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ORGANIZATION OF MONITORING AND OPTIMAL PREVENTIVE MEASURES OF A TRANSPORT ACCIDENT WITH THE SPECIFIED DEPENDABILITY LEVEL

The probability of a transport accident significantly depends on various factors and groups of factors. The paper investigates the economic expedience of monitoring a random factor and transport accident prevention. Several criteria for utility evaluation of monitoring and prevention for various strategies to minimize an accident risk are offered. The paper also presents an example of using these results in a train makeup.

Keywords: *monitoring, probability of a transport accident, optimal preventive measures, dependability, cost-effectiveness.*

1. Introduction

As is known from papers [1] and [2], various factors and groups of factors affect the occurrence of transport accidents and their consequences. At that, some factors affect the transport accident to a greater extent and the other – to a smaller extent. The characteristic of this effect are conditional probabilities of a transport accident under conditions of factors' occurrence [2]. Analytically, these probabilities cannot be defined. Therefore, the method of the probabilities' assessment based on observations of transport incident and realized values of factors is offered in the framework of the papers [1] and [2]. If simultaneous occurrence of factors and accidents can be estimated based on occurrences' report, then the conditional probability of accidents' occurrence at appearance of factors can be assessed only on the basis of the occurrence probability of factors that cannot be estimated only on the basis of accidents' protocols. Therefore, the organization of factors' appearance monitoring is urgent. In addition, these monitoring arrangements may require significant financial resources. Therefore, it is necessary to compare the money spent on monitoring with risk of accidents' occurrence (average loss in an accident).

Moreover, it is necessary to carry out addition preventive measures, aimed at reducing the impact of factors on the accident after monitoring arrangements of factors appearance. Such preventive measures also require financial resources. Therefore, the urgent task consists in assessing the resources needed for the organization of monitoring and preventive measures, as well as their comparison with the risk of accidents.

This paper considers the problem to minimize the influence of a random factor on the transport accident, using different criteria, in particular in the form of expectation. In addition, the paper presents the economic inexpediency of monitoring several special cases. This study also considers the task of finding the optimal level of a factor to which it should be brought taking into account the balance of the total cost and decision-making dependability. The economic effect value of preventive measures, insured at a given dependability level is calculated. The paper offers the succession of actions to organize monitoring of a factor appearance and optimal prevention of a transport accident. The example associated with a train makeup is presented.

2. Minimizing a factor impact on a transport accident

Let us state the task of minimizing the impact of a random factor on a transport accident.

Let A be a transport accident (such as rail accident), $P(A)$ – the probability of its occurrence, and c – the damage cost of the event occurrence, which is assumed to be known. We shall suppose that there is a factor F affecting the incident frequency. For example, if railway accident is under study, then you can take the number of cars in the train or the railroad bed displacement as F. Let F has a discrete set of numerical values f_k,, which are implemented with the corresponding probabilities $p_k, k = 1, \dots, N$.

At first, we shall formulate the simple task of assessing the expedience to carry out of monitoring the factor occurrence and the transport accident preventive measures implementation. Then we shall complicate the problem and consider it in a more general statement.

Let us find such a number of K of the value f_i for the factor F, where the conditional probability of the event A will be minimal, if the factor F has taken the value f_i

$$K = \arg \min_{i=1,\dots,N} P(A \mid F = f_i).$$

Then, if the implemented value f_i of the factor F is different from f_K it is reasonably to spend a certain amount of funds c_i to reduce the impact of the factor (that is, to bring it to the level f_K). Just, these actions should be understood as preventive measures, aimed at reducing the risk of a transport accident. Note that, on the one hand, during an accident prevention the transport risk decreases, on the other hand, there will be additional costs for the organization of preventive measures. Let us estimate the total costs arising from this strategy. Obviously, without observation (monitoring) of the factor F it is not possible to organize preventive measures. We shall assume that after the preventive measures the value of the actor F will be equal to f_K. Then the total costs related to the organization of the monitoring and preventive measures will consist of 3 values: a constant c_E related to the costs connected with monitoring of equipment installation and operation, random costs C_F^K directed to changing the value of the factor F (bringing it to the level f_K) and random costs C_A^K related to the potential damage of the transport accident A after carrying out of incident preventive measures.

