Method of recovery of priority vector for alternatives under uncertainty or incomplete expert assessment

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Abstract. Aim. *The so-called pair-wise comparison method is one of the most popular decision-making procedures owing to its efficiency, flexibility and simplicity. The primary disadvantage of this method in the context of expert evaluation of large numbers of alternatives or within a sufficiently wide field of knowledge is the impossibility to compare each element with each other, both due to the large number of such comparisons, random gaps and difficulties experienced by the expert while comparing some alternatives. The assessments are affected by gaps that complicate decision-making, as most statistical methods are not applicable to incomplete sets of data. The fairly popular algorithm for processing of pair-wise comparison matrices (the Saaty algorithm) cannot work with matrices that predominantly contain zero components. The purpose of the paper is to develop a method of processing comparison matrices in order to obtain weight coefficients (weights) of the considered alternatives that enable quantitative comparisons.* **Methods.** *In practice, there are several approaches to managing sets of data with gaps. The first, most easily implementable, approach involves the elimination of copies with gaps from the set with further handling of only complete data. This approach should be used in case gaps in data are isolated. Although even in this case there is a serious risk of "losing" important trends while deleting data. The second approach involves using special modifications of data processing methods that tolerate gaps in sets of data. And, finally, there are various methods of evaluation of missed element values. Those methods help to fill in the gaps in sets of data based on certain assumptions regarding the values of the missing data. The applicability and efficiency of individual approaches, in principle, depends on the number of gaps in data and reasons of their occurrence. In this paper, the pair-wise comparison matrix is considered in the form of a loaded graph, while the alternatives are the nodes and comparisons are the edges of the graph. Respectively, if a pair of alternatives occurs for which the expert could not specify a preference, the corresponding edge is absent. The paper considers a way of removing edges that correspond to the most controversial values, i.e. a cycle breakage algorithm that causes transformation of the initial graph to the spanning tree that allows for unambiguous comparison of any two alternatives. The algorithm of joint alignment of both the upper and lower boundaries of expert assessments is not considered in this paper.* **Results.** *The paper gives an example of practical application of the developed algorithm of processing incomplete matrices of pair-wise comparisons of ten objects obtained in a certain expert assessment. It also shows the efficiency of the suggested approach to priority recovery of compared alternatives, explores ways of automating computing and future lines of research.* **Conclusions.** *The proposed method can be used in a wide range of tasks of analysis and quantitative evaluation of risks, safety management of complex systems and objects, as well as tasks related to the verification of compliance with the requirements for such highly dependable elements as nuclear reactors, aviation and rocket technology, gas equipment components, etc., i.e. in cases when low (less than 0,01) probabilities of failure per given operation time are to be evaluated, while the failure statistics for such elements in operation is practically nonexistent. The proposed algorithm can be applied in expert assessment in order to identify the type and parameters of time to failure distribution of such highly dependable elements, which in turn will allow evaluating dependability characteristics with the required accuracy.*

Keywords: *missed data, expert assessment, loaded graph, spanning tree, pair-wise comparison graph, connectivity criterion.*

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Introduction

The decision-making procedures that involve experts in choosing the optimal variant(s) out of the allowed set are often used in a variety of areas for assessment, selection, definition of task priority, etc. Obviously, the comparison of various alternatives based on their preferability in terms of decision-making tasks in many cases is unfeasible using one criterion or one expert. Consequently, in most decision-making tasks there are procedures that allow combining the opinions of several experts regarding the alternatives presented to them [1, 2]. In most cases those procedures use the so-called pair-wise comparison method that assumes that an expert may prefer one alternative to another one while comparing them.

As each expert has a unique experience of solving specific problems, the opinions of various experts may significantly differ (indeed, there are many factors that affect an expert's preferences). This variety of expert assessments may cause a situation where some of them are unable to adequately express any degrees of preference by comparing two or more available alternatives. That may be caused by insufficient competence of the expert in an area of knowledge that pertains to the task or due to the fact that the expert is incapable of identifying the degree of preference of some of the presented variants over the others. In such situations such expert has to ensure fuzzy preference relation [3] or abandon the assessment of the presented pair of alternatives. A non-trivial task arises whereas missed data must be recovered in order to obtain acceptable results of expert assessment.

