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The mechanism of constructing an analytical solution for calculating the probability of no-failure of a cold standby system with heterogeneous elements

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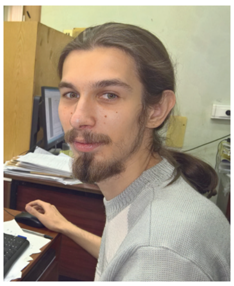

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Abstract. *The task related to the calculation of the probability of no-failure (PNF) of spacecraft onboard equipment is due to the fact that with the growth of the number of types and quantity of involved elements the process of dependability calculation becomes more complex and time-consuming. In the context of design for dependability, when the process of recalculation is performed repeatedly, this drawback is critical. In order to simplify the calculations, assumptions are made. For instance, in redundant systems heterogeneous elements are used. This approach does not allow evaluating the dependability of a system that features essentially different elements. In order to reduce the time of dependability calculation of the system under consideration, as well as to increase the accuracy of the results, the paper suggests a method of analytical solution for PNF calculation. It is suggested to use the system dependability time dependence function as the main dependability indicator, while for individual elements the respective failure rate is proposed. The authors look at the problem of consideration of the complexity of such function's construction for the cases of functional dependability calculation, when the elements of the system under consideration may not be homogeneous. For a system that includes any number of essentially different elements with cold redundancy, a method was developed and mathematically justified that allows representing in matrix form an analytic expression for calculation of probability of no-failure (PNF). The importance of considering the performance of the facilities that ensure redundancy of functional units is demonstrated in the context of design for dependability of spacecraft. A special attention is given to systems that include a random number of essentially different elements with cold redundancy. As one of the ways of solving the above problem, the paper shows that in this case a numeric evaluation of dependability is possible using rough computation with integration and differentiation. It is proposed to evaluate the degree of approximation of such calculations as both the accuracy of the computer itself and the complexity of the system under consideration. For that purpose, serial representation of the function of probability of no-failure is used for a system after the initiation of each next element under redundancy. The resulting function is formed by grouping of summands in particular order. The potential of replacing the differentiation and integration operations is shown. Under known matrix coefficients the application of the suggested algorithm will significantly improve the accuracy and speed of PNF computation. The practical details of the task related to ensuring spacecraft operational stability under environmental effects are characterized by the importance of the factor of prompt decision-making regarding the generation of control signal aimed at ensuring homoeostasis of the onboard systems performance. The analytic expression for calculation of PNF of a system comprised of a random number of elements can be used for mapping data in computer memory as part of decision support.*

Keywords: *design for dependability, technical systems, essentially different system components, cold redundancy, probability of no-failure, analytical expression, computation speedup, dependability estimation accuracy improvement, computer memory.*

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Introduction

Calculation of the probability of no-failure (PNF) is an integral stage of design for dependability. With the growth of the number of types and quantity of involved elements the process of dependability calculation becomes more complex and time-consuming.

In order to simplify the calculations, assumptions are made. For instance, in redundant systems heterogeneous elements are used. However this approach does not allow evaluating the dependability of a system that features essentially different elements. (Such tasks arise when it is required to calculate the probability of faultless function performance, i.e. estimation of functional dependability [1]). In this case a numeric evaluation of dependability is possible using rough computation with integration and differentiation.

The degree of approximation of such calculations is defined by both the accuracy of the computer itself and the complexity of the system under consideration [2]. In the context of design for dependability when the process of recalculation is performed repeatedly this drawback is critical.

In [4] it was shown that analytic solutions could be represented in matrix form, which is very convenient in terms of computer memory placement. In order to fully automate the PNF calculation it is required to develop a mechanism for matrix coefficient definition. For that purpose it is required to consider their changes under differentiation and integration according to the main calculation formulas.

Change of coefficients under differentiation

Let us examine a random system out of four elements.

$$P_4(t)=A(t)\cdot e^{-\lambda_1 t}+B(t)\cdot e^{-\lambda_2 t}+C(t)\cdot e^{-\lambda_3 t}+D(t)\cdot e^{-\lambda_4 t}.$$

The functions $A(t)$, $B(t)$, $C(t)$, $D(t)$ are polynomials in powers of t, of which the number of summands is defined by the number of respective identical system elements.

