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On the matter of evaluation of the variation coefficient of the time to failure based on low-level quantiles

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Abstract. *In the context of various tasks related to dependability estimation of systems by probabilistic physical methods the most important a priori information that ensures effective solutions is the information on the variation coefficient of the time to failure. Given the low failure statistics, the estimation of the variation coefficient of the time to failure is complicated due to significant sample censoring. In these cases, methods of variation coefficient evaluation with additional a priori information and the method of quantiles are used. The solution of a number of dependability-related tasks that require taking into consideration various failure distributions is significantly simplified if the functions of such distributions are tabulated in the relative operation time and variation coefficient parameters. An effective solution of dependability-related tasks with the use of tables of DN distribution function was first proposed for the parametrization of distribution in parameters x and v, where x is the scale parameter, relative operation time x = at; v is the shape parameter, variation coefficient v = V; a is the average degradation rate. That allowed performing tabulation out of real time, simplifying function tabulation and its use in a number of dependability-related tasks by method of quantiles. The paper analyzed the effectiveness of the method of quantiles in the estimation of the variation coefficient of the time to failure, that is at the same time the shape parameter of the DN distribution, under scarce failure statistics and based on it proposes a new, more effective, method. The method of estimation of the variation coefficient using low and ultralow-level quantiles is based on the behaviour analysis of function* $a_i = f(t)$ *obtained using the method of quantiles. It is considered that the best choice of the a priori value of v is a choice under which the dependence graph* $a_i = f(t)$ *is most accurately described by a straight horizontal line, which is in complete compliance with the hypothesis of constant degradation rate accepted in the context of DN distribution formalization. In cases when the dependence graph* $a_i = f(t)$ *does not easily allow concluding on the best choice of the a priori value v (it is especially difficult to make a choice based on the statistics of first failures), the following formal criterion can be used: the most acceptable a priori value of the shape parameter v lies within the range of values, where the sign of the trend of the average degradation rate (h) in graph* $a_i = f(t)$ *changes. Studies have established that the most significant errors in the estimation of the variation coefficient are associated with first failures. When processing the results of dependability tests it is assumed the first failures in a sample have the lowest information weight, as their occurrence is due to serious defects not detected by final quality inspection of products. The first failures normally "fall out" of the overall statistical pattern, and it is recommended to omit them from further analysis. The proposed method of estimation of the variation coefficient of the time to failure based on ultralow-level quantiles enables – in the context of limited failure statistics, when other methods are inefficient – for sufficiently accurate identification of not only the variation coefficient of the time to failure and DN distribution parameters, but also make conclusions regarding the feasibility and legitimacy of equalization (description) of the considered sample using this diffusion distribution, i.e. it can be used as a kind of criterion of compliance of the empirical failure distribution under consideration with the chosen theoretical dependability model. The described process of finding the truest values of the variation coefficient of the time to failure using the formal criterion can be computerized.*

Keywords: *method of quantiles, variation coefficient, low and ultralow-level quantiles, DN distribution.*

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1. Introduction

In the context of various tasks related to dependability estimation of systems by probabilistic physical methods [1] the most important a priori information that ensures effective solutions is the information on the variation coefficient of the time to failure. Given the low failure statistics the estimation of the variation coefficient of the time to failure is complicated due to significant sample censoring. In these cases methods of variation coefficient evaluation with additional a priori information [2-4] and method of quantiles are used [1].

The paper analyzed the effectiveness of the method of quantiles in the estimation of the variation coefficient of the time to failure (shape parameter of the *DN* distribution [5-8]) under scarce failure statistics and based on it proposes a new, more effective, method.

2. Method of quantiles

If the a priori value of the shape parameter v is known, whose consistent estimate is the variation coefficient of the degradation process V, the scale parameter of the *DN* distribution, i.e. the average degradation rate a, can be identified by solving equation [1]:

$$\Phi\left(\frac{at_\gamma - 1}{\nu\sqrt{at_\gamma}}\right) + \exp(2\nu^{-2})\Phi\left(-\frac{at_\gamma + 1}{\nu\sqrt{at_\gamma}}\right) = \hat{\gamma}, \quad (1)$$

where $\hat{\gamma} = r / N$ is the quantile calculated based on the ratio of the number of failures r to the sample size N submitted to tests; t_γ is the time of occurrence of the r-th failure.

