# Fuzzy cognitive maps in the dependability analysis of systems

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Abstract. Aim. Dependability simulation of a complex system starts with its structuring, i.e. partitioning into components (blocks, units, elements), for which probabilities of failure are known. The classical dependability theory uses the concept of structural function that allows ranking elements by their importance, which is required for optimal distribution of the resources allocated to ensuring system dependability. Man-machine systems are structured using an algorithmic description of discrete processes of operation, where the presence of clear boundaries between individual operations allows collecting statistical data on the probabilities of error that is required for modeling. Algorithmization is complicated in case of man-machine systems with continuous human activity, where the absence of clear boundaries between operations prevents the correct assessment of the probability of their correct performance. For that reason, the process of operation has to be considered as a single operation, whose correct performance depends on heterogeneous and interconnected human-machine system-related, technical, software-specific, managerial and other factors. The simulated system becomes a "black box" with unknown structure (output is dependability, inputs are contributing factors), while the problem of element ranking typical to the dependability theory comes down to the problem of factor ranking. Regression analysis is one of the most popular means of multifactor dependability simulation of man-machine systems. It requires a large quantity of experimental data and is not compatible with qualitative factors that are measured by expert methods. The "if - then" fuzzy rule is a convenient tool for expert information processing. However, regression analysis and fuzzy rules have a common limitation: they require independent input variables, i.e. contributing factors. Fuzzy cognitive maps do not have this restriction. They are a new simulation tool that is not yet widely used in the dependability theory. The Aim of the paper is to raise awareness of dependability simulation with fuzzy cognitive maps. Method. It is proposed based on the theory of fuzzy cognitive maps - to rank factors that affect system dependability. The method is based on the formalization of causal relationships between the contributing factors and the dependability in the form of a fuzzy cognitive map, i.e. directed graph, whose node correspond to the system's dependability and contributing factors, while the weighted edges indicate the magnitude of the factors' effect on each other and the system's dependability. The rank of a factor is defined as an equivalent of the element's importance index per Birnbaum, which, in the probabilistic dependability theory is calculated based on the structure function. Results. Models and algorithms are proposed for calculation of the importance indexes of single factors and respective effects that affect system dependability represented with a fuzzy cognitive map. The method is exemplified by the dependability and safety of an automobile in the "driver-automobile-road" system subject to the driver's qualification, traffic situation, unit costs of operation, operating conditions, maintenance scheduling, quality of maintenance and repair, quality of automobile design, quality of operational materials and spare parts, as well as storage conditions. Conclusions. The advantages of the method include: a) use of available expert information with no collection and processing statistical data; b) capability to take into account any quantitative and qualitative factors associated with people, technology, software, quality of service, operating conditions, etc.; c) ease of expansion of the number of considered factors through the introduction of additional nodes and edges of the cognitive map graph. The method can be applied to complex systems with fuzzy structures, whose dependability strongly depends on interrelated factors that are measured by means of expert methods.

**Keywords:** fuzzy cognitive map, system dependability, contributing factors, factor ranking, dependability and safety of automobiles.

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#### 1. Introduction

Successful simulation in the context of application tasks is largely defined by the choice of mathematics. The probability theory, that is at the foundation of the classic dependability theory, is poorly adapted to formalizing expert knowledge, that may prove to be useful as part of the decision-making process.

The Aim of the paper is to raise awareness of dependability simulation with fuzzy cognitive maps. It sets forth the primary formulas of the above and further proposes the method of ranking of factors that affect system dependability. The method is illustrated using the example of simulation of dependability and safety of automobiles subject to technical, human-machine system-related, environmental and managerial factors.

# 2. Structuring: from elements to factors

Dependability simulation of a complex system starts with its structuring, i.e. partitioning into components (blocks, units, elements), for which probabilities of failure are known.

The classical dependability theory [1] uses the concept of structural (logical) function that associates the logical condition of system operability (1, no failure, 0, failure) with the respective conditions for its elements. The transition from the structure function to the probabilistic dependability model is performed according to the rules of probabilistic logic calculation [2]. The structural function allows ranking elements by their importance, which is required for optimal distribution of the resources allocated to ensuring system dependability.

