF	U	HF	H	UH	$p_{uh}=p_{12}(T)$
3. Period with EF and HF	U	HF and EF	HE	UHE	$p_{uhe}=p_{13}(T)$
4. Period with EF	U	EF	E	UE	$p_{ue}=p_{14}(T)$
5. Down period	D	–	D	DD	$p_{dd}=p_{22}(T)$
6. Down period with EF	D	EF	E	DE	$p_{de}=p_{23}(T)$

Table 2. Probabilities of up and down states within periods of different types ($t \in [0;T]$)

Type of period	State transitions	Probabilities of states	
		up	down
1. UU	U→U	$p_u(t)=1$	$p_d(t)=0$
2. UH	U→H	$p_u(t)=\exp(-\lambda_h \times t)$	$p_d(t)=1-\exp(-\lambda_h \times t)$
3. UHE	U→H→E	$p_u(t)=\exp(-\lambda_h \times t)$	$p_d(t)=\frac{\lambda_h}{\lambda_h-\lambda_e}\cdot\left(\exp(-\lambda_e\cdot t)-\exp(-\lambda_h\cdot t)\right)$
4. UE	U→E	$p_u(t)=\exp(-\lambda_e \times t)$	$p_d(t)=0$
5. DD	D→H	$p_u(t)=0$	$p_d(t)=1$
6. DE	D→E	$p_u(t)=0$	$p_d(t)=\exp(-\lambda_e \times t)$

State durations within one period

Let the type of the period be known from the initial and final state. That means that the state transitions within a period occurred as specified in accordance with the conceptual model. The state transitions and probabilities of the state, in which the item is up or down, are shown in Table 2.

Note. In the up state there are no failures, in the down state there is only a hidden failure.

For an UHE period, the state probabilities

$$p_{11}(t)=\exp(-\lambda_h\cdot t);$$

$$p_{12}(t)=\frac{\lambda_h}{\lambda_h-\lambda_e}\cdot\left(\exp(-\lambda_e\cdot t)-\exp(-\lambda_h\cdot t)\right);$$

$$p_{13}(t)=1-\frac{\lambda_h\cdot\exp(-\lambda_e\cdot t)-\lambda_e\cdot\exp(-\lambda_h\cdot t)}{\lambda_h-\lambda_e}. \quad (10)$$

An inspection shows that $p_{11}(t)+p_{12}(t)+p_{13}(t)=1$, while with the probability $p_{13}(t)$ at the moment in time t the period will be interrupted, as an explicit failure is detected at the moment of its occurrence, while the probability that, at the moment t, the period continues will be

$$p_{11}(t)+p_{12}(t)=\frac{\lambda_h\cdot\exp(-\lambda_e\cdot t)-\lambda_e\cdot\exp(-\lambda_h\cdot t)}{\lambda_h-\lambda_e}. \quad (11)$$

It is obvious that the durations of states, as well as the duration of the period depend on the type of the period. An explicit failure is detected by the continuous monitoring system at the moment of its occurrence, upon which the item is submitted to recovery. For this reason, explicit failures reduce the duration of the period, however, hidden failures do not.

The average time of the item being in the j-th state, if the initial state is the i-th, within one period is calculated according to formula:

$$\theta_{ij}=\int_0^T p_{ij}(t)\mathrm{d}t. \quad (12)$$

The mean up time (θ_u) and down time (θ_d) within the periods of each type, as well as the mean durations of the periods, are calculated by integrating the respective probabilities. Those durations are shown in Table 3.

The sum of the times θ_u and θ_d is the mean time of the period. The mean times θ_u and θ_d are calculated by

Table 3. Mean time of up and down state within periods of different types

Type of period	θ_u	θ_d	Average period duration
1. UU	T	0	$T_{uu}=T$
2. RS	$\frac{1-\exp(-\rho_h)}{\rho_h}\cdot T$	$\frac{\rho_h-\left(1-\exp(-\rho_h)\right)}{\rho_h}\cdot T$	$T_{uh}=T$
3. UHE	$\frac{1-\exp(-\rho_h)}{\rho_h}\cdot T$	$\frac{\rho_h\left(1-\exp(-\rho_e)\right)-\rho_e\left(1-\exp(-\rho_h)\right)}{\left(\rho_h-\rho_e\right)\cdot\rho_e}\cdot T$	$T_{uhe}=\theta_c+\theta_и$
4. UE	$\frac{1-\exp(-\rho_e)}{\rho_e}\cdot T$	0	$T_{ue}=\frac{1-\exp(-\rho_e)}{\rho_e}\cdot T$
5. DD	0	T	$T_{dd}=T$
6. DE	0	$\frac{1-\exp(-\rho_e)}{\rho_e}\cdot T$	$T_{de}=\frac{1-\exp(-\rho_e)}{\rho_e}\cdot T$