In connection with mentioned above, the costs C_F^K for changing the factor F, and bringing it to the level f_K may be presented in the form of discrete random variable with a row of the distribution

C_F^K	c_1^K	c_2^K	$\dots$	c_N^K
P	p_1	p_2	$\dots$	p_N

where c_i^K is the value of funds that are needed to be spent in order to bring the value of the factor F from the level f_i *to* the level f_K , $i = 1, \dots, N$. It is obviously, that $c_K^K = 0$, since in this case preventive measures are not carried out.

In turn, the costs C_A^K for recoverinfg a possible damage at the occurrence of the event A after preventive measures can be represented by the following row of the distribution

C_A^K	c	0
P	$P_K(A)$	$1-P_K(A)$

where $P_K(A)$ is the probability of the event A occurrence after the reducing the factor F to the level f_K. In fact, $P_K(A)$ is the conditional probability of an event, provided that the preventive measures and the factor F have been brought to the level f_K. As after preventive measures the factor F can take only one value f_K with probability equal to one, then according to the formula of the total probability [3], we obtain:

$$P_K(A) = P(A \mid F = f_K)\sum_{i=1}^{N} p_i = P(A \mid F = f_K). \quad (1)$$

Note that the probability of the event A without preventive measures is equal to $P(A)$, that is to the unconditional probability of a transport accident. According to the formula of the total probability, we obtain that $P(A) = \sum_{i=1}^{N} P(A \mid F = f_i)p_i$.

However, according to our assumption $P(A \mid F = f_K) \le P(A \mid F = f_i)$ for all $i \ne K$, therefore we have the following:

$$P(A) \ge P(A \mid F = f_K)\sum_{i=1}^{N} p_i = P(A \mid F = f_K).$$

Thus, after preventive measures, the probability of an accident, and, consequently, the risk of that accident will be reduced. However, additional costs, which are associated with the event monitoring and its preventive measures will arise. Therefore, the cumulative loss of this strategy in case of monitoring system for preventive measures will make up the following:

$$\Phi = c_E + C_F^K + C_A^K.$$

Using the criterion in the form of expectation, we shall obtain the average costs associated with the system of monitoring and preventive measures, which will be equal to the following expression:

$$\overline{\Phi} \overset{def}{=} \mathrm{M}[\Phi] = c_E + \mathrm{M}\left[C_F^K\right] + \mathrm{M}\left[C_A^K\right].$$

Taking advantage of the expectation definition, we get the formula:

$$\overline{\Phi} = c_E + \sum_{i=1}^{N} c_i^K p_i + cP(A \mid F = f_K). \quad (2)$$

It should be noted that the average cost L (risk of an accident) without monitoring systems makes up what is presented below

$$L \overset{def}{=} \mathrm{M}[C_A] = cP(A), \tag{3}$$

where C_A is the accidental losses at incident A in case of no preventive measures, $P(A)$ is the probability of the event A if there is no preventive measures, c is the damage in case of an accident, which $P(A)$ is obtained under the formula of the total probability (1). Thus, we get

$$L = c\sum_{i=1}^{N} P\{A \mid F = f_i\} p_i. \tag{4}$$

If the average cost of using monitoring with preventive measures is not more than the cost of its absence, that is

$$\overline{\Phi} \le L, \tag{5}$$

it can be assumed that the monitoring system with the proposed preventive measures would be useful. Otherwise, its use should be recognized as inappropriate.

3. The optimal preventive measures of a transport accident

However, there is a possibility to use the available financial resources more efficiently. If the factor F is different from the optimal value f_K, it is not necessarily to lead the factor F to the level f_K, as it may require significant financial resources, and you can try to bring the factor to some other value of f_k, differing from f_K. In this case, the cost of changing factor may be reduced, and the cost of possible damage will change (may increase), but together they could be reduced, and, therefore, the average cost can be reduced. Let us formulate the corresponding task.

