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EMPERICAL TECHNIQUE OF FORECASTING OF LOADED MULTI-ELEMENT SYSTEM COLLAPSE

The paper presents a technique of lifetime reliability estimation for a loaded multi-element system, with the estimation being conducted during the failure of the system. The failure process is considered as quasi-static, and as a model example, the Daniels fiber bundle model is used.
The method is based on statistical analysis of the burst sequence, where burst is a simultaneous failure of a number of elements under current total load (package of destructions). As the destructions progresses, we register maximum frequencies for bursts of successive sizes (i.e., single, double etc.). This data is used (via "moving window" averaging technique) for the improvement of the statistical estimate of the time remaining to the avalanche and full system collapse.
Computer simulation is used to illustrate the performance of the proposed procedure. A comparison of the approach proposed with ones previously suggested by other researchers is presented, and the benefits of the new technique are shown.

Keywords: *quasi-static failure, multi-element system, Daniels fiber bundle, lifetime, avalanche, burst statistics, moving window.*

Introduction

The task of forecasting collapse coming (full destruction) for a loaded multi-element system is faced and found in its different variations in many areas of physics (models of non-uniform environments' destruction), technical equipment and engineering modeling, and on essentially various spatial and time scales (see, for example, [1]). Seismic and volcanic activity, various models of electromechanical systems, destruction of materials at operation of engineering systems and designs represent characteristic examples [2-5]. The situation when it is a question of forecasting a system resource at its designing stage is widely presented in the literature [6-7]. However, special interest is represented with cases when it is required to give the time forecasting of collapse coming directly in process of destruction progression based on supervision over system behavior.

Generally, the collapse is preceded with the period of gradual degradation of a system, expressed in consecutive destructions of separate elements. At the same time, there are both single and group destructions. The problem of localization of large group destructions (as potential sources of an avalanche origin) in investigated system can be solved, for example, by means of diverse methods of acoustical emission [8-9]. On the other hand, in some cases (for example, at fulfillment of any conditions of system uniformity), it is possible to attempt making conclusions about its affinity to a collapse based on the analysis of only one sequence of separate intermediate events of simultaneous destruction of elements, not being interested with spatial characteristics of these events. Many works relating to this subject has been devoted to studying of various discrete probabilistic models, among which modifications of a fiber bundle model are basic [10-12]. Models of continuous destruction [13] have been also investigated.

The present paper offers the technique related to dynamic, real time forecasting of the moment of a system final destruction coming under effect of increasing loading. The given technique is illustrated by an example of initial fiber bundle model [14], which is usually named as Daniels fiber bundle. The illustration is based on results of study of numerical experiments. Beside the description of the model and its numerical realization, the definition of characteristics used for the analysis of the system (current condition during the process of destruction stages on which there is an affinity to collapse system) and results of computer modeling are presented. The basic attention is given to the frequency change analysis of fixing group destructions of a small volume. Comparison of the offered technique with the approaches suggested earlier by other authors has been carried out.

Problem statement

First, let us give the brief description of considered classical model, Daniels fiber bundle, underlying in numerous models that are more complex. It is a question of fiber bundle mathematical model – the system consisting of N parallel fibers, fixed with both ends in such a manner that lengthening of all fibers under stretching loading imposed to a bundle are identical. The positive number, which reflects fiber strength, is appointed to each fiber. This number represents the minimal value of stretching longitudinal load under which the fiber collapses. Fibers are considered statistically identical and it is assumed, that the law of strength distribution as random variable is the same for all fibers in a bundle and it is described by some distribution function Fx, $x \geq 0$.

It is supposed that the stretching longitudinal load applied to the fiber bundle monotonously increases from zero value. By virtue of accepted assumptions loading is evenly distributed between fibers.

Actually, we deal with an abstract mathematical model in which external loading is evenly distributed between all efficient system elements. Further, for convenience we shall use the term “fiber bundle” for the designation of the specified model.

We shall name a fiber as overloaded one if the load in the fiber is equal to its strength or surpasses it. At modeling process of bundle destruction when the fiber appears overloaded, it is removed from the bundle (it collapses), and the total loading on the bundle is evenly redistributed between the remained fibers. Loading process is considered in quasi-static statement: removal of the overstressed (overloaded) fibers and load redistribution at constant current general load is considered as instant event and comes to an end, when there are no more overstressed fibers. Only after that, if there were not removed fibers, the monotonous load increases continue at a bundle.

The latter we shall name as a collapsed bundle when all fibers are removed from it. We shall name critical load a such value of bundle loading at which last fiber has collapsed. The value of critical load related to initial number of fibers N, we shall name critical specific load qbr.

