FOREWORD BY DZIRKAL E.V.

Serial control of reliability (and other similar parameters) boils down to the comparison of some value at each moment of observation with two boundaries – acceptance and rejection. Between the boundaries there is a region of uncertainty (testing continues when in there). The classical Wald approach does not imply any restrictions for testing time. As soon as the region of the Wald uncertainty is truncated, risks for a supplier and a customer grow considerably. And it is unknown at which level this region should be truncated, with decisions made in a completely arbitrary way. For example, risks are set to 0.1, but the outcome is 0.13 (and this outcome can be evaluated by some experts, including Demidovich with his method).

Demidovich N.O. developed a method that makes it possible to set boundaries in such a way that serial control using these boundaries will provide precise values of specified risks. Boundaries as to the Demidovich method can create an uncertainty region of any form including a closed one that does not require truncation (Demidovich himself used a triangular form for development). The method was acknowledged by ISO/IEC experts, adopted in Russia as the GOST R 27.402-95 standard and became part of the second version of the IEC 61124 draft standard to be issued.

It is worth noting that plans of N.E. Yarlykov with the same advantages appeared in the USSR before Demidovich.

It is obvious that the Demidovich plans are definitely better than the “classical” Wald plans, and the latter should be replaced in all standards and textbooks. Below you can find two first papers of Demidovich N.O. published in the journal Reliability and Quality Control in 1990 and 1991.

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Таблица 17

$\alpha = \beta = 0{,}1;\ T_\alpha/T_\beta = 2;\ t'_{\Sigma\alpha}/T_\alpha = 5{,}91;\ t'_{\Sigma\beta}/T_\alpha = 5{,}00$

r	$t_{\Sigma\alpha}/T_\alpha$	$t_{\Sigma\beta}/T_\alpha$	r	$t_{\Sigma\alpha}/T_\alpha$	$t_{\Sigma\beta}/T_\alpha$	r	$t_{\Sigma\alpha}/T_\alpha$	$t_{\Sigma\beta}/T_\alpha$
0	1,907	—	14	11,884	7,140	28	20,413	17,766
1	2,860	0,015	15	12,513	7,854	29	21,006	18,563
2	3,695	0,183	16	13,137	8,577	30	21,598	19,363
3	4,473	0,507	17	13,758	9,309	31	22,188	20,166
4	5,217	0,929	18	14,376	10,049	32	22,775	20,973
5	5,936	1,417	19	14,990	10,796	33	23,363	21,781
6	6,636	1,953	20	15,602	11,549	34	23,949	22,594
7	7,322	2,525	21	16,211	12,309	35	24,533	23,408
8	7,997	3,128	22	16,817	13,074	36	25,117	24,227
9	8,663	3,755	23	17,422	13,846	37	25,699	25,045
10	9,319	4,401	24	18,023	14,621	38	—	25,699
11	9,969	5,065	25	18,624	15,400			
12	10,612	5,744	26	19,223	16,186			
13	11,251	6,436	27	19,818	16,975			

1. **М. Б. Фарберман, Р. Д. Берштин, С. С. Дмитриченко** «Планирование объемов последовательных испытаний деталей машин на долговечность». Вестник машиностроения, № 8, 1987, С. 3—6.

2. Harter H. L., Moore A. H. An Evaluation of Exponentiol and Weibull Test Plans IEEE Transactions on Reliability, Vol R—25, N 2, June 1976, p. 100—104.

3. Надежность технических систем: Спр. **Ю. К. Беляев, В. А. Богатырев, В. В. Болотин** и др. Под ред. **И. А. Ушакова.** М.: Радио и связь, 1985.

