Dependability, vol. 20 no. 1, 2020 Original article
https://doi.org/10.21683/1729-2646-2020-20-1-20-24

On the consideration of progressive failure at the stage of design

Andrey I. Dolganov, *SEVERIN DEVELOPMENT Ltd., Russian Federation, Moscow*
dolganov-58@mail.ru

Andrey I. Dolganov

Abstract. *The stress that affects structures and their mechanical and geometrical parameters are random values. For that reason, the dependability of a construction facility (technical system) is generally evaluated in terms of the probability of no-failure over the estimated period of operation. The paper shows the feasibility of dependability analysis of building systems in the course of their design using logical and probabilistic methods, presents algorithms for regulating their dependability. It examines the feasibility of assuring the dependability of a construction project using the example of a double-span whole hinged beam. The paper also establishes the requirement of accounting for all possible destruction models of a building system. The dependability of a double-span whole hinged beam is estimated based on the probability of non-occurrence of all possible destruction models or one of a set of possible kinematic mechanisms. A kinematic mechanism forms a chain of plastic hinges or a chain of progressive failures of effective sections. In other words, the task of preventing progressive collapse comes down to ensuring the required dependability of both the building as a whole, and its individual members (effective sections) by adjusting qualitative and quantitative indicators of the dependability structure. The dependability of a member is understood as its ability to maintain internal force within the effective section at least as high as the external force. It is shown that correct design solutions, rational choice of materials and load non-exceedance probabilities enables specified dependability of a building system. In some cases that allows saving materials, in others enables lower probabilities of failure. Constructing the dependability structure of a technical system enables a quantitative estimation of the most hazardous design models of destruction, rational management of the choice of safety factors of load bearing members, redistribution of such safety factors, thus preventing progressive collapse. The introduced differential characteristics of the members' "weight", "significance", "contribution" and "specific contribution" allows demonstrating the distribution of the roles of each member within the specified structure in terms of specific problems, including accounting for the possibility of progressive collapse. The study has shown that the removal of undependable vertical load bearing structures does not solve the problem of dependability of a construction project, including protection against progressive collapse. It has been established that the design of structures, including in terms of considerations of progressive failure, must involve constructing a system dependability structure using kinematic analysis, identifying the most important and significant members of such structure and – using special adjustment techniques – obtaining the required structure dependability. That will enable significant resource saving and reduction of costs associated with the development of construction operations.*

Keywords: *probability, kinematic mechanism, dependability, plastic hinge, progressive collapse, destruction scheme, technical system.*

For citation: *Dolganov A.I. On the consideration of progressive failure at the stage of design. Dependability. 2020;1: 20-24. https://doi.org/10.21683/1729-2646-2020-20-1-20-24*

Received on: *28.08.2019* / **Revised on:** *15.02.2020* / **For printing:** *20.03.2020*

Initial observations

The paper examines the problem of accounting for progressive failure at the design stage. The requirement of progressive collapse calculation is set forth in Item 5.2.6 of GOST 27751-2014 Reliability for constructions and foundations. The calculations aim at preventing progressive (avalanche-type) destruction of buildings and structures.

According to the Guidelines for protection of tall building against progressive collapse and Guidelines for protection of monolithic residential buildings against progressive collapse developed by the Moscow City Architecture Committee in 2006 and 2005 respectively, as well as STO 008-02495342-2009 Prevention of progressive collapse of in-situ reinforced concrete building structures. Design and calculation, design should take into consideration the possible destruction (removal) of vertical structures of one (any) floor of a building:

1) two intersecting walls within the sections between the intersection (for instance, the building's corner) and the nearest aperture in each wall or vertical joint with a differently oriented wall (but with total length of the wall not more than 7 m);

2) freestanding column (pylon);

3) column (pylon) with sections of adjacent walls with the total length of 7 m.

At the same time, it is allowed to multiply standard characteristics of strength of materials by the extra factor of operating conditions of accidental limit state that is assumed to be from 1.1 to 1.25.

This approach to design allows doubling the span of flexible members and reducing the probability of non-exceedance of design strength.

Thus, by multiplying the standard strength (500 MPA) of A500 reinforcement steel by 1.15 we obtain 575 MPA, which is above the average value of 550 MPA. In other words, the resulting probability of non-exceedance of reinforcement steel strength is below 0.5.

