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Generation of PNF functions in matrix form for cold standby systems with heterogeneous elements

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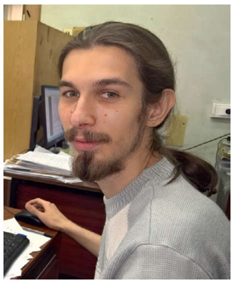

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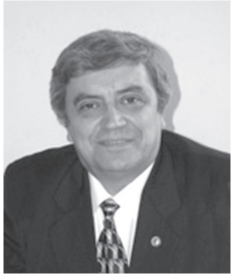

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The importance of considering the particular features of the facilities that ensure redundancy of functional units is demonstrated in the context of design for dependability. With the growth of the number of types and quantity of involved elements the process of dependability calculation becomes more complex and time-consuming. Therefore, in order to simplify the calculations, assumptions are made. For instance, in redundant systems heterogeneous elements are used. However, this approach does not allow evaluating the dependability of a system that features essentially different elements.
The paper considers systems that include a random number of essentially different elements with cold redundancy. As a possible solution to the above problem, a method was developed and mathematically justified that allows representing in matrix form an analytic expression for calculating the probability of no-failure. It is shown that in this case a numeric evaluation of dependability is possible using rough computation with integration and differentiation.
The degree of approximation of such calculations is proposed to be defined by both the accuracy of the computer itself and the complexity of the system under consideration. In the context of design for dependability, when the process of recalculation is performed repeatedly this drawback is critical. In order to reduce the time of dependability calculation of the system under consideration, as well as to increase the accuracy of the results, the paper suggests a method of analytical solution for PNF calculation. As a result, the design mechanism of cold standby systems can be simplified, while their dependability evaluation can be done more accurately.
Therefore, in order to calculate the PNF of systems with a random number of elements in general by means of the numerical method, it is proposed to perform the number of serial integrations of the product of the function and derivatives an entity less than the number of the system elements. Given the particular nature of computer calculation and algorithm recurrence, PNF calculation of a system of as much as 5 or more elements may take significant time, while cumulated calculation error is inevitable.
The practical details of the task related to ensuring spacecraft operational stability under environmental effects are characterized by the importance of the factor of prompt decision-making regarding the generation of control signal aimed at ensuring homoeostasis of the onboard systems performance. The paper mathematically substantiated a method of representing an analytic expression for PNF calculation for a system of any number of elements in cold standby. Such representation can be used for mapping data in computer memory. Under known matrix coefficients this representation will allow avoiding integration and differentiation in PNF calculation, which significantly reduces calculation time and increasing the accuracy of the results.

Keywords: *design for dependability, technical systems, essentially different system components, cold redundancy, analytical expression, matrix coefficients, computation speedup, dependability estimation accuracy improvement.*

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Introduction

Calculation of the probability of no-failure (PNF) is an integral stage of design for dependability. With the growth of the number of types and quantity of involved elements the process of dependability calculation becomes more complex and time-consuming.

In order to simplify the calculations, assumptions are made. For instance, in redundant systems heterogeneous elements are used. However, this approach does not allow evaluating the dependability of a system that features essentially different elements. (Such tasks arise when it is required to calculate the probability of faultless function performance, i.e. estimation of functional dependability [1]). In this case a numeric evaluation of dependability is possible using rough computation with integration and differentiation.

The degree of approximation of such calculations is defined by both the accuracy of the computer itself and the complexity of the system under consideration [2]. In the context of design for dependability, when the process of recalculation is performed repeatedly this drawback is critical.

In order to reduce the time of dependability calculation of the system under consideration, as well as to increase the accuracy of the results, the paper suggests a method of analytical solution for PNF calculation of cold standby systems, as those are some of the most dependable and most complexly calculated systems.

Method of numerical solution

The following recurrence formula is used to generally identify a cold standby system's PNF:

$$P_N(T)=P_{N-1}(T)+\int_0^T p_n(\tau,T)\cdot f_{N-1}(\tau)d\tau,$$

where $P_N(T)$ is the PNF of a system out of N elements over time T;

$P_{N-1}(T)$ is the PNF of a system out of (N-1) elements over time T;

$p_n(\tau,T)$ is the PNF of the n-th (initiated) element within the time period from τ to T;

$f_{N-1}(\tau)$ is the failure density distribution of the system out of (N-1) elements for the moment in time τ;

$$f_{N-1}(T)=-\frac{P_{N-1}(T)}{dT}.$$

Therefore, in order to generally numerically calculate the PNF of a system out of N elements it is required to perform (N–1) serial calculations of integrals of the function and derivatives $f_{N-1}(T)$. Given the particular nature of computer calculation and algorithm recurrence, PNF calculation of a system of as much as 5 or more elements may take significant time, while cumulated calculation error is inevitable.