In practice, there are several approaches to managing sets of data with gaps. The first, most easily implementable, approach involves the elimination of copies with gaps from the set with further handling of only complete data [4]. This approach should be used in case gaps in data are isolated. Although even in this case there is a serious risk of "losing" important trends while deleting data. In the same case, when the number of gaps is too high, the removal of the respective copies may cause a data deficiency or even impossibility of further processing. The second approach involves using special modifications of data processing methods that allow gaps in sets of data. In [5], the authors describe a number of modifications of classification and clustering methods for managing data that contain missed values. And, finally, the third, most common, approach is the use of methods of evaluation of missed element values. Those methods help to fill in the gaps in sets of data based on certain assumptions regarding the values of the missing data. The applicability and efficiency of individual approaches, in principle, depends on the number of gaps in data and reasons of their occurrence. In terms of the nature of data origins, the categories of gaps that are usually identified are set forth in [6].

Quite frequently, empirical research has to reject the results of expert polls if some data is missing [7].

In [8], the effects of the above sets of pair-wise comparisons are researched. The paper compared the results for complete pair-wise comparison matrices and incomplete ones that were obtained by removing known elements from complete ones. The findings of [8] have shown that "random removal of up to 50 percent of comparisons provides good results with no loss of accuracy". Nevertheless, as this process is based on a priori knowledge of the complete pair-wise comparison matrix it is not applicable in practice. Thus, [8] suggests – for the cases of incomplete pair-wise comparison matrices – using methods that allow "completing" the matrix. A strong argument in support of this approach is set forth in [9]: "as a rule, a scenario with missed values disrupts the rating more significantly than the same scenario with a value". A system that helps build fuzzy preference relations in a solution was suggested in [10]. In the decision-making group, procedures that correct the absence of knowledge in a specific expert using information provided by the other experts, along with some aggregation procedures can be found in [11] and [12]. Those approaches have a number of disadvantages some of which are noted by the authors of [13].

In Russian literature, there are also works related to the potential solutions of the above problems, e.g. [14], however the approaches used in them do not provide a clear solution.

1. Problem definition

In the classic Saaty setting [15] there is a certain set of objects $O_1, O_2, \ldots, O_N$ (possible actions, parameters, alternative solutions, etc.) with a certain hierarchy. An expert's quantitative judgement regarding a pair of objects (O_i, O_j) is represented with a matrix of size n×n: $A=(a_{ij})$, ($i, j=1, 2, \ldots, n$), where the numbers a_{ij}, of which the matrix consists correspond with the object's significance O_j compared to O_i and are non-negative. In order to identify the quantitative indicators of the relative significance of the compared objects the method suggests a scale of relative comparisons expressed in whole numbers from 1 to 9. Objects with equal significance are rated "1". The ratings along the main diagonal of the matrix are also "1" (objects are compared to themselves), i.e. $a_{ii}=1$. The Saaty matrix is antisymmetrical about the main diagonal, i.e. $a_{ij}=1/a_{ji}$.

Further, in [15], after the quantitative judgements regarding the pairs (O_i, O_j) have been formed in numeric expressions in terms of a_{ij} the task comes down to associating each of the compared objects numeric weights that would best match the stated expert judgements. In order to find the priority vector it is required to find the vector ω that fulfils the condition $A\omega=\lambda_{max}\omega$. According to the theorem on the existence and uniqueness, while solving the eigenvalue problem for the non-negative matrix as per [15, 16] the resultant eigenvector is found, which after normalization becomes the priority vector of the compared objects.

We are seeking the solution for the case when in the matrix A some assessments are not defined, i.e. $\exists a_{ij}:a_{ji}=NA$ (NA stands for Not Available).

2. Method description

The incomplete pair-wise comparison matrix $\overline{A}$ is easily transformed into the skew-symmetric matrix $\overline{\overline{A}}$ by taking logarithms of the elements of the matrix of coefficients.