We shall suppose that the conditional probability $P(A \mid F = f_i)$ increases monotonically $i = 1, \ldots, N$. Let the desired level of the factor F is f_k, then the total loss will be equal to

$$\Phi(k) = c_E + C_F^k + C_A^k,$$

where C_F^k is a random variable which represents the amount of funds that should be directed at bringing the factor F to the level f_k, and C_A^k is a random variable of costs in the accident A after preventive measures, that is, after the reduction of the factor F from the level f_i to the level f_k, if $i > k$. In case, $i \le k$, preventive measures is proposed not to be carried out, because by assumption the probability $P(A \mid F = f_i)$ should not be higher than the probability $P(A \mid F = f_k)$ for $i \le k$. Let the random variable C_F^k has a set of the following distribution

C_F^k	c_1^k	c_2^k	$\cdots$	c_i^k	c_{i+1}^k	$\cdots$	c_N^k
P	p_1	p_2	$\cdots$	p_i	p_{i+1}	$\cdots$	p_N

where c_i^k describes the amount of funds to be expended in order to reduce the value of the factor F from the level f_i to the desired level f_k, $i=1,\ldots,N$, and p_i, is the probability of the factor F appearance with the value f_i, $i=1,\ldots,N$. We also assume that the values f_i are monotonically increasing for $i=1,\ldots,N$. Note that the values $c_i^k=0$ for $i \le k$, because preventive measures is not carried out, if the value of the factor f_i is not greater than f_k, $i \le k$. If we still would bring the value of the factor F to the level f_k for $i \le k$, then the probability of a transport accident only would increase, as well as additional funds. Thus, after preventive measures, the random factor F can possess only k values and at that, F possesses the values f_i for $i<k$ with the probabilities p_i and the value f_k with the following probability

$$p_k^k \stackrel{def}{=} p_k + p_{k+1} + \ldots + p_N.$$

In other words, in order to avoid an unwanted increase of the probability of an accident and additional costs, only in p_k^k cases we shall change the value of the factor F to the desired level f_k.

It should be noted also that the probability of a random event A after preventive measures would change. Therefore, a range of loss distribution C_A^k after preventive measures will take the following form

C_A^k	c	0
P	$P_k(A)$	$1-P_k(A)$

where $P_k(A)$ is the conditional probability of the event after preventive measures, when the factor F is reduced to the level f_k. This probability can be calculated using the formula for the total probability

$$P_k(A)=\sum_{i=1}^{k-1} P(A \mid F=f_i)p_i + P(A \mid F=f_k)\sum_{i=k}^{N} p_i, k=1,\ldots,N,$$

Since preventive measures for $i \le k$ is not carried out, and the value of the factor F is reduced to the level of f_k for $i>k$. Therefore, the average total cost in case of using the monitoring system with preventive measures will make up the following:

$$\overline{\Phi}(k)=c_E+\mathrm{M}\left[C_F^k\right]+\mathrm{M}\left[C_A^k\right]=c_E+\sum_{i=k+1}^{N} c_i^k p_i + c\left(\sum_{i=1}^{k-1} P(A \mid F=f_i)p_i + P(A \mid F=f_k)\sum_{i=k}^{N} p_i\right). \quad (6)$$

It should be noted that the latter formula for $k=K=1$ coincides with mentioned-above formula (2). In case, $k=N$, then the probability $P_k(A)$ coincides with the probability $P(A)$ of the event A occurrence without preventive measures. Average costs without monitoring system (incident risk) as previously remain at level presented by the formula (4).

It is obvious that the average cost depends on the level of f_k, to which the given factor F is reduced. Let us formulate the problem of finding such a number k_* of the value f_k for the factor F, at which the average costs are minimal

$$k_* = \arg\min_{1 \le k \le N} \overline{\Phi}(k). \quad (7)$$

Note that if $\overline{\Phi}(k_*) \le L$, the monitoring system with preventive measures should be recognized useful. Otherwise, it is inappropriate to carry out monitoring and preventive measures.