Let us represent the matrix of initial coefficients as A, the matrix of failure rates as Λ and the matrix of degrees as T.

$$A=\begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix};$$

$$\Lambda=\begin{pmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4\end{pmatrix};$$

$$T=\begin{pmatrix} t^0 \\ t^1 \\ t^2 \\ t^3 \end{pmatrix}.$$

Let us also introduce the auxiliary function $F(A,t)$ that transforms the matrix according to the following rule:

$$B=F(A,t),\ \text{where}\ B_{ij}=e^{-t\cdot A_{ij}}.$$

Then the system's PNF that is characterized by the matrix A can always be defined using the formula:

$$P(t)=A^{\mathrm{T}}\cdot T\cdot F(\Lambda,t)^{T}.$$

In that case, the PNF calculation formula – if the fifth element is initiated – will be as follows:

$$P_5(T)=P_4(T)+\int_0^T p_5(\tau,T)\cdot f_4(\tau)d\tau,$$

$P_4(T)$ is the PNF of a system out of four elements over time T;

$p_5(\tau,T)$ is the PNF of the fifth (initiated) element within the time period from τ to T;

$f_4(\tau)$ is the failure density distribution of the system out of four elements for the moment in time τ.

In the calculation formula the distribution density must be defined.

$$f_4(t)=-\frac{P_4(t)}{dt}=$$

$$=-\left(A(t)\cdot e^{-\lambda_1 t}+B(t)\cdot e^{-\lambda_2 t}+C(t)\cdot e^{-\lambda_3 t}+D(t)\cdot e^{-\lambda_4 t}\right)\frac{1}{dt}=$$

$$=\left(-A(t)\cdot e^{-\lambda_1 t}\right)\frac{1}{dt}+\left(-B(t)\cdot e^{-\lambda_2 t}\right)\frac{1}{dt}+$$

$$+\left(-C(t)\cdot e^{-\lambda_3 t}\right)\frac{1}{dt}+\left(-D(t)\cdot e^{-\lambda_4 t}\right)\frac{1}{dt}.$$

Let us consider the summand individually.

$$\left(-A(t)\cdot e^{-\lambda_1 t}\right)\frac{1}{dt}=\left(-\left(a_0+a_1t+a_2t^2+a_3t^3\right)\cdot e^{-\lambda_1 t}\right)\frac{1}{dt}=$$

$$=\left(\left(\lambda_1a_0-a_1\right)+\left(\lambda_1a_1-2a_2\right)t+\left(\lambda_1a_2-3a_3\right)t^2+\left(\lambda_1a_3\right)t^3\right)\cdot e^{-\lambda_1 t}.$$

The other summands are differentiated similarly.

As previously, we will present the result of differentiation as a matrix.

	λ_1	λ_2	λ_3	λ_4
t^0	$\lambda_1a_0-a_1$	$\lambda_2b_0-b_1$	$\lambda_3c_0-c_1$	$\lambda_4d_0-d_1$
t^1	$\lambda_1a_1-2a_2$	$\lambda_2b_1-2b_2$	$\lambda_3c_1-2c_2$	$\lambda_4d_1-2d_2$
t^2	$\lambda_1a_2-3a_3$	$\lambda_2b_2-3b_3$	$\lambda_3c_2-3c_3$	$\lambda_4d_2-3d_3$
t^3	λ_1a_3	λ_2b_3	λ_3c_3	λ_4d_3

Let us represent the distribution densities of probability B as a coefficient matrix.

$$B=\begin{pmatrix} \lambda_1a_0-a_1 & \lambda_2b_0-b_1 & \lambda_3c_0-c_1 & \lambda_4d_0-d_1 \\ \lambda_1a_1-2a_2 & \lambda_2b_1-2b_2 & \lambda_3c_1-2c_2 & \lambda_4d_1-2d_2 \\ \lambda_1a_2-3a_3 & \lambda_2b_2-3b_3 & \lambda_3c_2-3c_3 & \lambda_4d_2-3d_3 \\ \lambda_1a_3 & \lambda_2b_3 & \lambda_3c_3 & \lambda_4d_3 \end{pmatrix}.$$

As the calculations will be performed using a computer, a common formula must be defined that can be used to calculate any element of the matrix B depending on its line and column index. For that purpose, let us complement matrix A with an extra line of zeroes.

$$A=\begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix} => \begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{pmatrix}.$$

Then, the calculation of matrix B will be transformed.

$$B=\begin{pmatrix} \lambda_1 a_0 - a_1 & \lambda_2 b_0 - b_1 & \lambda_3 c_0 - c_1 & \lambda_4 d_0 - d_1 \\ \lambda_1 a_1 - 2a_2 & \lambda_2 b_1 - 2b_2 & \lambda_3 c_1 - 2c_2 & \lambda_4 d_1 - 2d_2 \\ \lambda_1 a_2 - 3a_3 & \lambda_2 b_2 - 3b_3 & \lambda_3 c_2 - 3c_3 & \lambda_4 d_2 - 3d_3 \\ \lambda_1 a_3 - 4\cdot 0 & \lambda_2 b_3 - 4\cdot 0 & \lambda_3 c_3 - 4\cdot 0 & \lambda_4 d_3 - 4\cdot 0 \end{pmatrix} =$$