According to the above recommendations, it is allowed to multiply the standard strength of concrete by 1.25. For instance, for B40 cement the estimated strength will be: 29 × 1.25 = 36.25 MPA. A the same time, the average value under the variation coefficient is 0.1 is 34.69 MPA. In other words, the resulting probability of non-exceedance of estimated strength is 0.326.

Thus, a structure designed per the above recommendations has inherently unacceptably low dependability along with unjustifiable overspending of materials, e.g. reinforcement steel (this can be compared to a situation where an airliner loses a wing or its size is reduced mid-flight).

The subject matter

A considerable contribution to dependability estimation of complex technical systems and research of structural redundancy was made in [1–3]. In [4], the problem of dependability estimation and construction of its structure was solved using the recurrent logical and probabilistic method.

The point of the below method of estimation of the possibility of progressive failure consists in the rational management of the dependability of individual components of technical systems. A simple example of double-span whole beam (Fig. 1.) is considered. The logical and probabilistic method of orthogonalization is used for its clarity [1, 5].

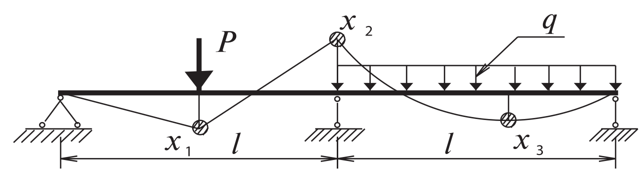

Figure 1 – The simplest technical system.

As kinematic analysis shows, the system (Fig. 1.) will fail in case of simultaneous failure of sections x_1 and x_2 or x_1 and x_3, or x_2 and x_3, i.e. in case of occurrence of one of the possible models of the kinematic mechanism. The above combinations of x_i (i = 1, 2, 3) are in fact the dependability structure of a technical system. Values x_i can be associated both with the probabilities of no-failure of system R_i, and probabilities of failure Q_i.

The matters of assignment of dependability levels for element x_i and methods of identifying the probabilities of no-failure of systems R_c are examined in [6, 7]. In other words, quantitatively system dependability is estimated using one of its indicators, i.e. probability non-occurrence of any possible models of destruction of a technical system or probability of development of one of a set of kinematic mechanisms.

As a kinematic mechanism forms a chain of plastic hinges (failures of members, effective sections), the task of prevention of a progressive collapse comes down to the assurance of the required dependability of its individual members (effective sections).

For cases when it must be decided with which section (member) dependability adjustment is to start in order to obtain the most rational technical system structure, special quantitative characteristics are used, i.e. "weight", "significance" and "contribution" of such member within the system's dependability structure. The above characteristics can allow identifying the "trouble spots" in a technical system, choosing optimal redundancy and rationally adjusting its dependability.

It is known that the initial dependability level of a technical system is defined by internal and external factors. For example, a structure's internal factors include the random nature of the geometrical parameters, mechanical characteristics of materials, etc. The external factors include the random nature of gravity, temperature loads, uneven settlement of undersoil, etc. For that reason dependability adjustment takes into consideration the independence of the external and internal factors. For instance, changes in the gravity loads do not modify the probability of non-exceedance of the mechanical characteristics of materials.

Below is the algorithm of dependability adjustment of a technical system. Formulas are given without derivations. The theoretical justification of formulas can be found in [1].

1. A structure diagram is constructed for dependability calculation of building systems

1.1. We identify all system components (effective sections), in which plastic hinges are allowed: x_i, i = 1, 2, 3 (Fig. 1).

1.2. Using kinematic analysis, all possible destruction models are specified for a technical system and written as conjunctions K_i:

$$y(x_1,x_2,x_3)=\begin{vmatrix}x_1x_2\\x_1x_3\\x_2x_3\end{vmatrix}=\begin{vmatrix}K_1\\K_2\\K_3\end{vmatrix}. \quad (1)$$

1.3. Using standard methods, expression (1) is transformed into orthogonal sum-of-products form (2):

$$y(x_1,x_2,x_3)=\begin{vmatrix}K_1\\K_1^{/}K_2\\K_1^{/}K_2^{/}K_3\end{vmatrix}=\begin{vmatrix}x_1x_2\\x_1x_2^{/}x_3\\x_1^{/}x_2x_3\end{vmatrix}, \quad (2)$$

where $K_1^{/}K_2=\begin{vmatrix}x_1^{/}\\x_1x_2^{/}\end{vmatrix}\left|x_1x_3\right|=\left|x_1x_2^{/}x_3\right|,$

$$K_1^{/}K_2^{/}K_3=\begin{vmatrix}x_1^{/}\\x_1x_2^{/}\end{vmatrix}\begin{vmatrix}x_1^{/}\\x_1x_3^{/}\end{vmatrix}\left|x_2x_3\right|=\left|x_1^{/}x_2x_3\right|.$$

