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Use of deduced Esary-Proschan assessments for evaluation of system dependability

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...When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind...

William Thomson (Lord Kelvin)

Abstract. *In [1-2] it is shown that the widely known Esary-Proschan assessments [3-6] (EPA) are NP-complete [7]. In the process of their calculation a mutual cross-over of those assessments occurs despite the fact that the procedure of enumeration of complete sets of simple chains (SChs) and simple cuts (SCus) is performed all the way. This is confirmed by special research of these paradoxical phenomena in EPA conducted in [8] that concludes that EPAs are not assessments, as assessments cannot be NP-complete. In [7] it is clearly stated that in general an enumeration of a complete set of SCh (or SCu) alone already is an NP-complete problem. It implies directly that any NP-complete method cannot be an assessment one. In [9-10] a number of problems are classified depending on the associated computational complexity. As we can see out of those presented the most favourable is the intellectual intensity, as it allows controlling the computational process in the most desirable way, i.e. allows implementing the forced interruption principle (FIP) in regards to the computational procedure that is assessed by a certain parameter. For example, the parameter of achieved relative computational error. It should be noted that the devices, mechanisms and other systems we deal with in real life are called automated because such man-machine systems implement the FIP at the discretion of the human operator. We deal much less with automatic systems.* **The aim** *of this paper is to set forth the formal rules that allows quite easily the conventional NP-complete Esary-Proschan assessments to be transformed to the class of intelligent (IN-class) assessment methods that implement the FIP. Complete sets of SCh and SCu do not need to be enumerated here. Expanding the class of existing [1-6, 8, 11-29] methods that in one way or another implement the FIP is without a doubt a relevant problem for experts involved in structural dependability analysis of complex systems. It is an axiom that any of the tools of such system analysis, of which the exhaustive events (EE) are the "delivery nurse", contributes to the design of structurally dependent systems, while developing at the same time the analysis tool system itself. Essentially, the problem consists in casting the classic EPAs in the form of logic symbol multiplication (LSM) of logical operands the method uses.* **The result** *consists in the fact that we remove the "hardships" of NP-completeness from the classic EPAs and obtain a sufficiently efficient analysis tool.*

Keywords: *dependability, probability, two-pole network, simple chain, simple cut, working condition, faulty condition.*

Abbreviations: *EPA – Esary-Proschan assessment; SCh – simple chain; SCu – simple cut; FIP – forced interruption principle; EE – exhaustive events; LSM – logic symbol multiplication; RG – random graph; BCN – boundary couple nodes; TPN – two-pole network; WC – working condition; FC – faulty condition; PC – probability of connectedness; LAM – logic algebraic multiplication; cEEc – completion EE conjunction.*

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1. Introduction

In [9–10] a number of problems are classified depending on the associated computational complexity: a) linear; b) polynomial; c) exponential; d) N-factorial; e) intellectual intensity (*IN*). Among experts involved in structural dependability analysis of complex systems the last one has the following definition: intellectual intensity is the principle of forced interruption of the computation process by a human being, when he is satisfied with the achieved error of function estimation by the given period time. As EPA contains logic multiplication of logical operands, describing the state of SCh and SCu excluding their structural interdependency, this article includes LSM developed rules of such operands taking into account this dependency. As a result, classic EPAs in computational complexity transform into N-class, as they become forcibly interruptible.

The article proposes formal rules that allow the conventional NP-complete Esary-Proschan assessments to be transformed to the convenient tool, excluding enumeration of complete sets of SCh and SCu. Let us name such assessments as <u>reduced EPA</u>.