Next, we shall name a package of destructions (or simply a package) a set of the fibers, which have collapsed simultaneously, that is at the same load applied to a bundle. Characteristics of a package are its sequence number (within the limits of current process of destruction), volume (amount of the collapsed fibers) and load value at which there was a destruction of fibers from the given package. Last package (after which the bundle is completely collapsed) we shall name an avalanche.

Let us formulate the following general problem: to estimate dynamically, i.e. on a course of a bundle destruction, time which has remained before an avalanche. More strictly, it is required to estimate a sequence number of an avalanche in sequences of packages and critical specific load according to some initial site of destruction process. It is evident, that the longer this initial site, then the obtained estimations, generally speaking, will be more accurate. At the same time, the signal about avalanche approach was to be received "not too late". The latter characteristic can be specified in various ways.

Planning of numerical experiment

Bundle volume of N and strength distribution function of fibers Fx are two parameters of the formulated problem. Further, results of the numerical analysis of considered model are presented in the work for various values N and the two types of strength distribution: uniform on a segment [0; 1] (for which average strength value and its dispersion are equal accordingly to 1/2 and 1/12); two-parametrical Weibull distribution with the same average strength value and dispersion. In the, we shall name these two types as the first type and the second type accordingly.

The following result has been obtained in the fundamental work [14]: critical specific load asymptotically at $N \to \infty$ normally represents distributed random variable with known average strength value and dispersion. The first is equal to maximal value of function

$$qx=x1-Fx,$$

and the second one asymptotically converges to 0. In particular, for two specified types of strength distribution corresponding to average values of critical specific load is an essence $q1^* = 0{,}25$ and $q2^* \approx 0{,}23$ accordingly.

The important, but elementary remark consists in the fact that at known N and Fx the specified task of forecasting of an avalanche is reduced to standard construction of a confidential interval. However in practice the basic interest represents research of a case when there is no opportunity of load measurement, and only effects of local destructions (packages) are registered. Besides, a bundle volume is also usually unknown.

At carrying out of numerical experiment, we shall be limited by the case when external load onto a bundle increases proportionally to time. At the same time we have the following from the given above definitions: sequence number value of a package or the load value corresponding to the given package can be chosen in considered statement as discrete analogue of time. In general, process of destruction is completely described by two sequences: volumes of consecutive packages and values of load corresponding to them. Accordingly, during numerical modeling these sequences make the output information on results of each separate experiment. Te analysis of statistical characteristics of destruction process is made on the accumulated data, and the basic attention is given to studying shares of packages' variability of various volume in process of development of destruction process and to search of possible regularities.

Realization and analysis of results

The course of numerical modeling represents the following logic sequence of steps.

1. Bundle fibers are bind with random strength value for the specified bundle volume N and bundle type (I or II).

2. Zero value is bind to the current full load on a bundle. Set of the destroyed fibers is assumed empty.

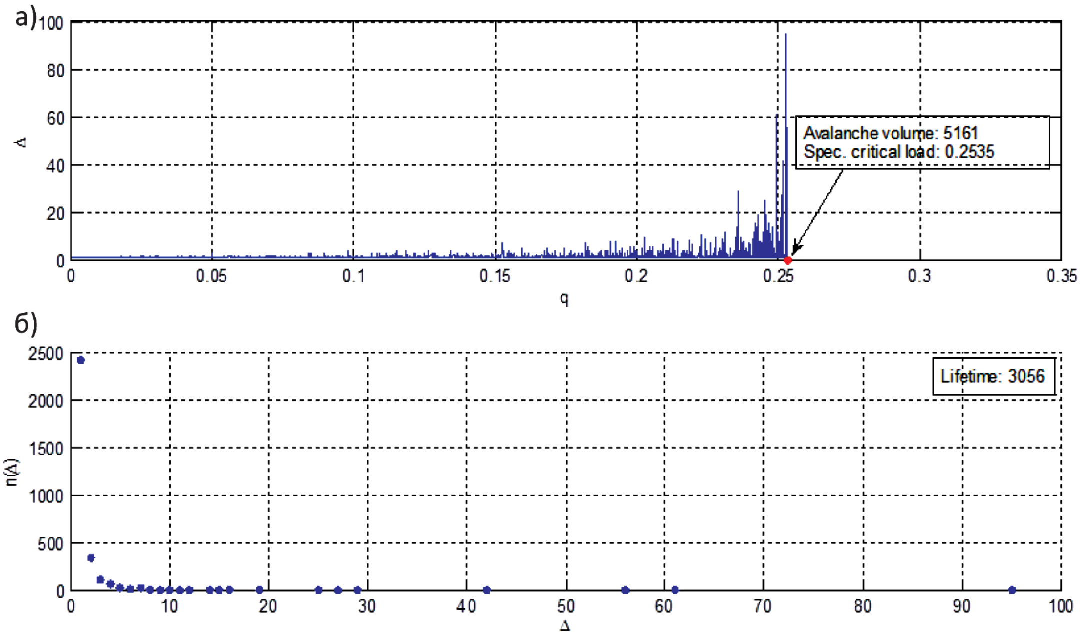

Fig. 1. Results of numerical modeling of destruction of one bundle:
volumes of consecutive packages (a), amount of recorded packages of the given volume (b)

