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ESTIMATIONS OF RESIDUAL LIFETIME OF ALTERNATING PROCESS. COMMON APPROACH TO ESTIMATIONS OF RESIDUAL LIFETIME

The paper describes a generalized method of estimating residual lifetime for an object with an arbitrary operating strategy. Asymptotic and non-asymptotic assessments of factors converging in the long-term object operation have been obtained. The main idea of the developed approach is to bring down an arbitrary renewal process to a well-known alternating renewal process. The method makes it possible to predict the residual lifetime of equipment at the operational stage, and it can be used in estimating resource characteristics of nuclear power plant (NPP) equipment.

Keywords: *residual lifetime, alternating process, operating strategy.*

Introduction

Nowadays, the so-called elementary model of repair is used in most cases when carrying out a probabilistic analysis of reliability characteristics. After each failure, a system is brought into sound condition in negligibly short time and immediately comes back into operation. However, the functioning of modern technical systems, as a rule, represents a more complicated process characterized by periodical or constant control of faults, schemes of fail detection, emergency and renewal works, etc.

In addition, it should be noted that when dealing with estimation or prediction of a residual resource, everything generally boils down not to calculations of durability characteristics but to analysis of reliability parameters, such as failure rate, probability of failure-free operation (PFO), availability factor, and based on the results, a conclusion is then made about the technical state of an object. To produce a more accurate study of equipment service life, it is necessary to develop methods for estimating durability characteristics per se. Also, it is necessary to take into account features and modes of equipment functioning that can considerably influence reliability as seen from experience.

As parameters of durability, resource and service life are one of the basic concepts of the reliability theory. Prediction of resource of objects at the operational stage is then of special importance. Subject to estimation is the residual resource which defines possible duration of the operation of an object from the given moment of time and up to the moment when the parameter of technical state achieves its limit value.

The present work presents methods of statistical estimation of residual resource which allow us to take into account the peculiarities of functioning and maintenance of elements as part of complex technical systems.

1. Non-asymptotic estimations of average residual time for alternating process

There are a number of publications in which a lot of attention is paid to such resource characteristics as direct residual time (DRT), reverse residual time (RRT), as well as average direct residual time (ADRT) and average reverse residual time (ARRT). The study [1], for example, considers how to define residual operating time (ADRT) for non-recoverable equipment. In the majority of scientific publications [2, 3] on this issue, the definition of DRT and ADRT (and, accordingly, formulas and calculations) are provided for the case of simple renewal process (it is supposed that time of renewal can be neglected). So, it is necessary to develop methods for estimating residual time for more complicated alternating processes.

A simple process of renewal represents a sequence of times to failure of an object under consideration, with the time of renewal set to zero. In this case, ADRT is an expectation of the remained operating time of system to a next failure, starting from the time point t in which the system was efficient [2]:

$$V(t) = \mathrm{M}\sum_{i=0}^{\infty}(\tau_{i+1} - t)\cdot I\{\tau_i \le t < \tau_{i+1}\}; \tag{1}$$

where τ_i is the time point of *i-th* failure, $I\,\{A\}$ is function-indicator of the argument validity which is equal to one, if A is true, and to zero if not.

Let's extend the concept "DRT" for the case of any renewal process. In the work [5], ADRT is treated as an expectation of the remained operating time of system to a next failure, starting from the time point t in which the system was efficient:

$$V(t) = \mathrm{M}\sum_{i=0}^{\infty}(\tau_{i+1} - t)\cdot I\{\nu_i \le t < \tau_{i+1}\}; \tag{2}$$

Here v_i and τ_i are time points of the i-th renewal and i-th failure, $v_0 = 0$, $v_i \le \tau_{i+1} \le v_{i+1}$. It should be noted that in the formula (2) there is a condition of the object being efficient at the time point t.

Let's show the general approach to obtain estimations of residual time for the strategy of functioning described by means of alternating process of object renewal. Such a process is in detail investigated in the theory of renewal [2]. It is considered that at the initial time point $t_0 = 0$ an object is in sound condition. The object functions till the moment of failure τ_i, then the emergency renewal is carried out till the moment of renewal v_i. After renewal the object continues to work till the next moment of failure, and then there is renewal and transition into efficient state. Random variables α_i and β_i are duration of the *i-th* interval of availability and unavailability respectively. Such a cycle is reproduced again till the chosen moment of time t. The given process is represented in figure 1:

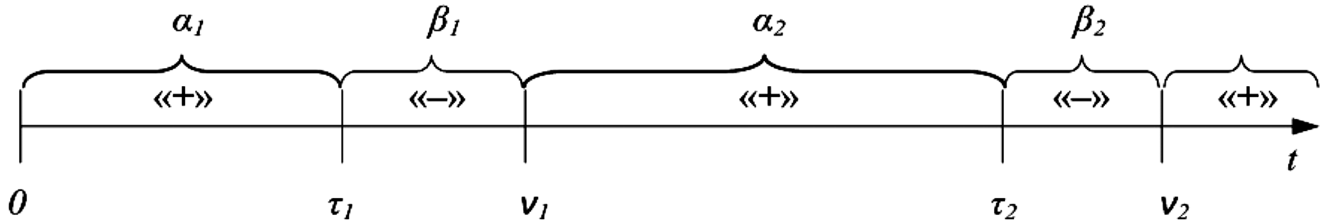

Figure 1. Alternating process of renewal

The formula (2) for the given process can be written down in the form of:

$$V(t) = \mathrm{M}\sum_{i=0}^{\infty}(\nu_i + \alpha_{i+1} - t)\cdot I\{\nu_i \le t < \nu_i + \alpha_{i+1}\}; \tag{3}$$

Alternating process is a special case examined in [5], and based on works [4, 5], it is enough just to obtain non-asymptotic estimation of ADRT in the form of integrated Volterra equation of the second kind in images of the Laplace transform:

$$V(p) = \frac{G_\alpha(p)}{1 - f_\alpha(p) f_\beta(p)}; \tag{4}$$

where $G_\alpha(p) = \dfrac{M\alpha - P_\alpha(p)}{p}$;

And in space of originals (hereinafter the symbol "*" designates operation of integrated convolution):

$$V(t) = G_\alpha(t) + (V * f_\alpha * f_\beta)(t); \tag{5}$$

where $G_\alpha(t)$ is a free member of the equation (5), such that:

$$G_\alpha(t) = \int_0^\infty x g_\alpha(t;x)dx = \int_0^\infty x f_\alpha(t+x)dx = \int_t^\infty P_\alpha(x)dx = M\alpha - \int_0^t P_\alpha(x)dx = M\alpha - t + \int_0^t F_\alpha(x)dx; \tag{6}$$

where $f_\alpha(t)$, $f_\beta(t)$ is the density of durability distribution of the *i-th* interval of availability and unavailability respectively.

Note that such characteristic as reverse residual time (RRT) is poorly studied. By definition RRT is a time from the moment of the last failure till the moment of time *t*. We shall show how to deduce non-asymptotic estimation for an average reverse residual time (ARRT) similar to (5). We shall designate ARRT as *R (t)* similar to [4-5], so we shall obtain:

$$R(t) = \mathrm{M}\sum_{i=0}^{\infty}(t - \nu_i)\cdot I\{\nu_i \le t < \nu_i + \alpha_{i+1}\} = \sum_{i=0}^{\infty}\varphi_i(t); \tag{7}$$

where

$$\varphi_i(t) = \mathrm{M}[(t-\nu_i)\cdot I\{\nu_i \le t < \nu_i + \alpha_{i+1}\}] = \int_0^\infty\int_0^\infty (t-y)\cdot I\{y \le t < y+x\} f_\alpha(x) f_{\nu_i}(y)\,dxdy =$$

$$= \int_0^t (t-y) f_{\nu_i}(y) \int_{t-y}^\infty f_\alpha(x)\,dxdy = \int_0^t (t-y) f_{\nu_i}(y) P_\alpha(t-y)\,dy = (f_{\nu_i} * A_\alpha)(t);$$

where $A_\alpha(t)=xP_\alpha(x)$; (8)

Let's apply the Laplace transform to $\varphi_i(t$, so we will get the following:

$$\varphi_i(p) = f_{\nu_i}(p) A_\alpha(p);$$

The Laplace representation for (7) will look as follows:

$$R(p) = \sum_{i=0}^\infty \varphi_i(p) = \sum_{i=0}^\infty f_{\nu_i}(p) A_\alpha(p) = A_\alpha(p) \sum_{i=0}^\infty f_{\nu_i}(p);$$

It should be noted that

$$\nu_i = \sum_{j=1}^i (\alpha_j + \beta_j),\ i = 1, 2, \ldots,\ \nu_0 = 0.$$

Hence

$$\sum_{i=0}^\infty f_{\nu_i}(p) = \frac{1}{1 - f_\alpha(p)\cdot f_\beta(p)};$$

We obtain

$$R(p) = A_\alpha(p) + R(p) f_\alpha(p) f_\beta(p);$$

Moving to originals, we shall write down the required expression of ADRT:

$$R(t) = A_\alpha(t) + (R * f_\alpha * f_\beta)(t) = tP_\alpha(t) + (R * f_\alpha * f_\beta)(t). \quad (9)$$

By analogy with ARRT and ADRT, we shall introduce and investigate new resource characteristics:

W (t) is average direct residual time of unavailability (ADRTU) – the expectation of remained time of object unavailability before the next renewal, from the time point *t* in which the system is in unavailable state.

Q (t) is average reverse residual time of unavailability (ARRTU) – the expectation of object unavailability time from the moment of the last failure till the moment of time *t* in which the system is unavailable.

