Antsupov A.V., Antsupov A.V. (jr), Antsupov V.P.

METHODOLOGY OF MACHINE ELEMENTS' RELIABILITY PREDICTION BY MEANS OF VARIOUS CRITERIA

The paper offers a general methodological approach to reliability estimation of loaded elements of mechanical systems at the design stage in the form of sequence of construction procedure stages of a physical and probabilistic model for generating process of their parametrical failures due to various criteria. The methodology of durability prediction of parts by criterion of kinetic strength is presented and the example of its realization is shown.

Keywords: *methodology, prediction, reliability, durability, failure, damageability, γ-percentile life.*

One of the most important problems of the reliability theory is behavior prediction (change of states) of mechanical systems' components and units in assumed conditions of external loading when parameters of their reliability and durability is possible to estimate with necessary degree of validity at the design stage. The basic method of prediction is mathematical modeling. In this case the estimation of system elements' behavior and change of their parameters at the future operation is carried out on dynamic, physical and probabilistic models [1, 2].

This paper defines the universal methodological approach to reliability prediction of mechanical system elements on the basis of the analysis of known models for estimation of the reliability of technical systems. The basic concepts of technical diagnostics (GOST 20911-89), reliability theory (GOST 27.002-89) as well as the probability theory [1, 2] are used in this study for solving the stated problem.

Understanding "methodology" as the doctrine, set of methodical methods, rules or principles according to which process of knowledge of objective natural phenomena is carried out [3, 4] (in these paper an objective natural phenomenon is considered as the objective process of failure generation of technical products at their damageability (degradation) in the field of external influences), the offered approach is represented as sequence of interconnected operations.

In general the approach can be stated as sequence of logic procedure stages for estimation of parameters of reliability and durability. This is the process of physical and mathematical model construction by various criteria for parametric reliability of both operating and newly designed technical objects.
Here it is stated in the probability form and represents package of seven consecutive stages [5].

I. The choice of base parameter of an object state

Parameter X_t (as a random variable) is selected for product type being under development according to the standard (GOST 20911-89) definition of the concept "object state". Change of parameter X_t simulates product behavior (change of states) during the whole operation time in known conditions of external influence.

II. Formulation of an object state equation

Derivation or choice of random function that is the dependence describing increasing (+) or decreasing (-) change of parameter X_t in time and modeling state change of a product at its ageing (degradation) while in service:

$$X_t = X_0 \pm \int_0^t \dot{X}_t \cdot dt\,, \qquad \text{(I)}$$

where X_0 is the distribution of parameter X_t during time point $T = t_0$, describing initial state of an object;

$\dot{X}_t = dX_t / dt$ is the urrent distribution of random variable of rate of object damageability during the time point $T = t$.

If the random variable of rate of object damageability does not change in time, then $\dot{X}_t = \dot{X} = const$ and therefore the expression (1) can be written down in the form of:

$$X_t = X_0 \pm \dot{X} \cdot t\,. \qquad \text{(I.a)}$$

The equation (I) models a process of object degradation in time.

III. Formulation of condition of object operability

In conformity with the standard (GOST 27.002-89) definition "object operability", its condition of operability mathematically is formulated in the form of one of possible inequalities:

$$X_t = X_0 + \int_0^t \dot{X}_t \cdot dt < x_{np} \text{ or } X_t = X_0 - \int_0^t \dot{X}_t \cdot dt > x_{np}, \qquad \text{(II)}$$

where x_{np} is the limiting value of parameter X_t specified in normative-technical documentation (NTD) or set according to operating experience of similar objects.

If random variables in the expression (I) are distributed under the normal law, and $\dot{X}_t = \dot{X} = const$, then the conditions of operability (II) can be written down in the form of:

$$X_t = X_0 + \dot{X} \cdot t < x_{np} \text{ and } X_t = X_0 - \dot{X} \cdot t > x_{np}. \qquad \text{(II.a)}$$