3. If in a bundle, there are not destroyed fibers; values of their strength are ordered according to their increase. Current value of full load on a bundle is increased abruptly up to ν·σm, where ν is the current number of not destroyed fibers, σm is the minimal of strength values from not destroyed fibers. If not destroyed bundle fibers are not present, process of destruction modeling is considered to be over.

4. While not destroyed fibers are available among overstressed ones, the latter are removed from a bundle (i.e. they are joined to the set of destroyed fibers) with simultaneous recalculation of current specific load on other fibers. Full load on a bundle within the limits of this cycle remains constant. When there are no overstressed fibers, you should come back to the step 3.

On results of modeling the information on critical specific stress, consecutive volumes of the packages formed on step 4, and values of specific load corresponding to packages are saved.

Let us represent graphically a trajectory of arising random process, setting aside on a course of bundle destruction the value of packages' volumes along Y-axis, and corresponding values of specific load along abscissa axis.

As an example, the typical trajectory of arising point random process for a bundle of the first type (along abscissa axis – current specific stress, and along Y-axis – volumes of registered packages) is shown in Fig. 1, a. In this case the value of critical specific load has appeared equal to 0,2535 and the avalanche volume has made 5161 fibers at bundle volume equal to N=10 000 fibers,. The frequency diagram for volumes of packages recorded during destruction is presented in Fig. 1, b; their total number in the given experiment has made Kb=3056.

It should be noted that relation KbN in all fulfilled experiments has appeared with rather small variability close to the certain value. For example, for bundles of the first type this value has made approximately 31%. If the subsequent researches of model with various other types of strength distributions of fibers lead to similar results, then corresponding empirical estimations of total amount of packages can be used for forecasting of an avalanche affinity in the problem with known function of strength distribution. The author of this paper could not find the description of corresponding analytical or numerical result in the available literature.

Representation of the frequency diagram (Fig. 1, b) in logarithmic coordinates shows good conformity with known asymptotic relation [15]

$$n\ (\Delta)\ N \propto \Delta{--}5/2,$$

where n (Δ) is the total number of recorded packages of volume Δ on a course of destruction.

The following approach to forecasting time of an avalanche occurrence is offered in the investigation [16]. Authors mark that at approach to collapse (i.e. on the so-called prior to avalanche segment) the above-stated asymptotic relation is transformed into the following expression

$$n\ (\Delta)\ N \propto \Delta{--}3/2,$$

where n (Δ) is the total number of recorded packages of volume Δ recorded on prior to avalanche segment. They suggest to consider change of exponent estimation in expression Δα from values close to α1 =--5/2, onto the values close to α2 =--3/2 as the indication of avalanche approach.

It is necessary to note, however, that strict definition of a prior to avalanche segment in the specified in the mentioned above work is absent, and in the numerical calculations authors take as this segment such a segment in the beginning of which specific load made 90 % from its critical value. As the concept of specific load is not defined in conditions when the bundle volume is unknown, the given approach cannot be directly used for a quantitative forecasting of the moment of an avalanche approach in any problem with unknown amount of elements.

Let us try to modify this approach, using the estimation α on sliding time window (i.e. according to last packages s, where s is some number set beforehand). Fig. 2 presents the results of numerical modeling of bundle destruction of the second type. The bundle consists of fibers equal to N=100 000, the width of a supervision window makes s=1 000 packages.