The solution of a number of dependability-related tasks that require taking into consideration various failure distributions is significantly simplified if the functions of such distributions are tabulated. An effective solution of dependability-related tasks with the use of tables of the *DN* distribution function was first proposed in [9], where the *DN* distribution function was parametrized and tabulated in the x and v parameters. The use of the relative operation time $at = x$ as the distribution parameter allowed performing tabulation out of real time, simplifying function tabulation and its use in a number of dependability-related tasks by method of quantiles.

$$\Phi\left(\frac{x_\gamma - 1}{\nu\sqrt{x_\gamma}}\right) + \exp(2\nu^{-2})\Phi\left(-\frac{x_\gamma + 1}{\nu\sqrt{x_\gamma}}\right) = \hat{\gamma}, \quad (2)$$

where $x_\gamma = at_\gamma$.

With the use of *DN* distribution tables [1] and input data on $\hat{\gamma}$ and v the value of x_γ is identified, then formula $a = \frac{x_\gamma}{t_\gamma}$ is used to calculate the value of average degradation rate a.

If, in the course of estimation of the scale parameter of the *DN* distribution a by method of quantiles, the a priori value of the shape parameter v is chosen (based on the most general considerations of physics of failure [10]) that is definitely higher than the actual value of V, the predicted average degradation rate is underestimated. On the contrary, if the a priori value of the shape parameter v is definitely lower that the actual value V, the prediction results are overestimated. And only if the chosen a priori estimation of the shape parameter is close to the actual value of the variation coefficient of the entire assembly $\hat{V}$, estimates a_i obtained by method of quantiles are around the mean estimate $\hat{a}$ with minimal dispersion and are dependence graph $a_i = f(t)$ that is as close as possible to the horizontal line around the true average.

It is recommended to average estimates a_i obtained by method of quantiles by omitting the first failures and accepting for averaging the final, most linearized section of the dependence $a_i = f(t)$, or to use the weighted average formula proposed in [1]. It must be taken into consideration that the use of statistical information on first failures causes significant errors in the estimation of the scale parameter of the *DN* distribution. No steady pattern has been identified, so estimates a_i obtained from the first failures can be both overestimated and underestimated with respect to $\hat{a}$ obtained for the entire assembly.

By using the above patterns the following method of small-sample estimation of variation coefficient of the time to failure can be formulated.

3. Method of estimation of variation coefficient based on low-level quantiles

The process of electronics degradation, along with monotone realizations (mechanical destruction in the course of thermoelectric cycling) as the result of the electric phenomena, has non-monotone realizations. Therefore, in the general case the degradation of such products is commonly considered as a process with non-monotone realizations (Figure 1). In this case the slope ratio of the average value of the determining parameters of the degradation process (inclined solid line on the graph) that occur in the product is a constant value equal to the average rate of the generalized degradation process.

$$tg\alpha = \hat{a} = \text{const.} \quad (3)$$

The formalization of the *DN* distribution assumes that the degradation process for a set of same-type products is uniform, i.e. its average rate, mean square deviation of the rate and, subsequently, rate variation coefficient are constant (Figure 2).

The method of estimation of the variation coefficient using low-level quantiles is based on the behavior analysis of dependence graphs $a_i = f(t)$ obtained using the method of quantiles. It is considered that the best choice of the a priori value v is a choice under which the dependence graph $a_i = f(t)$ is most accurately described by a straight horizontal line, which is in compliance with the hypothesis of constant degradation rate accepted at the formalization of the *DN* distribution [4, 9] (figure 2).