The weights W_i are transformed into V_i=ln(W_i), the pair-wise comparison coefficients matrix $\overline{S}_{ij} = \ln(S_{ij})$, while the residuals matrix $F_{ij} = \left(\frac{S_{ij} \times W_j}{W_i} - 1\right)$ is transformed into the functionals matrix $\overline{F}_{ij} = \left(\overline{S}_{ij} + V_j - V_i\right)$ and becomes skew-symmetric.

$$W_i, S_{ij}, F_{ij} = \left(\frac{S_{ij} \times W_j}{W_i} - 1\right) \to V_i = \ln\left(W_i\right),$$
$$\overline{S}_{ij} = \ln\left(S_{ij}\right), \overline{F}_{ij} = \left(\overline{S}_{ij} + V_j - V_i\right) \quad (1)$$

Having identified the values V_i, we perform a backward transformation:

$$V_i \to W_i = \frac{\exp(V_i)}{\sum_{j=1}^{N} \exp(V_j)} \quad (2)$$

in order to fulfil the Saaty weight normalization requirement, i.e.

$$\sum_{i=1}^{N} W_i = 1, W_i > 0 \left(i = 1, \ldots, N\right).$$

The matrix may be diagonalized in order to make it positive above the main diagonal. However, it should be noted that such diagonalization is not always possible even in case of a single expert and admittedly unnecessary as only the matrix graph connectivity matters. For a group of experts diagonalization is necessary, as depending on the vertex degree the upper and lower assessments swap places. It is assumed that for a group of experts total agreement is unachievable, while their preference coefficient $\overline{S}_{ij}$ is within a certain range:

$$B_{ij} \le \overline{S}_{ij} \le T_{ij} \quad (3)$$

The lower bound B_{ij} (Bottom) corresponds to the minimal value, while the upper bound T_{ij} (Top) corresponds to the maximum value. In this paper we examine the implementation of the algorithm for the upper estimates $T_{i,j}$ or one expert $\overline{S}_{i,j}$.

Due to the particular properties of the matrix T_{ij} the initial values of weights can be random constant values in order not to be confused by the weight increment signs. The value of change depends on the type of line/column. The type equals to "-1" when in the j-th line of the matrix ($E_1 - \ldots - N_1; \ldots; E_n - \ldots - N_n$) above the main diagonal there are only indeterminate values (*NA*). The type equals to "1" when in the i-th column of the matrix ($E_1 - \ldots - N_1; \ldots; E_n - \ldots - N_n$) above the main diagonal there are only indeterminate values (*NA*). In those cases when actual data are present both in the j-th line and in the i-th column it is obvious that the type of the first object always equals to "1", while the type of the last object equals to "-1".

Initially the matrix $\left\|\overline{F}_{ij}\right\|$ equals to the matrix $\left\|\overline{S}_{ij}\right\|$. The arbitrary choice of the weights is due to the fact that the result does not depend on the choice of the initial approximation

$$\overline{F}_{ij} = \left(\overline{S}_{ij} + \left[V_j + const\right] - \left[V_i + const\right]\right) = \left(\overline{S}_{ij} + \left[V_j\right] - \left[V_i\right]\right). \quad (4)$$

The first important stage is finding the hardest contradiction. We seek the optimal weight displacement for all lines through the optimization of the weight V_i that involves both the j-th line and the i-th column. We chose lines in the descending order (i=N,…,1) until <u>all displacements</u> become zero[1].

Then, we take

$$\left|\Delta V_i\right| \le \varepsilon C = 1{,}0e^{-7}. \quad (5)$$

Weight change happens step by step in accordance with

$$V_i = V_i + \Delta V_i. \quad (6)$$

After the next step ΔV_i becomes zero.

The value ΔV_i is chosen using the algorithm based on the modulus optimization in the matrix $\left\|\overline{F}_{ij}\right\|$, but only as regards the i-th step (i-th line).

We minimize the maximum moduli of the following values

$$\max_{\square}\left\{\left|R^i_{\max} - \Delta^i\right|, \left|R^i_{\min} - \Delta^i\right|, \left|C^i_{\max} + \Delta^i\right|, \left|C^i_{\min} + \Delta^i\right|\right\} \to \min_{\Delta}\square \quad (7)$$

for all i=1,…,N, where N is the dimension of the problem; $R^i_{\max}$ is the maximum value above the diagonal in the i-th line; $R^i_{\min}$ is the minimum value above the diagonal in the i-th line; $C^i_{\max}$ is the maximum value above the diagonal in the i-th column; $C^i_{\min}$ is the minimum value above the diagonal in the i-th column.