Aligning and normalizing quantities X_t and x_{np}, inequalities (II.a) can be written down with use of corresponding fractiles in the form of:

$$U_t < u_{np(t)} \text{ or } U_t > u_{np(t)}. \qquad \text{(II.б)}$$

Developing the current value of reciprocal normal rated distribution (NRD) of the parameter, it is possible to represent the conditions (II.б) in the following form:

$$\frac{X_t-\left(\overline{x}_0+\overline{\dot{x}}\cdot t\right)}{\sqrt{\sigma_{x\,0}^2+\sigma_{\dot{x}}^2\cdot t^2}}>u_{np(t)} \text{ or } \frac{X_t-\left(\overline{x}_0-\overline{\dot{x}}\cdot t\right)}{\sqrt{\sigma_{x\,0}^2+\sigma_{\dot{x}}^2\cdot t^2}}<u_{np(t)}, \qquad \text{(II.в)}$$

where $u_{np(t)}=\dfrac{x_{np}-\left(\overline{x}_0\pm\overline{\dot{x}}\cdot t\right)}{\sqrt{\sigma_{x\,0}^2+\sigma_{\dot{x}}^2\cdot t^2}}$ is the current limiting value of fractile NRD of random variable $X_t=x_{np}$;

$\overline{x}_0=(x_{0\max}+x_{0\min})/2$; $\sigma_{x\,0}=(x_{0\max}-x_{0\min})/6$ are numerical characteristics (mean and standard) of random parameter $X_t=X_0$ of tribological conjunction condition for the initial time point $T=t_0$; $x_{0\max}$, $x_{0\min}$ are the maximal and minimal values of object parameter X_0, set as initial condition; $\overline{\dot{x}}$ and $\sigma_{\dot{x}}$ are numerical characteristics of random parameter $\dot{X}$.

The equations (II) reflect area of all possible states of object operability.

IV. Derivation of equations for estimation of object reliability parameters.

With use of the basic concept of the probability theory – "distribution function", dependences for estimation of probability of object operation reliability, i.e. probabilities of conditions' fulfillment of operability (II) for any fixed time point of the future operation are formulated:

$$P(t)=P(X_t<x_{np})=P((X_0+\int_0^t\dot{X}_t\cdot dt)<x_{np})=F(x_{np})$$

or

$$P(t)=P(X_t>x_{np})=1-P((X_0-\int_0^t\dot{X}_t\cdot dt)<x_{np})=1-F(x_{np}). \qquad \text{(III)}$$

If the normal distribution of parameter $\dot{X}_t$ is invariable in time ($\dot{X}_t=\dot{X}=const$), the basic parameter of reliability is expressed with the help of normal rated distribution (NRD) function $F\,(u_{np(t)})=F_t(x_{np})$ or Laplace function $\Phi(u_{np(t)})$:

$$P(t)=P(X_t<x_{np})=F\,(u_{np(t)})=\Phi(u_{np(t)})=\Phi\left(\frac{x_{np}-\left(\overline{x}_0+\overline{\dot{x}}\cdot t\right)}{\sqrt{\sigma_{x\,0}^2+\sigma_{\dot{x}}^2\cdot t^2}}\right)$$

or

$$P(t)=P(X_t>x_{np})=1-F\,(u_{np(t)})=1-\Phi(u_{np(t)})=1-\Phi\left(\frac{x_{np}-\left(\overline{x}_0-\overline{\dot{x}}\cdot t\right)}{\sqrt{\sigma_{x\,0}^2+\sigma_{\dot{x}}^2\cdot t^2}}\right). \qquad \text{(III.a)}$$

The equations (III.a) define the law of object reliability in integral (or differential $f(t) = -dP(t)/dt$) form – the law of generation of gradual failures of object at solving direct problem in the reliability theory. According to the expression [2], it is asymmetric and does not conform to normal distribution.

V. Formulation of the equation of object transition into marginal state (state of parametrical failure)

In conformity with the standard (GOST 27.002-89) definitions "marginal state" and "parametrical failure", conditions of conjugation transition into the state of parametrical failure in the form of obtaining limiting value by parameter X_t are formulated:

$$X_t = X_0 \pm \int_0^t \dot{X}_t \cdot dt = x_{np}. \tag{IV}$$

If normal distribution of parameter $\dot{X}_t$ is invariable in time that is $\dot{X}_t = \dot{X} = const$, then the expressions (IV) look like:

$$\frac{X_t - \left(\bar{x}_0 + \bar{\dot{x}} \cdot t\right)}{\sqrt{\sigma_{x0}^2 + \sigma_{\dot{x}}^2 \cdot t^2}} = u_{np(t)} \text{ or } \frac{X_t - \left(\bar{x}_0 - \bar{\dot{x}} \cdot t\right)}{\sqrt{\sigma_{x0}^2 + \sigma_{\dot{x}}^2 \cdot t^2}} = u_{np(t)}. \tag{IV.a}$$

The equations (IV) reflect the area of all possible marginal states of object.