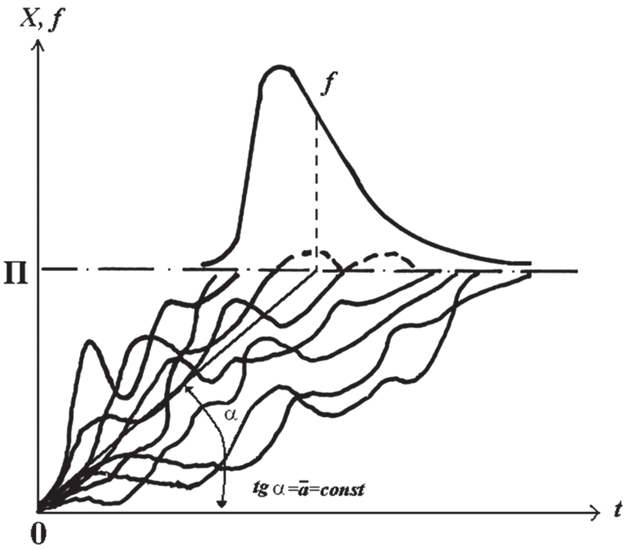

Figure 1. Graph of the *DN* distribution density formation for a product (L is the limiting value of the determining parameter that marks the onset of object failure)

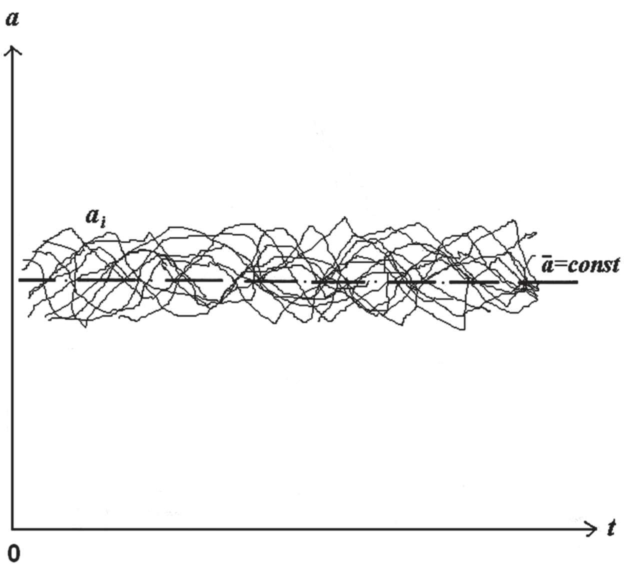

Figure 2. Graph of the theoretical dependence $a_i = f(t)$ for a set of products

In cases when the dependence graph $a_i = f(t)$ does not enable an easy conclusion regarding the best choice of a priori value v (it is especially difficult to make a choice based on the statistics of first failures), the following formal criterion can be used.

Fitting criterion of the a priori value of the shape parameter The most acceptable a priori value of the shape parameter v lies within the range of values that enable the change of the sign of the trend of the average degradation rate (h) on the graph $a_i = f(t)$.

$$h = \frac{a_n - a_1}{\overline{a}_i}, \tag{4}$$

where a_1, a_n are the estimates of the product's degradation rate obtained based on the quantiles of the minimum and maximum levels respectively; $\overline{a}_i$ is the average value of the degradation rate estimates obtained using the method of quantiles

$$\overline{a}_i = \frac{\sum_{i=1}^{n} a_i}{n}. \tag{5}$$

Let us illustrate the efficiency of this criterion with the example of full-scale durability tests of product samples, whose failure statistics are well described by the *DN* distribution.

An example. As an example, let us consider fatigue endurance tests of product samples made of the V-95 aluminum alloy [10]. It is required to assess the variation coefficient of the time to failure based on small samples and using the proposed method.

The first elements of the sample with the size $N = 463$ with the respective quantiles within the range from 0.0021 to 0.3131 are given in Table 1. In Table 1, the following notations are used: r, accumulated failure count to moment of time t_γ; t_γ, test time that corresponds to the accumulated failure count; γ, empirical failure probability.

Table 1. Data table

r	γ	t_γ, 10^3 cycle
1	0,0021	44
5	0,0107	49
10	0,0215	57
15	0,0323	59
20	0,0431	63
25	0,0539	66
30	0,0647	68
35	0,0755	73
40	0,0863	75
45	0,0971	78
50	0,1079	79
55	0,1187	82
60	0,1295	84
65	0,1403	86
70	0,1511	89
75	0,1619	91
80	0,1727	93
85	0,1835	95
90	0,1943	97
95	0,2051	99
100	0,2159	102
105	0,2267	102
110	0,2375	105
115	0,2483	106
120	0,2591	107
125	0,2699	108
130	0,2807	109
135	0,2915	111
140	0,3023	113
145	0,3131	114