$$=\begin{pmatrix} \lambda_1 a_0 - 1a_1 & \lambda_2 b_0 - 1b_1 & \lambda_3 c_0 - 1c_1 & \lambda_4 d_0 - 1d_1 \\ \lambda_1 a_1 - 2a_2 & \lambda_2 b_1 - 2b_2 & \lambda_3 c_1 - 2c_2 & \lambda_4 d_1 - 2d_2 \\ \lambda_1 a_2 - 3a_3 & \lambda_2 b_2 - 3b_3 & \lambda_3 c_2 - 3c_3 & \lambda_4 d_2 - 3d_3 \\ \lambda_1 a_3 - 4a_4 & \lambda_2 b_3 - 4b_4 & \lambda_3 c_3 - 4c_4 & \lambda_4 d_3 - 4d_4 \end{pmatrix};$$

$$B=\begin{pmatrix} a_0' & b_0' & c_0' & d_0' \\ a_1' & b_1' & c_1' & d_1' \\ a_2' & b_2' & c_2' & d_2' \\ a_3' & b_3' & c_3' & d_3' \end{pmatrix}.$$

Note. A similar result is obtained if coefficients a_4, b_4, c_4, d_4 are present in the respective polynomials $A(t)$, $B(t)$, $C(t)$, $D(t)$.

Thus, the differentiation process takes the following form:

$$f_4(t) = -\frac{P_4(t)}{dt} = -\frac{A^{\mathrm{T}}\cdot T\cdot F(\Lambda,t)^T}{dt} = B^{\mathrm{T}}\cdot T\cdot F(\Lambda,t)^T;$$

$$A=\begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{pmatrix} => B=\begin{pmatrix} a_0' & b_0' & c_0' & d_0' \\ a_1' & b_1' & c_1' & d_1' \\ a_2' & b_2' & c_2' & d_2' \\ a_3' & b_3' & c_3' & d_3' \end{pmatrix}.$$

The differentiation is substituted with the calculation of the coefficients of matrix B out of matrix A and matrix elements, which is a much simpler operation from the machine computation point of view. The common formula for calculation of the elements of matrix B is:

$$B_{i,j} = \Lambda_j \cdot A_{i,j} - i\cdot A_{i+1,j},\ i=1..N,\ j=1..N,$$

i is the line index in the respective matrix;
j is the column index in the respective matrix;
N is the number of elements in the initial system.

Change of coefficients under integration

The next step of calculation is the integration:

$$\int_0^t p_5(\tau,t)\cdot f_4(\tau)d\tau =$$

$$= \int_0^t e^{-\lambda_5(t-\tau)} \cdot \begin{pmatrix} A'(\tau)\cdot e^{-\lambda_1\tau} + B'(\tau)\cdot e^{-\lambda_2\tau} + \\ +C'(\tau)\cdot e^{-\lambda_3\tau} + D'(\tau)\cdot e^{-\lambda_4\tau} \end{pmatrix} d\tau =$$

$$= e^{-\lambda_5 t}\cdot\int_0^t A'(\tau)\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau + e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau +$$

$$+ e^{-\lambda_5 t}\cdot\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau + e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau.$$

Two distinctly different cases are possible: λ_5 can either numerically match one of the four failure rates, or not match it. Depending on that, the expression under the integral sign significantly transforms.

Let us consider the case when the failure rates of the initiated element matches the failure rate of one of the elements of the system under consideration, e.g. the first one.

$$\lambda_5 = \lambda_1$$

Then the respective integral will be simplified.

$$e^{-\lambda_5 t}\cdot\int_0^t A'(\tau)\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau = e^{-\lambda_1 t}\cdot\int_0^t A'(\tau)d\tau =$$

$$= e^{-\lambda_1 t}\cdot\int_0^t a_0' + a_1'\tau + a_2'\tau^2 + a_3'\tau^3 d\tau =$$

$$= \left(a_0't + \frac{a_1'}{2}t^2 + \frac{a_2'}{3}t^3 + \frac{a_3'}{4}t^4\right)\cdot e^{-\lambda_1 t}.$$

Further, let us examine the remaining integrals.

The PNF of a system out of 5 elements is the combination of the sum of the PNFs of the system out of 4 elements and the result of integration. Let us write how the coefficients will change in the case of matching for one type of element types (λ_1):

$$P_5(t) = P_4(t) + \int_0^t p_5(\tau,t)\cdot f_4(\tau)d\tau =$$

$$= A(t)\cdot e^{-\lambda_1 t} + e^{-\lambda_5 t}\cdot\int_0^t A'(\tau)\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau +$$

$$+ B(t)\cdot e^{-\lambda_2 t} + e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau +$$

$$+ C(t)\cdot e^{-\lambda_3 t} + e^{-\lambda_5 t}\cdot\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau +$$

$$+ D(t)\cdot e^{-\lambda_4 t} + e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau =$$

$$= \left(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4\right)\cdot e^{-\lambda_1 t} +$$