2. Problem definition

2.1. Generalities. Let it be a simple edge random graph (RG) (Fig. 1) [11–14, 30–31] G: <u>simple</u> excludes loops, parallels and isolated nodes; <u>random</u> means that its elements are either present in the graph with probability of p or absent with probability of $q = 1 - p$ (as $p + q = 1$, then presence and absence of the edge in the graph compose EE denoted as symbol I [31]); <u>edge</u> means that only edges are unreliable in the graph (this assumption is not essential, but simplifies the paradigm of the statement). Graph G contains a node set $V=\{v_i\}$ with the power $m_v=|V|$ and edges $L=\{l_{i,j}\}$ with the power $m_L|L|$, where the function of incidence and type adjacency $\Phi(l_{i,j})=v_i\&v_j$ reflects their interdependence: an edge is incident to its boundary couple nodes (BCN) where BCN are adjacent to each other by the edge $l_{i,j}$.

Let us assume two-pole network (TPN) be set on the graph G, whose pole nodes are denoted as s (source) and t (drain) with the parameters: $m_V=4$ and $m_L=5$. For example, as it is shown in Figure 1 (experts often define such structure as <u>*«bridge»*</u> [16]). It should be noted that currently TPN has transit nodes in the following form: $V^T_{s,t}=V\setminus\{s,t\}$. In this situation: $V^T_{s,t}=\{2,3\}$ (Fig. 1).

Figure 1b shows a graph with numbers renumbered by edges (using the method of «arc crossing» [16]), ranging from (m_V+1) with step 1 to (m_V+m_L), and Figure 1c presents the same renumbered RG, but here symbol «*l*» on edges is omitted for the purity of the picture. We will often resort to Figure 1. Thus, «continuous» numbering of RG elements is presented in Figure 1c. It should be noted that symbols «*v*» and «*l*» will be used rarely when creating any structures on RG (for example, SCh and SCu).

Let us form in TPN (Fig. 1c) a set of its SCh according to the principle: **«Every *<u>parent</u>*-node chooses a *<u>child</u>*-node from nodes adjacent with it and not occupied, that has the lowest number *k*** [17]»:

$$F_{s,t}=\left\{\begin{matrix} f_1=\{5,7,9\}, f_2=\{5,8\},\\ f_3=\{6,7,8\}, f_4=\{6,9\}\end{matrix}\right\}\Rightarrow\left(m_F=\left|F_{s,t}\right|=4\right). \qquad (1)$$

own in Figure 1, every transit node was both «*parent*» and «*child*», but s (source) was only «*parent*» and t (drain) – only «*child*».

With the same principle let us form SCu set (method of «continuous start» [17]):

$$R_{s,t}=\left\{\begin{matrix} r_1=\{5,6\}, r_2=\{6,7,8\},\\ r_3=\{5,7,9\}, r_4=\{8,9\}\end{matrix}\right\}\Rightarrow\left(m_R=\left|R_{s,t}\right|=4\right). \qquad (2)$$

SCh is in good order when all its edges are in working condition (WC): $\overline{\overline{f}}_n=\forall l_k\in f_n\left[\bar{\bar{l}}_k\right]$, and is in fault order when even one of its edges is in faulty condition (FC): $\overline{f}_n=\exists l_k\in f_n\left[\bar{l}_k\right]$ (here $\forall$ is an *«generality»* quantifier, $\exists$ is an *«existential»* quantifier, and $\in$ means *«belongs to»*