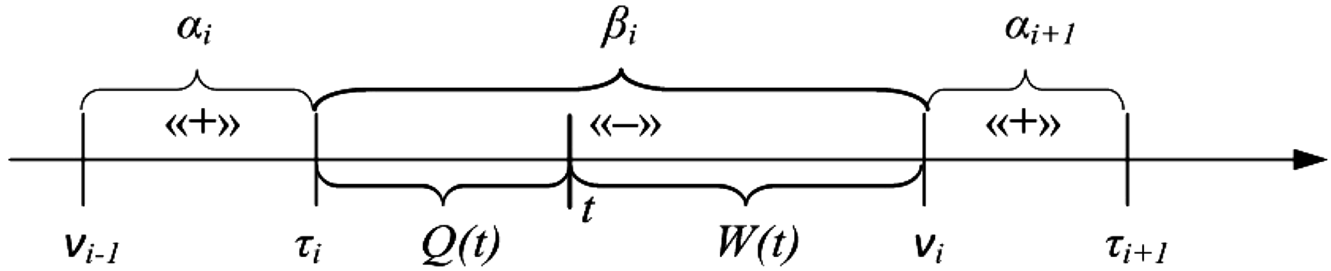

Figure 2. ADRTU and ARRTU

Similarly to (3) and (7), formulas for ADRTU and ARRTU can be written down in the following form:

$$W(t) = \mathrm{M}\sum_{i=0}^{\infty}(\nu_i - t)\cdot I\{\tau_i \le t < \tau_i + \beta_i\};$$

$$Q(t) = \mathrm{M}\sum_{i=0}^{\infty}(t - \tau_i)\cdot I\{\tau_i \le t < \tau_i + \beta_i\};$$

Let's obtain non-asymptotic estimation for *W (t)* similarly to the way how the estimation of ADRT has been obtained above:

$$W(p) = G_\beta(p)\sum_{i=1}^{\infty} f_{\tau_i}(p);$$

where $G_\beta(p)$ is the Laplace image of the function $G_\beta(t)$:

$$G_\beta(t) = M\beta - \int_0^t P_\beta(x)dx; \tag{10}$$

It should be noticed that

$$\tau_i = \alpha_i + \sum_{j=1}^{i-1}(\alpha_j + \beta_j);$$

$$f_{\tau_i}(p) = f_\alpha(p)\cdot(f_\alpha(p)f_\beta(p))^{i-1};$$

$$\sum_{i=1}^{\infty} f_{\tau_i}(p) = f_\alpha(p)\sum_{i=1}^{\infty}(f_\alpha(p)f_\beta(p))^{i-1} = \frac{f_\alpha(p)}{1 - f_\alpha(p)\cdot f_\beta(p)};$$

Final estimation in space of images:

$$W(p) = G_\beta(p) f_\alpha(p) + W(p) f_\alpha(p) f_\beta(p);$$

In space of originals:

$$W(t) = (G_\beta * f_\alpha)(t) + (W * f_\alpha * f_\beta)(t). \tag{11}$$

The formula (11) can be written down in some other form. Applying Laplace's transformation to (10), we have:

$$G_\beta(p) = \frac{M\beta - P_\beta(p)}{p}; \tag{12}$$

Then:

$$G_\beta(p) f_\alpha(p) = \frac{M\beta - P_\beta(p)}{p} f_\alpha(p) = M\beta \frac{f_\alpha(p)}{p} - P_\beta(p)\frac{f_\alpha(p)}{p};$$

Moving into the space of originals:

$$(G_\beta * f_\alpha)(t) = M\beta F_\alpha(t) - (P_\beta * F_\alpha)(t); \tag{13}$$

The formula (11) becomes as the following:

$$W(t) = M\beta F_\alpha(t) - (P_\beta * F_\alpha)(t) + (W * f_\alpha * f_\beta)(t). \tag{14}$$

Let's more briefly dwell upon the conclusion of non-asymptotic estimation for ARRTU:

$$Q(p) = A_\beta(p) \sum_{i=1}^{\infty} f_{\tau_i}(p);$$

where $A_\beta(p)$ is an image of function $A_\beta(t) = xP_\beta(x)$;

The required equation in images:

$$Q(p) = A_\beta(p) f_\alpha(p) + Q(p) f_\alpha(p) f_\beta(p);$$

And after transition to originals:

$$Q(t) = (A_\beta * f_\alpha)(t) + (Q * f_\alpha * f_\beta)(t). \tag{15}$$

Note 1:

All the four obtained equations (4, 7, 14, 15) of non-asymptotic estimations of investigated resource characteristics are Volterra equations of the second kind with various free members, but with the same kernel – the function $(f_\alpha * f_\beta)(t)$ representing the integrated convolution of distribution densities of object availability and unavailability intervals. In some cases it is convenient to introduce a random variable $\omega_i = \alpha_i + \beta_i$ describing duration of one cycle of investigated object work and its related density of distribution $f_\omega(t) = (f_\alpha * f_\beta)(t$ which further is considered as the kernel of integral equations.