Note: The formulas are used for practical calculation of dependability with the assumption that redundant elements do not lose dependability when switched off.

Analytical solutions

Let us consider a number of cases of cold redundancy in order to obtain analytical solutions and analyze the results.

A system out of 1 element

Let the element's failure rate be λ_1, then the system's PNF is:

$$P_1(T)=e^{-\lambda_1 T}.$$

The time function of PNF is as follows:

$$P_1(t)=e^{-\lambda_1 t}.$$

A system out of 2 elements

Two essentially different cases are possible.

The failure rate of the 2-nd initiated element is λ_1, then the system's PNF is:

$$P_2(T)=P_1(T)+\int_0^T p_2(\tau,T)\cdot f_1(\tau)d\tau,$$

$$p_2(\tau,T)=e^{-\lambda_1(T-\tau)},$$

$$f_1(\tau)=-\frac{P_1(\tau)}{d\tau}=-\frac{e^{-\lambda_1\tau}}{d\tau}=\lambda_1\cdot e^{-\lambda_1\tau},$$

$$P_2(T)=e^{-\lambda_1 T}+\int_0^T e^{-\lambda_1(T-\tau)}\cdot\lambda_1\cdot e^{-\lambda_1\tau}d\tau=$$

$$=e^{-\lambda_1 T}+\lambda_1\cdot e^{-\lambda_1 T}\cdot\int_0^T 1d\tau=e^{-\lambda_1 T}+\lambda_1\cdot e^{-\lambda_1 T}\cdot T=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}.$$

The failure rate of the 2-nd initiated element is λ_1, then the system's PNF is:

$$P_2(T)=P_1(T)+\int_0^T p_2(\tau,T)\cdot f_1(\tau)d\tau,$$

$$p_2(\tau,T)=e^{-\lambda_2(T-\tau)},$$

$$f_1(\tau)=-\frac{P_1(\tau)}{d\tau}=-\frac{e^{-\lambda_1\tau}}{d\tau}=\lambda_1\cdot e^{-\lambda_1\tau},$$

$$P_2(T)=e^{-\lambda_1 T}+\int_0^T e^{-\lambda_2(T-\tau)}\cdot\lambda_1\cdot e^{-\lambda_1\tau}d\tau=e^{-\lambda_1 T}+\lambda_1\cdot e^{-\lambda_2 T}\cdot$$

$$\cdot\int_0^T e^{(\lambda_2-\lambda_1)\tau}d\tau=e^{-\lambda_1 T}+\lambda_1\cdot e^{-\lambda_2 T}\cdot\frac{1}{\lambda_2-\lambda_1}\cdot\left(e^{(\lambda_2-\lambda_1)T}-1\right)=$$

$$=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}.$$

The time function of PNF is as follows:

in case of matching elements:

$$P_2(t)=(1+\lambda_1\cdot t)\cdot e^{-\lambda_1 t};$$

in case of non-matching elements:

$$P_2(t)=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 t}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 t}.$$

A system out of 3 elements

The case when two identical elements are connected with a similar third one.

The element's failure rate is λ_1, then the system's PNF is:

$$P_3(T)=P_2(T)+\int_0^T p_3(\tau,T)\cdot f_2(\tau)d\tau,$$

$$P_2(T)=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T},$$

$$p_3(\tau,T)=e^{-\lambda_1(T-\tau)},$$

$$f_2(\tau)=-\frac{P_2(\tau)}{d\tau}=-\frac{(1+\lambda_1\cdot\tau)\cdot e^{-\lambda_1\tau}}{d\tau}=\lambda_1^{\ 2}\cdot\tau\cdot e^{-\lambda_1\tau},$$

$$P_3(T)=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}+\int_0^T e^{-\lambda_1(T-\tau)}\cdot\lambda_1^{\ 2}\cdot\tau\cdot e^{-\lambda_1\tau}d\tau=$$
$$=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}+\lambda_1^{\ 2}\cdot e^{-\lambda_1 T}\cdot\int_0^T\tau d\tau=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}+$$
$$+\lambda_1^{\ 2}\cdot e^{-\lambda_1 T}\cdot\frac{T^2}{2}=\left(1+\lambda_1\cdot T+\frac{\lambda_1^{\ 2}}{2}\cdot T^2\right)\cdot e^{-\lambda_1 T}.$$

The case when two identical elements are connected with an element of another type.