However, the problem (7) minimizes only average losses and at that the probability that the cost of the monitoring system will be repaid, may be very small. Let us consider the probabilistic formulation of the problem of monitoring, that is, estimate the probability

$$P_L(k) \overset{def}{=} P\{\Phi(k) \le L\} \tag{8}$$

of such an event, in which the costs of monitoring with preventive measures will not exceed the average costs without monitoring. Let us formulate the problem

$$k_L = \arg\max_{1\le k\le N} P_L(k), \tag{9}$$

consisting in finding the optimum number k_L of the level f_k for the factor F, at which the probability (8) under consideration is maximal. For this purpose, we construct a series of a random variable distribution of total costs $\Phi(k)$, preliminarily noting that the set of values of this random variable is finite and consists of $c_E, c_E + c_i^k, c_E + c, c_E + c_i^k + c$, where $i = 1,\ldots,N$. The value c_E is obtained if preventive measures are not carried out and, in addition, the event A does not occur. The value $c_E + c$ is obtained if preventive measures are carried out, and the event A takes place. Preventive measures are not carried out when $i \le k$, that is, when $F \le f_k$. Therefore

$$P\{\Phi(k) = c_E\} = P(\overline{A}\cdot\{F \le f_k\}) = \sum_{i=1}^{k}(1 - P(A \mid F = f_i))p_i, \tag{10}$$

$$P\{\Phi(k) = c_E + c\} = P(A\cdot\{F \le f_k\}) = \sum_{i=1}^{k}P(A \mid F = f_i)p_i. \tag{11}$$

The values $c_E + c_i^k$ are obtained if the event A does not occur, and preventive measures are carried out in which the factor F is reduced from level f_i to the level $f_k, i > k$. The values $c_E + c_i^k + c$ are obtained if the event A occurs after preventive measures. And since $P\{C_F^k = c_i^k\} = P\{F = f_i\} = p_i$, then

$$P\{\Phi(k) = c_E + c_i^k\} = \big(1 - P(A \mid F = f_k)\big)p_i, i = k+1,\ldots,N, \tag{12}$$

$$P\{\Phi(k) = c_E + c_i^k + c\} = P(A \mid F = f_k)p_i, i = k+1,\ldots,N. \tag{13}$$

Thus, we get a number of distributions

$\Phi(k)$	c_E	$c_E + c_i^k$	$c_E + c$	$c_E + c_i^k + c$
P	$\sum_{i=1}^{k}(1 - P(A \mid F = f_i)\ p_i$	$\big(1 - P(A \mid F = f_k)\big)p_i$	$\sum_{i=1}^{k}P(A \mid F = f_i)p_i$	$P(A \mid F = f_k)p_i$

Based on the obtained distribution series it is possible to find the probability $P_L(k)$. Solving the problem (9), one can find such a number k_L of the value f_k for the factor *F,* at which the probability is maximal for such an event in which the incidental expenses $\Phi(k)$ will be no greater than the average loss *L* at transport accident without monitoring.

However, a situation can occur when the probability $P_L(k)$ for some k would be great and it makes no sense to continue its maximizing. In such a situation, the magnitude of cost-effectiveness of the monitoring system, insured for a given level of dependability α is of particular interest. In this regard, we shall consider the quantile formulation of the monitoring problem and estimate the guaranteed cost-effectiveness from monitoring with preventive measures

$$\varphi_\alpha(k) \overset{def}{=} \max\{\varphi : P\{L - \Phi(k) \geq \varphi\} \geq \alpha\}. \tag{14}$$

We shall solve the problem

$$k_\alpha = \arg\max_{1 \leq k \leq N} \varphi_\alpha(k). \tag{15}$$

Then at the found number k_α of the level f_k for the factor *F,* the cost-effectiveness from monitoring with preventive measures will make up $\varphi_\alpha(k_\alpha)$ and it is guaranteed with the probability α.

Let us select the dependability level from economic reasons. We shall find such a level α_* of dependability α, at which the cost-effectiveness value by using quantile strategy will not be negative

$$\alpha_* = \max\{\alpha : \varphi_\alpha(k_\alpha) \geq 0\}. \tag{16}$$

Thus, in α_* cases the losses will be absent, and the cost-effectiveness value will make up $\varphi_{\alpha_*}(k_{\alpha_*})$. Note that if the solution of this problem does not exist, then for each in terms of quintile we obtain losses for the organization of monitoring and preventive measures. This means that the monitoring and preventive measures are unreasonable in principle.

4. The analysis of the obtained ratios

Let us consider some special cases of the ratios we have obtained.