Man-machine systems are structured using the algorithmic description of the operating processes [3, 4]. In this case, the given data for dependability calculation is the probabilities of correct performance of basic, check and diagnostic operations. The rules of transition from logical algorithmic description of a system in the language of algorithmic algebra by V.M. Glushkov [5] to probabilistic and fuzzy dependability models are suggested in [6, 7].

Algorithmic description is a natural method of formalization of systems with discrete processes of operation, e.g. automated data processing and control systems, assembly lines, etc., where the presence of clear boundaries between individual operations allows collecting statistical data on the probabilities of errors that is required for modeling.

Algorithmization is complicated in case of man-machine systems with continuous human activity that is dominated by operations of supervision and decision-making. Examples include control systems of the transportation, chemical and nuclear industries and other high-risk systems, where human errors cause catastrophic consequences.

The absence of clear boundaries between operations prevents a correct estimation of the probability of their correct performance. For that reason the process of operation

has to be considered as a single operation, whose correct performance depends on heterogeneous and interconnected human-machine system-related, technical, software-specific, managerial and other factors. The simulated system is a "black box" with unknown structure: output is dependability, inputs are contributing factors. In this case, the conventional problem of the dependability theory – the ranking of elements – becomes a problem of factor ranking. For instance, in [8] it is noted, that the difficulty of taking into account the contributing factors makes it impossible to accurately predict the probability of failure, which undermines the confidence in the dependability calculations.

Regression analysis is the most popular means of multifactor dependability simulation of man-machine systems (see e.g. [9]). It requires a large quantity of experimental data and is not compatible with qualitative factors that are measured by expert methods. The "if – then" fuzzy rules are a convenient tool for expert information processing [10]. Regression analysis and fuzzy rules have a common limitation: they require independent input variables, i.e. contributing factors. Fuzzy cognitive maps (FCM) [10] do not have this restriction. They are a new simulation tool that is not yet widely used in the dependability theory.

Set forth below are the primary FCM formulas and proposed method of ranking the factors that affect system dependability and safety. The method is illustrated with the "driver-automobile-road" system.

### 3. Primary concepts and formulas

#### 3.1. General observations

FCM were introduced by B. Kosko [11] as a generalization of R. Axelrod's binary cognitive maps [12], intended for simulating the dynamics of the causal relationships

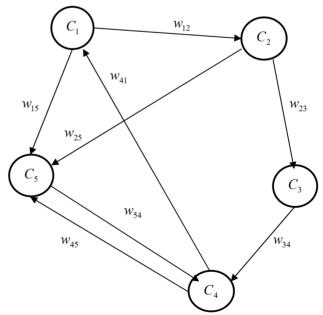


Figure 1 – An example of a fuzzy cognitive map.

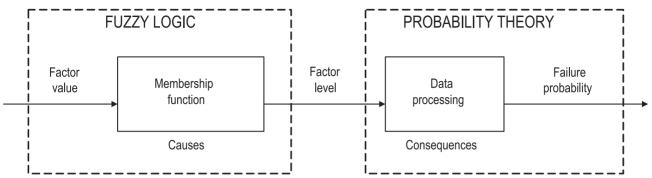


Figure 2 – Interrelation between the probability theory and fuzzy logic in dependability estimation.

in sociopolitical systems. FCM is a directed graph with weighted edges, of which an example is shown in Fig. 1. Graph nodes  $C_i$  called *concepts* correspond to the input and output variables that are taken into consideration in the model. Weighted edges of the graph reflect the *magnitude of the effect*  $w_{ij}$  of the changes of one variable  $C_i$  on the changes of another variable  $C_i$ .

The term "cognitive" implies, that the initial data for simulation consists of subjective opinions of an expert expressed as, e.g. "increases" or "decreases", for instance: "increasing  $C_i$  causes the decrease of  $C_j$ ". In binary cognitive maps [12], an "increase" is estimated as "+1", while a "decrease" is estimated as "-1".