The element's failure rate is λ_1, then the system's PNF is:

$$P_3(T)=P_2(T)+\int_0^T p_3(\tau,T)\cdot f_2(\tau)d\tau,$$
$$P_2(T)=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T},$$

$$p_3(\tau,T)=e^{-\lambda_2(T-\tau)},$$

$$f_2(\tau)=-\frac{P_2(\tau)}{d\tau}=-\frac{(1+\lambda_1\cdot\tau)\cdot e^{-\lambda_1\tau}}{d\tau}=\lambda_1^{\ 2}\cdot\tau\cdot e^{-\lambda_1\tau},$$

$$P_3(T)=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}+\int_\sigma^T e^{-\lambda_2(T-\tau)}\cdot\lambda_1^2\cdot\tau\cdot e^{-\lambda_1\tau}d\tau=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}+$$
$$+\lambda_1^{\ 2}\cdot e^{-\lambda_2 T}\cdot\int_0^T e^{(\lambda_2-\lambda_1)\tau}\cdot\tau d\tau=(1+\lambda_1\cdot T)\cdot e^{-\lambda_1 T}+\lambda_1^{\ 2}\cdot e^{-\lambda_2 T}\times$$
$$\times\left(\frac{1}{\lambda_2-\lambda_1}\cdot T\cdot e^{(\lambda_2-\lambda_1)T}-\frac{1}{(\lambda_2-\lambda_1)^2}\cdot e^{(\lambda_2-\lambda_1)T}+\frac{1}{(\lambda_2-\lambda_1)^2}\right)=$$
$$=\left(\frac{\lambda_2^{\ 2}-2\lambda_1\lambda_2}{(\lambda_2-\lambda_1)^2}+\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot T\right)e^{-\lambda_1 T}+\frac{\lambda_1^{\ 2}}{(\lambda_2-\lambda_1)^2}\cdot e^{-\lambda_2 T}.$$

The case when two non-identical elements are connected with a matching element.

The element's failure rate is λ_1, then the system's PNF is:

$$P_3(T)=P_2(T)+\int_0^T p_3(\tau,T)\cdot f_2(\tau)d\tau,$$

$$P_2(T)=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T},$$

$$p_3(\tau,T)=e^{-\lambda_1(T-\tau)},$$

$$f_2(\tau)=-\frac{P_2(\tau)}{d\tau}=-\frac{\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1\tau}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2\tau}}{d\tau}=$$
$$=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1\tau}+\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2\tau},$$

$$P_3(T)=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}+\int_0^T e^{-\lambda_1(T-\tau)}\cdot$$
$$\cdot\left(\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1\tau}+\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2\tau}\right)d\tau=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+$$
$$+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}+e^{-\lambda_1 T}\cdot\int_0^T\left(\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}+\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\cdot e^{(\lambda_1-\lambda_2)\tau}\right)d\tau=$$
$$=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}+e^{-\lambda_1 T}\times$$
$$\times\left(\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot T+\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)^2}\cdot e^{(\lambda_1-\lambda_2)T}-\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)^2}\right)=$$
$$=\left(\frac{\lambda_2^{\ 2}-2\lambda_1\lambda_2}{(\lambda_2-\lambda_1)^2}+\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot T\right)e^{-\lambda_1 T}+\frac{\lambda_1^{\ 2}}{(\lambda_2-\lambda_1)^2}\cdot e^{-\lambda_2 T}.$$

Note: The last two cases show that the formula of the PNF function does not depend on the order of element initiation in a cold standby system. It rather depends only on the type of the element.

The case when two non-identical elements are connected with a non-matching element.

The element's failure rate is λ_3, then the system's PNF is:

$$P_3(T)=P_2(T)+\int_0^T p_3(\tau,T)\cdot f_2(\tau)d\tau,$$

$$P_2(T)=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T},$$

$$p_3(\tau,T)=e^{-\lambda_3(T-\tau)},$$

$$f_2(\tau)=-\frac{P_2(\tau)}{d\tau}=-\frac{\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1\tau}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2\tau}}{d\tau}=$$
$$=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1\tau}+\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2\tau},$$