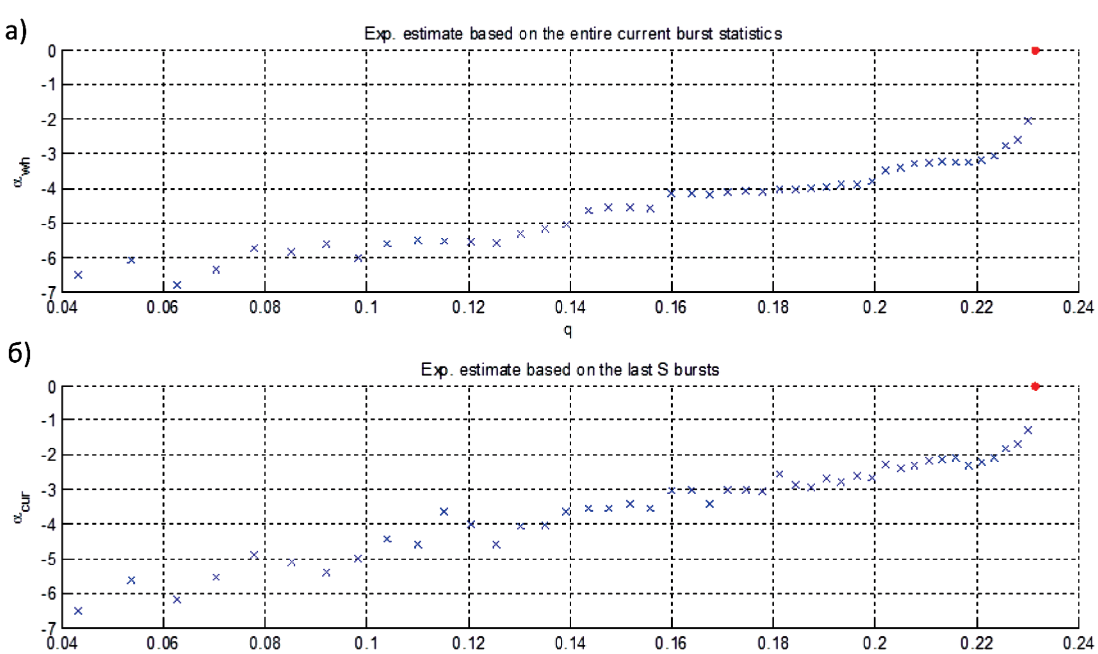

Fig. 2. Estimation of an exponent in the law of distribution of volumes of packages in one experiment: on all fixed{recorded} to a present situation of time to packages (), on the last s to the fixed{recorded} packages ()

At the same time the estimation of an exponent shown in Fig. 2,a is received in view of total amount of the packages recorded from the very beginning of destruction process, and in Fig. 2,b – in view of amount of recorded packages within the limits of current window. It is obvious, that stabilization of estimations near to the value α1 practically are absent, and in the first case the situation is essentially worse. Results of numerical experiments show, that at larger values of a bundle volume the stabilization is going on slowly if at all. Besides, transition to exponent values close to α2, occurs without characteristic step jump which would allow to localize the moment of the beginning of prior to avalanche segment. It is important, that the described picture does not depend on the size s of a used window. For example, corresponding diagrams for the same realization of destruction process are shown in Fig. 3which are presented in Fig. 2, but at other value of the window size: s=10 000.

Offered technique of an avalanche forecasting

The following approach to the statistical analysis of destruction process appears more effective and simultaneously simpler. Let us set some sufficiently great value of width s for a sliding window. Beginning with package under number s through every s/2 packages we shall calculate portions of packages of volume 1, 2 and 3, registered in the current window. In parallel, we shall trace an estimation of above-mentioned exponent according to the same last s packages.

The numerical analysis shows, that obtained diagrams (the role of time as it was mentioned above, plays the current value of specific load on a bundle) possess feature of universality in the following sense. Their statistical averaging, i.e. according to results of independently carried out numerical experiments, (at the fixed bundle type) is a trajectory, not dependent neither from bundle volume nor from the size of used sliding window. Results of calculations according to outcomes of five numerical experiments for each bundle of volume N=20 000, 50 000 and 100 000 accordingly are shown on fig. 4, 5 and 6. Diagrams of change of the exponent α are shown on corresponding fig. (a). Diagrams of change of packages' portions for individual volume concerning the total number of packages recorded in the current window are shown in corresponding Fig. (b). Diagrams of change of packages' portions for unit volume packages in

а)