The term "fuzzy" implies that FCM [11] use various levels of "increase" and "decrease". They are defined by numbers from the intervals [0, 1] and [-1, 0], which corresponds to the terms "weak", "average", "strong", etc. from the fuzzy set theory [10].

From the point of view of the identification theory [13, 14] that involves restoring patterns based on experimental data, FCM is an approximator of the "inputs/outputs" dependence with interrelated outputs. As any approximator, e.g. regression, fuzzy rules, neural network, etc., FCM contains configurable parameters that are to be estimated through minimization of the disparity between the model and experimental output values. If the experimental data "inputsoutputs" is not available, the quality of the whole model depends on the expert's qualification. The art of simulation consists in compensating for the missing experimental data through high quality of expert estimates.

It would be relevant comparing FCM and Markovian chains (processes) familiar to the dependability experts. Both types of models are weighted directed graphs. The basic difference between FCM and Markovian dependability models consists in the fundamental difference between the fuzzy logic (causes) and probability theory (effects) shown in Fig. 2: the Markovian models reflect the dynamics of system state probabilities accounting for failures and restorations; FCM simulate the level dynamics of interrelated factors that cause failures and affect their probability.

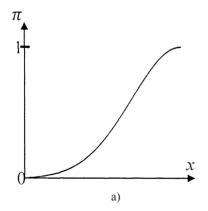
# 3.2. Concepts

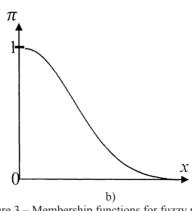
Let  $C=\{C_1, C_2, ..., C_n\}$  be a known set of concepts, i.e. variables used in the model. According to [11], each concept  $C_i \in C$  is evaluated with value  $A_i \in [0,1]$ , that defines the level of the concept and is based on expert opinion. Value  $A_i$  is to be obtained as follows.

We will assume each concept  $C_i \in C$  to be a linguistic variable [10], that is estimated with value  $x_i$  on a universal set, i.e. interval  $\left[\underline{x}_i, \overline{x}_i\right]$ , where  $\underline{x}_i\left(\overline{x}_i\right)$  is the lower (upper) boundary. We will estimate concept  $C_i \in C$  with the use of the fuzzy terms "perfection of concept  $C_i$ ", that is denoted as  $PC_i$  and is a fuzzy set

$$PC_{i} = \int_{\left[\underline{x}_{i}, \overline{x}_{i}\right]} \pi(x_{i})/x_{i},$$

where  $\pi(x_i)$  is the membership function of variable  $x_i$  in the notion of "perfection of concept  $C_i$ ". Using this function, each absolute estimate  $x_i \in [\underline{x}_i, \overline{x}_i]$  is associated with





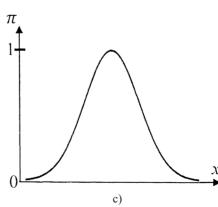


Figure 3 – Membership functions for fuzzy perfection.

number  $A_i = \pi(x_i) \in [0,1]$ , that characterizes the proximity of the value of concept  $C_i \in C$  to a certain ideal: 0 is the lowest perfection, 1 is the highest perfection. "Fuzzy perfection" is synonymous with "fuzzy correctness", for which the membership functions were considered in [15]. Possible fuzzy boundaries between perfect and non-perfect values of variable x are shown in Fig. 3, where, as the value of x grows, the following transitions take place:

- a) "non-perfect" (1) "perfect" (0),
- b) "perfect" (1) "non-perfect" (0),
- c) "non-perfect" (0) "perfect" (1) "non-perfect" (0).