2. Asymptotic estimations of average residual time for alternating process

Let's show how to obtain asymptotic estimation for alternating process of ADRT:

$$V_{асимп} = \lim_{t\to\infty} V(t); \tag{16}$$

The first method:

In the limit (16), we shall pass in space of images taking into account (4):

$$\lim_{t\to\infty} V(t) = \lim_{p\to 0} V(p)p = \lim_{p\to 0} p[\frac{G_\alpha(p)}{1 - f_\alpha(p) f_\beta(p)}] = \lim_{p\to 0} \frac{M\alpha - P_\alpha(p)}{1 - f_\alpha(p) f_\beta(p)};$$

Note that:

$$\lim_{p\to 0}(M\alpha - P_\alpha(p)) = M\alpha - \lim_{p\to 0}\int_0^\infty e^{-pt} P_\alpha(t)dt = M\alpha - \int_0^\infty P_\alpha(t)dt = 0;$$

$$\lim_{p\to 0} f_\alpha(p) = \lim_{p\to 0}\int_0^\infty e^{-pt} f_\alpha(t)dt = \int_0^\infty f_\alpha(t)dt = 1;$$

To get rid of uncertainty of the kind $\frac{0}{0}$, we shall apply L'Hôpital's rule:

$$\lim_{p\to 0} V(p)p = \lim_{p\to 0} \frac{(M\alpha - P_\alpha(p))'}{(1 - f_\alpha(p)\cdot f_\beta(p))'} = \lim_{p\to 0} \frac{P'_\alpha(p)}{f_\alpha{}'(p)\cdot f_\beta(p) + f_\alpha(p)\cdot f_\beta{}'(p)};$$

Note that:

$$\lim_{p\to 0} P'_\alpha(p) = -\lim_{p\to 0}\int_0^\infty te^{-pt} P_\alpha(t)dt = -\int_0^\infty tP_\alpha(t)dt = -(\frac{(M\alpha)^2 + D\alpha}{2});$$

$$\lim_{p\to 0} f_\alpha(p) = -\lim_{p\to 0}\int_0^\infty te^{-pt} f_\alpha(t)dt = -\int_0^\infty tf_\alpha(t)dt = -M\alpha;$$

Final estimation for ADRT has the following form:

$$V_{асимп} = \lim_{p\to 0} V(p)p = \frac{(M\alpha)^2 + D\alpha}{2(M\alpha + M\beta)} = \frac{M\alpha^2}{2(M\alpha + M\beta)}; \tag{17}$$

The second method:

An alternating process can be considered as generalization of classical process of renewal [2], then we shall obtain asymptotic estimation for ADRT:

$$V_{асимп} = V_\alpha P_\alpha;$$

where P_α is the probability of any moment of time *t* falling into the interval of availability, V_α is the asymptotic estimation of ADRT for the classical process consisting of a sequence of time between failures $\{\alpha_i\}$.

Further we have:

$$P_\alpha = \frac{M\alpha}{M\alpha + M\beta};$$

According to [3], an asymptotic estimation for classical process:

$$V_\alpha = \frac{M\alpha^2}{2M\alpha} = \frac{(M\alpha)^2 + D\alpha}{2M\alpha};$$

where $M\alpha^2$ is the 2-nd time moment of a random variable α.

The final estimation takes the following form:

$$V_{асимп} = \frac{M\alpha^2}{2(M\alpha + M\beta)};$$

The given expression completely coincides with the earlier received estimation (17).

Thus, the problem of asymptotic estimation for ADRT is generally reduced to an estimation of the moments of random variables – duration of intervals of object availability and unavailability. By similar reasoning, we can obtain:

$$R_{асимп} = V_{асимп} = \frac{M\alpha^2}{2(M\alpha + M\beta)}; \tag{18}$$

$$W_{асимп} = Q_{асимп} = \frac{M\beta^2}{2(M\alpha + M\beta)}; \tag{19}$$

3. Estimations of residual time for an arbitrary process

In any system it is possible to carry out technical renewal actions. Such actions can be carried out at moments appointed in advance and at random moments. Types of considered strategy reflect presence of various preventive maintenance types (for example, scheduled/corrective, emergency/preventive), control, and various types of failures, renewal, and other various conditions of operation. All aforesaid generates a huge amount of equipment maintenance strategies and even more of mathematical models for the description of occurring events.

Process of renewal for an arbitrary maintenance strategy can be generally reduced to alternating process. By designating as α_i and β_i the general durations of object availability and unavailability intervals respectively, we shall receive the renewal process similar to that in Figure 1. To apply the described mathematical tools, it is necessary to describe distributions of random variables α and β.