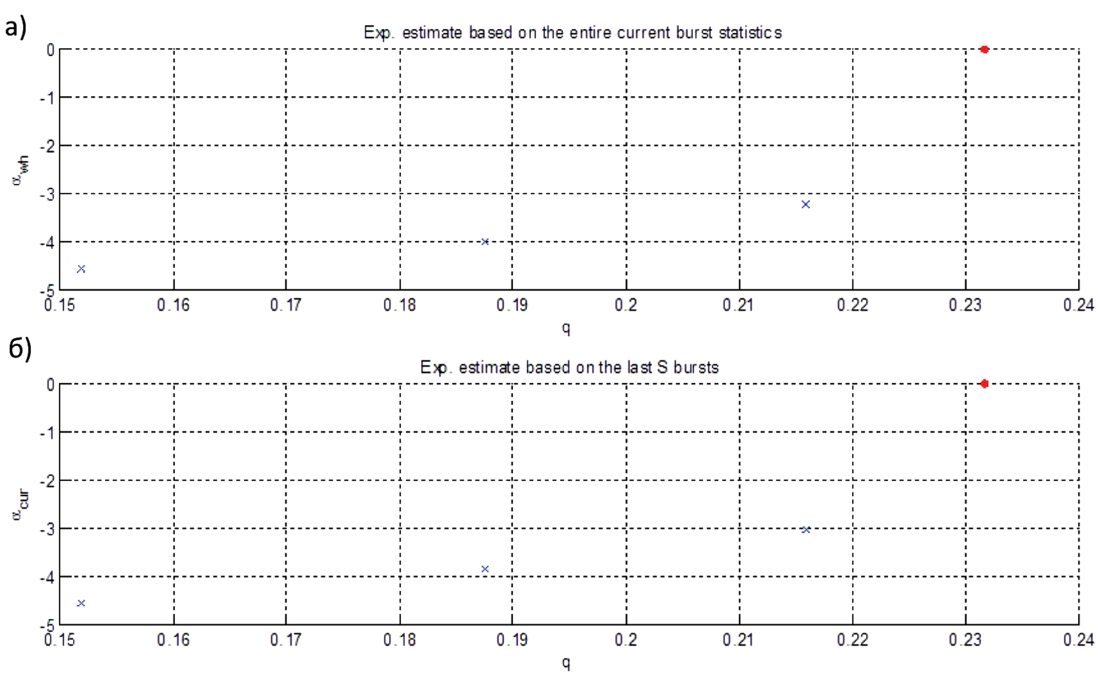

Fig. 3. Estimation of an exponent in the law of distribution of volumes of packages in one experiment: on all fixed{recorded} to a present situation of time to packages (), on the last s to the fixed{recorded} packages ()

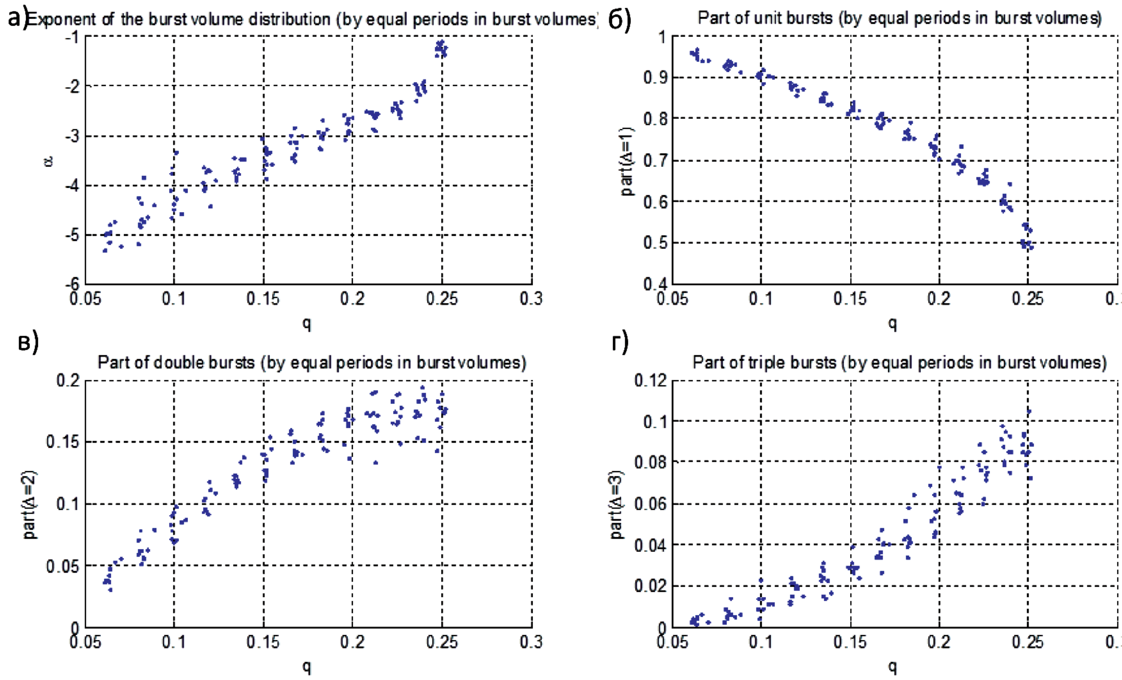

Fig. 4. Results of numerical modelling of destruction of 10 bundlees{beams} with uniform distribution of durability of fibres: Estimation of an exponent in distribution of packages (),
Share of packages of volume 1 (), a share of packages of volume 2 (), a share of packages of volume 3 ()