1.4. The final formula of dependability structure for the considered technical system is written:

$$R_c = R_1R_2 + R_1Q_2R_3 + Q_1R_2R_3. \quad (3)$$

The matters of assignment of dependability levels for element x_i and methods of identifying the probabilities of no-failure of technical systems R_c are examined in [6, 7]. In the examined example, we will assign identical dependabilities of sections (probabilities of no-failure): $R_1 = R_2 = R_3 = 0.9$. Then, system dependability will be:

$$R_c = R^2 \times (1 + 2 \times Q) = 0.9^2 \times \times [1 + 2 \times (1 - 0.9)] = 0.972.$$

2. Parameters of the structure diagram are defined: "weight", "significance" and "contribution"

2.1. The "weight" of elements

$$g_{x_i}=\frac{G\{\Delta_{x_i}y(x_1,\ldots,x_n)\}}{2^n}=\sum_{j=1}^{l}2^{-(r_j-1)}-\sum_{f=1}^{k}2^{-(r_f-1)}, \quad (4)$$

where r_j is the rank of conjunction with x_i; r_f is the rank of conjunction with $x_i^{/}$.

For the considered example, the standard "weight" of element x_i, i = 1, 2, 3 is:

$$g_{x_1}=\frac{G\{\Delta_{x_1}y(x_1,x_2,x_3)\}}{2^3}=\frac{4}{8}=0,5,$$

$$g_{x_2}=\frac{G\{\Delta_{x_2}y(x_1,x_2,x_3)\}}{2^3}=\frac{4}{8}=0,5,$$

$$g_{x_3}=\frac{G\{\Delta_{x_3}y(x_1,x_2,x_3)\}}{2^3}=\frac{4}{8}=0,5.$$

The example shows that the "weights" of elements x_i, i = 1, 2, 3 are identical. Therefore, the "weight" of elements in the dependability structure of the system is identical. The "weight" of an element characterizes the relative number of such critical up states of a system, in which the failure of such element causes the failure of the system (and vice versa, its recovery causes the recovery of the system).

2.2. The "significance" of elements

The "significance" shows the effect of the element on the system's dependability.

$$\zeta_{x_i}=\frac{\partial P\{y(x_1,\ldots,x_n)=1\}}{\partial P\{x_i=1\}}=\frac{\partial R_c}{\partial R_i}. \quad (5)$$

For element 1, "significance" is:

$$\zeta_{x_1}=\frac{\partial y_c}{\partial x_1}=x_2+x_2^{/}x_3-x_2x_3=0,1+0,9\cdot 0,1-0,1^2=0,180.$$

2.3. "Contribution" of the elements

The "contribution" of element x_i in system $y(x_1, \ldots, x_n)$ is the product of the probability of no-failure of element R_i and its "significance", i.e.:

$$B_{x_i}=R_i\frac{\partial R_c}{\partial R_i}. \quad (6)$$

For element 1 the "contribution" is:

$$B_{x_i}=x_i^{/}\frac{\partial y_c}{\partial x_i}=0,9\cdot 0,18=0,162.$$

The criterion of "contribution" characterizes the increment of system dependability after the recovery of element x_1 from down or conditionally down state into up state with actual probability of no-failure of R_i.

2.4. "Specific contribution" of elements

The "specific contribution" of element x_i in system $y(x_1, \ldots, x_n)$ is the standardized "contribution" of such element, i.e.

$$b_{x_i}=B_{x_i}/\sum_{i=1}^{n}B_{x_i}=0,162/(3\times 0,162)=0,333. \quad (7)$$

The criterion of "contribution" enables rational definition of the priority of elements' recovery in the system.