integrating the probabilities of up and down times within the respective period. The mean duration of a period of type UHE:

$$T_{\text{uhe}} = \theta_h + \theta_d = \frac{\rho_u^2 \cdot \left(1 - \exp(-\rho_e)\right) - \rho_e^2 \cdot \left(1 - \exp(-\rho_h)\right)}{\left(\rho_h - \rho_e\right) \cdot \rho_h \cdot \rho_e} \cdot T.$$

State diagram

In accordance with the conceptual model, the item operation time consists of the period between inspections, the inspections themselves and the recovery. Fig. 2 shows the state diagram of operation. The states are numbered and designated with notional indexes: 1U is the up state of an item at the beginning of a period; 2UU is the up state of an item at the end of a period; 3UH is the state of an item at the end of a period with a hidden failure; 4UHE is the state of an item with a hidden and explicit failure; 5UE is the state of an item with a hidden and explicit failure; 6D is the down state of an item at the beginning of a period as the result of an inspection errors of the II kind; 7DD is the down state of an item at the end of a period; 8DE is the down state of an item with an explicit failure; 9IU and 10ID is the inspection of an up and down item at the end of a period; 11IE is the inspection of an item with an explicit failure (detection of explicit failure); 12R is item recovery.

Out of the above diagram follows that the up state is the initial one after recovery or latest inspection of an up item. The following period may be up (transition 1→2), with a hidden failure (transition 1→3) or with an explicit failure (transitions 1→4 and 1→5). Thus, up state 2 and down state 3 are the states at the end of a period, while states 4 and 5 are the states, whose duration is shorter that a period.

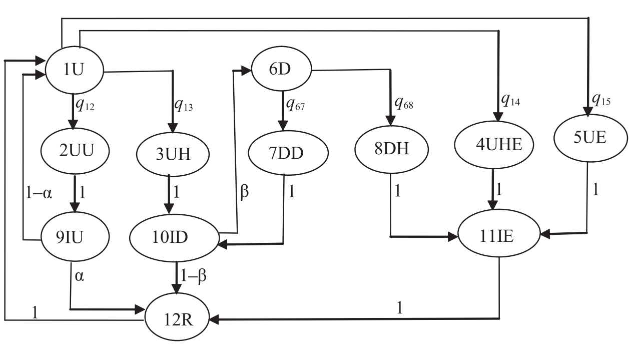

Fig. 2. State diagram of item operation

The diagram shows state transitions as the result of inspection errors: transition 9→12 as the result of an inspection error of the I kind with probability α and transition 10→6 as the result of an inspection error of the II kind with probability β. Transitions designated with probability 1 occur reliably.

After states 2 and 3, as part of technical state monitoring, the up and down items are inspected respectively. If the item is up, then, after the inspection, it is returned into operation with the probability 1 – α, while if it is down, it is submitted to recovery with the probability 1 – β. Upon recovery, the item is returned into operation in the up state.

The diagram shows the state transitions and the respective passage probabilities. The passage probability is a characteristic of the respective period:

$$q_{12} = p_{\text{uu}};\ q_{13} = p_{\text{uh}};\ q_{14} = p_{\text{uhe}};\ q_{15} = p_{\text{ue}};\ q_{67} = p_{\text{dh}};\ q_{68} = p_{\text{de}}. \quad (14)$$

State transitions are described using a passage probability matrix. A passage probability matrix at the whole set of states is as follows:

$$Q = \begin{pmatrix} 0 & q_{12} & q_{13} & q_{14} & q_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{67} & q_{68} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1-\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 1-\beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Relative frequencies of states

Lest us divide the set of states into two subsets: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $V=\{12\}$. As the result of such division, an item's operation can be represented as in the form of sequential transitions between such subsets: $U \to V \to U \to V \ldots$ An item being in the states of subset U followed by it being in the states of subset V will be referred as a cycle. The diagram in Fig. 1 follows that subset U always starts with state 1.

