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COMPUNATIONAL AND EXPERIMENTAL METHOD FOR ESTIMATING RELIABILITY INDICATORS OF TECHNOLOGICAL COMPLEX BASED ON THE RESULTS OF ITS TESTING USING PRIOR INFORMATION ON RELIABILITY DERIVED FROM TESTING OF ITS COMPONENTS

The paper offers a computational and experimental method for estimating reliability indicators of complex chemical technological systems based on the results of tests using available prior information on reliability derived from testing of components. As a result, it has been found that when estimating and controlling the reliability of system using prior information, it allows us to reduce substantially the amount of its required testing.

Keywords: *estimation of reliability indicators, computational and experimental method for estimating the reliability of complex systems using prior information, results of tests.*

This computational and experimental method is used to estimate lower confidence bounds of FFP (fault-free probability) and mean time between failures of technological complex (system) based on the results of system reliability tests using prior information about the reliability of system derived from the results of independent tests of constituents (components). As prior information on the reliability of system taken into account, FFP estimator is used in the form of confidence interval $\left[P_{H0}(t_H),1\right]$ with the confidence probability γ_0, where $P_{H0}(t_H)$ is a prior estimator of the FFP lower confidence bound of system. Such an estimator can be obtained by means of a computational and experimental method for estimating reliability indicators of technological complex upon results of independent testing of its components.

However, in the literature dedicated to methods of applying prior information for estimating the reliability of complex systems [1-6], this very important in practical terms case when a FFP interval estimator is known due to results of preliminary in-situ tests, which takes place in case of statistical processing of limited experimental material, has been discussed only with the assumption that for a confident interval known a priori, the confidence is equal to one [2, 4].

We have developed a method of applying prior information in the form of a FFP interval estimator for any given value of confidence probability.

Let us consider a sequence of N independent tests of system, in which of them for testing duration t_H there are two outcomes possible: A and $\overline{A}$ (success and failure). The probabilities of the outcomes are equal to $P(t_H)$ and $Q(t_H)$ respectively, with $P(t_H)=1-Q(t_H)$. For each testing, the probability is $P(t_H)=P=const$. Tests using such a scheme are called trials of the Bernoulli model.

If N trials provide m failures, then the lower bound P_H of confident interval is found by the classical expression [7]:

$$1-\gamma=\sum_{k=0}^{m}\binom{N}{k}P^{N-k}(1-P)^k,$$

where $\binom{N}{k}=\frac{N!}{(N-k)!k!}$.

In case there are no failures during trials [8]:

$$P_H=\sqrt[N]{1-\gamma}.$$

Let $P\in[0,1]$ and it be known that upon results of preliminary independent reliability tests of system, a prior FFP estimator is obtained as:

$$\Pr\{P\in[P_{H0},1]\}=\gamma_0,$$

where P_{H0} is a prior lower bound P, and γ_0 is a confident probability. It is then obvious that

$$\Pr\{P\in[0,P_{H0}]\}=1-\gamma_0,$$

and prior probability density writes down as [9]:

$$\phi(P)=\begin{cases}\frac{1-\gamma_0}{P_{H0}} & \forall P\in[0,P_{H0}]\\ \frac{\gamma_0}{1-P_{H0}} & \forall P\in[P_{H0},1]\end{cases}. \quad (1)$$

A probability density function in the form (1) is chosen on the basis of using a distribution with the most variance for available prior information, which makes the computation justified.

Posterior density for trials made according to the Bernoulli model [5-6] can be written as follows [2]:

$$f(P)=\frac{\phi(P)\cdot P^{N-m}(1-P)^m}{\int_0^1 P^{N-m}(1-P)^m\cdot\phi(P)dP}. \quad (2)$$

Expression (2) with equation (1) taken into account yields:

$$f(P)=\begin{cases}(1-\gamma_0)\cdot P^{N-m}(1-P)^m(P_{H0}C)^{-1}, & \forall P\in[0,P_{H0}]\\ \gamma_0 P^{N-m}(1-P)^m(P_{H0}C)^{-1}, & \forall P\in[P_{H0},1]\end{cases}$$

where:

$$C=\frac{1-\gamma_0}{P_{H0}}I_1+\frac{\gamma_0}{1-P_{H0}}I_2;\ I_1=\int_0^{P_{H0}}P^{N-m}(1-P)^m\,dP;\ I_2=\int_{P_{H0}}^{1}P^{N-m}(1-P)^m\,dP.$$