3. The structural components of system dependability are defined

3.1. The "qualitative" components of dependability

The "qualitative" $\Delta R_c, q$ structural components of a technical system's dependability include the quality of materials, state of technology, probability of non-exceedance of design strengths of materials, loads, etc.

$$\Delta R_{c,q} = \sum_{i\in M_1} \frac{\partial R_c}{\partial R_i}\Delta R_i + \sum_{i,j\in M_2} \frac{\partial^2 R_c}{\partial R_i \partial R_j}\Delta R_i \Delta R_j + \ldots$$
$$\ldots + \rightarrow + \sum_{i,j,\ldots,k\in M_l} \frac{\partial^l R_c}{\partial R_i \partial R_j \ldots \partial R_k}\Delta R_i \Delta R_j \ldots \Delta R_k + \ldots$$
$$\ldots + \Delta R_1 \Delta R_2 \ldots \Delta R_n \qquad (8)$$

or in case of equal increments of dependability in effective sections ΔR_i

$$\Delta R_{c,q} = \sum_{j=1}^{n} C_n^j \frac{\partial^j R_c}{\partial R_i \ldots \partial R_k}(\Delta R_j)^j. \qquad (9)$$

Let us perform a qualitative progressive increment of the dependability of sections, e.g. up to 0.99. That can be achieved, for instance, by reducing the design loads. The difference between the specified, new (0.99) and initial (0.9) levels of dependability of sections ΔR_i, $i = 1, 2, 3$ will be: $\Delta R_1 = \Delta R_2 = \Delta R_3 = 0.99 - 0.9 = 0.09$.

According to formula (8) or (9), let us identify the qualitative increment of dependability:

$\Delta R_c, q = \Delta R \times [2R(1+2Q-R)] + \Delta R^2 \times [(1-2R) + 2\times(Q-R)] + \Delta R^3 \times (-2) =$

$= 0.09 \times [2 \times 0.9 \times (1 + 2 \times 0.1 - 0.9)] + 0.09^2 \times [1 - 2 \times 0.9) + 2 \times (0.1-0.9)] + 0.09^3 \times (-2) = 0.0277$.

System dependability subject to the qualitative increment became: 0.972 + 0.0277 = 0.9997.

We will get the same result if we substitute the required dependabilities of sections (0.99) into formula (3):

$R_c = 0.99^2 \times [1 + 2 \times (1 - 0.99)] = 0.9997$.

If we follow the Guidelines for protection of tall buildings against progressive collapse and Guidelines for protection of monolithic residential buildings against progressive collapse developed by the Moscow City Architecture Committee in 2006 and 2005 respectively, as well as STO 008-02495342-2009 Prevention of progressive collapse of in-situ reinforced concrete building structures. Design and calculation, we reduce the probabilities of non-exceedance of materials strength when we multiply them by the extra factor of conditions of operation for the accidental limit state. For that reason, the qualitative increment of dependability in this case is negative: – 0.0277. In other words, the dependability of sections will become: 0.9 – 0.0277 = 0.8723. System dependability will become $R_c = 0.8723^2 \times [1 + 2 \times (1-0.8723)] = 0.9552$.

3.2. The "quantitative" components of dependability

The "quantitative" components $\Delta R_c, v$ of the dependability structure of a technical system are materials reservation, reinforcing laps, additional pylons, connections, etc.

In case of quantitative variation of dependability, e.g. in case of duplication of the i-th element with same-type element x_i, the dependability of such group increases by ΔR_z [1]:

$$\Delta R_z = (2R_i - R_i^2) - R_i, \qquad (10)$$

the dependability of the whole system increases by R_c, ν:

$$\Delta R_{c,v} = \frac{\partial R_c}{\partial R_i}\Delta R_z = \frac{\partial R_c}{\partial R_i} R_i Q_i, \qquad (11)$$

where $Q_i = 1 - R_i$ is the probability of failure of the i-th section.

It is evident from (11) that a quantitative increment of system dependability depends on the "significance" and dependability of the duplicating element.

In the general case, in case of duplication of several elements up to the maximum possible number n, we will obtain

$$\Delta R_{c,v} = \sum_{i\in M_1} R_i Q_i \frac{\partial R_c}{\partial R_i} + \sum_{i,j\in M_2} R_i R_j Q_i Q_j \frac{\partial^2 R_c}{\partial R_i \partial R_j} + \ldots$$
$$\ldots + \sum_{i,j,\ldots,k\in M_l} R_i R_j \ldots R_k Q_i Q_j \ldots \rightarrow$$
$$\rightarrow \ldots Q_k \frac{\partial^l R_c}{\partial R_i \partial R_j \ldots \partial R_k} + \ldots + R_1 R_2 \ldots R_n Q_1 Q_2 \ldots Q_n. \qquad (12)$$

A quantitative variation of dependability can be achieved, for instance, by means of adjusting structural redundancy. For instance, as it was mentioned above, that may include reinforcing laps, addition of pylons or ties.