The passage probability matrix on subset U, Q_{UU} is obtained by removing the 12-th row and 12-th column from matrix Q. Out of matrix Q_{UU} the relative frequency matrix N_U is calculated according to formula (3). As state 1 always is the initial one in transition $V \to U$, we will calculate only the first row of matrix N_U. The elements of inverse matrix N_U can be calculated in a number of ways (it is recommended to use computer mathematics).

Let us express the passage probabilities in terms of reduced rates:

$$q_{12} = p_{\text{uu}} = \exp(-\rho);\ q_{13} = p_{\text{uh}} = \exp(-\rho_e) - \exp(-\rho);$$

$$q_{14} = p_{uhe} = \frac{\rho_h \cdot \left(1 - \exp(-\rho_e)\right) - \rho_e \cdot \left(\exp(-\rho_e) - \exp(-\rho)\right)}{\rho} = \frac{\rho_h + \rho_e \cdot \exp(-\rho) - \rho \cdot \exp(-\rho_e)}{\rho};$$

$$q_{15}=p_{ue}=\frac{\rho_e\cdot\left(1-\exp(-\rho)\right)}{\rho};$$

$$q_{67}=p_{\text{dd}}=\exp(-\rho_e);\ q_{68}=p_{\text{dh}}=1-\exp(-\rho_e). \quad (15)$$

Let us reduce the elements of the first row of matrix N_U by expressing them in terms of the passage probabilities and reduced rates:

$$n(1,1)=\frac{1}{\Delta 1};\ n(1,2)=\frac{q_{12}}{\Delta 1}=\frac{\exp(-\rho)}{\Delta 1};$$

$$n(1,3)=\frac{q_{13}}{\Delta 1}=\frac{\exp(-\rho_e)-\exp(-\rho)}{\Delta 1};$$

$$n(1,4)=\frac{q_{14}}{\Delta 1}=\frac{\rho_h\cdot\left(1-\exp(-\rho_e)\right)-\rho_e\cdot\left(\exp(-\rho_e)-\exp(-\rho)\right)}{\rho\cdot\Delta 1};$$

$$n(1,5)=\frac{q_{15}}{\Delta 1}=\frac{\rho_e\cdot\left(1-\exp(-\rho)\right)}{\rho\cdot\Delta 1};$$

$$n(1,6)=\frac{\beta\cdot q_{13}}{\Delta 1\cdot\Delta 2}=\frac{\beta\cdot\left(\exp(-\rho_e)-\exp(-\rho)\right)}{\Delta 1\cdot\Delta 2};$$

$$n(1,7)=\frac{\beta\cdot q_{13}\cdot q_{67}}{\Delta 1\cdot\Delta 2}=\frac{\beta\cdot\left(\exp(-\rho_e)-\exp(-\rho)\right)\cdot\exp(-\rho_e)}{\Delta 1\cdot\Delta 2};$$

$$n(1,8)=\frac{\beta\cdot q_{13}\cdot q_{68}}{\Delta 1\cdot\Delta 2}=\frac{\beta\cdot\left(\exp(-\rho_e)-\exp(-\rho)\right)\cdot\left(1-\exp(-\rho_e)\right)}{\Delta 1\cdot\Delta 2};$$

$$n(1,9)=\frac{q_{12}}{\Delta 1}=\frac{\exp(-\rho)}{\Delta 1};$$

$$n(1,10)=\frac{q_{13}}{\Delta 1\cdot\Delta 2}=\frac{\exp(-\rho_e)-\exp(-\rho)}{\Delta 1\cdot\Delta 2};$$

$$n(1,11)=\frac{q_{14}+q_{15}}{\Delta 1}+\frac{\beta\cdot q_{13}\cdot q_{68}}{\Delta 1\cdot\Delta 2}=\frac{1-\exp(-\rho_e)}{\Delta 1}+\frac{\beta\cdot\left(1-\exp(-\rho_h)\right)\cdot\left(1-\exp(-\rho_e)\right)\cdot\exp(-\rho_e)}{\Delta 1\cdot\Delta 2}, \quad (16)$$

where $\Delta 1=1-(1-\alpha)\cdot q_{12}=1-(1-\alpha)\cdot\exp(-\rho)$; $\Delta 2=1-\beta\cdot q_{67}=1-\beta\cdot\exp(-\rho_e)$.