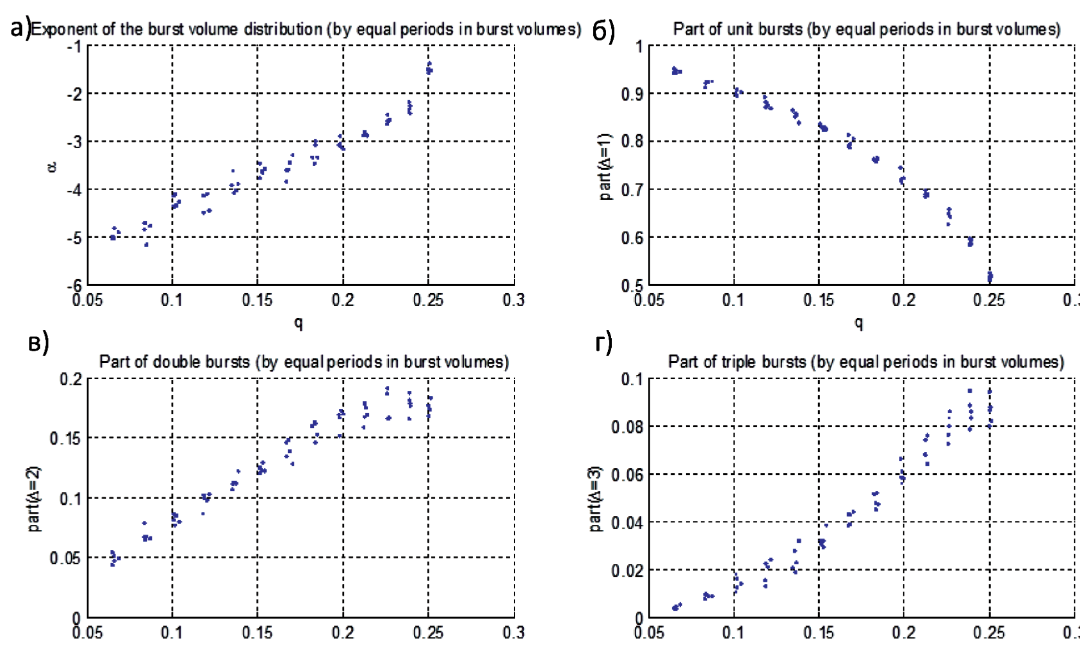

Fig. 5. Results of numerical modelling of destruction of 5 bundlees{beams} with uniform distribution of durability of fibres: Estimation of an exponent in distribution of packages (),
Share of packages of volume 1 (), a share of packages of volume 2 (), a share of packages of volume 3 ()

relation of volume Δ=2 are shown in corresponding Fig. (c). Diagrams of change of packages' portions for individual volume packages in relation of volume Δ =3 are shown in corresponding Fig. (d). Each point in Fig. 4 – 6 has been obtained by results of recording in "observation window" from s=1000, 2000 and 3000 consecutive packages accordingly, thus shift between windows makes s/2 packages. Values of load corresponding to the last packages in the current window are put on X-axis.

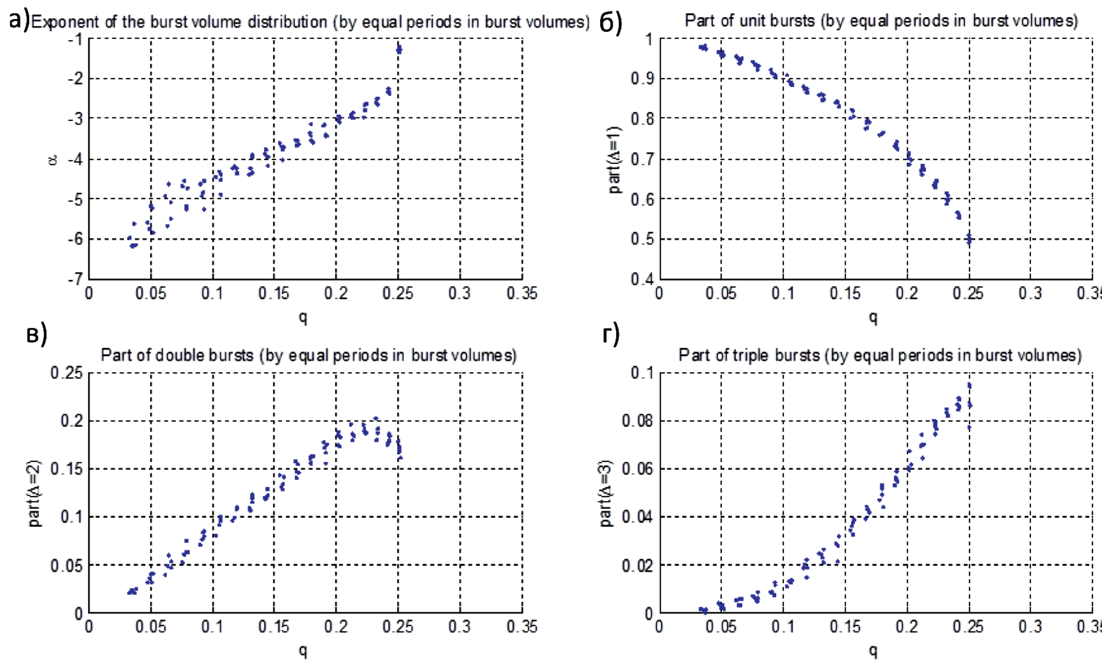

Fig. 6. Results of numerical modelling of destruction of 5 bundlees{beams} with uniform distribution of durability of fibres: Estimation of an exponent in distribution of packages (),
Share of packages of volume 1 (), a share of packages of volume 2 (), a share of packages of volume 3 ()