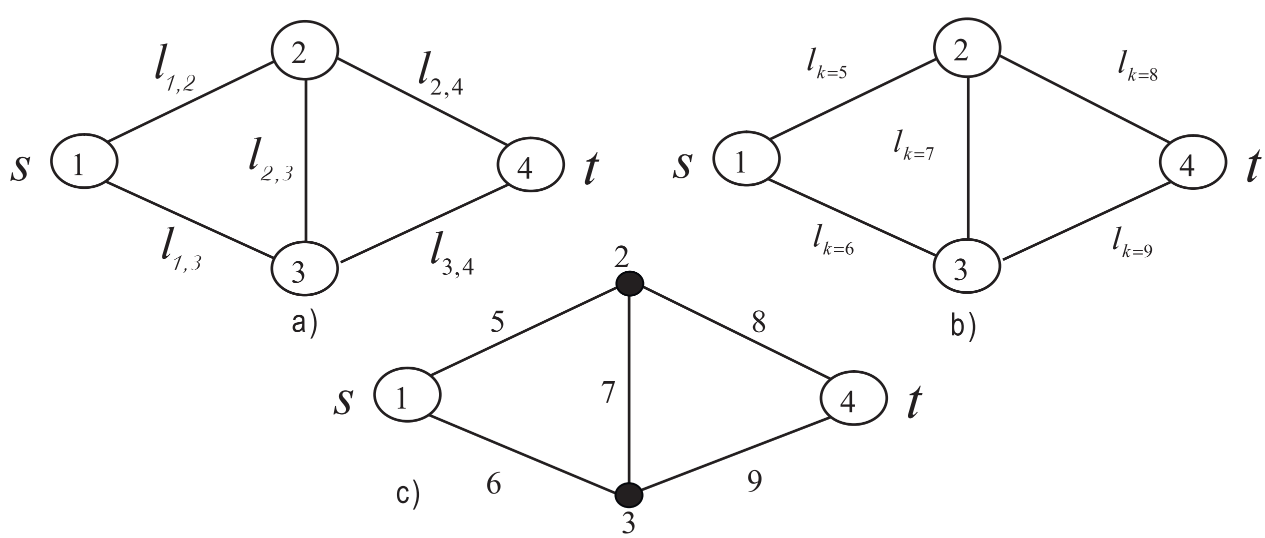

Figure 1. Renumbered simple random edge graph

[30–33]). SCu is in good order when all its edges are in FC: $\bar{\bar{r}}_n = \forall l_k \in r_n\left[\bar{l}_k\right]$, and is in fault order when even one of its edges is in WC: $\bar{r}_n = \exists l_k \in r_n\left[\bar{\bar{l}}_k\right]$.

2.2 Explaining example. Let us take initial data on AF of RG TPN edges:

$$(L \neq \varnothing) \Rightarrow (p_5 = 0{,}5,\ p_6 = 0{,}6,\ p_7 = 0{,}7,\ p_8 = 0{,}8,\ p_9 = 0{,}9), \quad (3)$$

where $\Rightarrow$ is the «*sequence*» symbol [31–33].

Let us calculate an exact value of probability of connectedness (PC) of our TPN. Firstly, based on (1), let us describe complete event of connectedness (CoC) of TPN (when TPN nodes-poles are connected with even one working SCh), denoted as symbol $E_{s,t}$:

$$E_{s,t} = 5\bullet 7\bullet 9 + 5\bullet 8\bullet \overline{7\bullet 9} + \\ + 6\bullet 7\bullet 8\bullet \bar{5} + 6\bullet 9\bullet\left(\bar{5}\bullet\overline{7\bullet 8} + 5\bullet\bar{7}\bullet\bar{8}\right), \quad (4)$$

where $\bullet$ is the symbol of logic algebraic multiplication (LAM) (* is the symbol of LSM, in which in contrast to LAM, structural dependence of the multiplied logical operands is taken into account).

It should be noted that here (and hereafter) we don't «load» the edge WC with double bar over elements of operands as it shows that the edge is in faulty condition.

Double negation is the edge WC (bar cancels bar), i.e. «faulty» and «working» are synonyms. Let us write calculation formula for $P_{s,t}$, based on the result (4):

$$P_{s,t} = p_5 \cdot p_7 \cdot p_9 + p_5 \cdot p_8 \cdot \overline{p_7 \cdot p_9} + \\ + p_6 \cdot p_7 \cdot p_8 \cdot q_5 + p_6 \cdot p_9 \cdot \left(q_5 \cdot \overline{p_7 \cdot p_8} + p_5 \cdot q_7 \cdot q_8\right). \quad (5)$$

Let us expand in (5) our initial data (3) and deduce that the exact (within the accuracy of initial data (3)) PC value of our TPN will be equal to:

$$P_{s,t} = 0{,}315 + 0{,}148 + 0{,}168 + 0{,}136 = \underline{\underline{0{,}766}}. \quad (6)$$

Now we have some «criterion» in the form of $P_{s,t}$=0,766, that allows us to make a comparison between «old» and «new», i.e. offered.