# 3.3. Associations between concepts

The weight  $w_{ij}$  of the edge that connects concepts  $C_i$  and  $C_j$  indicates the magnitude of the effect of  $C_i$  on  $C_j$ . Let concepts  $C_i$  and  $C_j$  be characterized by variables  $x_i$  and  $x_j$ , while – as the result of the experiment – dependence  $x_j = \varphi(x_i)$  was achieved. Then, the weight  $w_{ij}$  is defined as the derivative  $w_{ij} = dx_i/dx_i$  that can have three forms (Fig. 4):

 $w_{ij} > 0$ , if the increase (decrease) of value  $x_i$  causes the increase (decrease) of value  $x_j$  (positive effect of  $C_i$  on  $C_j$ );  $w_{ij} > 0$  if the increase (decrease) of value  $x_i$  causes the decrease (increase) of value  $x_j$  (negative effect of  $C_i$  on  $C_i$ );

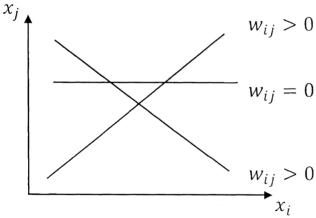


Figure 4 – Types of effects between concepts.

Table 1. Methods of estimating the magnitude of an effect.

Thermom- eter scale	Linguistic estimations	Quantitative estimations		
	Positive maximum Positive above average Positive average Positive under average Not available Negative under average Negative average Negative above average Negative maximum	1 0,75 0,5 0,25 0 -0,25 -0,5 -0,75		

 $w_{ij} = 0$  if value  $x_j$  does not depend on value  $x_i$  (no effect  $C_i$  on  $C_i$ ).

The magnitude of effect  $(w_{ij})$  is estimated expertly by means of linguistic terms and thermometer scale (Table 1). If several expert opinions are taken into consideration, the value  $w_{ij}$  is estimated as the weighted average:

$$w_{ij} = \frac{\alpha_1 w_{ij}^1 + \alpha_2 w_{ij}^2 + \dots + \alpha_m w_{ij}^m}{\alpha_1 + \alpha_2 + \dots + \alpha_m},$$

where  $w_{ij}^p$  is the estimate of the magnitude of the effect of the *p*-th expert;  $\alpha_p$  is the weight of the *p*-th expert, p = 1, 2, ..., m; *m* is the number of experts.

In order to reduce the subjectivity of expert estimates, the method of the least effect proposed in [16] can be used.

#### 3.4. Recurrence equations

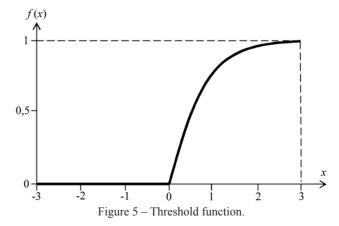
According to [11, 17], the dynamics of concept values variation in FCM are defined by formula

$$A_i^{k+1} = f\left(\sum_{\substack{j=1\\j\neq i}}^n A_j^k w_{ji} + cA_i^k\right), k = 0, 1, 2, \dots$$
 (1)

where  $A_i^{k+1}$  is the value of concept  $C_i$  at step k+1;  $A_i^k$  and  $A_j^k$  is the value of concept  $C_i$  and  $C_j$  at step k respectively,  $w_{ji}$  is the magnitude of the effect of concept  $C_j$  on concept  $C_i$ ; c is the parameter that takes into consideration the history, i.e. the contribution of the concept's value at the preceding step,  $c \in [0,1]$ ; f is the threshold function, due to which the value of the concept does not exceed one.

In this paper, it is assumed that c = 1, while for the threshold function is used the positive part of the hyperbolic tangent (Fig. 5):

$$f(x) = \begin{cases} \tanh(x) \text{ при } x \ge 0; \\ 0 \text{ при } x < 0, \end{cases} \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



#### 3.5. Matrix model

The recurrence equation (1) can be represented in matrix form

$$A^{k+1} = f(A^k W_0 + cA^k), k = 0, 1, 2, ...,$$
 (2)

where  $A^{k+1}$ ,  $A^k$  k = 0, 1, 2, ... are  $(1 \times n)$  state vectors of FCM, whose elements define the values of concept at steps k+1 and k respectively;

$$W_{0} = \begin{bmatrix} 0 & w_{12} & \dots & w_{1n} \\ w_{21} & 0 & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & 0 \end{bmatrix}$$
(3)

is the  $(n \times n)$  matrix of the magnitude of mutual effects of concepts  $C_i$ , in which diagonal elements are equal to zero.