It is visible, that portion of unit volume packages significantly decreases in due course while portion of packages with volume 2 and 3 increase in process of system approach to the total collapse. The form of diagrams for Δ =2 (Fig. (b)) is especially characteristic. One can observe steady presence of the marked maximum of double packages' portion shortly before an avalanche. Time from passage of this maximum before an avalanche occurs makes on the average 10 % from full duration of destruction process for fiber bundles of the first type.

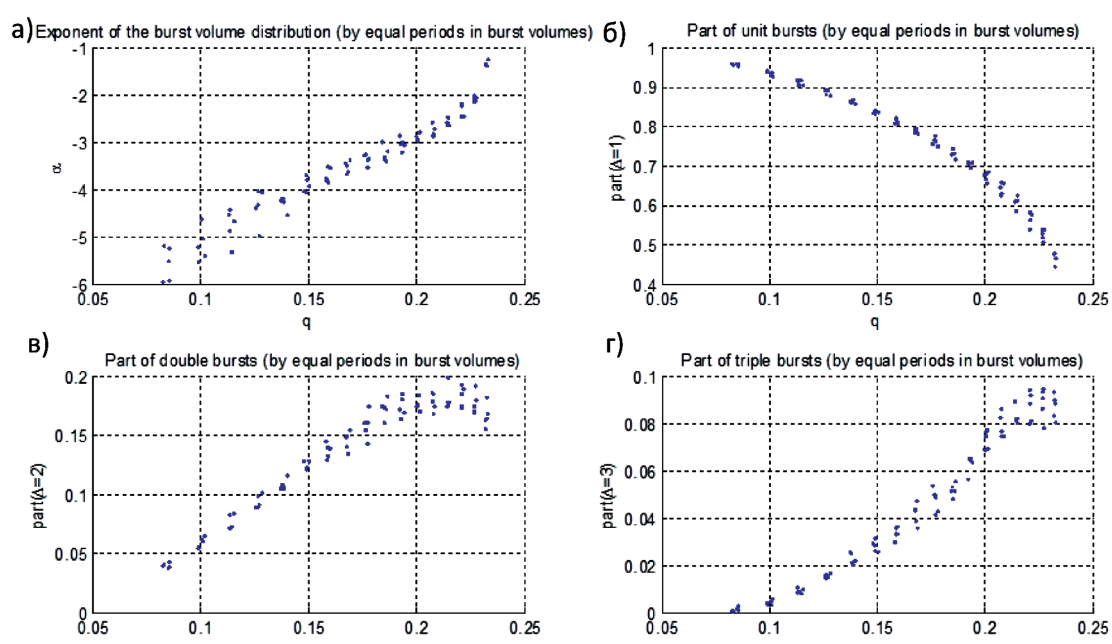

Fig. 7. Results of numerical modelling of destruction of 5 bundlees{beams} with вейбулловским distribution of durability of fibres: an estimation of a parameter
Degrees in distribution of packages (), a share of packages of volume 1 (),
Share of packages of volume 2 (), a share of packages of volume 3 ()

It is important to note, that characteristic forms of corresponding trajectories are also typical for fiber bundles of the second type. Similar dependences for the case of Weibull distribution are presented in Fig. 7. Properties of trajectories' monotony and relative size of time pause between passage of maximum of double packages' portion and the moment of system collapse are held true for fiber bundles of the second type.

Under the described universality, the given empirical observation can serve as one of criteria of an avalanche forecasting in considered bundle model. At receiving in real time (on a course of destruction) points of discrete trajectory for $\Delta=2$, it is offered to initiate warning about avalanche approach immediately after passage of a local maximum of double packages' portion. In addition to that, absolute size of time which has remained up to an avalanche, is offered to estimate (in conditions when N is not known, and estimation of destruction process duration a priori is impossible) by comparison of the form of trajectories for $\Delta=1, 3$, received in real time, with the form of trajectories obtained from modeling examples. Based on the comparison it is necessary to make scaling of X-axis on discussed diagrams.

As to sharp jump of estimated value α, which is really observed (Fig. 4,a; 5,a; 6,a), it occurs directly ahead of an avalanche and cannot help to forecast the moment of system collapse approach with reserve of time.