$$P_3(T)=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}+\int_0^T e^{-\lambda_3(T-\tau)}\cdot$$
$$\cdot\left(\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1\tau}+\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2\tau}\right)d\tau=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+$$
$$+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}+e^{-\lambda_3 T}\times\times\int_0^T\left(\begin{array}{l}\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot e^{(\lambda_3-\lambda_1)\tau}+\\+\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\cdot e^{(\lambda_3-\lambda_2)\tau}\end{array}\right)d\tau=$$
$$=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 T}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 T}+e^{-\lambda_3 T}\times$$
$$\times\left(\begin{array}{l}\frac{\lambda_1\lambda_2}{(\lambda_2-\lambda_1)\cdot(\lambda_3-\lambda_1)}\cdot e^{(\lambda_3-\lambda_1)T}-\frac{\lambda_1\lambda_2}{(\lambda_2-\lambda_1)\cdot(\lambda_3-\lambda_1)}+\\+\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)\cdot(\lambda_3-\lambda_2)}\cdot e^{(\lambda_3-\lambda_2)T}-\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)\cdot(\lambda_3-\lambda_2)}\end{array}\right)=$$
$$=\frac{\lambda_2\lambda_3}{(\lambda_2-\lambda_1)\cdot(\lambda_3-\lambda_1)}\cdot e^{-\lambda_1 T}+\frac{\lambda_1\lambda_3}{(\lambda_1-\lambda_2)\cdot(\lambda_3-\lambda_2)}\cdot e^{-\lambda_2 T}+$$
$$+\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_3)\cdot(\lambda_2-\lambda_3)}\cdot e^{-\lambda_3 T}.$$

Let us consider in detail the resulting PNF functions.
The time function of PNF is as follows:
- one element with the rate of λ_1

$$P_1(t)=e^{-\lambda_1 t};$$

- for two elements:
two elements with the rate of λ_1

$$P_2(t)=(1+\lambda_1\cdot t)\cdot e^{-\lambda_1 t};$$

one element with the rate of λ_1, the second one with the rate of λ_2

$$P_2(t)=\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot e^{-\lambda_1 t}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot e^{-\lambda_2 t};$$

- for three elements:
three elements with the rate of λ_1

$$P_3(t)=\left(1+\lambda_1\cdot t+\frac{\lambda_1^2}{2}\cdot t^2\right)\cdot e^{-\lambda_1 t};$$

two elements with the rate of λ_1, the third one with the rate of λ_2

$$P_3(t)=\left(\frac{\lambda_2^2-2\lambda_1\lambda_2}{(\lambda_2-\lambda_1)^2}+\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\cdot t\right)e^{-\lambda_1 t}+\frac{\lambda_1^2}{(\lambda_2-\lambda_1)^2}\cdot e^{-\lambda_2 t};$$

three elements with the rates of λ_1, λ_2, λ_3, therefore

$$P_3(t)=\frac{\lambda_2\lambda_3}{(\lambda_2-\lambda_1)\cdot(\lambda_3-\lambda_1)}\cdot e^{-\lambda_1 t}+\frac{\lambda_1\lambda_3}{(\lambda_1-\lambda_2)\cdot(\lambda_3-\lambda_2)}\cdot e^{-\lambda_2 t}+$$
$$+\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_3)\cdot(\lambda_2-\lambda_3)}\cdot e^{-\lambda_3 t}.$$

The dependency shows that if «repeating» elements are present, the degree of polynomial ascends under the corresponding exponential. When a «non-repeating» element is initiated, the degree of polynomial does not ascend, yet the coefficients in the formulas rearrange. In order to represent the formulas in computer memory, let us develop the method of their representation.

Algorithm of analytic solution representation

In order to produce the general algorithm let us consider a random system out of 4 elements with the failure rate of λ_1, λ_2, λ_3, λ_4. As it was shown above, in general, the formula for calculating the probability of no failure can be made as a series containing exponents of the numbers $-\lambda_1 t$, $-\lambda_2 t$, $-\lambda_3 t$, $-\lambda_4 t$.

$$P_4(t)=A(t)\cdot e^{-\lambda_1 t}+B(t)\cdot e^{-\lambda_2 t}+C(t)\cdot e^{-\lambda_3 t}+D(t)\cdot e^{-\lambda_4 t}$$

The functions $A(t)$, $B(t)$, $C(t)$, $D(t)$ that are polynomials in powers of t, of which the number of summands is defined by the number of respective identical system elements. I.e. if in a system there are 2 elements with the failure rate of λ_1, the polynomial $A(t)$ contains 2 summands:

$$A(t)=a_0+a_1 t$$

In general, the number of identical elements in a system is unknown. When elements are connected to the system, if a new type of element appears, a new exponential appears in the formula, while if the type of the initiated element matches the one of one of the already connected elements, the corresponding polynomial gets a new summand. For instance, for a system out of 4 elements, two «edge» cases are possible:

- 4 different elements. All the polynomials contain exactly one non-zero summand and have all types of exponentials;
- 4 identical types of elements. There is only one polynomial containing 4 elements and one type of exponential.