VI. Derivation of equations for estimation of durability parameters (life-time characteristics) of object

According to the definition of γ-percentile life t_γ under GOST 27.002-89, dependences for its estimation are derived by solving the equation (IV) relating $t = t_\gamma$ for the specified admissible value of reliability probability $[P(t)] = \gamma$ and a corresponding value of fractile $[u_{np(\gamma)}]$ distributions of random variable X_t under the known law. In general this dependence can be presented by some function:

$$t_\gamma = f(\bar{x}_0, \sigma_{x0}, \bar{\dot{x}}, \sigma_{\dot{x}}, [u_{np(\gamma)}]). \tag{V}$$

If the normal distribution of parameter $\dot{X}_t$ is invariable in time that is $\dot{X}_t = \dot{X} = const$, then the γ-percentile life is defined by solving the equations (IV.a) relating $t = t_\gamma$ by substitution in it $X_t = x_{np}$ and $u_{np(t)} = [u_{np(\gamma)}]$ in the following form:

$$t_\gamma = \frac{(\pm\bar{\dot{x}}) \cdot \Delta\bar{x}_{np} - \sqrt{(\Delta\bar{x}_{np} \cdot \bar{\dot{x}})^2 - ([u_{np(\gamma)}]^2 \cdot \sigma_{\dot{x}}^2 - \bar{\dot{x}}^2) \cdot ([u_{np(\gamma)}]^2 \cdot \sigma_{x0}^2 - \Delta\bar{x}_{np}^2)}}{\bar{\dot{x}}^2 - [u_{np(\gamma)}]^2 \cdot \sigma_{\dot{x}}^2}, \tag{V.a}$$

where the value $[u_{np(\gamma)}]$ is defined by the specified admissible value of reliability probability $[P(t)] = \gamma$, and the quantity $\Delta\bar{x}_{np} = x_{np} - \bar{x}_0$ is the expectation estimation of limiting change ΔX_t of parameter X_t.

VII. Derivation of kinetic equation of object damageability

On the basis of any theory, concept or experimental researches, the kinetic equation is deduced for object damageability rate estimation $\dot{X}_t$ depending on its geometrical and microgeometrical d_i, Δ_t characteristics, properties σ_i of material, force and kinematic parameters, F, V external impacts, time t and other factors.

In general this equation can be presented in the form of some random function.

The condition (VI) is the kinetic equation of damageability of object.

$$\dot{X}_t = f(d_i, \Delta_t, F, V, \sigma_i, t, ...)\,. \tag{VI}$$

The methodological approach stated above (I) – (VI) can be formulated in the form of individual methodology of reliability estimation (prediction) of specific groups of technical objects according to one of possible criteria – conditions (II) of reliability [1, 2]:

- Static or kinetic durability;
- Wear resistances;
- Rigidity;
- Bearing ability;
- Heat resistance, etc.

At the chosen parameter X_t of specific product (component, unit) state and at well-known following data:

- about the law of its distribution;
- about the boundary conditions describing the scheme of object loading, its property and an initial state in expected conditions of operation;
- about the equation (VI) and its damageability for estimation of $\dot{X}_t$,

the specified sequence of stages degenerates into sequence of operations (probabilistic technique) for quantitative estimation of reliability parameters $P(t) = P(X_t < x_{np})$ or $P(t) = P(X_t > x_{np})$ and durability parameters t_γ of investigated object or at its designing, or at operation.

In particular, in the study [5] the given approach is realized in the form of methodology of prediction of parametrical reliability of the big group of objects – "stationary" tribological assemblies, by criterion of wear resistance of their elements.

Below the methodology of reliability prediction of other group of objects – the loaded elements of machines and constructions has been formulated on the basis of the offered approach by criterion of kinetic durability.

Nowadays, the problem of durability of solids under loading is examined using the kinetic approach [6-8]. From this point of view a destruction process is represented as developing in time process of damages' accumulation of material structure. When material structure current density of defects in any local volume (its damageability) achieves limit value, a microcrack occurs which propagates with the speed of sound on the most loaded volumes of the whole material and that leads to its break into parts (destruction). Damage level of local volumes' structures of a component material during any fixed time point t is quantitatively estimated by density value of potential energy of defects $u_e(t) = u_{e\,t}$ which is defined by conditions of external loading and properties of material [8].

Considering the above, methodology stages for prediction of reliability and durability parameters of loaded components can be formulated, according to (I) – (VI), in the form of sequence of the following operations.