The modified weight is calculated using the following formulas:

$$W^i = W^i + \Delta^i. \quad (8)$$

The respective solutions for any line are identified using the formulas given in Table 1.

The next step is the removal of cells. If it is allowable according to the connectivity criterion[2], we remove the first edge belonging to the right-hand side of the «floater» against which in the line there is an edge from the «left-hand» side of the «floater», when the edge (i,j) is the only edge that connects non-overlapping node (object) subset.

Upon removal of the edge the solution restarts from the initial conditions.

It should be noted that it is important to track possible changes in the type of the "line/column". Usually the change

[1] For instance, for a matrix with the dimension of 10 the number of required iterations is around 15.

[2] The connectivity criterion is important in cases when the spanning graph may be discontinued.

Table 1

Type	Condition	Calculation formula
0	There are elements in the line and elements in the adjacent column	
	if $R_{\max} > \max\{\lvert R_{\min}\rvert, \lvert C_{\min}\rvert, C_{\max}\}$, then	$\Delta^{(I)} = \frac{1}{2}\cdot\min\{R_{\max}+R_{\min}; R_{\max}-C_{\max}\}$
	if $C_{\max} > \max\{\lvert R_{\min}\rvert, \lvert C_{\min}\rvert, R_{\max}\}$, then	$\Delta^{(II)} = \frac{1}{2}\cdot\min\{-C_{\max}-C_{\min}; R_{\max}-C_{\max}\}$
	if $-R_{\max} > \max\{\lvert C_{\min}\rvert, C_{\max}, R_{\max}\}$, then	$\Delta^{(III)} = \frac{1}{2}\cdot\min\{R_{\max}+R_{\min}; R_{\min}-C_{\min}\}$
	if $-C_{\max} > \max\{\lvert R_{\min}\rvert, C_{\max}, R_{\max}\}$, then	$\Delta^{(IV)} = \frac{1}{2}\cdot\min\{-C_{\max}-C_{\min}; R_{\min}-C_{\min}\}$
	otherwise	$\Delta^{(V)} = 0$
1	All components of the column are equal to NA	$\Delta V_i = \frac{(R^i_{\max}+R^i_{\min})}{2}$
-1	All components of the line are equal to NA	$\Delta V_i = \frac{-(C^i_{\max}+C^i_{\min})}{2}$

is from “0” to “1”, i.e. when the last significant element in the respective column disappears.

The next stage is the procedure that consists of two embedded cycles. The external involves finding the next “boundary cycle” in the remaining matrix. The internal cycle involves finding the edges of which the “boundary cycle” consists.

The end of the external cycle is the condition:

$$\left| sup \left\| F_{ij} \right\| \right| < \varepsilon_F = 1{,}0 \cdot e^{-5} \tag{9}$$

The resulting solution will be the solution of the problem for the single expert case. For the group of exerts case, this solution is for reference only ($V_i^{\text{ref.}}$).

It is required to take into consideration the remoteness of the lower bounds defined by the matrix $\left\| B_{ij} \right\|$.

For that purpose, let us recover the initial configuration of the graph.

Let us calculate the resulting matrix $\left\| R_{ij} \right\|$:

$$R_{ij} = -(B_{ij} + V_j - V_i) \tag{10}$$

and remove all negative elements.

Some edges can complement the reference configuration.

Then, let us perform the following procedure that also has two cycles.

In the external cycle we define the most critical edge that can be dropped. For that purpose, the scale of single displacement for each weight U_i is calculated, the size of single displacement step for each edge (i,j) D_{ij} and the size of the step h using the formula

$$h = \min_{i,j}\left(\frac{R_{ij}}{D_{ij}}\right). \tag{11}$$

In this paper we omit the formulas for D_{ij} and U_i.