We shall assume that the transport accident *A* "weakly" depends on the factor *F*, if

$$P(A \mid F = f_i) \approx p, i = 1, \ldots, N,$$

that is, the conditional probability of the event *A* varies little from one value of the factor *F* to another. Note that if $P(A \mid F = f_i) = p, i = 1, \ldots, N$, and then there is a complete absence of this dependence of the transport accident *A* on the factor *F.* In the case of a complete absence lack of this dependence, the average costs for monitoring and preventive measures will be equal to the following

$$\overline{\Phi}(k) = c_E + \sum_{i=k+1}^{N} c_i^k p_i + cp, k = 1, \ldots, N.$$

Due to the fact that the values are non-negative; the minimum loss will be achieved at $k = N$, i.e.

$$\Phi^* = \overline{\Phi}(N) = c_E + cp.$$

The obtained result shows that the desired value of the factor is found on the level f_N. This suggests that no any actions to change the factor are required if the transport accident does not depend on the factor *F*. Furthermore, because the operation costs of monitoring system with rare exception are equal to zero, therefore the average damage (accident risk) L in this case is equal to $L = cp$, and it is less than the losses when using the monitoring system. This means that the factor *F* monitoring does not make sense, if the relationship between transport accident and the factor completely absent. In case of "weak" dependence, the result is the same.

Now we shall assume that the transport accident *A* "strongly" depends on the factor *F*, if

$$P\{A \mid F = f_i\} = t^{i-1} p, i = 1, \ldots, N, \tag{17}$$

for some $t \in T \overset{def}{=} \{t \in \mathbb{R}^1 : t > 1, t^{N-1} p \le 1\}$ and $p \in (0,1)$. If $i = 1$, then $P(A \mid F = f_1) = p$. We also shall assume for simplicity that the factor *F* levels are equiprobable, that is,

$$p_i \overset{def}{=} P\{F = f_i\} = \frac{1}{N}, i = 1, \ldots, N.$$

Then, in this case, according to (6), we obtain

$$\overline{\Phi}(k) = c_E + \frac{1}{N}\sum_{i=k+1}^{N} c_i^k + c\left(\frac{p}{N}\sum_{i=1}^{k-1} t^{i-1} + \frac{t^{k-1} p(N-k+1)}{N}\right), k = 1, \ldots, N. \tag{18}$$

According to the formula for the sum of a geometric progression

$$\sum_{i=1}^{k-1} t^{i-1} = \frac{t^{k-1} - 1}{t - 1}, \tag{19}$$

the value of the average total loss defined by the formula (18) is equal to

$$\overline{\Phi}(k) = c_E + \frac{1}{N}\sum_{i=k+1}^{N} c_i^k + c\left(\frac{p}{N}\frac{t^{k-1} - 1}{t - 1} + \frac{t^{k-1} p(N-k+1)}{N}\right), k = 1, \ldots, N. \tag{20}$$

Average damage (risk of an accident) with no monitoring system in case of "strong" dependence of the event *A* on the factor *F*, according to (4) and (19) is equal to

$$L = c\sum_{i=1}^{N} \frac{1}{N} p t^{i-1} = \frac{cp}{N}\sum_{i=1}^{N} t^{i-1} = \frac{cp}{N}\frac{t^N - 1}{t - 1}. \tag{21}$$

Let us consider the "critical" case where the conditional probability $P(A \mid F = f_N)$ of the event *A* occurrence is equal to unity, i.e., according to (17) when

$$t^{N-1} p = 1 \tag{22}$$

Let preventive measures consists in reducing the factor F to the level f_1. Since, by assumption, the conditional probabilities $P(A \mid F = f_i)$ increase monotonically for $i = 1, \ldots, N$, then in this case the number K defined in section 2, is equal to one. Therefore, the average loss in such a preventive measures will be determined by the formula (2) for $K = 1$. We shall find such values of the parameter p for a fixed number N of factor F values, at which the average total cost of monitoring with preventive measures were not more than the average cost without monitoring, that is, according to (20) and (21) from the condition

$$B(p,N) \overset{def}{=} \overline{\Phi} - L = c_E + \frac{1}{N}\sum_{i=2}^{N} c_i^1 + cp - \frac{cp}{N}\frac{t^N - 1}{t-1} \le 0. \tag{23}$$