As an example, let's consider one strategy of object functioning typically met in practice. The system is deemed to be available at the initial moment of time t_0, and the factor of system availability at the point $t_0 = 0$ is equal to one. The system foresees scheduled preventive maintenance which is carried out periodically in the time interval T (the period of preventive maintenance) and lasts for the random time η_m (see Figure 2). If before the moment of scheduled preventive maintenance there is a failure during the casual moment of time ξ, then the emergency renewal of the system is carried out whose casual time is equal to η_f. After system emergency renewal there is a re-planning of the moment of preventive maintenance that is the current period of preventive maintenance moves to the right at the time interval $\xi + \eta_f$. After renewal the system continues to work till the moment ξ, if there was a failure or till the next moment of preventive maintenance T if a failure did not occur. Further there is a corresponding renewal and a similar cycle of work starts over again. The described strategy of functioning is presented in Figure 3:

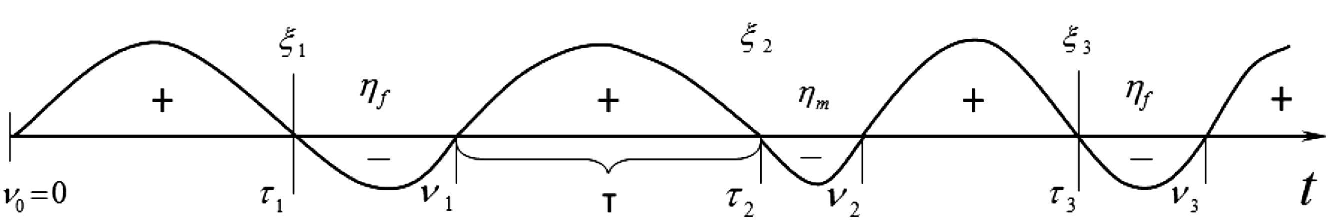

Figure 3. The strategy which takes into account the built-in control with preventive maintenance

ξ_i means the moments of failure, τ_i means time intervals from the beginning of system operation till the moment of the next failure of system, and ν_i means time intervals from the beginning of system operation till the end of the next recovery.

According to [6] the moments of renewal and failure are defined as follows:

$$\begin{cases} \tau_i = \nu_0 + T \cdot I\{\xi_i > T\} + \xi_i I\{\xi_i < T\} + \sum_{j=1}^{i-1} I\{\xi_j < T\}(\xi_j + \eta_f) + I\{\xi_j > T\}(T + \eta_m); \\ \nu_i = \nu_0 + I\{\xi_i < T\}(\xi_i + \eta_f) + I\{\xi_i > T\}(T + \eta_m) + \sum_{j=1}^{i-1} I\{\xi_j < T\}(\xi_j + \eta_f) + I\{\xi_j > T\}(T + \eta_m); \end{cases}$$

Hence, the *i-th* interval of availability can be presented as

$$\alpha_i = \tau_i - \nu_{i-1} = T \cdot I\{\xi_i > T\} + \xi_i I\{\xi_i < T\};$$

Similarly, we shall obtain the expression for the *i-th* interval of unavailability:

$$\beta_i = \nu_i - \tau_i = \eta_m I\{\xi_i > T\} + \eta_f I\{\xi_i < T\};$$

According to (18-19), in order to obtain asymptotic estimations of ADRT, it is necessary to find the first and second central moments of random variables α and β. For non-asymptotic estimations, we generally need distribution functions and densities of random variables of mean times between failures and time of renewal. Let's investigate random variables α and β. Note that

$$F_\alpha(t) = \begin{cases} F_\xi(t), & t < T \\ 1, & t \geq T \end{cases};$$

Hence

$$M\alpha = \int_0^T P_\xi(t)dt; \tag{20}$$

To obtain $M\alpha^2$, we shall write down the distribution function in the form of:

$$F_\alpha(x) = H(T-x) \cdot F_\xi(x) + H(x-T); \tag{21}$$

where $H(x)$ is the Heaviside function, then:

$$f_\alpha(x) = F'_\alpha(x) = H(T-x) \cdot f_\xi(x) - \delta(T-x) \cdot F_\xi(x) + \delta(x-T); \tag{22}$$

where $\delta(x)$ is the Dirac delta function.
So, we obtain:

$$M\alpha^2 = \int_0^\infty x^2 f_\alpha(x)dx = \int_0^\infty x^2 H(T-x) \cdot f_\xi(x)dx - \int_0^\infty x^2 \delta(T-x) \cdot F_\xi(x)dx + \int_0^\infty x^2 \delta(x-T)dx =$$

$$= \int_0^T x^2 f_\xi(x)dx - T^2 F_\xi(T) + T^2; \tag{23}$$

The mathematical expectation of the value β is

$$M\beta = M(\eta_m I\{\xi_i > T\}) + M(\eta_f I\{\xi_i < T\}) = M\eta_m P\{\xi_i > T\}) + M\eta_f P\{\xi_i < T\} =$$

$$= M\eta_{mr}(1 - F_\xi(T)) + M\eta_{fr} F_\xi(T); \tag{24}$$

To compute $M\beta^2$, we shall find the function of distribution of β:

$$F_\beta(x) = \mathrm{P}(\beta < x) = \mathrm{P}\left(\{\beta < x\} \cap \{\xi \ge T\}\right) + \mathrm{P}\left(\{\beta < x\} \cap \{\xi < T\}\right) =$$
$$= \mathrm{P}\left(\{\eta_m < x\} \cap \{\xi \ge T\}\right) + \mathrm{P}\left(\{\eta_f < x\} \cap \{\xi < T\}\right) = F_{\eta_m}(x)\left(1 - F_\xi(T)\right) + F_{\eta_f}(x) F_\xi(T);$$