If instead of matrix (3) an  $(n \times n)$  matrix is used,

$$W = \begin{bmatrix} c & w_{12} & \dots & w_{1n} \\ w_{21} & c & \dots & w_{2n} \\ \vdots & \vdots & \dots & \vdots \\ w_{n1} & w_{n2} & \dots & c \end{bmatrix}, \tag{4}$$

in which all elements on the main diagonal are equal to parameter  $c \in [0,1]$ , then we will write formula (2) as

$$A^{k+1} = f(A^k W), k = 0, 1, 2, ...,$$
 (5)

that is similar to the recurrence equation for a Markovian chain, if we take f(x) = x. The fundamental difference consists in the fact that a Markovian chain simulates the dynamics of event probability variation, while FCM simulates the dynamics of the level of causes, i.e. factors that lead to such states or events (see Fig. 2).

The initial state of an FCM is defined by vector

$$A^{0} = \left[ A_{1}^{0}, A_{2}^{0}, ..., A_{n}^{0} \right], \tag{6}$$

whose elements reflect the values of concepts at step k = 0. As the result of interaction between concepts FCM enters the steady mode, that corresponds with one of the types of stability [18].

#### 4. Ranking of concepts

The allocation of system dependability resources is based on quantitative estimates (ranks) of its elements' importance. In the statistical dependability theory, Birnbaum's importance index of an element is the most widely used [1]. It is defined based on the system's dependability function

$$P_S = f_S(P_1, ..., P_i, ...),$$
 (7)

where  $P_S$  and  $P_i$  are the system's probability of no-failure and its *i*-the element respectively.

The first derivative in (7) is the importance index of the system's *i*-th element according to Birnbaum, that is calculated as follows [1]:

$$I_{i} = \frac{\partial P_{S}}{\partial P_{i}} = P_{S} (P_{1}, ..., P_{i-1}, 1, P_{i+1}, ..., P_{n}) - P_{S} (P_{1}, ..., P_{i-1}, 0, P_{i+1}, ..., P_{n}).$$
(8)

The second derivative in (7) is the importance index of the joint effect of the *i*-th and *j*-th elements (*joint reliability importance*), that was introduced in [19, 20].

In our case the elements of the model include the input concepts, i.e. the factors that affect the output level of system dependability. That explains the requirement to calculate the importance indices of FCM concepts.

# 4.1. Definition of importance indices

In the set of concepts  $C=\{C_1, C_2, ..., C_n\}$  we will assume the following:

 $C_n$  is the output concept that defines the level of system dependability and is estimated with number  $A_n \in [0,1]$ ;

 $C_1, C_2, ..., C_{n-1}$  are the input concepts that correspond with the interconnected factors affecting system dependability and estimated by levels  $A_i \in [0,1]$ , i = 1, ..., n-1.

The value of concept  $C_n$  at the *l*-th step is the function of the elements of vector (6), i.e.

$$A_n^l = F(A_1^0, A_2^0, ..., A_n^0). (9)$$

It is assumed that  $A_n^l$  is the value of concept  $C_n$  in the steady state, i.e. at such step l, when  $A_n^l$  is close to  $A_n^{l-1}$ . Formula (9) is equivalent to (7), which allows proceeding to the definition of concept ranks based on derivatives.

Let  $I(C_j)$  be the importance index of concept  $C_j$ , while  $I(C_p, C_k)$  be the index of combined importance of concepts  $C_j$  and  $C_k$ . Following (8) and [19, 20], let us identify such importance indices as:

$$I\left(C_{j}\right) = \frac{\partial A_{n}^{l}}{\partial A} = \frac{F\left(1_{j}, 0\right) - F\left(0\right)}{1 - 0} = F\left(1_{j}, 0\right), \quad (10)$$

$$I(C_{j}, C_{k}) = \frac{\partial^{2} A_{n}^{l}}{\partial A_{j} \partial A_{k}} = \frac{F(1_{j}, 1_{k}, 0) - F(0)}{(1 - 0)(1 - 0)} = F(1_{j}, 1_{k}, 0), \quad (11)$$

where  $F(1_j,0)$  is the value of function (9), when  $A_j^0 = 1$  are equal to zero; F(0) is the value of function (9), when all arguments are equal to zero (it is assumed that F(0) = 0);  $F(1_j,1_k,0)$  is the value of function (9), when  $A_j^0 = A_k^0 = 1$ , while all the other arguments are equal to zero.