$$+ \left(a_0't + \frac{a_1'}{2}t^2 + \frac{a_2'}{3}t^3 + \frac{a_3'}{4}t^4\right)\cdot e^{-\lambda_1 t} +$$

$$+B(t)\cdot e^{-\lambda_2 t}+e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau+$$

$$+C(t)\cdot e^{-\lambda_3 t}+e^{-\lambda_5 t}\cdot\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau+$$

$$+D(t)\cdot e^{-\lambda_4 t}+e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau=$$

$$=\begin{pmatrix} a_0+(a_1+a_0')t+\left(a_2+\dfrac{a_1'}{2}\right)t^2+ \\ +\left(a_3+\dfrac{a_2'}{3}\right)t^3+\left(a_4+\dfrac{a_3'}{4}\right)t^4 \end{pmatrix}\cdot e^{-\lambda_1 t}+$$

$$+B(t)\cdot e^{-\lambda_2 t}+e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau+$$

$$+C(t)\cdot e^{-\lambda_3 t}+e^{-\lambda_5 t}\cdot\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau+$$

$$+D(t)\cdot e^{-\lambda_4 t}+e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau.$$

I.e. the initial matrix A transforms in such a way that some matrix elements are combined with some summands.

$$A=\begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{pmatrix} \Rightarrow A=\begin{pmatrix} a_0 & b_0 & c_0 & d_0 \\ a_1+a_0' & b_1 & c_1 & d_1 \\ a_2+\dfrac{a_1'}{2} & b_2 & c_2 & d_2 \\ a_3+\dfrac{a_2'}{3} & b_3 & c_3 & d_3 \\ a_4+\dfrac{a_3'}{4} & b_4 & c_4 & d_4 \end{pmatrix}.$$

The common formula of coefficients calculation in the matching part is as follows:

$$A_{i+1,j}=A_{i+1,j}+\frac{B_{i,j}}{i},\ \text{if}\ \Lambda_j=\lambda_k,\ i=1..N,\ j=1..N,$$

i is the line index in the respective matrix;
j is the column index in the respective matrix;
k is the index of the initiated element of matrix Λ;
N is the number of elements in the initial system.

Let us consider the case when the failure rate of the initiated element does not match any of the failure rates of the system's elements.

$$\lambda_5\neq\lambda_1,\ \lambda_5\neq\lambda_2,\ \lambda_5\neq\lambda_3,\ \lambda_5\neq\lambda_4.$$

Then, the integration will become more complex:

$$e^{-\lambda_5 t}\cdot\int_0^t A'(\tau)\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=e^{-\lambda_5 t}\cdot\int_0^t a_0'\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau+$$

$$+e^{-\lambda_5 t}\cdot\int_0^t a_1'\tau\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau+$$

$$+e^{-\lambda_5 t}\cdot\int_0^t a_2'\tau^2\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau+e^{-\lambda_5 t}\cdot\int_0^t a_3'\tau^3\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau.$$

Let us take each integral in sum individually:

■ $$e^{-\lambda_5 t}\cdot\int_0^t a_0'\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=a_0'\cdot e^{-\lambda_5 t}\cdot\int_0^t e^{(\lambda_5-\lambda_1)\tau}d\tau=$$

$$=a_0'\cdot e^{-\lambda_5 t}\cdot\left(\frac{1}{\lambda_5-\lambda_1}\cdot e^{(\lambda_5-\lambda_1)t}-\frac{1}{\lambda_5-\lambda_1}\right)=$$

$$=\frac{a_0'}{\lambda_5-\lambda_1}\cdot e^{-\lambda_1 t}-\frac{a_0'}{\lambda_5-\lambda_1}\cdot e^{-\lambda_5 t};$$

■ $$e^{-\lambda_5 t}\cdot\int_0^t a_1'\tau\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=a_1'\cdot e^{-\lambda_5 t}\cdot\int_0^t \tau\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=$$

$$=a_1'\cdot e^{-\lambda_5 t}\cdot\left(\frac{1}{\lambda_5-\lambda_1}\cdot t\cdot e^{(\lambda_5-\lambda_1)t}-\frac{1}{(\lambda_5-\lambda_1)^2}\cdot e^{(\lambda_5-\lambda_1)t}+\frac{1}{(\lambda_5-\lambda_1)^2}\right)=$$

$$=\frac{a_1'}{\lambda_5-\lambda_1}\cdot t\cdot e^{-\lambda_1 t}-\frac{a_1'}{(\lambda_5-\lambda_1)^2}\cdot e^{-\lambda_1 t}+\frac{a_1'}{(\lambda_5-\lambda_1)^2}\cdot e^{-\lambda_5 t};$$