Next, in the internal cycle we identify the virtual optimal solution using the formulas:

$$V_i^{\text{opt.}} = V_i^{\text{ref.}} - h \times U_i, \ (i = 1, \ldots, N) \tag{12}$$

$$R_{ij}^{\text{opt.}} = R_{ij}^{\text{ref.}} - h \times D_{ij}, \ (i=1,\ldots,N-1;\ j=i+1,\ldots,N) \tag{13}$$

If the emerged zero elements $R_{ij}^{\text{opt.}}$ do not cause the graph to lose connectivity, the edge of the first one can be omitted. If, on the contrary, the output matrix loses connectivity, the

Table 2. Initial coefficient values

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10
O1	1	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{3}$	NA	NA	NA	$\frac{1}{4}$
O2	$\frac{3}{1}$	1	$\frac{1}{5}$	NA	$\frac{1}{5}$	$\frac{1}{4}$	NA	NA	NA	$\frac{1}{3}$
O3	$\frac{5}{1}$	$\frac{5}{1}$	1	$\frac{3}{1}$	NA	$\frac{3}{1}$	$\frac{3}{1}$	$\frac{5}{1}$	$\frac{5}{1}$	$\frac{3}{1}$
O4	$\frac{3}{1}$	NA	$\frac{1}{3}$	1	$\frac{1}{3}$	NA	NA	NA	NA	$\frac{1}{3}$
O5	$\frac{7}{1}$	$\frac{5}{1}$	NA	$\frac{3}{1}$	1	$\frac{3}{1}$	$\frac{5}{1}$	$\frac{5}{1}$	$\frac{5}{1}$	$\frac{3}{1}$
O6	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{1}{3}$	NA	$\frac{1}{3}$	1	NA	NA	NA	$\frac{1}{4}$
O7	NA	NA	$\frac{1}{3}$	NA	$\frac{1}{5}$	NA	1	NA	NA	$\frac{1}{5}$
O8	NA	NA	$\frac{1}{5}$	NA	$\frac{1}{5}$	NA	NA	1	NA	$\frac{1}{5}$
O9	NA	NA	$\frac{1}{5}$	NA	$\frac{1}{5}$	NA	NA	NA	1	$\frac{1}{5}$
O10	$\frac{4}{1}$	$\frac{3}{1}$	$\frac{1}{3}$	$\frac{3}{1}$	$\frac{1}{3}$	$\frac{4}{1}$	$\frac{5}{1}$	$\frac{5}{1}$	$\frac{5}{1}$	1

Table 3. Logarithms of coefficient values

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10
O1	0	-1,09861	-1,60944	-1,09861	-1,94591	-1,09861	NA	NA	NA	-1,38629
O2	1,098612	0	-1,60944	NA	-1,60944	-1,38629	NA	NA	NA	-1,09861
O3	1,609438	1,609438	0	1,098612	NA	1,098612	1,098612	1,609438	1,609438	1,098612
O4	1,098612	NA	-1,09861	0	-1,09861	NA	NA	NA	NA	-1,09861
O5	1,94591	1,609438	NA	1,098612	0	1,098612	1,609438	1,609438	1,609438	1,098612
O6	1,098612	1,386294	-1,09861	NA	-1,09861	0	NA	NA	NA	-1,38629
O7	NA	NA	-1,09861	NA	-1,60944	NA	0	NA	NA	-1,60944
O8	NA	NA	-1,60944	NA	-1,60944	NA	NA	0	NA	-1,60944
O9	NA	NA	-1,60944	NA	-1,60944	NA	NA	NA	0	10,65705
O10	1,386294	1,098612	-1,09861	1,098612	-1,09861	1,386294	1,609438	1,609438	1,609438	0

Table 4. Reordering of objects

	O5	O3	O10	O6	O2	O4	O1	O7	O8	O9
O5	0	NA	1,0986	1,0986	1,7041	1,0986	1,9459	1,7041	1,7041	1,7041
O3	NA	0	1,0986	1,0986	1,7041	1,0986	1,7041	1,0986	1,7041	1,7041
O10	-1,0986	-1,0986	0	1,3863	1,0986	1,0986	1,3863	1,7041	1,7041	1,7041
O6	-1,0986	-1,0986	-1,3863	0	1,3863	NA	1,0986	NA	NA	NA
O2	-1,7041	-1,7041	-1,0986	-1,3863	0	NA	1,0986	NA	NA	NA
O4	-1,0986	-1,0986	-1,0986	NA	NA	0	1,0986	NA	NA	NA
O1	-1,9459	-1,7041	-1,3863	-1,0986	-1,0986	-1,0986	0	NA	NA	NA
O7	-1,7041	-1,0986	-1,7041	NA	NA	NA	NA	0	NA	NA
O8	-1,7041	-1,7041	-1,7041	NA	NA	NA	NA	NA	0	NA
O9	-1,7041	-1,7041	-1,7041	NA	NA	NA	NA	NA	NA	0

last successful attempt is "memorized" as a real optimal value for the weights $R_{ij}^{opt.}$.