2.3. Task definition. Firstly, let us write according to [3] the formal representation of EPA for general cases:

$$\left.\begin{aligned} E^B_{s,t} &= I - \prod_{n=1}^{m_F} \bullet \bar{f}_n = I - \prod_{n=1}^{m_F} \bullet \left(I - \prod_{e=1}^{m_n} \bullet \bar{\bar{l}}_{k_e} \right) = \\ &\quad I - \prod_{n=1}^{m_F} \bullet \left(\coprod_{e=1}^{m_n} \bullet \bar{\bar{l}}_{k_e} \right) = \coprod_{n=1}^{m_F} \bullet \left(\coprod_{e=1}^{m_n} \bullet \bar{\bar{l}}_{k_e} \right) \\ E^H_{s,t} &= \prod_{n=1}^{m_R} \bullet \bar{r}_n = \prod_{n=1}^{m_R} \bullet \left(I - \prod_{e=1}^{m_n} \bullet \bar{l}_{k_e} \right) = \prod_{n=1}^{m_R} \bullet \left(\coprod_{e=1}^{m_n} \bullet \bar{l}_{k_e} \right) \end{aligned}\right\}. \quad (7)$$

where $\prod$ is the conjunction (logical (or arithmetical) products); $\coprod$ is the complement of EE (or I) conjunction (logical (or arithmetical)) (cEEc).

If we exclude the intermediate transformation, then EPA should be written as follows:

$$\left.\begin{aligned} E^B_{s,t} &= I - \prod_{n=1}^{m_F} \bullet \bar{f}_n = \coprod_{n=1}^{m_F} \bullet \left(\coprod_{e=1}^{m_n} \bullet \bar{\bar{l}}_{k_e} \right) \\ E^H_{s,t} &= \prod_{n=1}^{m_R} \bullet \bar{r}_n = \prod_{n=1}^{m_R} \bullet \left(\coprod_{e=1}^{m_n} \bullet \bar{l}_{k_e} \right) \end{aligned}\right\}. \quad (8)$$

In relation to our RG example (Fig. 1), these events (according to (1), (2) and (8)) can be graphically represented as shown in Fig. 2.

The redundant TPN CoC on SCh is read as**:** «The complement of EE conjunction of the complement of EE conjunction of WC edges included in n-th SCh». At the same time, insufficient description of TPN CoC on SCu reads differently: «The conjunction of complement of EE conjunctions of FC edges included in n-th SCu».

If TSE PC TPN have been calculated, then the calculation of approximate estimate of $P^{\approx}_{s,t}$, relative error of $\Delta_{s,t}$ (a priori) and absolute error of $W_{s,t}$ (a posteriori) is simple:

$$P^{\approx}_{s,t} = \frac{P^B_{s,t} + P^H_{s,t}}{2},\ \Delta_{s,t} = \frac{\left|P^B_{s,t} - P^H_{s,t}\right|}{2}, \\ W_{s,t} = \left|P_{s,t} - P^{\approx}_{s,t}\right| \text{ and } W_{s,t} < \Delta_{s,t}. \quad (9)$$

1: It is required to prove (the first in (8)), that eliminating of structural interdependency of logical operands of conjunction leads to the redundancy in description of TPN CoC.

2. It is required to prove (the second in (8)), that eliminating of structural interdependency of logical operands of cEEc leads to insufficiency in description of TPN CoC.