Таблица 5

Планы контроля средних показателей надежности по последовательному методу для экспоненциального распределения

T_α/T_β	a	r_0	$r_{ус}$	t_0/T_α	t'_Σ/T_α
		$\alpha = \beta = 0{,}10$			
21,70	6,74	0,713	1	0,106	0,099
7,30	3,17	1,110	2	0,349	0,408
5,00	2,49	1,370	3	0,549	0,735
4,83	2,43	1,400	3	0,574	0,780
4,00	2,16	1,590	4	0,732	1,090
3,83	2,11	1,640	4	0,776	1,180
3,50	2,00	1,750	5	0,879	1,410
3,29	1,92	1,850	5	0,960	1,600
3,00	1,82	2,000	6	1,100	1,960
2,94	1,80	2,040	6	1,130	2,040
2,70	1,71	2,210	7	1,290	2,480
2,53	1,65	2,380	8	1,440	2,940
2,50	1,64	2,400	9	1,460	3,010
2,39	1,60	2,520	9	1,580	3,380

Продолжение табл. 5

T_α/T_β	a	r_0	$r_{ус}$	t_0/T_α	t_Σ/T_α
2,28	1,55	2,660	10	1,710	3,840
2,19	1,52	2,810	11	1,840	4,310
2,12	1,49	2,930	12	1,970	4,780
2,06	1,47	3,050	13	2,080	5,240
2,00	1,44	3,160	15	2,190	5,690
1,96	1,42	3,280	15	2,310	6,200
1,79	1,36	3,770	20	2,770	8,420
1,67	1,31	4,270	25	3,270	11,100
1,60	1,28	4,660	30	3,650	13,400
1,50	1,23	5,420	40	4,390	18,600

2.0, 0.1=0.1, 1.44 and 3.16, acceptance is 1.44к – 3.16 = 1.44*10 – 3.16 = 11.24, rejection is 17.56

Demidovich N.O.

QUALITY CRITERIA FOR PLANS OF FAILURE-FREE PERFORMANCE CONTROL

In order to control such parameters of failure-free performance as TTF of recoverable products and MTBF of non-recoverable products, the standards [1,2] specify two basic types of plans applied usually with the assumption of the exponential distribution of random values, i.e. times between failures and times to failures – plans with limited duration or with a limited number of failures (type I) and Wald's truncated serial plans (type II).

Further on we'll consider plans with replacement or recovery of failed products, although the results can refer to plans without replacement (recovery) as well.

As criteria for selecting plans, we use maximum duration of testing (for plans of type I, it is minimal) or average duration (for plans of type II, it is close to a minimal one "with precision up to truncation").

When truncating serial plans, values of parties' risks increase compared to the nominal values. This increase can be substantial (up to 30 per cent), and it should not be always neglected.

Besides the abovementioned criteria, we'll also consider other criteria of testing plans that are overlooked in theoretical researches but are still important for practice. Let us tentatively note that when comparing the boundaries of plans of types I and II (Fig. 1), we can conclude about some "empty" regions in plans of type I, which faintly influence generation of wrong decisions. Such regions (upper and lower ones) are lined in the figure. It is characteristic of them that stepwise lines of failure process realization occurred in these regions come out on the corresponding boundaries of type I plan with the probability close to unity for any of hypotheses.

The most important thing that should be taken into account while preparing plans of testing is that there is a break-in period practically for all types of products whose MTBF or TTF are approximated by an exponential distribution. The standardized plans do not take this into account. A break-in period can vary both in terms of duration and character depending on the state of technological process etc. A break-in factor results in that moments of failure occurrence are distributed not evenly as per duration of testing as it should be in case of exponential distribution but are more concentrated at the initial stage of testing. While the lines of failure process realization instead of a lineal character take on a tendency to a curve that by its convex part points to the boundary of non-conformity of type II plan, crosses it and then come beyond the boundary of conformity of plans of types I and II, as shown in Fig. 1. As a result, a manufacturer's risk increases uncontrollably when products acceptable in terms of reliability can be rejected. This circumstance prevents from wide application of type II plans in practice, in spite of their economical benefits.