■ $$e^{-\lambda_5 t}\cdot\int_0^t a_2'\tau^2\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=a_2'\cdot e^{-\lambda_5 t}\cdot\int_0^t \tau^2\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=$$

$$=a_2'\cdot e^{-\lambda_5 t}\cdot\begin{pmatrix}\dfrac{1}{\lambda_5-\lambda_1}\cdot t^2\cdot e^{(\lambda_5-\lambda_1)t}-\dfrac{2}{(\lambda_5-\lambda_1)^2}\cdot t\cdot e^{(\lambda_5-\lambda_1)t}+ \\ +\dfrac{2}{(\lambda_5-\lambda_1)^3}\cdot e^{(\lambda_5-\lambda_1)t}-\dfrac{2}{(\lambda_5-\lambda_1)^3}\end{pmatrix}=$$

$$=\frac{a_2'}{\lambda_5-\lambda_1}\cdot t^2\cdot e^{-\lambda_1 t}-\frac{2a_2'}{(\lambda_5-\lambda_1)^2}\cdot t\cdot e^{-\lambda_1 t}+$$

$$+\frac{2a_2'}{(\lambda_5-\lambda_1)^3}\cdot e^{-\lambda_1 t}-\frac{2a_2'}{(\lambda_5-\lambda_1)^3}\cdot e^{-\lambda_5 t};$$

■ $$e^{-\lambda_5 t}\cdot\int_0^t a_3'\tau^3\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=a_3'\cdot e^{-\lambda_5 t}\cdot\int_0^t \tau^3\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=$$

$$=a_3'\cdot e^{-\lambda_5 t}\cdot\begin{pmatrix}\dfrac{1}{\lambda_5-\lambda_1}\cdot t^3\cdot e^{(\lambda_5-\lambda_1)t}-\dfrac{3}{(\lambda_5-\lambda_1)^2}\cdot t^2\cdot e^{(\lambda_5-\lambda_1)t}+ \\ +\dfrac{6}{(\lambda_5-\lambda_1)^3}\cdot t\cdot e^{(\lambda_5-\lambda_1)t}-\dfrac{6}{(\lambda_5-\lambda_1)^4}\cdot e^{(\lambda_5-\lambda_1)t}+\dfrac{6}{(\lambda_5-\lambda_1)^4}\end{pmatrix}=$$

$$=\frac{a_3'}{\lambda_5-\lambda_1}\cdot t^3\cdot e^{-\lambda_1 t}-\frac{3a_3'}{(\lambda_5-\lambda_1)^2}\cdot t^2\cdot e^{-\lambda_1 t}+\frac{6a_3'}{(\lambda_5-\lambda_1)^3}\cdot t\cdot e^{-\lambda_1 t}-$$

$$-\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\cdot e^{-\lambda_1 t}+\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\cdot e^{-\lambda_5 t}.$$

Let us sum up the results of integration by grouping the summands:

$$e^{-\lambda_5 t}\cdot\int_0^t A'(\tau)\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau=$$

$$=\begin{pmatrix}\left(\frac{a_0'}{\lambda_5-\lambda_1}-\frac{a_1'}{(\lambda_5-\lambda_1)^2}+\frac{2a_2'}{(\lambda_5-\lambda_1)^3}-\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)+\\+\left(\frac{a_1'}{\lambda_5-\lambda_1}-\frac{2a_2'}{(\lambda_5-\lambda_1)^2}+\frac{6a_3'}{(\lambda_5-\lambda_1)^3}\right)\cdot t+\\+\left(\frac{a_2'}{\lambda_5-\lambda_1}-\frac{3a_3'}{(\lambda_5-\lambda_1)^2}\right)\cdot t^2+\frac{a_3'}{\lambda_5-\lambda_1}\cdot t^3\end{pmatrix}\cdot e^{-\lambda_1 t}+$$

$$+\left(-\frac{a_0'}{\lambda_5-\lambda_1}+\frac{a_1'}{(\lambda_5-\lambda_1)^2}-\frac{2a_2'}{(\lambda_5-\lambda_1)^3}+\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)\cdot e^{-\lambda_5 t}.$$

Integrals with other polynomials are calculated in a similar way.

The PNF of a system out of 5 elements is the combination of the sum of the PNFs of the system out of 4 elements and the result of integration. Let us write how the coefficients will change in the case of no-matching for one of the element types (λ_1):

$$P_5(t)=P_4(t)+\int_0^t p_5(\tau,t)\cdot f_4(\tau)d\tau=$$

$$=A(t)\cdot e^{-\lambda_1 t}+e^{-\lambda_5 t}\cdot\int_0^t A'(\tau)\cdot e^{(\lambda_5-\lambda_1)\tau}d\tau+B(t)\cdot e^{-\lambda_2 t}+$$

$$+e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau+C(t)\cdot e^{-\lambda_3 t}+e^{-\lambda_5 t}\cdot$$