At the first stage we accept density of potential energy of defects u_{et} as parameter X_t of a component state in which under effects of external loadings at temperature T, there are internal stresses σ. Density of potential energy of defects u_{et} characterizes the current level of structure damages of local volumes of a component's material [8]. In addition to that, distribution of a random variable $X_t = u_{et}$ at any time point t as the parameter depending on a set of independent random factors, we shall assume as normal, according to the central limit theorem of probability theory [2]. Besides for simplification of mathematical expressions, we shall operate with its mean value $\overline{u}_{et}$.

At the second stage we shall formulate the equation of the loaded element states:

$$\overline{u}_{et} = \overline{u}_{e0} + \int_0^t \dot{\overline{u}}_{et} \cdot dt. \tag{1}$$

where $\overline{u}_{e0}$ is the mean density of the potential (latent) energy of a component material in initial state (at $t = 0$) which, according to [8], is possible to define in function of mean value of Vickers hardness HV:

$$\overline{u}_{e0} = \frac{\left((0,071 \cdot HV)^{1,2}\right)^2}{6 \cdot G(T) \cdot \left(6,47 \cdot 10^{-6} \cdot HV + 0,12 \cdot 10^{-2}\right)^2}; \tag{1.a}$$

where $\dot{\overline{u}}_{et} = d\overline{u}_{et} / dt$ is the average rate of damageability (accumulation of damages) of component material structure during the moment in time point t.

At the third stage we shall formulate operability condition of the loaded component:

$$\overline{u}_{et} = \overline{u}_{e0} + \int_0^t \dot{\overline{u}}_{et} \cdot dt < u_{e*}, \tag{2}$$

where $u_{e*} = u_* - u_T$ is the critical density of defects' energy of structure in local volumes of the loaded component material [8];

u_* is the critical density of internal energy (critical energy intensity) of material, equal to enthalpy of its melting in solid ΔH_{TB} or liquid ΔH_S state;

$u_T = \int_0^T \rho \cdot c dT$ is a thermal component of internal energy density of material of the loaded component at the set temperature T; ρ, c are the density and thermal capacity of a component's material.

At the fourth stage we shall write down the expression for reliability estimation of the loaded element during time point t, using safety margin (safety factor) as a parameter, calculated on mean value $\overline{u}_{et}$ of state parameter [1, 2]:

$$n_t = u_{e*} / \overline{u}_{et} = u_{e*} / (\overline{u}_{e0} + \int_0^t \dot{\overline{u}}_{et} \cdot dt). \tag{3}$$

At the fifth stage we shall formulate the equation of transition of the loaded component into limit state (state of parametrical failure):

$$\overline{u}_{e\,t} = \overline{u}_{e\,0} + \int_0^t \overline{\dot{u}}_{e\,t} \cdot dt = u_{e*}. \tag{4}$$

At the sixth stage, using expression (5), we formulate the equation, whose solution relating to $t = \overline{t}_{np}$, will allow to define average limiting life-time to failure (destruction) of an element:

$$\int_0^{t_{np}} \overline{\dot{u}}_{e\,t} \cdot dt = u_{e*} - \overline{u}_{e\,0}. \tag{5}$$

At the seventh stage it is possible to use V.V. Feodorov's dependence constructed by him in work [8] with use of thermodynamic criterion of long-term durability. This dependence is used as the kinetic equation of damageability for estimation of average rate $\overline{u}_{et}$ of damageability of the element which is being under loaded state. This rate is a member of the equations (1) – (5) of general layout of process of the element failure formation. In general this equation represents some function of the following form:

$$\overline{\dot{u}}_{e\,t} = f(\sigma, T, u_{e\,t}, t, ...). \tag{6}$$

At performance of engineering calculations one of the simplified variants of this dependence for definition of time constant value $\overline{\dot{u}}_{e\,t} = \overline{\dot{u}}_e$ [8] is used:

$$\overline{\dot{u}}_e = M_R^2 \cdot k_\sigma^2 \cdot \sigma^2 \cdot A_0 \,/\, (6 \cdot G \cdot \nu), \tag{6.a}$$

where $M_R^2 = \left((1+r)^2 + (1-r)^2\right)/4$ is a factor of equivalence of the non-stationary stress state (transfer of the non-stationary stress state with factor of asymmetry $r = \sigma_{\min}/\sigma_{\max}$ into equivalent stationary state with stress $\sigma = \sigma_a$); $\sigma_{\min}$, $\sigma_{\max}$, σ_a are minimal, maximal and amplitude stress of a cycle;

$k_\sigma = 1/\left(6{,}47 \cdot 10^{-6} \cdot HV + 0{,}12 \cdot 10^{-6}\right)$ is a factor of overstress of interatomic connections;

G and v are shear modulus and factor of non-uniformity of internal energy distribution over volume of the loaded component which value is selected under recommendations of work [8].