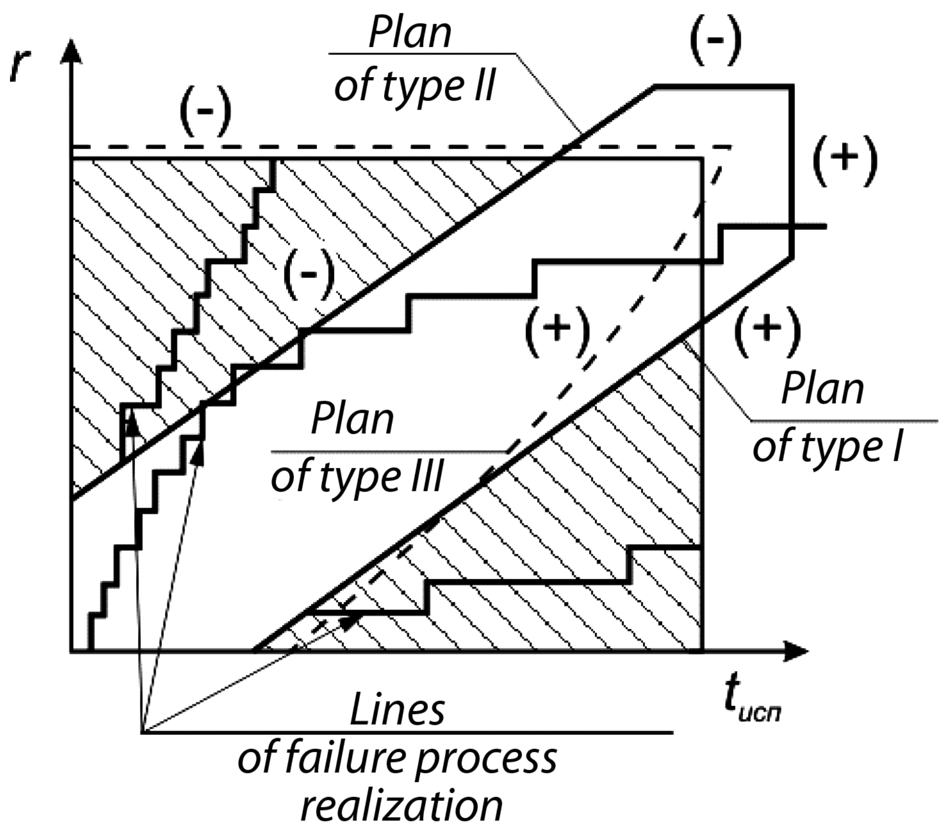

Fig. 1. Comparison of the boundaries of testing plans of various types. The regions of type I plan that feebly influence the values of risks are lined. The symbol (+) denotes the conformity boundaries, the non-conformity boundaries of plans

Type I plans are not sensitive to possible break-in factors, as the boundary of non-conformity there is a right line parallel to the X-axis. In other words, of two regions lined in Fig. 1 the lower region is always empty, and the upper one is not in case of break-in.

The above arguments justify the reasonability of considering, as an alternative to type II plans, new serial plans (of type III), which are obtained not by truncating Wald's plans but excluding a lower lined region from the respective plans of type I. The general character of the boundaries of such plans is shown in Fig. 1 as a dash line. As it will be shown further, they combine the advantages of plans of types I and II and are free from their disadvantages.

The retention of an upper region in fact means setting one of the plan boundaries (non-conformity), which leads to the necessity of more detailed consideration of such characteristics as an average duration of testing. In this respect we should present the actual attitude of the parties towards the duration of testing bearing in mind their possible completion with two contrary outcomes.

A manufacturer does not tend to quickly complete testing with a negative outcome, while there is a longer duration provided for and one can expect better results, i.e. he is not inclined to make a "premature" decision about non-conformity, notwithstanding the costs that testing incurs. The interests of a consumer are more neutral in this respect, or he also suffers from some inconvenience related to making a negative decision, as it can lead to some delay of delivery etc. That's why instead of average duration as to all (positive and negative) decisions one should also consider average duration of testing only for positive decisions made (about conformity) as a plan characteristic. This parameter more reflects the real interests of the parties.

Its application instead of a traditional average duration also has some technical advantage, since all calculation difficulties in defining a plan's average duration are related to negative decisions. Functions of distribution of testing duration till negative decision making are complicated, and approximated formula are not so efficient.

In cases when positive decisions are made, the average duration of testing is defined in a simple way

$$T_{cp} = \sum_{i=0}^{r_{np}} \frac{p_i}{\sum_{i=0}^{r_{np}} p_i} t_i ,$$

where p_i is the probability of testing completion with i failures; t_i is the duration of testing with i failures; r_{np} is the ultimate acceptable number of failures when reaching this, a decision about non-conformity is made.