In order to ensure a better understanding of the obtained results, let us refer to some formulas that confirm the correctness of the above findings:

1) the product of the first row of matrix N_U and matrix $(E-Q_{UU})$ equals the row, whose first element is equal to 1, the remaining elements are equal to 0;

2) active states 1 are distributed between states 2, 3, 4, 5: $n(1,2)+n(1,3)+n(1,4)+n(1,5)=n(1,1)$;

3) a state with an explicit failure within one cycle originates from states 4, 5, 8: $n(1,4)+n(1,5)+n(1,8)=n(1,11)$.

Item dependability indicators

Let us proceed to calculating the dependability indicators taking into consideration the adopted conditions and assumptions. In accordance with the conceptual model, the mean up and down time within one cycle is defined by such times in states 2, 3, 4, 5, 7, 8.

The mean up time within one cycle:

$$t_u=\left[\begin{array}{l} n(1,2)+n(1,3)\cdot\dfrac{1-\exp(-\rho_h)}{\rho_h}+n(1,4)\cdot \\ \cdot\dfrac{1-\exp(-\rho_h)}{\rho_h}+n(1,5)\cdot\dfrac{1-\exp(-\rho_e)}{\rho_e}\end{array}\right]\cdot T. \quad (17)$$

The mean down time within one cycle:

$$t_d=\left[\begin{array}{l} n(1,3)\cdot\dfrac{\rho_h-\left(1-\exp(-\rho_h)\right)}{\rho_h}+n(1,4)\cdot \\ \cdot\dfrac{\rho_h\left(1-\exp(-\rho_e)\right)-\rho_e\left(1-\exp(-\rho_h)\right)}{\left(\rho_h-\rho_e\right)\cdot\rho_e}+ \\ +n(1,7)+n(1,8)\cdot\dfrac{1-\exp(-\rho_e)}{\rho_e}\end{array}\right]\cdot T. \quad (18)$$

The mean cycle duration:

$$t_c=t_u+t_d. \quad (19)$$

Availability coefficient C_a and non-availability coefficient C_{na}

$$C_a=t_u/t_c;\ C_{na}=t_d/t_c. \quad (20)$$

Results

Set forth below are the calculation data for the mean cycle duration and non-availability coefficient under various failure rates and various probabilities of inspection errors. Those calculations are presented in table form with specific numerical values. Tables show the changes in the dependability indicators and predicted values of dependability indicators.

Table 4 shows the expected values of such indicators under various values of inspection error probability and reduced rates $\rho_h=0.005$ and $\rho_e=0.05$ and inspection frequency $T=1$ hour, while Table 5 shows similar calculations under $\rho_h=0.05$; $\rho_e=0.005$; $T=1$ hour.

The above calculations show that the mean cycle duration significantly depends on the probability of inspection errors of the I kind, as this probability defines the average number of up periods, i.e. the higher is the probability of inspection errors of the I kind, the lower is the average number of periods within one cycle. The calculations clearly indicate that

Table 4. Values of the non-availability coefficient if ρ_e = 0.005; ρ_e = 0.05; T = 10 h.

α \ β		0	0.001	0.01	0.1	0.2	0.3	0.4	0.5
0	t_c	18.7	18.7	18.7	18.7	18.7	18.7	18.8	18.8
	C_u	$1.3·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.2·10^{-3}$	$3.9·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.01	t_c	15.9	15.9	15.9	15.9	15.9	15.9	16.0	16.0
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$3.9·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.05	t_c	9.9	9.9	9.9	9.9	99	9.9	10.0	10.0
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$3.9·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.1	t_c	6.7	6.7	6.7	6.7	6.8	6.8	6.8	6.8
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$3.9·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.2	t_c	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$3.9·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.3	t_c	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$4.0·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.4	t_c	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$4.0·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$
0.5	t_c	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9
	C_u	$1.2·10^{-5}$	$2.2·10^{-5}$	$1.1·10^{-4}$	$1.0·10^{-3}$	$2.3·10^{-3}$	$4.0·10^{-3}$	$6.0·10^{-3}$	$8.9·10^{-3}$

Table 5. Values of the non-availability coefficient if ρ_e = 0.05; ρ_e = 0.005; T = 10 h.