Factor A_0 of influence of spherical part of stress tensor on energy of activation of interatomic connections' destruction according to [8]:

$$A_0 = \frac{\nu \cdot U(\sigma, T)}{h \cdot N_0} \exp\left[-\frac{U(\sigma, T)}{R \cdot T}\right], \tag{6.б}$$

where h is Planck's constant; N_0 is Avogadro number; R is universal gas constant;

$U(\sigma, T)$ is the energy of activation of interatomic connections' destruction at the given stress σ and temperature T:

$$U(\sigma_a, T) = U_0 - \Delta U(T) - \left(M_R^2 \cdot k_\sigma^2 / (18 \cdot \nu \cdot K)\right) \cdot \sigma^2, \tag{6.в}$$

U_0 – free energy of process activation at $T = 0$ and $\sigma = 0$;

$\Delta U(T) = \frac{3}{2} \cdot \alpha_0 \cdot K^{-1} \cdot T$ – a share of activation energy, defined by temperature;

$K = E/(3 \cdot (1 - 2 \cdot \mu))$ – Factor of all-round a material compression;

α_0, μ, E – linear thermal expansion coefficient; Poisson's ratio and modulus of elasticity of material.

Set of the equations (6.а-в) represents a mathematical model of process of stationary damageability of elements of machines and constructions in which under effects of external loadings and constant temperature T there are internal static or dynamic stresses σ.

For stationary process of damageability of the loaded element (with constant rate $\bar{\dot{u}}_{et} = \bar{\dot{u}}_e$, defined by expressions (6.а-в), average value of reliability margin for any time point t is defined under on the simplified expression (3):

$$n_t = u_{e*} / \bar{u}_{et} = u_{e*} / (\bar{u}_{e0} + \bar{\dot{u}}_e \cdot t). \tag{3.a),}$$