The idea of forming a side boundary of type III plan consists in the following. For any of its arbitrary forms (a set of values $\{t_i\}$) there is its own set of probabilities $\{p_i\}$, and vice versa, a set of numerical values of probabilities $\{p_i\}$ specified for an arbitrary fixed value of time to failure T_0 unmistakably defines a side boundary of a plan, i.e. a set of values $\{t_i\}$.

If the main hypothesis ($T_0 = T_\alpha$) is true, the sum of probabilities is equal to complement up to unity of a manufacturer's risk:

$$\sum_{i=0}^{r_{np}-1} p_i(T_\alpha) = 1 - \alpha ,$$

and if the alternative ($T_0 = T_\beta$) is true, the sum of these probabilities is equal to a consumer's risk:

$$\sum_{i=0}^{r_{np}-1} p_i(T_\beta) = 1 - \beta .$$

If we assume the value of time to failure (mean time to failure) equal to rejection level T_β and consider the n sets of probabilities $\{p_i\}_k$, $k = 1\ldots.n$, such that their sum is equal to the set value of a consumer's risk β, then each set will define a side boundary of the plan $\{t_i\}_k$. All n of plans with such side boundaries of conformity and one and the same boundary of non-conformity r_{np} will have the same consumer risk but different other parameters: a manufacturer's risks, maximum and average durations etc. The number of such plans is unlimited, and among them there is also an unlimited set of plans with the identical risks for a manufacturer, i.e. a set of plans corresponding to one set of initial data T_α/T_β, α, β.

To select among this set of plans a particular plan appropriate in terms of a set of quality characteristics, a manufacturer shall have some way of searching them in some order. The basis for this search can be some principle of distribution ("quota" allotment) of a consumer's risk β as to the r_{np} of possible testing durations. The rational way of such construction and the algorithm of calculation of plans require a separate exposition.

For further consideration of quality criteria of plans, Figure 2 and the Table provide two examples of type III plans with initial data complying with the initial data of the most widely used standardized plan of type II: $T_\alpha/T_\beta = 2$, $\alpha = \beta = 0.128$ (the risks of type II plan are higher than the nominal ones α = β = 0.1 due to truncation).

Fig. 3 shows graphs of average testing duration till making a positive decision as to the compared plans. The graphs show that within a working range of possible TTF values (from rejection level T_β up to acceptance level T_a) new plans on the average are considerable less longer till making a positive decision than the standardized plan by 14-27%. In particular, the entire side boundary *a* is positioned to the left of the side boundary of the standardized plan.

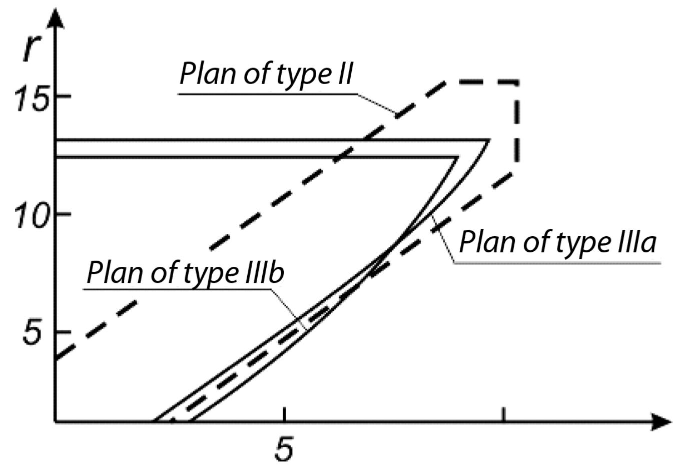

Fig. 2. The standard plan of type II for initial data T_a/T_β=2; $a = \beta$ = 0.128 (dash line) and two corresponding plans of types *IIIa* and *IIIb* (continuous lines)