Table 2. Data table

r	γ	t_γ, 10^3 cycle	$v = 0,6$		$v = 0,5$		$v = 0,4$		$v = 0,3$	
			a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h
1	0,0021	44	4,545	0,1106	5,681	0,1503	7,272	0,0358	9,318	**-0,0397**
5	0,0108	49	5,102		6,327		7,959		9,796	
10	0,0215	57	5,088		6,614		7,544		8,947	

Table 3. Data table

r	γ	t_γ, 10^3 cycle	$v = 0,6$		$v = 0,5$		$v = 0,4$		$v = 0,3$	
			a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h
15	0,0323	59	5,254	0,0092	6,44	**-0,0119**	7,797	**-0,0288**	8,915	**-0,0143**
20	0,0431	63	5,397		6,349		7,619		8,889	
25	0,0539	66	5,303		6,364		7,576		8,788	

Table 4. Data table

r	γ	t_γ, 10^3 cycle	$v = 0,6$		$v = 0,5$		$v = 0,4$		$v = 0,3$	
			a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h	a_i, 10^{-6} cycle^{-1}	h
15	0,0323	59	5,254	0,0148	6,44	**-0,0273**	7,797	**-0,0615**	8,915	**-0,0747**
20	0,0431	63	5,397		6,349		7,619		8,889	
25	0,0539	66	5,303		6,364		7,576		8,788	
30	0,0647	68	5,441		6,47		7,647		8,823	
35	0,0755	73	5,342		6,164		7,260		8,356	
40	0,0863	75	5,333		6,267		7,333		8,267	

Let us verify the method of variation coefficient estimation using low-level quantiles. As input data, let us take quantiles of levels from 0.0021 to 0.0215. For different values of parameter v let us define, using the method of quantiles, values a_i based on γ and r data and calculate the value of criterion h using formula (4). The values are given in Table 2.

The experimental dependence graph $a_i = f(t)$ for quantiles from 0.0021 to 0.0215 is shown in Figure 3.

Conclusions regarding parameter estimation. The change of sign of trend h occurred when $0.3 < v < 0.4$, therefore

$$\nu = \frac{0,3+0,4}{2} = 0,35; \ \overline{a_i} = 8,473 \cdot 10^{-6} \text{ cycle}^{-1};$$

$$\delta_\nu = \frac{0,56-0,35}{0,56} = 0,375;$$

$$\delta_a = \frac{8,473 \cdot 10^{-6} - 5,9 \cdot 10^{-6}}{5,9 \cdot 10^{-6}} = 0,436.$$

The estimation error of both the shape parameter and the scale parameter are quite significant in the case of first failures. As it is known, when processing the results of dependability tests it is assumed that the first failures in a sample have the lowest weight, as their occurrence is due to serious defects not detected by final quality inspection of products. The first failures normally "fall out" of the overall statistical pattern, therefore for the purpose of further analysis we will omit them and continue the research of the effectiveness of the variation coefficient evaluation method based on quantiles of the level from 0.0323 to 0.0539. The values of a_i and h are given in Table 3.

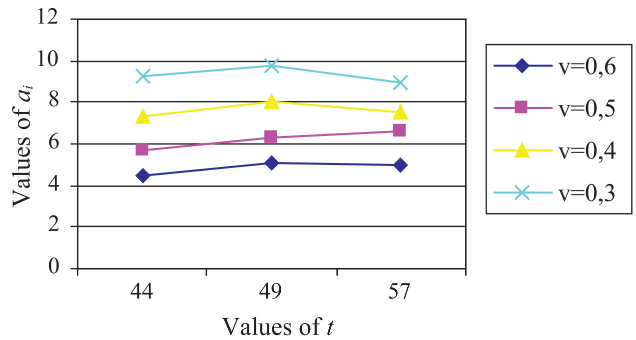

Figure 3. Experimental dependence graph $a_i = f(t)$ for quantiles from 0.0021 to 0.0215

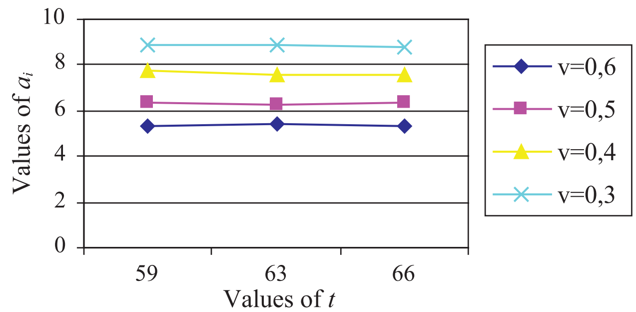

Figure 4. Experimental dependence graph $a_i = f(t)$ for quantiles from 0.0323 to 0.0539

The experimental dependence graph $a_i = f(t)$ for quantiles from 0.0323 to 0.0539 is shown in Figure 4.