Hence

$$f_\beta(x) = f_{\eta_m}(x)\left(1 - F_\xi(T)\right) + f_{\eta_f}(x) F_\xi(T);$$

$$M\beta^2 = \int_0^\infty x^2 f_\beta(x)dx = \left(1 - F_\xi(T)\right) M{\eta_m}^2 + F_\xi(T) M{\eta_f}^2; \tag{25}$$

Considering (20), (23), (24), (25), the asymptotic estimations become suitable for numerical calculations:

$$R_{асимп} = V_{асимп} = \frac{M\alpha^2}{2(M\alpha + M\beta)} = \frac{\int_0^T x^2 f_\xi(x)dx - T^2 F_\xi(T) + T^2}{2(\int_0^T P_\xi(t)dt + M\eta_m(1 - F_\xi(T)) + M\eta_f F_\xi(T))};$$

$$W_{асимп} = Q_{асимп} = \frac{M\beta^2}{2(M\alpha + M\beta)} = \frac{\left(1 - F_\xi(T)\right) M{\eta_m}^2 + F_\xi(T) M{\eta_f}^2}{2(\int_0^T P_\xi(t)dt + M\eta_m(1 - F_\xi(T)) + M\eta_f F_\xi(T))};$$

To obtain non-asymptotic estimations of characteristics studied in this paper, it is necessary to solve integral equations (4, 7, 14, 15). According to Note 1, a kernel of the equations in all four cases is the function $f_\omega(t)$, the density of distribution of a random variable ω. For the strategy under consideration, we have:

$$\omega = \begin{cases} \xi + \eta_f, & \xi < T \\ T + \eta_m, & \xi \ge T \end{cases};$$

Let's find the function of distribution of this random variable:

$$F_\omega(t) = P(\omega < t) = P(I\{\xi \ge T\}(T + \eta_m) + I\{\xi < T\}(\xi + \eta_f) < t) =$$

$$= P(\eta_m < t - T;\ \xi > T) + P(\eta_f < t - \xi;\ \xi < T) =$$

$$= \iint\limits_{\left\{\substack{\xi > T \\ 0<\eta_m<t-T}\right\}} f_\xi(x) f_{\eta_m}(y) dy dx + \iint\limits_{\left\{\substack{\xi < t \wedge T \\ 0<\eta_f<t-\xi}\right\}} f_\xi(x) f_{\eta_f}(y) dy dx =$$

$$= \int_T^\infty f_\xi(x) \int_0^{t-T} f_{\eta_m}(y) dy dx + \int_0^{t\wedge T} f_\xi(x) \int_0^{t-x} f_{\eta_m}(y) dy dx = \left(1 - F_\xi(T)\right) F_{\eta_m}(t-T) + \int_0^{t\wedge T} f_\xi(x) F_{\eta_f}(t-x) dx;$$

We shall obtain the required density of distribution by differentiation:

$$f_\omega(t) = \frac{dF_\omega(t)}{dt} = \left(1 - F_\xi(T)\right) f_{\eta_m}(t-T) + \int_0^{t\wedge T} f_\xi(x) f_{\eta_f}(t-x) \; dx; \tag{26}$$

Next we shall find free members of the integral equations. For ADRT the free member of the equation is defined (6), the function of distribution and the expectation of the random variable α are defined in (20), (21). For ARRT the free member of the equation is defined in (8). For ADRTU it looks like in (13). For ARRTU it is necessary to find $(A_\beta * f_\alpha)(t)$. This problem becomes complicated by the fact that in the found formula (22) there is a Dirac delta function, thus making the correct numerical integration $f_\alpha(x)$ impossible. Considering (22) and parity of delta function, we shall write down the density of distribution in the following form:

$$f_\alpha(x) = H(T-x) f_\xi(x) + \delta(x-T) P_\xi(x) = C(x) + \delta(x-T) P_\xi(x);$$

where $C(x) = H(T-x) f_\xi(x);$

Then

$$(A_\beta * f_\alpha)(t) = \int_0^t A_\beta(t-x) C(x) dx + \int_0^t A_\beta(t-x) \delta(x-T) P_\xi(x) dx =$$

$$= \int_0^t A_\beta(t-x) C(x) dx + A_\beta(t-T) P_\xi(T) H(t-T); \tag{27}$$

where $A_\beta(t) = t(1 - F_\beta(t)) = t P_\beta(t);$

It is already possible to calculate numerically a free member under the formula (27).