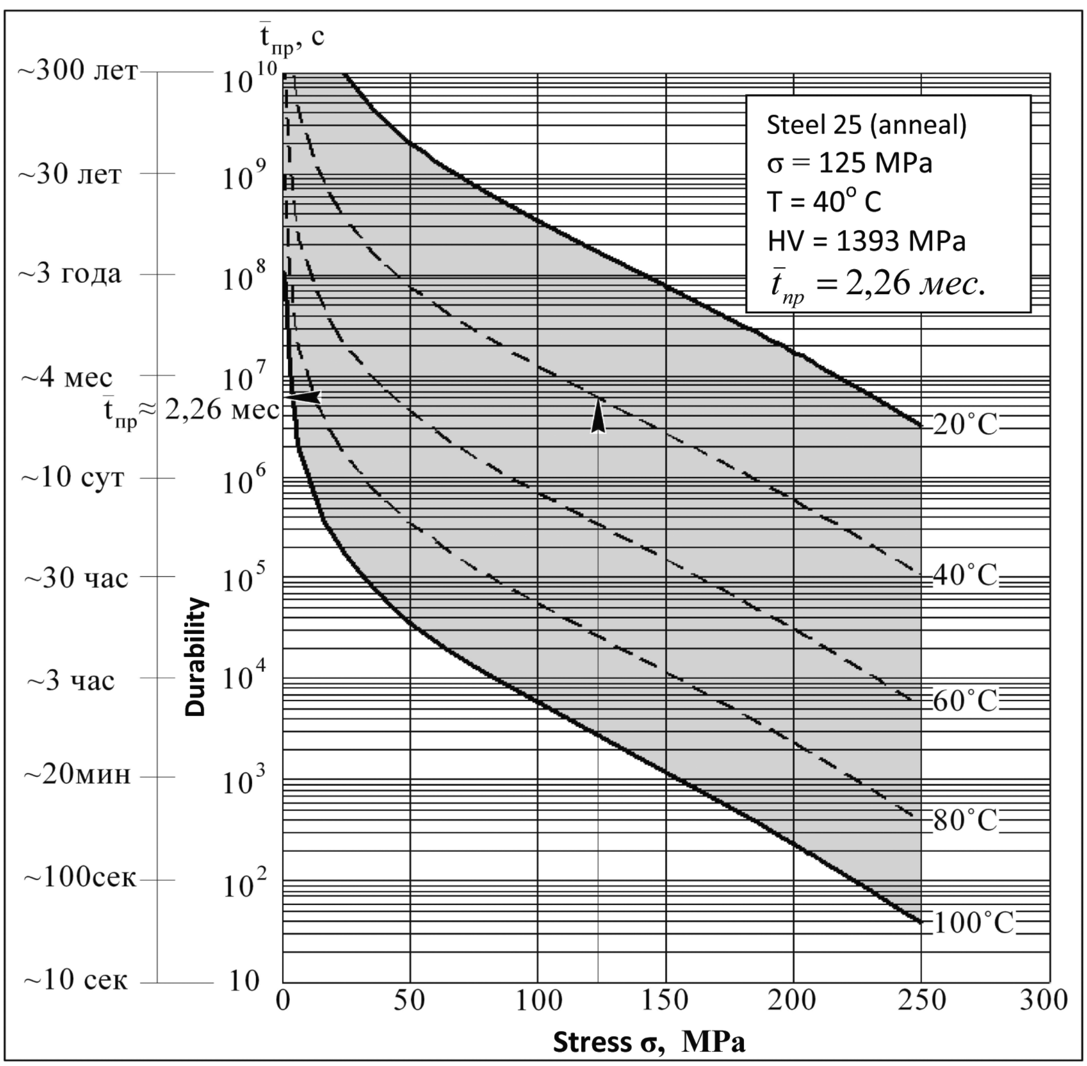

Fig. 1. Nomograph for estimation of durability of loaded elements

and limiting life-time is determined by solving of the simplified expression (5) in relation of $t = \overline{t}_{np}$ in the following form:

$$t_{np} = (u_{e*} - \overline{u}_{e0}) / \overline{\dot{u}}_e. \qquad (5.a)$$

Set of the equations (3.a), (5.a) – (6.a-в) represents a physical and probabilistic model of formation process of gradual failures of machines' components in which under effects of external loadings and constant temperature T internal static or dynamic stresses σ take place.

The sequence of mathematical operations constructed on the basis of this model for an estimation of the expected average life-time $\overline{t}_{np}$ defines the technique of prediction of their durability and is a theoretical justification of a known S.N. Zhurkov experimental equation [6, 7].

The example of realization of the offered technique is shown in Fig.1. The figure shows a graphic interpretation of the solution of equation (5.a), in view of (6.a-в), for a 25 steel rod with physical and mathematical characteristics set according to [8], subjected to static one-axle stretching at various temperatures. In particular, at $\sigma = 125\, МПа$ and temperature $T = 40^\circ C$ its predicted limiting duration, i.e. an average limiting resource ("life time') in the given conditions, makes up $\overline{t}_{np} = 2,26\, мес$.

The offered approach, in our opinion, allows to predict durability of the various loaded parts on values of maximal pressure σ, temperatures T and physical and mechanical characteristics of material, and as well as to analyze possible ways of increase of their resource at the design and operational stages. For example, the nomograph shows that the durability of a rod can be essentially raised by reduction in its temperature, change of physical and mechanical characteristics, and also other parameters which are part of equations (5.a) – (6.a-в).

References

1. Mechanical engineering. Encyclopedia in forty volumes. Ed. by K.V.Frolov et al. M.: M38 Mechanical engineering. Reliability of equipment. T.IV-3/B.B. Klyuyev, V.V.Bolotin, etc.; 2003. – p.592.

2. **Pronikov A.S.** Parametrical reliability of equipment. – M.: Publishing house of MGTU N.E.Bauman, 2002. – p.560.

3. New encyclopedic dictionary.-M.: Bolshaya Russian encyclopedia: RIPOL classic, 2008.-p.1456.

4. **Bulyko A.N.** Modern dictionary of foreign words. – M.: "Martin", 2005. – p.848.

5. **Antsupov A.V., Antsupov A.V. (jr.), Antsupov V.P.** Methodology of reliability prediction of tribologic conjunctions // Friction and greasing in machines and mechanisms.-2012. – №2. – Pp..3-9.

6. **Zhurkov S.N., Nurzullaev B.N.** Time dependence of durability at various load modes // Journal of technical physics. 1953, Issue.10. – Pp..1677-1689.

7. **Regel V.R., Slutsker A.I., Tomashevsky E.E.** Kinetic nature of durability of solid objects. – "Science ", 1974. P.560.

8**. Feodorov V.V.** Kinetics of damageability and destructions of solid objects. – Tashkent: Publishing house "Fan" UzSSR, 1985. P.165.