To describe such system, let us complement all the polynomials with zero coefficients so that they contain the maximum number of elements.

$$A(t)=a_0+a_1 t+a_2 t^2+a_3 t^3$$

$$B(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3$$

$$C(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$D(t) = d_0 + d_1 t + d_2 t^2 + d_3 t^3$$

For clarity, let us represent the coefficients in matrix form:

$$\begin{array}{c|cccc} & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ & - & - & - & - \\ t^0 & a_0 & b_0 & c_0 & d_0 \\ t^1 & a_1 & b_1 & c_1 & d_1 \\ t^2 & a_2 & b_2 & c_2 & d_2 \\ t^3 & a_3 & b_3 & c_3 & d_3 \end{array}$$

Let us represent the matrix of initial coefficients as A, the matrix of failure rates as Λ and the matrix of degrees as T

$$A = \begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix},$$

$$\Lambda = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{pmatrix},$$

$$T = \begin{pmatrix} t^0 \\ t^1 \\ t^2 \\ t^3 \end{pmatrix}.$$

Let us also introduce the auxiliary function $F(A,t)$ that transforms the matrix according to the following rule:

$$B = F(A,t), \text{ where } B_{ij} = e^{-t \cdot A_{ij}}.$$

Then the system's PNF that is characterized by the matrix A can always be defined using the formula:

$$P(t) = A^{\mathrm{T}} \cdot T \cdot F(\Lambda, t)^T.$$

Example. The introduced PNF functions $P(t)$ are characterized by the following matrices with the respective auxiliary matrices T and Λ:

$$P_1(t) = e^{-\lambda_1 t},$$

$$A = (1),$$

$$P_2(t) = (1 + \lambda_1 \cdot t) \cdot e^{-\lambda_1 t},$$

$$A = \begin{pmatrix} 1 & 0 \\ \lambda_1 & 0 \end{pmatrix},$$

$$P_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 - \lambda_2} \cdot e^{-\lambda_2 t},$$

$$A = \begin{pmatrix} \frac{\lambda_2}{\lambda_2 - \lambda_1} & \frac{\lambda_1}{\lambda_1 - \lambda_2} \\ 0 & 0 \end{pmatrix},$$

$$P_3(t) = \left(1 + \lambda_1 \cdot t + \frac{\lambda_1^{\ 2}}{2} \cdot t^2\right) \cdot e^{-\lambda_1 t},$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ \lambda_1 & 0 & 0 \\ \frac{\lambda_1^{\ 2}}{2} & 0 & 0 \end{pmatrix},$$

$$P_3(t) = \left(\frac{\lambda_2^{\ 2} - 2\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)^2} + \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} \cdot t \right) e^{-\lambda_1 t} + \frac{\lambda_1^{\ 2}}{(\lambda_2 - \lambda_1)^2} \cdot e^{-\lambda_2 t},$$

$$A = \begin{pmatrix} \frac{\lambda_2^{\ 2} - 2\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)^2} & \frac{\lambda_1^{\ 2}}{(\lambda_2 - \lambda_1)^2} & 0 \\ \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$P_3(t) = \frac{\lambda_2\lambda_3}{(\lambda_2 - \lambda_1) \cdot (\lambda_3 - \lambda_1)} \cdot e^{-\lambda_1 t} + \frac{\lambda_1\lambda_3}{(\lambda_1 - \lambda_2) \cdot (\lambda_3 - \lambda_2)} \cdot e^{-\lambda_2 t} + \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_3) \cdot (\lambda_2 - \lambda_3)} \cdot e^{-\lambda_3 t},$$

$$A = \begin{pmatrix} \frac{\lambda_2\lambda_3}{(\lambda_2 - \lambda_1) \cdot (\lambda_3 - \lambda_1)} & \frac{\lambda_1\lambda_3}{(\lambda_1 - \lambda_2) \cdot (\lambda_3 - \lambda_2)} & \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_3) \cdot (\lambda_2 - \lambda_3)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Conclusions

The paper has developed and mathematically substantiated a method of representing an analytic expression for PNF calculation for a system of any number of elements in cold standby. Such representation can be used for mapping data in computer memory. Under known matrix coefficients this representation will allow avoiding integration and differentiation in PNF calculation, which significantly reduces calculation time and increasing the accuracy of the results.

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