If the matrix $R_{ij}^{opt.}$ matches the connectivity matrix of the matrix $R_{ij}^{ref.}$, the single displacement U_i=1 (type i=1); U_i=0 (type i=–1).

After node run in $R_{ij}^{opt.}$ the optimal solution takes its final form. It connects to the lower bound and is the optimal solution deduced using the higher bound data. It is assumed that the conflict of interests is at the higher bound where each expert wants to define his/her own priorities.

3. Example of practical application of the method

Let us assume there is a pair-wise comparison matrix filled by experts using the Saaty method.

The matrix is not complete (missing assessments are marked *NA*), because the experts could not express their preferences while comparing some pairs of objects (e.g. O_1 and O_7, O_2 and O_4, etc.).

Let us transform the incomplete pair-wise comparison matrix into a skew-symmetric matrix by taking logarithms of the elements of the matrix of coefficients.

The matrix after diagonalization that we perform so that the matrix is positive above the main diagonal is shown in Table 4.

Let us examine the implementation of the algorithm[1] for upper estimates $T_{i,j}$ or single expert $\overline{S}_{i,j}$.

Due to the particular properties of the matrix T_{ij} the initial values of weights (column B, Table 5) can be random constant value in order not to be confused by the weight increment signs. The value of change (column C, Table 5), as we said above, depends on the type of line/column.

In order to find the hardest contradiction we seek the optimal weight displacement for all lines through the optimization of the weight V_i that involves both the j-th line and the i-th column. We select lines in the descending order (i=N,…,1) until **all displacements** in column C (Table 5) become zero.

Using the above algorithm, we find the boundary modulo cycle for the initial conditions. The result is given in table 6.

Which cell must be removed? On the right-hand side of the «floater» that are (2,3), (3,4), (4,5), (5,7), on the left-hand side that is only the edge (2,7). The edges on the right-hand side indicate that the 2-nd object is better that the 3-rd one, the 3-rd object is better than the 4-th one, the 4-th one is better that the 5-th one, the 5-th one is better than the 7-the one $e^{0,8029}$=2,3632 times.

[1] As the matrix is skew-symmetric let us omit the part below the main diagonal.

Table 5. Input data

A	B	C	D	E	F	G	H	I	J	K	L	M	N
i	V_j	V_j	Тип	1	2	3	4	5	6	7	8	9	10
1	10,00000	1,70280	1	0	NA	1,2528	1,3863	1,7918	1,2528	2,1528	1,7118	1,7918	1,6247
2	10,00000	1,48430	1		0	1,3863	1,3863	1,7047	1,0968	1,8718	1,2528	1,7047	1,8718
3	10,00000	0,24275	0			0	1,5041	1,2528	1,3863	1,5041	1,7918	1,8718	1,7047
4	10,00000	0,05265	0				0	1,6094	NA	1,2528	NA	NA	NA
5	10,00000	-0,20275	0					0	NA	1,3863	NA	NA	NA
6	10,00000	-0,06675	0						0	1,2528	NA	NA	NA
7	10,00000	-1,70280	-1							0	NA	NA	NA
8	10,00000	-1,52230	-1								0	NA	NA
9	10,00000	-1,78825	-1									0	NA
10	10,00000	-1,74825	-1										0

Table 6

A	B	C	D	E	F	G	H	I	J	K	L	M	N
i	V_j	V_j	Тип	1	2	3	4	5	6	7	8	9	10
1	10,81469	0,00	1	0	0	0,5956	0,0279	-0,373	-0,354	-0,5956	-0,176	-0,3619	-0,3686
2	10,74094	0,00	1		0	0,8029	0,1016	-0,387	-0,436	-0,8029	-0,5612	-0,3753	-0,0477
3	10,15750	0,00	0			0	0,8029	-0,255	0,4365	-0,5871	0,5612	0,3753	0,3686
4	9,45626	0,00	0				0	0,8029	0	-0,1372	0	0	0
5	8,64972	0,00	0					0	0	0,8029	0	0	0
6	9,20767	0,00	0						0	0,1114	0	0	0
7	8,06628	0,00	-1							0	0	0	0
8	8,92692	0,00	-1								0	0	0
9	8,66097	0,00	-1									0	0
10	8,82140	0,00	-1										0