α \ β		0	0.001	0.01	0.1	0.2	0.3	0.4	0.5
0	t_c	18.7	18.7	18.7	18.9	19.1	19.5	20.0	20.0
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.01	t_c	15.9	15.9	15.9	16.0	16.3	16.5	16.9	17.4
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.05	t_c	9.9	9.9	9.9	10.0	10.2	10.3	10.6	10.9
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.1	t_c	6.7	6.8	6.8	6.8	6.9	7.0	7.2	7.4
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.2	t_c	4.1	4.1	4.1	4.2	4.2	4.3	4.4	4.5
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.3	t_c	3.0	3.0	3.0	3.0	3.0	3.1	3.2	3.2
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.4	t_c	2.3	2.3	2.3	2.3	2.4	2.4	2.5	2.5
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$
0.5	t_c	1.9	1.9	1.9	1.9	1.9	2.0	2.0	2.1
	C_u	$1.2·10^{-3}$	$1.3·10^{-3}$	$2.2·10^{-3}$	$1.2·10^{-2}$	$2.5·10^{-2}$	$4.1·10^{-2}$	$6.2·10^{-2}$	$8.9·10^{-2}$

the average cycle duration changes by an order of magnitude in case of changes to the probability of errors of the I kind. It is also evident that the average cycle duration practically does not depend on the probability of error of the II kind, as the contribution of such error to down states is negligibly small as compared with the cycle duration. The explanation is simple. Down states as the result of inspection errors of the II kind can occur only after a hidden failure, while the probability of a hidden failure within a single period is a sufficiently small value. It should be noted that the mean cycle duration defines the recovery rate that can serve as the foundation for the calculation of the scope of recovery operations. The rate or percentage of false recoveries in the total scope of recovery activities.

Out of the above model follows that the non-availability coefficient practically does not depend on the probability of errors of the I kind, as the mean relative rates of all states are equally proportional to the parameter that depends on the probability of errors of the I kind (this parameter is designated $\Delta 1$). However, the non-availability coefficient significantly depends on the probability of inspection errors of the II kind. This indicator may vary several times and even 2 to 3 orders of magnitude in case of sufficient changes to the probability of errors of the II kind. That is due to the fact that the duration of down states may significantly vary subject to changes to the probability of errors of the II kind, although such values make up an insignificant part of the mean cycle duration.

The dependability level also depends on the correlations between the hidden and explicit failures, i.e. the higher is the share of hidden failures in the overall failure flow, the higher is the non-availability coefficient that may grow 1 to 2 orders of magnitude depending on this factor. Thus, the growing share of hidden failures may significantly reduce the dependability level.

Discussion

The specificity of this model consists in the fact that the technical state inspections as part of the monitoring system are conducted with a constant frequency. In the analytical models used for describing technical system monitoring, random inspection frequency is often used. Using different ways of defining inspection frequency in models may have a significant effect on the expected values of dependability indicators. For instance, in [8], it is shown that the difference between the unavailability coefficient under different ways of defining inspection frequency may amount to several orders of magnitude under various failure rates.

Another particular trait of the above model consists in the fact that time count starts from the observed events that include explicit failures, technical state inspections, completed recovery. It should be noted that, within the monitoring system, time count from a hidden failure is impossible, as such event is not observable. In Markovian model-based research, it is assumed that the duration of states is random and is distributed exponentially with a constant strength. At the same time, such choice is rarely substantiated. The meaning of the “raty of state end” parameter is normally not explained.

The advantage of the model is its compatibility with computer simulation tools, e.g. Mathcad and Matlab. The use of matrix methods provides simple calculation algorithms in those systems.

Conclusion

While designing new and improving the maintenance procedures of complex systems, explicit and hidden errors, inspection frequency, inspection errors are to be taken into consideration. At the same time, it is required to predict and calculate not only the availability or non-availability coefficients, but also the temporal characteristics associated with dependability. In actual monitoring systems, there is a great diversity in states, transitions and numeric values of input data. The presented model allows calculating and predicting such indicators subject to the influencing factors. For instance, the model allows analyzing the mean down time within one cycle and the components of this indicator that depend on the hidden failures and the probability of inspection errors of the II kind. The effect of explicit failures on the mean down time within one cycle can be taken into consideration as well.

Using the above model enables substantiated predictions of the dependability level taking into consideration the requirements for the monitoring system.

References

1. Viktorov V.S., Stepaniants A.S. [Models and methods of technical systems dependability calculations]. Moscow: Lenand; 2014. (in Russ.)