The conclusion

The investigation based on statistical processing of numerical experiments' results on modeling process of Daniels fiber bundle destruction some regularities relating change in time (in process of load increase) of destruction packages' portion of small volume in total of registered packages have been revealed. It is empirically shown, that at sufficiently big size of a bundle, change diagrams in packages' portion of volume $\Delta=2$ in relation to the total number of packages recorded in the current observation window, have the marked maximum shortly before an avalanche. Preliminary estimations show, that the section from a point of maximum till the moment of avalanche beginning makes about 10 % of the total destruction process duration. In addition to that there are bases to assume universality of the given criterion of an avalanche forecasting in fiber bundle model as this value has appeared identical to both types of considered strength distributions of fibers – uniform and Weibull distributions, and it is realized for bundles of various volume and at use of observation windows of various length.

An interesting problem consists in check of hypothesis that at unlimited growth of a bundle volume, maxima will arise also on diagrams of packages' portion of volume $\Delta > 2$, and the bigger the value Δ is, the closer we are to the moment of an avalanche. Confirmation of this hypothesis would allow building similarity of the hierarchical plan for system collapse forecasting.

Among other possible directions of further work, it is possible to single out checking of similar regularities in more complex models, in particular, bundle models with other types of load redistribution, models of non-uniform bundle (i.e. containing fibers with various statistical properties) [17], hierarchical models of tree type.

References

1. Statistical Models for the Fracture of Disordered Media // Ed. H. J. Herrmann and R. Roux. Amsterdam: North-Holland. 1990.368 p.

2. **Zapperi S., Ray P., Stanley H.E., Vespignani A.** Avalanches in breakdown and fracture processes // Phys. Rev. E. 1999. V. 59, No. 5. P. 5049-5057.

3. **Gabrielov A., Zaliapin I., Newman W.I., Keilis-Borok V.I.** Colliding cascades model for earthquake prediction // Geophys. J. Int. 2000. V. 143. P. 427-437.

4. **Newman W.I., Turcotte D.L., Gabrielov A.M.** Log-periodic behavior of a hierarchical failure model with applications to precursory seismic activation // Phys. Rev. E. 1995. V. 52. No. 5. P. 4827-4835.

5. **Moreno Y., Gomez J.B., Pacheco A.F.** Instability of scale-free networks under node-breaking avalanches // Eur. Phys. Lett. 2002. V. 58. P. 630.

6. **Bolotin V.V.** Resource forecasting of machinery and engineering designs. -M.: Mechanical engineering, 1984. – 312 p.

7. **Bogdanoff J., Kozin F.** Probabilistic models of damages' accumulation: Tr. from English – M.: the World, 1989. – 344 p.

8. **Braginsky A.P.** About forecasting structural reorganizations in materials according to features of collective behavior of acoustical emission sources // Physical aspects of forecasting of destruction and deformation of heterogeneous materials: collection of publications, 1987. p. 55-75.

9. **Erenkov O.Y., Mazin R.S., Khalin A.A., Sushkov P.S.** Experimental research of kinetics process of polymeric materials' destruction by acoustical emission method. // Proceedings of USGU, # 1 (2011). p. 7-15.

10. **Onischenko D.A.** Probabilistic modeling of multiscale destruction. // Mechanics of rigid bodies, # 5 (1999). p. 27-48.

11. **Hemmer P.C., Hansen A., Pradhan S.** Rupture processes in fiberbundle model // Lect. Notes Phys. 2007. V. 705. P. 27-55.

12. **Kun F., Raischel F., Hidalgo R.C., Herrmann H.J.** Extensions of FiberBundle Models // Lect. Notes Phys. 2007. V. 705. P. 57-92.

13. **Hidalgo R.C., Kun F., Herrmann H.J.** Bursts in a fiber model with continuous damage. / Phys. Rev. E 2001. V. 64. No. 6 (pt. 2).

14. **Daniels H.E.** The Statistical Theory of the Strength of Bundles of Threads. I. // Proc. Roy. Soc. London 1945 V. A183. P. 405.

15. **Kloster M., Hansen A., Hemmer P.C.** Burst avalanches in solvable models of fibrous materials <http://pre.aps.org/abstract/PRE/v56/i3/p2615_1> // Phys. Rew. E 1997. V. 56. P. 2615.

16. **Pradhan S., Hansen A., Hemmer P.C.** Burst Statistics as a Criterion for Imminent Failure // in F.M. Borodich (ed.) IUTAM Bookseries, 1, vol. 10, IUTAM Symposium on Scaling in Solid Mechanics, Springer, 2009.

17. **Herrmann H.J., Kun F., Raischel F.** Some New Results on FiberModels // Scaling in Solid Mechanics // Ed. F.M. Borodich. Springer Science, 2009. P. 263-272.