For the convenience of computation of integrals I_1 and I_2, let us consider their representation in the general form, and application of Newton's binominal series, after integration, yields:

$$\int_a^b P^{N-m}(1-P)^m\,dP=\sum_{i=0}^{m}\frac{(-1)^i\binom{m}{i}}{N-m+i+1}\left(b^{N-m+i+1}-a^{N-m+i+1}\right) \tag{3}$$

If in expression (3) $a=0$ and $b=1$, we have a beta function [10]:

$$B(N-m+1,m+1)=\frac{\Gamma(N-m+1)\Gamma(m+1)}{\Gamma(N+2)}\quad, \tag{4}$$

where $\Gamma(\cdot)$ is Gamma function defined in case of positive integers ϑ by equation $\Gamma(\vartheta)=(\vartheta-1)$. Considering that, expression (4) looks like:

$$B(N-m+1,m+1)=\frac{(N-m)!m!}{(N+1)!}=\frac{1}{N(+1)\binom{N}{m}}.$$

Using formula (3), we can write the following expression for C:

$$C=\frac{\gamma_0}{1-P_{H0}}\left[\frac{1}{(N+1)\binom{N}{m}}-\left(1-\frac{1-P_{H0}}{\gamma_0}\right)P_{H0}^{N-m}\sum_{i=0}^{m}\frac{(-1)^i\binom{m}{i}P_{H0}^i}{N-m+i+1}\right].$$

The estimator of the lower confidence probability bound P_{H1} of system fault-free operation, with prior information taken into account, yields from the condition:

$$\gamma_1=\int_{P_{H1}}^{1}f(P)dP, \tag{5}$$

where γ_1 is a posterior confident probability.

After substituting the corresponding value $f(P)$ for $P_{H1} \geq P_{H0}$ into equation (5) and integrating, there is:

$$\gamma_1 = \frac{1-(N+1)\binom{N}{m}P_{H1}^{N-m+1}\sum_{i=0}^{m}\frac{(-1)^i\binom{m}{i}P_{H1}^i}{N-m+i+1}}{1-(N+1)\binom{N}{m}\left(1-\frac{1-P_{H0}}{\gamma_0}\right)P_{H0}^{N-m}\sum_{i=0}^{m}\frac{(-1)\binom{m}{i}P_{H0}^i}{N-m+i+1}} . \quad (6)$$

Using the function of binominal distribution B_i [5], equation (6) can be represented in a way more convenient for practical application:

$$B_i\left(N+1, P_{H1}, m\right) = 1-\gamma_1', \quad (7)$$

where $\gamma_1' = \gamma_1\left[1-\frac{1}{P_{H0}}\left(1-\frac{1-P_{H0}}{\gamma_0}\right)B_i\left(N+1, P_{H0}, m\right)\right]$.

The root of equation (7) $P_{H1} = f_1\left(N+1, m, \gamma_1'\right)$ representing the value of the lower confidence interval bound for the parameter P with prior information taken into account is tabulated in [5], which permits to quite easily obtain an interval estimator of system reliability considering the current experimental data (N, m), where N is a total number of trials (functioning phases), m is a number of trials wherein a failure is registered, and prior information in the form of some confident level $\left[P_{H0}, 1\right]$ with confident probability γ_0.

In practice, as regards such technological complexes as, for example, facilities for destroying chemical weapon, equipment is used in single copies which as such represent a general population of products. In this case we speak about combining homogenous information on reliability (the same components in independent and integrated forms), so we may not carry out verification of statistical compatibility of prior and current information upon results of trials.

In case of using information on reliability of components and analogous systems, verification of statistical compatibility of prior and current information upon results of trials is carried out based on the theory of verification of statistical hypotheses [3]. An initial hypothesis when applying this method is the hypothesis H_0 that consists in that $P_0 = P_T = P$ where P_0, P_T, P is a prior current and true value of system FFP respectively. If the hypothesis H_0 is rejected, then one of the two competing hypotheses $H_1\left\{P_0 < P_T\right\}$ or $H_2\left\{P_0 > P_T\right\}$ is accepted.