Note that according to (22) $t = p^{-1/(N-1)}$.
Therefore, proceeding to the limit in (23), we obtain

$$B_*(p) = \lim_{N\to\infty} B(p,N) = c_E + \mathrm{M}[C_F^1] + cp + \frac{c - cp}{\ln(p)} \le 0, \tag{24}$$

since by L'Hopital rule [4]

$$\lim_{N\to\infty} N(p^{-1/(N-1)} - 1) = \lim_{N\to\infty} \frac{p^{-1/(N-1)} - 1}{1/N} = \lim_{N\to\infty} \frac{-N^2 \ln(p) p^{-1/(N-1)}}{(N-1)^2} = -\ln(p)$$

and, moreover, $\lim_{N\to\infty} \frac{1}{N}\sum_{i=2}^{N} c_i^1 = \mathrm{M}[C_F^1]$.

Solving the inequality (24) with respect to p, it is possible to find such a value p_1 of the parameter p, at which the average costs for the preventive measures and monitoring do not exceed the risk of the accident L. Note that from (24) it is possible to find all p for any sufficiently large N much easier than to calculate (2), (4) and (5), under which the strategy to minimize the impact factor will be reasonable.

5. An example

Let us consider an example of the use of obtained ratios. We shall understand train derailment as a transport accident A, and consider the number of cars in the train formation as a factor F. We shall use hypothetical data based on data from the U.S statistics [5]. Let us assume that some customers require from 61 to 80 cars and at that the number of cars in the order is random and their number is equiprobable. We shall assume that the longer the train formation, the greater the probability of its derailment, and at that in geometrical progression with some denominator t and the numerator $p,$ that is,

$$P(A \mid F = f_i) = pt^{i-1},$$

where the parameters $p = 2 \cdot 10^{-6}$ and $t = 1.8$. Let the cost of damage in case of train derailment makes up $c = 1500000$ \$; the cost of shipping one car of cargo to the customer by another mode of transportation makes up $c_0 = 1500$ \$. In other words, if $i-k$ of cars should be uncoupled from train formation for preventive measures of an accident (to decrease the train length to the desired number), the economic losses arise, which are equal to $c_i^k = c_0(i-k)$. Note that according to (1) and (19) the probability of a train derailment without monitoring and preventive measures is equal to

$$P(A) = \sum_{i=1}^{20} \frac{1}{20} pt^{i-1} = 10^{-7} \sum_{i=1}^{20} t^{i-1} \approx 0.0159.$$

The average damage without monitoring system (accident risk), in this example according to formula (3) is equal to $L = cP(A) \approx 23903\$$.

In this case, monitoring consist in counting the number of cars in the train formation, so it is natural to assume that the magnitude of the costs associated with monitoring is zero, that is $c_E = 0$. Therefore, according to (6), the average total costs will make up

$$\overline{\Phi}(k) = \frac{1}{20} \sum_{i=k+1}^{20} c_i^k + c\left(\frac{p}{20} \sum_{i=1}^{k-1} t^{i-1} + \frac{t^{k-1} p(20-k+1)}{20} \right), k = 1,\ldots,20. \tag{25}$$

Note that in the last formula the values $c_i^k = c_0(i-k)$ describe the cost of shipping *i-k* cars of cargo to the customer by another mode of transportation if the train will be formed only of $60 + k$ cars. Therefore

$$\overline{\Phi}(k) = \frac{1}{20} \sum_{i=k+1}^{20} 1500(i-k) + c\left(\frac{p}{20} \frac{t^{k-1}-1}{t-1} + \frac{t^{k-1} p(20-k+1)}{20} \right), k = 1,\ldots,20. \tag{26}$$

Based on (19) and (25) we obtain the following values for the average total loss (in U.S. dollars) for the preventive measures on reducing the length of the train from 60+*i* up to 60+*k* cars (Table 1).