Conclusions regarding parameter estimation. The last change of sign of trend h occurred when $0.5 < v < 0.6$, therefore

$$\nu = \frac{0,5+0,6}{2} = 0,55;\ \overline{a_i} = 5,851 \cdot 10^{-6}\ \text{cycle}^{-1};$$

$$\delta_\nu = \frac{0,56-0,55}{0,56} = 0,018;$$

$$\delta_a = \frac{5,9\cdot 10^{-6} - 5,851\cdot 10^{-6}}{5,9\cdot 10^{-6}} = 0,008.$$

Analyzing the absolute values of trends 0.0092 and 0.0119, an additional conclusion can be made that the true value of the shape parameter v is closer to 0.6.

Let us increase the quantity of statistical information on failures to quantiles of level 0.0863. The values of a_i and h are given in Table 4.

The experimental dependence graph $a_i = f(t)$ for quantiles from 0.0323 to 0.0863 is shown in Figure 5.

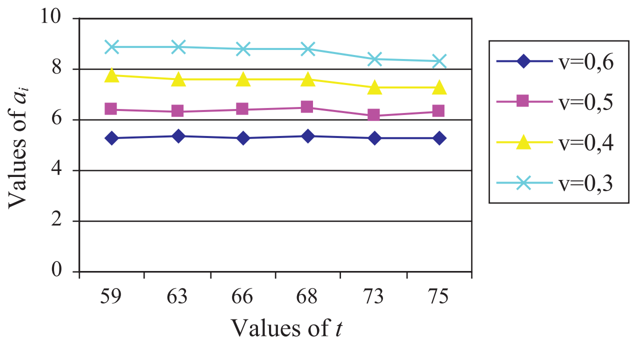

Figure 5. Experimental dependence graph $a_i = f(t)$ for quantiles from 0.0323 to 0.0864

Conclusions regarding parameter estimation. The sign of trend h did not change when the failure statistics grew and occurred again when $0.5 < \nu < 0.6$. Therefore

$$\nu = \frac{0,5+0,6}{2} = 0,55;\ \overline{a_i} = 5,844 \cdot 10^{-6}\ \text{cycle}^{-1};$$

$$\delta_\nu = \frac{0,56-0,55}{0,56} = 0,018;$$

$$\delta_a = \frac{5,9\cdot 10^{-6} - 5,844\cdot 10^{-6}}{5,9\cdot 10^{-6}} = 0,009.$$

Under the current discreteness of variation of the a priori value of v equal to 0.1 further growth of the failure statistics does not result in more precise estimation of the shape parameter. If we assume the discreteness is equal to 0.05, the value of v could be estimated even more accurately. It must be noted that the above described process of finding the most true value of the sample estimate of the shape parameter v using the formal criterion is sufficiently algorithmic and can be successfully computerized.

Let us take a look at the graph of the average degradation rate in case the statistical information is increased to low-level quantiles of 0.3131.

In [1, 10], data are given that were obtained as the result of processing of a complete sample of V-95 products: $N = 463$, $\hat{V} = 0,56$, $\hat{S} = 169 \cdot 10^3$ cycle, $\hat{a} = 5,9 \cdot 10^{-6}$ cycle^{-1}.

The estimates of the variation coefficient of the time to failure per low-level quantiles using the proposed formal criterion are very close ($v = 0.55$; $\overline{a_i} = 5.844 \cdot 10^{-6}$ cycle^{-1}) to the estimates obtained experimentally using a complete sample.

Figure 6 shows the dependence graph of the DN distribution scale parameter estimate obtained per quantiles from 0.0021 to 0.3131 for $v = 0.57$.