2. Egunov M.M., Shuvalov V.P. Reservation and recovery in telecommunication networks. *Vestnik SibGUTI* 2012;2:3-9. (in Russ.)

3. Zverev G.Ya. [Dependability estimation of an entity in the course of operation]. Moscow: URSS; 2010. (in Russ.)

4. Zelentsov B.P. [Matrix methods of simulating homogeneous Markovian processes]. Palmarium Academic Publishing; 2017. (in Russ.)

5. Zelentsov B.P. Matrix models of functioning of telecommunication equipment. *Vestnik SibGUTI* 2015;4;62-73. (in Russ.)

6. Zelentsov B.P. Method of relative frequencies for probabilistic systems modeling. *Vestnik SibGUTI* 2017;2:51-63. (in Russ.)

7. Zelentsov B.P. Cyclic functioning of long used systems. *Vestnik SibGUTI* 2017;4:3-14. (in Russ.)

8. Zelentsov B.P., Trofimov A.S. Research models of reliability calculation with different ways of task the periodic inspection. *Reliability & Quality of Complex Systems* 2019;1:35-44. (in Russ.)

9. Zubilevich A.L., Sidnev S.A., Tsarenko V.A. [Identifying the efficiency of application of the predictive FOCL maintenance strategy]. In: Proceedings of the XIII Interna-

tional industry science and technology conference Technology of the Information Society. Vol. 1. Moscow: ID Media Publisher; 2019. (in Russ.)

10. Ivchenko G.I., Kashtanov V.A., Kovalenko I.N. [Mass service theory]. Moscow: Vysshaya Shkola; 2012. (in Russ.)

11. Kelbert M.Ya., Sukhov Yu.M. [Probability and statistics in examples and problems. Vol. 2: Markovian chains as the foundation of the theory of random processes]. Moscow: MCCME; 2009. (in Russ.)

12. Koroliuk V.S., Turbin A.F. [Semi-Markovian processes and their applications]. Kiev: Naukova dumka; 1982. (in Russ.)

13. Lubkov N.V., Spiridonov I.B., Stepaniants A.S. [The effect of inspection characteristics on the dependability indicators of systems]. *Trudy MAI* 2016;85:1-27. (in Russ.)

14. Makhitko V.P., Zaskanov M.V., Savin M.V. Methods of products reliability evaluation based on the results of tests and operation. *Izvestia of Samara Scientific Center of the Russian Academy of Sciences* 2011:293-299. (in Russ.)

15. Ostreykovsky V.A. [Dependability theory]. Moscow: Vysshaya Shkola; 2003. (in Russ.)

16. Polovko A.M., Gurov S.M. [Introduction into the dependability theory]. Saint Petersburg: BHV-Peterburg; 2006. (in Russ.)

17. Rahman P.A. Reliability indices of repairable systems with predefined threshold of emergency shutdown. *International Journal of Applied and Fundamental Research* 2015;9:146-153. (in Russ.)

18. Silvestrov D.S. [Semi-Markovian processes with discrete set of states]. Moscow: Sovietskoye Radio; 1980. (in Russ.)

19. Trofimov A.S. [Functional model of relay protection of power systems]. *Elektroenergiya. Peredacha and raspredelenie* 2016;6:110-114. (in Russ.)

20. Chekmarev Yu.V. [Dependability of information systems]. Moscow: Dik Press; 2012. (in Russ.)

21. Shaykhutdinov D.V. Methods for dynamic complex technical systems monitoring and diagnosis based on imitation simulation. *Modern High Technologies* 2018;11(1):146-153. (in Russ.)

22. Shalin A.I. [Dependability and diagnostics of relay protection of power systems]. Novosibirsk: NSTU Publishing; 2003. (in Russ.)

23. Shneerzon E.M. [Digital relay protection]. Moscow: Energoatomizdat; 2007. (in Russ.)

About the author

Boris P. Zelentsov, Doctor of Engineering, Professor of the Department of Further Mathematics, Siberian State University of Telecommunications and Information Sciences, Russian Federation, Novosibirsk, e-mail: zelentsovb@mail.ru

The author’s contribution

Zelentsov B.P. developed an analytical model of item dependability based on matrix methods for situations of technical state monitoring with constant inspection frequency and subject to inspection errors and of failures of various kinds in terms of detection features.

Conflict of interests

The author declares the absence of a conflict of interests.