Table 1. The average losses from preventive measures carried out in accordance with the number of cars

$L = \overline{\Phi}(20)$	$\overline{\Phi}(1)$	$\overline{\Phi}(2)$	$\overline{\Phi}(3)$	$\overline{\Phi}(4)$	$\overline{\Phi}(5)$	$\overline{\Phi}(6)$	$\overline{\Phi}(7)$	$\overline{\Phi}(8)$	$\overline{\Phi}(9)$
23903	14253	12831	11485	10216	9027	7921	6903	5981	5169
$\overline{\Phi}(10)$	$\overline{\Phi}(11)$	$\overline{\Phi}(12)$	$\overline{\Phi}(13)$	$\overline{\Phi}(14)$	$\overline{\Phi}(15)$	$\overline{\Phi}(16)$	$\overline{\Phi}(17)$	$\overline{\Phi}(18)$	$\overline{\Phi}(19)$
4450	3978	3688	3705	4152	5201	7075	10014	14160	19257

Solving the problem (7) it is possible to find the optimal number k_*=12 from the given table. This means that if the customer needs more than 60 +12 = 72 cars, it is necessary to form a train of only 72 cars, while the rest of the cargo to be transported, necessary to transport by some other way. This result is related to the fact that the cost of transporting by not railway transport monotonically decreasing according to *k,* however, a significant increase of the probability of a train derailment is observed, which increases the potential damage in the accident.

Now we shall solve the problem (15) for the different dependability levels α. As a result, we obtain the following data (Table 2).

Table 2. Assured cost-effectiveness from carrying out preventive measures, depending on the dependability level

α	0.95	0.99	0.995	0.999	0.9995	0.9999	0.99997	0.99999
k_α	20	18	16	12	11	8	5	3
$\varphi_\alpha(k_\alpha)$	23903	20903	17903	11903	10403	5903	1403	-1598

In accordance with (16) we obtain that $\alpha_* = 0.99997$. Therefore, the optimal strategy decision taking $k_{\alpha_*} = 5$ (i.e. the train should be formed of no more than out of 60 + 5= 65 cars). Moreover, in 0.99997 cases a guaranteed positive cost-effectiveness equal to 1403 \$ will be obtained from the monitoring system with preventive measures. The table shows that by increasing the train length in α cases the cost-effectiveness increases, but at that, in (1 – α) of adverse events the losses also grow. For example, when $k_\alpha = 12$ (which corresponds to the solution of the problem (7), while minimizing the average cost) in 999 cases out of a thousand events the cost-effectiveness from preventive measures will make up of more than 11903 \$, and in one case out of a thousand losses may be unacceptably high. Therefore, the recommended value for this example is equal to $k_\alpha = 5$, with only in three cases out of 100 000 there are losses, and the cost-effectiveness of the monitoring and carrying out preventive measures will be at least 1403 \$.

6. The algorithm for monitoring and carrying out preventive measures

In view of the mentioned above, we shall describe the recommended sequence of actions in the carrying out monitoring and preventive measures.

1. Install sensors to measure the frequency of a factor $F = f_i$ occurrence (for example, the railway bed shift).
2. Calculate the frequency of appearance of the *i*-th value f_i of the factor *F*, that is, the value $\tilde{p}_i$ of the probability $p_i = P\{F = f_i\}, i = 1,\dots,N$ is defined.
3. Refine the value $\tilde{p}_i$ based on logistic data processing circuit on a small sample [2].
4. Process accident reports by which the probabilities $P(A \cdot \{F = f_i\})$ of simultaneous occurrence of an event *A* and a factor *F* with the value $f_i, i = 1,\dots,N$ are estimated. This again uses logistic data processing circuit.
5. Calculate the conditional probability of the event *A* under condition that the factor *F* possessed the value f_i, which, by definition [3] is equal to

$$P(A \mid F = f_i) = \frac{P(A \cdot \{F = f_i\})}{P\{F = f_i\}}, i = 1,\dots,N.$$

6. According to (10) – (13), define a series of distribution accidental costs $\Phi(k)$ for carrying out monitoring and preventive measures, aimed at reducing the factor *F* bringing to the level $f_k, k = 1,\dots,N$.
7. By using (14), (15) determine the optimal number k_α of the level f_k, to which it is necessary to reduce the factor *F*.

8. Solve the problem (16), where the optimized value of dependability α_* is selected for decision making to conduct preventive measures.

9. Carry out preventive measures, if the factor F possessed the value f_i, where $i > k_{\alpha_*}$. If the factor F has the value f_i with number $i \le k_{\alpha_*}$, then preventive measures is not performed.

10. In this case, the cost-effectiveness of the monitoring system with preventive measures will make up not less than that determined by the formula (14) for $\alpha = \alpha_*$.

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