*Note.* The zero values of input concepts (except one in (10) and two in (11) that are equal to one) are selected in order to eliminate the possibility of them having an effect on the output concept through transitive connections.

# 4.2. Algorithm of importance index calculation

Step 1. Specifying the initial vector (6). For importance index  $I(C_i)$ , the initial vector is specified as follows

$$A^{0} = [A_{i}^{0} = 1, A_{i}^{0} = 0, i = 1, 2, ..., n, i \neq j],$$
 (12)

while for importance index  $I(C_i, C_k)$  it is specified as

$$A^{0} = \left[ A_{j}^{0} = A_{k}^{0} = 1, A_{i}^{0} = 0, \ i = 1, 2, ..., n, \ i \neq j, k \right].$$
 (13)

Step 2. Using recurrence equation (5), finding the FCM state vector

$$A^{l} = \begin{bmatrix} A_1^{l}, A_2^{l}, ..., A_n^{l} \end{bmatrix}$$
 (14)

in steady-state operating conditions, i.e. at such step l, whereas  $\left|A_i^l - A_i^{l-1}\right| < \varepsilon$ , where  $\varepsilon$  is a small positive number, i = 1, 2, ..., n.

Step 3. Elements  $A_n^I$  of vector (14) obtained under initial vectors (12) and (13) respectively shall be considered to be importance indices  $I(C_i)$  and  $I(C_i, C_k)$ .

# 5. An example

# 5.1. Concepts and effects

Let us examine the automobile dependability and safety model in the "driver-automobile-road" system. The fuzzy cognitive map of the system is shown in Fig. 6, where the concepts have the following contents:  $C_1$  is the driver's

Table 2. Values of concepts in steady state for various initial vectors.

Step	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_9$	$A_{10}$
1	1	0	0	0	0	0	0	0	0	0
3040	0,022	0,000	0,000	0,000	0,187	0,000	0,000	0,000	0,000	0,68579
1	0	1	0	0	0	0	0	0	0	0
774	0,000	0,044	0,000	0,365	0,747	0,000	0,000	0,000	0,000	0,94834
1	0	0	1	0	0	0	0	0	0	0
3717	0,000	0,000	0,020	0,000	0,000	0,000	0,000	0,000	0,000	0,22707
1	0	0	0	1	0	0	0	0	0	0
3014	0,000	0,000	0,000	0,022	0,335	0,000	0,000	0,000	0,000	0,79115
1	0	0	0	0	1	0	0	0	0	0
5324	0,000	0,000	0,000	0,000	0,017	0,000	0,000	0,000	0,000	0,33491
1	0	0	0	0	0	1	0	0	0	0
3196	0,000	0,000	0,000	0,000	0,186	0,022	0,000	0,000	0,000	0,68912
1	0	0	0	0	0	0	1	0	0	0
4953	0,000	0,000	0,000	0,000	0,000	0,000	0,017	0,000	0,000	0,30912
1	0	0	0	0	0	0	0	1	0	0
2742	0,000	0,000	0,000	0,000	0,321	0,000	0,000	0,023	0,000	0,77418
1	0	0	0	0	0	0	0	0	1	0
3086	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,022	0,18667

Table 3. Importance indices of combined effect of factors.