In the joint of section 2 (Fig. 1), let us make a provision for additional cover plates on beams or reinforcing lap (for reinforced concrete beams) able to withstand the ultimate moment. Thus, we duplicate element 2. We will calculate the quantitative increment of system dependability using formula (11):

$$\Delta R_{c,v} = \frac{\partial R_c}{\partial R_i}\Delta R_z = \frac{\partial R_c}{\partial R_i} R_i Q_i = 0{,}9997 \times$$
$$\times (1 + 0{,}0003 - 0{,}9997) \times 0{,}9997 \times 0{,}0003 = 1{,}8 \times 10^{-7}.$$

As calculations show, the addition of pylons, reinforcing lap or other ties does not result in a significant increment of dependability for the considered system.

Additionally, extra reinforcement of concrete structures (the case of concrete failure) may cause the inverse effect, i.e. reduced dependability. That is caused by the fact that in case of extra reinforcement the element's operation involves only one material, i.e. concrete. The variation coefficient of the strength of concrete is much higher than that of reinforcement steel.

System dependability subject to the qualitative and quantitative increment will be: $R_c = 0.972 + 0.0277 + 0.00000018 = 0.9997$. That is about 3.43 of the standard value, which complies with current expectations regarding the implications of dependability for building systems. Currently, dependability of sections of engineering structures of normal criticality projects is about 0.99865 [6].

Conclusion

According to current regulatory documents on the calculation of progressive collapse, the removal of vertical load-bearing members and associated reservation of additional materials, e.g. reinforcement steel, does not solve the problem of assuring required dependability of technical systems. Subsequently, such actions are useless, while this method of dependability adjustment is self-deceitful.

Progressive collapse can be prevented by rationally adjusting the selected safety factors of the load-bearing members and their redistribution through the construction of the dependability structure of a technical system as an obligatory part of its design. Additionally, the construction of the dependability structure of a technical system enables a quantitative estimation of the most hazardous design models of destruction, demonstration of the distribution of the role of all members within the specified structure as part of progressive collapse problem solution.

It is also shown that, as the dependability of a member deteriorates, its significance and contribution to the dependability structure of the considered technical system grow, and vice versa. System dependability does not change proportionally to the changes in the dependability of member (sections).

Thus, the design of structures, including in terms of consideration of progressive failure, must involve (along with the calculations per two groups of limit states) constructing a technical system dependability structure, identifying the most important and significant members of such structure and – using special adjustment techniques – obtaining the required structure dependability. That will enable significant resource saving and reduction of costs.

References

[1] Riabinin I.A., Cherkesov G.N. [Logical and probabilistic methods of dependability study of structurally complex systems]. Moscow: Radio i sviaz; 1981. (in Russ.)

[2] Schäbe H., Shubinsky I.B. Limit reliability of structural redundancy. Dependability. 2016;16(1):9-13. DOI: 10.21683/1729-2646-2016-16-1-3-13.

[3] Shubinsky I.B. [Structural dependability of information systems. Methods of analysis]. Ulianovsk: pechatny dvor; 2012. (in Russ.)

[4] Dolganov A.I. [Dependability assessment of site-cast multi-floor buildings]. Industrial and civil engineering. 2010;8:50-51. (in Russ.)

[5] Dolganov A.I. [Dependability of framed concrete structures]. Magadan: MAOBTI; 2001. (in Russ.)

[6] Dolganov A.I. Sakharov A.V. On the assignment of dependability level. Dependability. 2018;3:18-21.

[7] Dolganov A.I. [On the dependability of mass-built structures]. Industrial and civil engineering. 2010;11:66-68. (in Russ.)

About the author

Andrey I. Dolganov, Doctor of Engineering, Technical Director, SEVERIN DEVELOPMENT, Russian Federation, Moscow, e-mail: dolganov-58@mail.ru

The author’s contribution

Dolganov A.I. analyzed the construction accidents that occurred in Russia between 2001 and 2018, developed a method that is based on a probabilistic model and allows adjusting the dependability of technical systems, thus preventing progressive collapse with the probability adopted in industrial and civil engineering. The paper shows the applicability of the method to any building system. It also proposes developing the indicator of probability of no-failure for the purpose of technical system design using such parameters as significance, importance and contribution of each member (structure) within the dependability structure of a building.