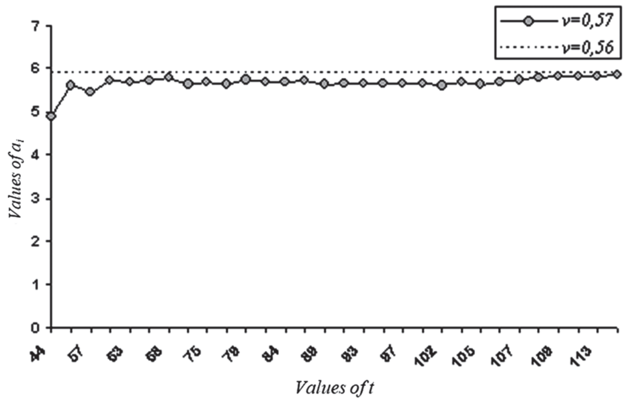

Figure 6. Experimental dependence graph $a_i = f(t)$ for quantiles from 0.0021 to 0.3131

As it can be seen, while the a priori value of the shape parameter $v = 0.57$ is almost completely identical to the sample estimate of the variation coefficient $\hat{V}$=0,56, the dependence graph $a_i = f(t)$ is a sufficiently straight line slightly below estimate $\hat{a} = 5,9 \cdot 10^{-6}$ cycle^{-1} obtained per the complete sample. As it was expected, the exception is the first failures that underestimate the average degradation rate and do not match the general trend.

4. Conclusions

The proposed method of quantile-based estimation of the variation coefficient of the time to failure enables – in the context of limited failure statistics – using ultralow level quantiles for sufficiently accurate identification of not only the variation coefficient of the time to failure and *DN* distribution parameters, but also make conclusions regarding the feasibility and legitimacy of equalization (description) of the considered sample using this diffusion distribution, i.e. it can be used as a kind of criterion of compliance of the empirical failure distribution under consideration with the chosen theoretical dependability model. The above described process of finding the most true values of the variation coefficient of the time to failure using the formal criterion can be computerized.

References

[1]. Strelnikov VP, Fedukhin AV. Otsenka i prognozirovanie nadezhnosti elektronnykh elementov i sistem [Estimation and prediction of the dependability of electronic elements and systems]. Kiev: Logos; 2002 [in Russian].

[2]. Savchuk VP. Bayesovskie metody statisticheskogo otsenivaniya nadezhnosti tekhnicheskikh obektov [Bayesian methods of statistical evaluation of technical objects dependability]. Moscow: Nauka; 1989 [in Russian].

[3]. Prokhorenko VD, Golikov VF. Uchet apriornoy informatsii pri otsenke nadezhnosti [Accounting for a priori information in dependability estimation]. Moscow: Nauka i teknika; 1978 [in Russian].

[4]. GOST 27.201-81. Reliability in the equipment. Assessment of indicators of reliability at small number of observations with the use of additional information. Moscow: Izdatelstvo standartov; 1981 [in Russian].

[5]. Strelnikov VP. Prognozirovanie resursa izdeliy elektronnoy tekhniki [Predicting of electronic products' lifetime]. Dependability 2014;4(12):43-48 [in Russian].

[6]. Strelnikov VP. Metodicheskie pogreshnosti rascheta nadezhnosti sistem [Systematic errors of system dependability calculation]. Dependability 2005;3(12):41-46 [in Russian].

[7]. Strelnikov VP. Metodicheskie pogreshnosti rascheta nadezhnosti elektronnykh elementov i sistem [Systematic errors of electronic components and systems dependability estimation]. Dependability 2009;2:27-32 [in Russian].

[8]. Strelnikov VP. Zakonomernosti izmeneniya narabotki mezhdu otkazami tekhnicheskikh sistem v protsesse ekspluatatsii [Change trends of time between failures in technical systems in operation]. Dependability 2011;1(36):17-22 [in Russian].

[9]. Pogrebinsky SB, Strelnikov VP. Proektirovanie i nadezhnost mnogoprotsessornykh EVM [Design and dependability of multiprocessor computers]. Moscow: Radio i svyaz; 1988 [in Russian].

[10]. GOST 27.005-97. Industrial product dependability. Failure models. General concepts. Kiev: Gosstandart Ukrainy; 1997 [in Russian].

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