Thus, we have found a kernel (26) and free members (6, 8, 13, 27) of integral equations (4, 7, 14, 15) respectively, in the form that is suitable for numerical solution.

4. An example of calculation

Let's compute asymptotic and non-asymptotic estimations of ADRT, ARRT, ADRTU, and ARRTU for the strategy of object functioning described in Section 3. The period of preventive maintenance will be set to 7 arbitrary units (a.u.) of time. As the law of distribution for time to failure and scheduled preventive maintenance, we shall take a two parameter Weibull distribution, and for emergency renewal we shall take a gamma distribution. Parameters of the distribution law are specified in Table 1:

Table 1 Parameters of the law of distribution of random variables

Random variable	Designation	Parameter of form, a.u.	Parameter of scale, a.u.
Time to failure	ξ	5	6
Duration of scheduled preventive maintenance	η_m	2	2
Duration of emergency renewal	η_f	3	5

Figures 4-5 represent asymptotic and non-asymptotic estimations of ADRT and ARRT, ADRTU and ARRTU respectively. The given figures show convergence of asymptotic (dotted line) and non-asymptotic (continuous line) estimations:

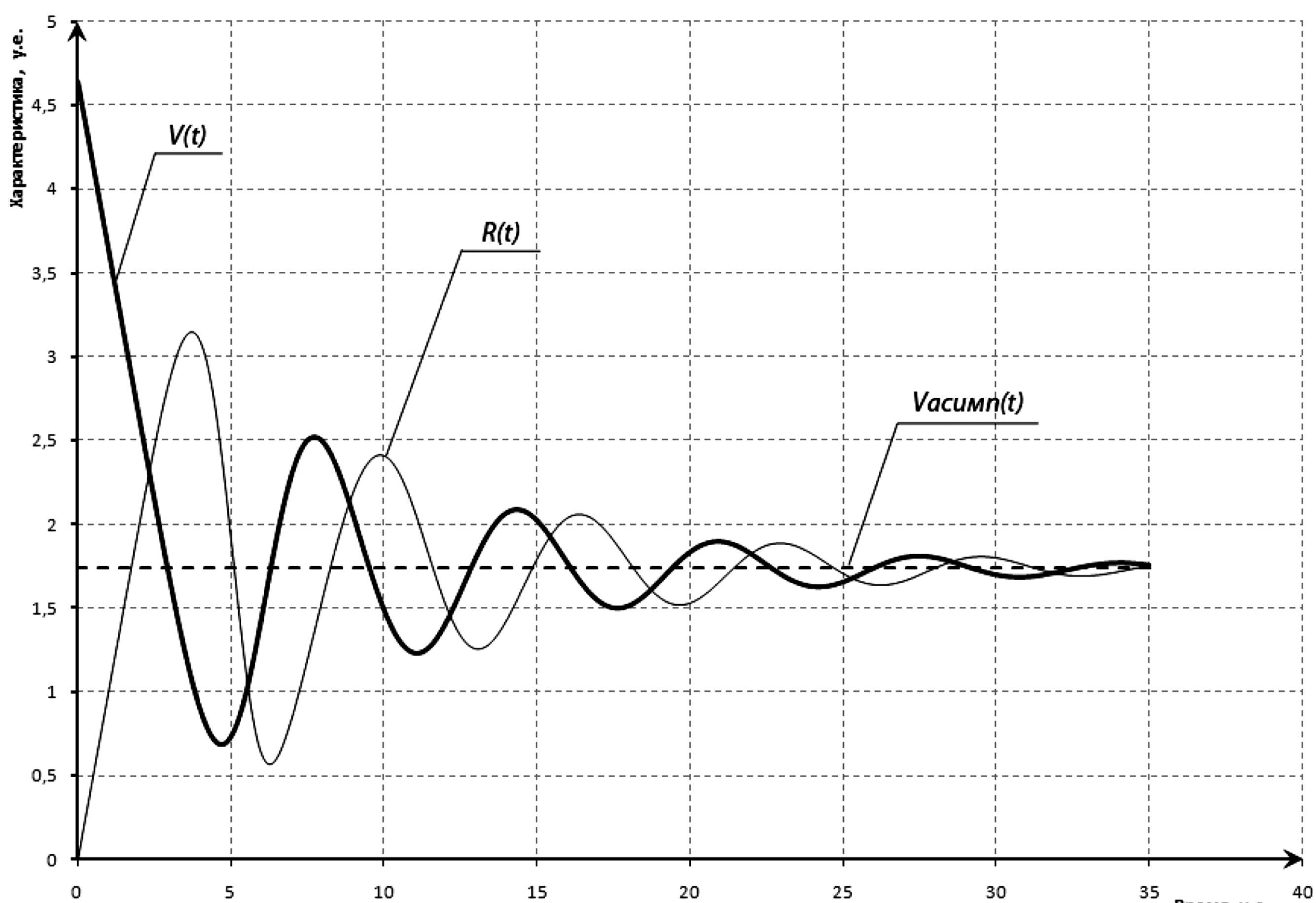

Figure 4. Graph of behavior of estimations of ADRT and ARRT expectations

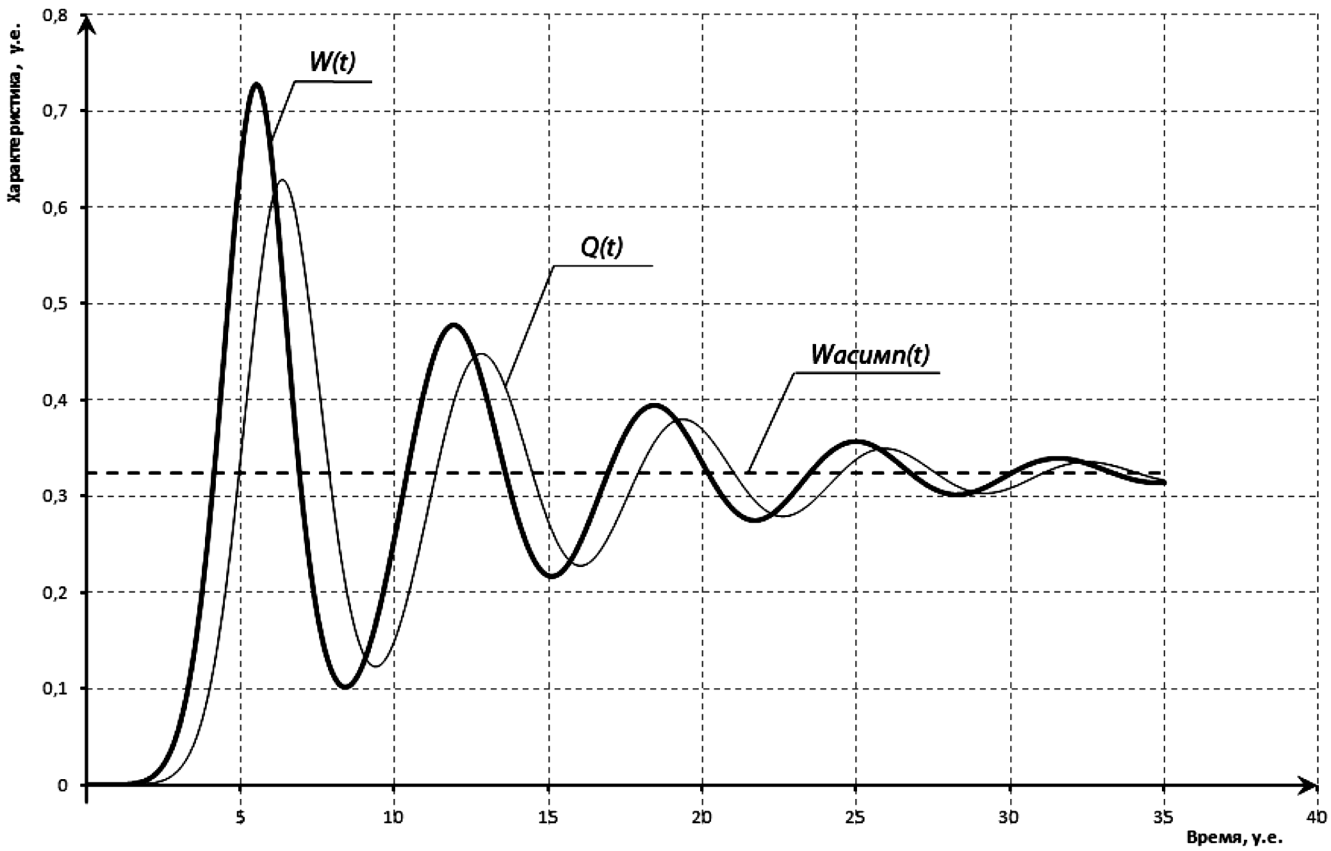

Figure 5. Graph of behavior of estimations of ADRTU and ARRTU expectations

5. Conclusion

We have developed a generalized method for calculating the following resource characteristics for an object with a random strategy of functioning:

1) Average direct residual time;

2) Average reverse residual time;

3) Average direct residual time of unavailability;

4) Average reverse residual time of unavailability.

We have obtained both asymptotic and non-asymptotic estimations of the given parameters. The analysis of estimations shows that the difference between asymptotic and non-asymptotic estimations at the initial time interval can be a significant quantity, whereas in case of long-term functioning of an object the given estimations converge.

The method can be applied for estimating resource characteristics of NPP equipment, including technical systems with periodical or constant control of faults, schemes of fail detection, emergency and renewal works, etc. inherent in them. The method allows us to predict a residual resource of objects at the stage of operation that defines possible duration of the operation of an object from the given moment of time and up to the moment when the parameter of technical state achieves its limit value.

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