Table 7

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	V_i	ΔV_j	W_i	Тип V_i	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
v_1	2,688766	0,00000	0,162651	1	0	0	0	0	0	0	0	0	$-1e^{-16}$	$-2e^{-16}$
v_2	2,229765	0,00000	0,102782	1		0	0	0	0	0	0	$-1e^{-16}$	0	0
v_3	2,768765	0,00000	0,176197	1			0	0	0	0	$-2e^{-16}$	$-1e^{-16}$	$1e^{-6}$	$1e^{-6}$
v_4	2,517465	0,00000	0,137044	1				0	0	0	$-2e^{-16}$	0	0	0
v_5	2,650965	0,00000	0,156618	1					0	0	$2e^{-16}$	0	0	0
v_6	2,517465	0,00000	0,137044	1						0	$-2e^{-16}$	0	0	0
v_7	1,264665	0,00000	0,039154	-1							0	0	0	0
v_8	0,976965	0,00000	0,029365	-1								0	0	0
v_9	0,896966	0,00000	0,027107	-1									0	0
v_{10}	1,064066	0,00000	0,032038	-1										0

As the result, the 2-nd object is better that the 7-th object $(2{,}3632)^4=31{,}1890$ times. But cell (2,7) shows the opposite, i.e. the 2-nd object is worse than the 7-th one 2.3632 times. Thus, a contradiction arises. On the one hand the 2-nd object is 31.1890 times better that the 7-th one, on the other hand its is worse 2.3632 times. But most importantly the cycle can not be improved. Attempting to modify the weights of objects in the cycle automatically increases the modulus.

Therefore, the edge (2,3) must be removed. That is because from 0.8029 to 0.1016 (edge (2,4) the reduction is the greatest.

Following the algorithm we implement the embedded cycles (external and internal). The resulting solution (column B, Table 7) is the solution of the problem for the single expert case.

In order to account for the remoteness of the lower bounds defined by the matrix $\|B_{ij}\|$ let us recover the initial configuration of the graph (Table 8).

Next, let us calculate using (10) the resultant matrix R_{ij} and remove all negative elements (Table 9).

Some edges, e.g. (1,8) can complement the reference configuration. Next, in the external cycle we define the most

Table 8

	w_{cp}			w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
w_1	2,688766			0	0	-0,773	-0,745	-1,3485	-0,745	-0,3677	0,2077	0,1824	0,1206
w_2	2,229765				0	-1,232	-0,981	-1,8075	-1,204	-0,539	0,3365	-0,1713	-0,2206
w_3	2,768765					0	-1,002	-0,5754	-0,665	0,4073	0,4055	0,3677	0,4519
w_4	2,517465						0	-1,3863	0	0,5596	0	0	0
w_5	2,650965							0	0	0,47	0	0	0
w_6	2,517465								0	0,5596	0	0	0
w_7	1,264665									0	0	0	0
w_8	0,976965										0	0	0
w_9	0,896966											0	0
w_{10}	1,064066												0

Table 9

	w_{cp}			w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
w_1	2,688766			0	0	0	0	0	0	0	0,2077	0,1824	0,1206
w_2	2,229765				0	0	0	0	0	0	0,3365	0	0
w_3	2,768765					0	0	0	0	0,4073	0,4055	0,3677	0,4519
w_4	2,517465						0	0	0	0,5596	0	0	0
w_5	2,650965							0	0	0,47	0	0	0
w_6	2,517465								0	0,5596	0	0	0
w_7	1,264665									0	0	0	0
w_8	0,976965										0	0	0
w_9	0,896966											0	0
w_{10}	1,064066												0