No. of failures	Boundaries of plans of types II и III			
	II (accord. to [1])		IIIa	III*b*
	(rejection)	(+, i.e. acceptance)	(+)	(+)
0	—	2,20	2,03	**2,27**
1	—	2,89	2,78	**3,03**
2	—	3,59	3,47	3,69
3	0,35	4,25	4,13	4,31
4	1,04	4,97	4,77	4,90
5	1,74	5,67	5,40	5,47
6	2,43	6,36	6,02	6,02
7	3,12	7,05	6,64	6,56
8	3,82	7,75	7,27	7,09
9	4,51	8,44	7,89	7,62
10	5,20	9,13	8,51	8,13
11	5,90	9,83	9,14	8,64
12	6,59	10,30	9,76	—
13	**7,28**	10,30	—	—
14	**7,97**	10,30	—	—
15	**8,67**	10,30	—	—

In case of break-in period, the advantage of new plans also consists in that the values of testing completion probabilities for *i* failures (compared to the values of probabilities corresponding to an exponential distribution) are changed such that there is an increase of a testing share coming to completion with high values of failed products when lines of failure process realization cross side boundaries of plans in their upper region. In Fig. 2 this region is marked by a wavy line, and the plan ensuring a quicker completion of testing in this particular part of the side boundary can be mostly preferred by a manufacturer. In Fig.2

such plan is plan *b*. The standardized plan of type II in this respect seems the worst one, since its side boundary is away to the right to the maximum.

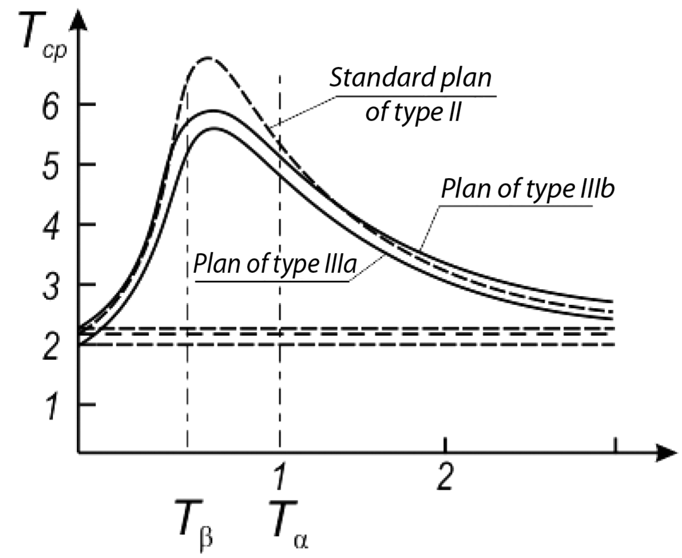

Fig. 3. Dependency of the average duration of testing till making a positive decision (about conformity) T_{cp} on time to failure of tested products T_0 of plans with initial data $T_a/T_\beta = 2$; $\alpha = \beta = 0.128$ in scale T_α

Analogous results are also obtained for other initial data. Summing up the consideration of quality criteria (characteristics) of safety performance control plans, it is necessary to make the following conclusions:

A manufacturer who takes into account the specifics of failure-free performance testing is interested by a number of characteristics of plans: maximum duration, risk immunity to possible breaks-in, average duration till making a positive decision, in particular in the field of high values of acceptable failure rates;

The average duration till making a negative decision is not of interest for the parties, and therefore, such characteristic wide spread in theoretical researches as total average duration is the less important one for practice;

The task of selecting an optimal plan as to a set of specified characteristics is not stated mathematically, i.e. there is no optimal solution in this case, so the task should be solved by providing a manufacturer an opportunity of selecting at his discretion a required plan among a set of plans of type III with identical initial data and varying, presumably rational correlations of quality characteristics;

Type III plans combine benefits of standardized plans of types I and II and are free from their drawbacks;

In terms of a consumer, all plans of type III with identical initial data are equivalent and ensure checking the set hypotheses with specified risks of the parties.

The presented approach with necessary changes can be also applied for plans without replacement (recovery) of failed products or for types of distribution different from an exponential distribution and (in a slightly lesser degree) for plans of controlling parameters like “probability”.

References

1. IEC 605-7. Reliability testing of equipment. P.7. Plans of control testing for defining failure rates and mean time between failures with the assumption of constant failure rates. — M.: Publishing house VNII-electrostandard, 1980.

2. GOST 27.410-87. Reliability in equipment. Methods for controlling reliability parameters and plans of reliability control testing.