Concepts	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	<i>C</i> <sub>7</sub>	$C_8$	$C_9$
$C_1$	0,949	0,686	0,801	0,686	0,730	0,335	0,786	0,255
$C_2$	_	0,948	0,948	0,948	0,950	0,949	0,950	0,948
$C_3$	_	_	0,791	0,335	0,689	0,309	0,774	0,254
$C_4$	_	_	_	0,791	0,803	0,703	0,823	0,782
$C_5$	_	_	_	_	0,689	0,309	0,774	0,187
$C_6$	_	_	_	_	_	0,356	0,788	0,294
$C_7$	_	_	_	_	_	_	0,309	0,323
$C_{8}$	_	_	_	_	_	_	_	0,763

qualification,  $C_2$  is the road conditions,  $C_3$  is the unit costs of operation,  $C_4$  is the operating conditions,  $C_5$  is the frequency of maintenance operations,  $C_6$  is the quality of service and repair,  $C_7$  is the quality of automobile's design,  $C_8$  is the quality of operational materials and spare parts,  $C_9$  is the storage conditions,  $C_{10}$  is the dependability and safety of the automobile.

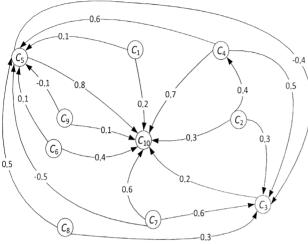


Figure 6 – Fuzzy cognitive map for dependability and safety estimation.

Matrix W(4) with expert estimates of the magnitude of effect, that assumes that c = 1, is as follows

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0,1 & 0 & 0 & 0 & 0,5 \\ 0 & 1 & 0,3 & 0,4 & 0 & 0 & 0 & 0 & 0 & 0,3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0,2 \\ 0 & 0 & 0,5 & 1 & 0,6 & 0 & 0 & 0 & 0 & 0,7 \\ 0 & 0 & -0,4 & 0 & 1 & 0 & 0 & 0 & 0 & 0,8 \\ 0 & 0 & 0 & 0 & 0,1 & 1 & 0 & 0 & 0 & 0,4 \\ 0 & 0 & -0,6 & 0 & -0,5 & 0 & 1 & 0 & 0 & 0,6 \\ 0 & 0 & 0,3 & 0 & 0,5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0,1 & 0 & 0 & 0 & 1 & 0,1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

# 5.2. Importance indices of concepts

Table 2 contains nine pairs of vectors associated with the calculation of the importance indices of concepts  $C_1, \ldots, C_9$ . Each pair contains the initial vector (12) and vector (14) in steady-state operating conditions. The last element of the second vector in each pair corresponds to the importance index of the concept, i.e.  $I(C_1) = 0.686$ . The last column in Table 2 shows the step-by-step change of the level of dependability and safety of an automobile  $(A_{10})$  in case of activation of one of the factors  $(A_i, i=1, \ldots, 9)$ . The diagram of the importance indices of concepts is shown in Figure 7. The results of calculation of the importance indices of the combined effect of concepts are shown in Table 3, i.e.  $I(C_1, C_2) = 0.949$ .

It should be noted that concept  $C_7$  can be detailed subjects to the conclusions of [21].

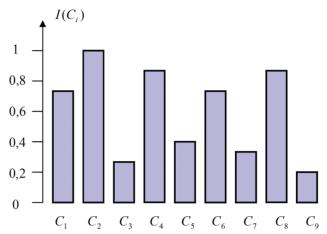


Figure 7 – Diagram of importance indices of factors.

#### 6. Conclusion

The paper proposes and demonstrates with an example of a man-machine system a method of ranking of factors that affect its dependability. The method is based on the formalization of causal relationships between the contributing factors and the dependability in the form of a fuzzy cognitive map, i.e. directed graph, whose nodes correspond to the system dependability and contributing factors, while the weighted edges indicate the magnitude of the factors' effect on each other and the system's dependability.

The proposed method may be regarded as an equivalent to Birnbaum's ranking of system components in the probabilistic dependability theory. The advantages of the method include:

- use of available expert information with no collection and processing of statistical data;
- capability to take into consideration any qualitative and qualitative factors associated with people, technology, software, quality of service, operating conditions, etc.; In particular, individual concepts can characterize various types of redundancy (structural, algorithmic, etc.), that are used to improve dependability;
- easily scalable number of considered factors through the introduction of new nodes and edges of a directed graph.

The method can be applied to complex systems with fuzzy structures, whose dependability strongly depends on interrelated factors that are measured by means of expert methods.

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