Table 10

	V_i	W_i		w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
w_1	2,48106	0,1611740		0	0	0	0	0	0	0	$1e^{-6}$	0	0
w_2	2,02867	0,1025226			0	0	0	0	0	0	0,1354	0	0
w_3	2,56767	0,1757537				0	0	0	0	0,2062	0,2044	0,1666	0,2508
w_4	2,31637	0,1366993					0	0	0	0,3585	0	0	0
w_5	2,44987	0,1562230						0	0	0,2689	0	0	0
w_6	2,11637	0,1119200							0	0,1585	0	0	0
w_7	1,26467	0,0477550								0	0	0	0
w_8	0,97697	0,0358156									0	0	0
w_9	0,89697	0,0330620										0	0
w_{10}	1,06407	0,0390751											0

critical edge that can be dropped. For that purpose, the scale of single displacement for each weight U_i is calculated, the size of single displacement step for each edge (i, j) D_{ij} and the size of the step using the formula (11), in the internal cycle we define the virtual optimal solution using the formulas (12) and (13).

In the considered example the matrix $R_{ij}^{opt.}$ matches the connectivity matrix of the matrix $R_{ij}^{ref.}$, the single displacement U_i=1 (type i=1); U_i=0 (type i=–1).

After (1,10) and (1,9) nodes run in $R_{ij}^{opt.}$ the optimal solution takes its final form (Table 10).

The solution connects with the edge (1,8) to the lower bound and is the optimal solution deduced using the higher bound data. It is assumed that the conflict of interests is at the higher bound where each expert wants to define his/her own priorities.

References

1. Evangelos T. Multi-criteria decision making methods: a comparative study. Dordrecht: Kluwer Academic Publishers; 2000.

2. Fodor J, Roubens M. Fuzzy preference modelling and multicriteria decision support. Dordrecht: Kluwer Academic Publishers; 1994.

3. Xu ZS. Goal programming models for obtaining the priority vector of incomplete fuzzy preference relation. International Journal of Approximate Reasoning 2004;36:3:261–270.

4. Little RJA, Rubin DB. Statistical analysis with missing data. Moscow: Financy i statistika; 1991.

5. Garcia-Laencina PJ, Sanco-Gomez J-L, Figueiras-Vidal AR. Pattern classification with missing data: a review. London: Springer-Verlag Limited; 2009.

6. Schafer JL, Graham JW. Missing data: Our view to the state of the art. Psychological methods 2002;7(2):147–177.

7. Millet I. The effectiveness of alternative preference elicitation methods in the analytic hierarchy process. J. Multi-Criteria Decis. Anal. 1997l;6(1):41–51.

8. Carmone FJ, Kara Jr A, Zanakis SH. A Monte Carlo investigation of incomplete pairwise comparison matrices in AHP. Eur. J. Oper. Res. 2001;102(3):533–553.

9. Ebenbach DH, Moore CF. Incomplete information, inferences, and individual differences: The case of environmental judgements. Org. Behav. Human Decis. Process. 2000;81(1):1–27.

10. Alonso S, Cabrerizo FJ, Chiclana F, Herrera F, Herrera-Viedma E. An interactive decision support system based on consistency criteria. J. Mult.-Valued Log. Soft Comput. 2008;14(3–5):371–386.

11. Kim JK, Choi SH. A utility range-based interactive group support system for multiattribute decision making. Comput. Oper. Res. 2001;28(5):485–503.

12. Kim JK, Choi SH, Han CH, Kim SH. An interactive procedure for multiple criteria group decision making with incomplete information. Comput. Ind. Eng. 1998;35(1/2):295–298.

13. Chiclana F, Herrera-Viedma E, Alonso S. A Note on Two Methods for Estimating Missing Pairwise Preference Values. IEEE Transactions On Systems, MAN, and Cybernetics – Part B: Cybernetics 2009;39(6):1628-1633.

14. Karlov IA. Vosstanovlenie propushchennukh dannykh pri chislennom modelirovanii slozhnykh dynamicheskikh system [Recovery of missing data in computational modeling of complex dynamic systems]. SPbSPU Journal. Computer Science. Telecommunication and Control Systems 2013;6(186):137–144 [in Russian].

15. Saaty T. Decision making. Analytic hierarchy method. Moscow: Radio i sviaz; 1993.

16. Ashmanov SA. Matematicheskie modeli i metody [Mathematical models and methods]. Moscow: Moscow State University publishing, 1980 [in Russian].

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