Possible methods for solving problems of such kind are described in [11]. Let us briefly consider the solution of the given problem taking into account the peculiarities of available data. A set of all possible outcomes defined by a number of fault-free trials with N trials consists of discrete points, $R = \left\{0, 1, 2, ..., N\right\}$. Divide this set by points r_H and r_B into three subsets defining respectively: the area of lower critical values $R_H = \left\{0, 1, 2, ..., r_H\right\}$; the area of acceptable values $R_Д = \left\{r_H+1, r_H+2, ..., r_{B-1}\right\}$; the area of upper critical values $R_B = \left\{r_B, r_{B+1}, ..., N\right\}$.

Let us set a significance level $\alpha = \alpha_H + \alpha_B$, from which we could define bounds of the area of acceptable values for outcomes of trials with the hypothesis H_0.

By definition, the lower critical area will be chosen in compliance with a given significance level, if the condition is fulfilled

$$P\left(r \le r_H / H_0\right) \le \alpha_H . \tag{8}$$

In relation to the upper critical area, we can also set inequality

$$P\left(r \ge r_B / H_0\right) \le \alpha_B . \tag{9}$$

These inequalities with the hypothesis H_0 being true guarantee that outcomes of trials will fall into one of the critical areas with a probability not more than α.

In case of independent testing outcomes, relations (8) and (9) can be defined on the basis of the binominal law of distribution in the form:

$$\sum_{i=0}^{r_H} C_N^i P_T^i \left(1 - P_T\right)^{N-i} \le \alpha_H \tag{10}$$

$$\sum_{i=r_B}^{N} C_N^i P_T^i \left(1 - P_T\right)^{N-i} \le \alpha_B \tag{11}$$

Comparing relations (10) and (11) with the corresponding relations for computing confidence intervals, we come to conclusion that a solution of the given problem can be based on corresponding tables for defining confidence intervals [5]. In this case entries into tables should be P_T and N which with a given value α provide selection between r_H and r_B. Based on these values, we can define the limits $P_H = r_H / N$ and $P_B = r_B / N$, wherein a prior reliability value should be enclosed in order that for a chosen α we can assert convergence of prior and current trial data. A significance level of α can then be chosen within 0.2 – 0.05, with $\alpha_H = \alpha_B = \alpha / 2$.

Of practical interest is the application of formula (7) for the case when m =0, that is

$$\gamma_1 = \frac{1 - P_{H1}^{N+1}}{1 - P_{H0}^{N}\left(1 - \dfrac{1 - P_{H0}}{\gamma_0}\right)},$$

whence

$$P_{H1} = \sqrt[N+1]{1 - \gamma_1\left[1 - P_{H0}^{N}\left(1 - \frac{1 - P_{H0}}{\gamma_0}\right)\right]} . \tag{12}$$

As seen from expression (12), the estimator of the FFP lower confidence bound P_{H1} obtained in case of having N trials will be higher than P_{H0}, i.e. the accuracy of interval reliability estimation is higher.

Increasing the accuracy of estimation in case of a limited amount of trials is equivalent to reducing an amount of trials with a given accuracy.

Using equation (12), we can find the amount of trials N required to verify specified requirements for reliability of system for the case m =0 provided that a prior value of FFP confidence interval obtained with confidence $\gamma_0 = 0$ at the stage of preliminary testing is equal to $\left[P_{H0},1\right]$.

Therefore, expression (12) can be used for estimating the reliability of a complex system upon results of its testing (for m =0) with prior information taken into account as well as for planning of reliability testing of a complex system with prior information taken into account.

Let us consider the developed method using some example. Let upon results of preliminary testing of system components there be an estimator of the lower confidence interval bound of prior FFP for the time t_H =2h $P_{HO}(t_H)$ =0.87 with the prior confidence probability γ_0 =0.8. It is required to define a minimum amount of trials in the form of N cycles of trials for t_H =2h to confirm the given value of the FFP lower bound of system $P_{H1}(2) = 0,912$ with confidence probability γ_1 =0.8, with prior information taken into account. Since we speak about a minimum amount of trials, planning of testing is made for the case m =0, using formula (12). Substituting initial data into expression (12) and treating it as inequality, we have

$$0,912 \le \sqrt[N+1]{1-0,8\left[1-0,87^N\left(1-\frac{1-0,87}{0,8}\right)\right]},$$

whence we obtain N =10 trials.

Note that a minimum number of trials of system to confirm a reliability level specified in the example without a prior information taken into account, according to [12], looks like N =17 fault-free trials.

As we can see, application of prior information for estimating and controlling the reliability of system makes it possible to considerably reduce an amount of its current trials.

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