$$\cdot\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau+D(t)\cdot e^{-\lambda_4 t}+e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau=$$

$$=\left(a_0+a_1t+a_2t^2+a_3t^3\right)e^{-\lambda_1 t}+$$

$$+\begin{pmatrix}\left(\frac{a_0'}{\lambda_5-\lambda_1}-\frac{a_1'}{(\lambda_5-\lambda_1)^2}+\frac{2a_2'}{(\lambda_5-\lambda_1)^3}-\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)+\\+\left(\frac{a_1'}{\lambda_5-\lambda_1}-\frac{2a_2'}{(\lambda_5-\lambda_1)^2}+\frac{6a_3'}{(\lambda_5-\lambda_1)^3}\right)\cdot t+\\+\left(\frac{a_2'}{\lambda_5-\lambda_1}-\frac{3a_3'}{(\lambda_5-\lambda_1)^2}\right)\cdot t^2+\frac{a_3'}{\lambda_5-\lambda_1}\cdot t^3\end{pmatrix}\cdot e^{-\lambda_1 t}+$$

$$+\left(-\frac{a_0'}{\lambda_5-\lambda_1}+\frac{a_1'}{(\lambda_5-\lambda_1)^2}-\frac{2a_2'}{(\lambda_5-\lambda_1)^3}+\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)\cdot e^{-\lambda_5 t}+$$

$$+B(t)\cdot e^{-\lambda_2 t}+e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau+C(t)\cdot e^{-\lambda_3 t}+e^{-\lambda_5 t}\cdot$$

$$\cdot\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau+D(t)\cdot e^{-\lambda_4 t}+e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau=$$

$$=\begin{pmatrix}\left(a_0+\frac{a_0'}{\lambda_5-\lambda_1}-\frac{a_1'}{(\lambda_5-\lambda_1)^2}+\frac{2a_2'}{(\lambda_5-\lambda_1)^3}-\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)+\\+\left(a_1+\frac{a_1'}{\lambda_5-\lambda_1}-\frac{2a_2'}{(\lambda_5-\lambda_1)^2}+\frac{6a_3'}{(\lambda_5-\lambda_1)^3}\right)\cdot t+\\+\left(a_2+\frac{a_2'}{\lambda_5-\lambda_1}-\frac{3a_3'}{(\lambda_5-\lambda_1)^2}\right)\cdot t^2+\left(a_3+\frac{a_3'}{\lambda_5-\lambda_1}\right)\cdot t^3\end{pmatrix}\cdot e^{-\lambda_1 t}+$$

$$+\left(-\frac{a_0'}{\lambda_5-\lambda_1}+\frac{a_1'}{(\lambda_5-\lambda_1)^2}-\frac{2a_2'}{(\lambda_5-\lambda_1)^3}+\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)\cdot e^{-\lambda_5 t}+$$

$$+B(t)\cdot e^{-\lambda_2 t}+e^{-\lambda_5 t}\cdot\int_0^t B'(\tau)\cdot e^{(\lambda_5-\lambda_2)\tau}d\tau+C(t)\cdot e^{-\lambda_3 t}+e^{-\lambda_5 t}\cdot$$

$$\int_0^t C'(\tau)\cdot e^{(\lambda_5-\lambda_3)\tau}d\tau+D(t)\cdot e^{-\lambda_4 t}+e^{-\lambda_5 t}\cdot\int_0^t D'(\tau)\cdot e^{(\lambda_5-\lambda_4)\tau}d\tau.$$

As the result of initiation of an additional element with the failure rate of $\lambda_5\neq\lambda_1$, the coefficients realigned in a certain way according to exponential $-\lambda_1 t$. The degree of polynomial $A(t)$ did not increase. Additionally, the integration created a summand that will be in the sum of the coefficient for the new exponential $-\lambda_5 t$. Let us write the complete result of summation in matrix form:

$$P_5(t)=P_4(t)+\int_0^t p_5(\tau,t)\cdot f_4(\tau)d\tau;$$

$$P_5(t)=A^T\cdot T\cdot F(\Lambda,t)^T;$$

$$\Lambda=\begin{pmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5\end{pmatrix};$$

$$T=\begin{pmatrix}t^0\\t^1\\t^2\\t^3\end{pmatrix};$$

$$A=\begin{pmatrix}
a_0+\frac{a_0'}{\lambda_5-\lambda_1}-\frac{a_1'}{(\lambda_5-\lambda_1)^2}+\frac{2a_2'}{(\lambda_5-\lambda_1)^3}-\frac{6a_3'}{(\lambda_5-\lambda_1)^4} & b_0+\frac{b_0'}{\lambda_5-\lambda_2}-\frac{b_1'}{(\lambda_5-\lambda_2)^2}+\frac{2b_2'}{(\lambda_5-\lambda_2)^3}-\frac{6b_3'}{(\lambda_5-\lambda_2)^4} & c_0+\frac{c_0'}{\lambda_5-\lambda_3}-\frac{c_1'}{(\lambda_5-\lambda_3)^2}+\frac{2c_2'}{(\lambda_5-\lambda_3)^3}-\frac{6c_3'}{(\lambda_5-\lambda_3)^4} & d_0+\frac{d_0'}{\lambda_5-\lambda_4}-\frac{d_1'}{(\lambda_5-\lambda_4)^2}+\frac{2d_2'}{(\lambda_5-\lambda_4)^3}-\frac{6d_3'}{(\lambda_5-\lambda_4)^4} & \begin{pmatrix}\left(-\frac{a_0'}{\lambda_5-\lambda_1}+\frac{a_1'}{(\lambda_5-\lambda_1)^2}-\frac{2a_2'}{(\lambda_5-\lambda_1)^3}+\frac{6a_3'}{(\lambda_5-\lambda_1)^4}\right)+\\ +\left(-\frac{b_0'}{\lambda_5-\lambda_2}+\frac{b_1'}{(\lambda_5-\lambda_2)^2}-\frac{2b_2'}{(\lambda_5-\lambda_2)^3}+\frac{6b_3'}{(\lambda_5-\lambda_2)^4}\right)+\\ +\left(-\frac{c_0'}{\lambda_5-\lambda_3}+\frac{c_1'}{(\lambda_5-\lambda_3)^2}-\frac{2c_2'}{(\lambda_5-\lambda_3)^3}+\frac{6c_3'}{(\lambda_5-\lambda_3)^4}\right)+\\ +\left(-\frac{d_0'}{\lambda_5-\lambda_4}+\frac{d_1'}{(\lambda_5-\lambda_4)^2}-\frac{2d_2'}{(\lambda_5-\lambda_4)^3}+\frac{6d_3'}{(\lambda_5-\lambda_4)^4}\right)\end{pmatrix} \\
a_1+\frac{a_1'}{\lambda_5-\lambda_1}-\frac{2a_2'}{(\lambda_5-\lambda_1)^2}+\frac{6a_3'}{(\lambda_5-\lambda_1)^3} & b_1+\frac{b_1'}{\lambda_5-\lambda_2}-\frac{2b_2'}{(\lambda_5-\lambda_2)^2}+\frac{6b_3'}{(\lambda_5-\lambda_2)^3} & c_1+\frac{c_1'}{\lambda_5-\lambda_3}-\frac{2c_2'}{(\lambda_5-\lambda_3)^2}+\frac{6c_3'}{(\lambda_5-\lambda_3)^3} & d_1+\frac{d_1'}{\lambda_5-\lambda_4}-\frac{2d_2'}{(\lambda_5-\lambda_4)^2}+\frac{6d_3'}{(\lambda_5-\lambda_4)^3} & 0 \\
a_2+\frac{a_2'}{\lambda_5-\lambda_1}-\frac{3a_3'}{(\lambda_5-\lambda_1)^2} & b_2+\frac{b_2'}{\lambda_5-\lambda_2}-\frac{3b_3'}{(\lambda_5-\lambda_2)^2} & c_2+\frac{c_2'}{\lambda_5-\lambda_3}-\frac{3c_3'}{(\lambda_5-\lambda_3)^2} & d_2+\frac{d_2'}{\lambda_5-\lambda_4}-\frac{3d_3'}{(\lambda_5-\lambda_4)^2} & 0 \\
a_3+\frac{a_3'}{\lambda_5-\lambda_1} & b_3+\frac{b_3'}{\lambda_5-\lambda_2} & c_3+\frac{c_3'}{\lambda_5-\lambda_3} & d_3+\frac{d_3'}{\lambda_5-\lambda_4} & 0
\end{pmatrix}.$$

Thus, the integration process in case of initiation of elements can be reduced to the transformation of matrix A: its expansion and associated recalculation of the coefficients. The common formula for calculation of the coefficients for the non-matching part of the matrix is:

$$A_{i,j}=A_{i,j}+\frac{B_{i,j}}{\Lambda_k-\Lambda_j}+\sum_{m=1}^{N-i}\frac{(-1)^m\cdot B_{i+m,j}\cdot(m+i-1)!}{\left(\Lambda_k-\Lambda_j\right)^{m+1}\cdot(i-1)!},$$

if $\Lambda_j\neq\lambda_k$, $i=1..N$, $j=1..N$,
i is the line index in the respective matrix;
j is the column index in the respective matrix;
k is the index of the initiated element of matrix Λ;
N is the number of elements in the initial system.

A common formula is also required for the summands that will make up a new coefficient for the exponential of the initiated element failure rate, i.e. for the formula:

$$-\frac{a_0'}{\lambda_5-\lambda_1}+\frac{a_1'}{(\lambda_5-\lambda_1)^2}-\frac{2a_2'}{(\lambda_5-\lambda_1)^3}+\frac{6a_3'}{(\lambda_5-\lambda_1)^4}.$$

$$A_{i,k}=A_{i,k}+\sum_{m=1}^{N}\frac{(-1)^m\cdot B_{m,j}\cdot(m-1)!}{\left(\Lambda_k-\Lambda_j\right)^m},$$

if $\Lambda_j\neq\lambda_k$, $i=1..N$, $j=1..N$,
i is the line index in the respective matrix;
j is the column index in the respective matrix;
k is the index of the initiated element of matrix Λ;
N is the number of elements in the initial system.

Algorithmic diagram of the analytic solution

By combining the obtained formulas for the cases when the initiated element matches or does not match one of the types of the elements in the initial system, an analytic solution for this iteration can be obtained using the diagram shown below.

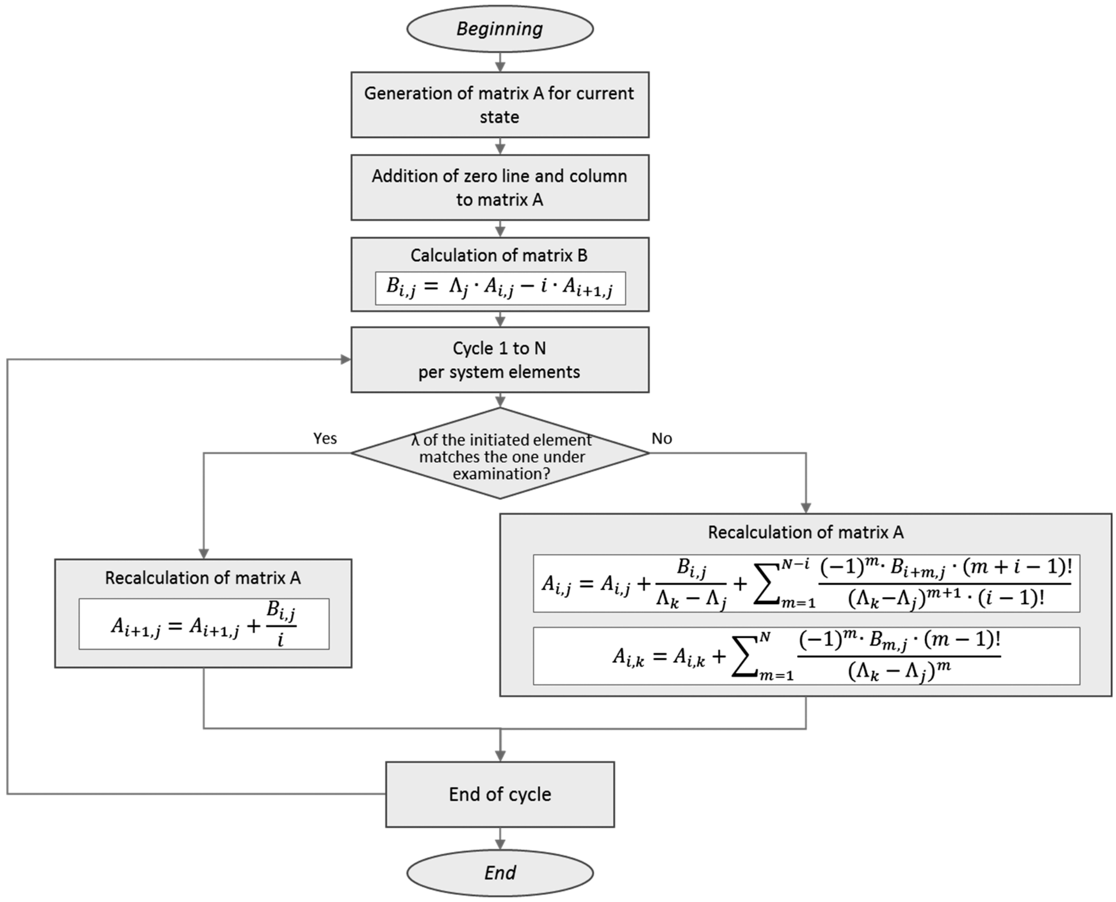

Figure 1. Diagram of analytic solution algorithm

i is the line index in the respective matrix;
j is the column index in the respective matrix;
k is the index of the initiated element of matrix Λ;
N is the number of elements in the initial system.

Conclusions

The paper develops and mathematically substantiates an algorithm that enables recurrent development of analytic expression for PNF calculation for a system of any number of elements in cold standby. The solution process consists in the recalculation of matrix coefficients using generalized formulas instead of numeric derivation and integration, which allows significantly reducing calculation time and increasing the accuracy of the results. In terms of data representation the algorithm is adapted for computer calculation.

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