3. The proved statements should be illustrated by graphical and numerical examples.

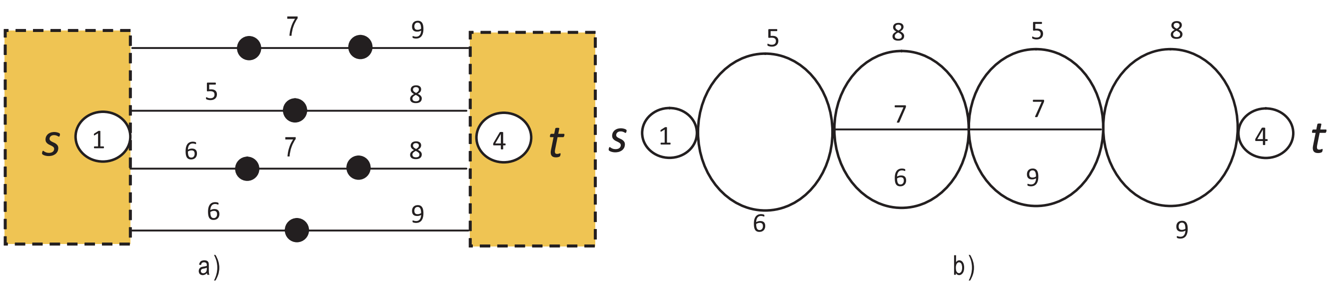

Figure 2. “Bridge” in terms of EPA elemental events

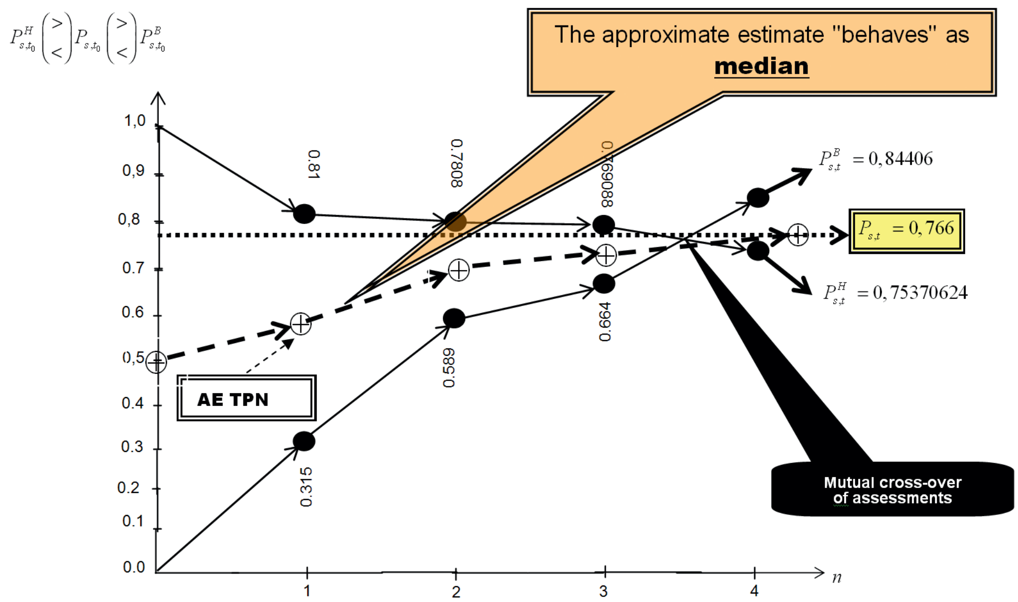

Figure 3. Graph of the dual image of the behavior of the PC TSE for the "bridge" under the adopted input data $\forall l_{(5\le\nu\le9)}\in L\left[p_{(5\le\nu\le9)}=0,(5\le\nu\le9)\right]$

3. Task solution

In accordance with the rules (7-8) and sets of SCh (1) and SCu (2) sets EPA in relation to the «bridge» (Fig. 1 and Fig. 2) looks the following:

$$\left.\begin{aligned}E^U_{s,t}&=\overline{\overline{5\bullet7\bullet9}\bullet\overline{5\bullet8}\bullet\overline{6\bullet7\bullet8}\bullet\overline{6\bullet9}}\\E^L_{s,t}&=\overline{\overline{5}\bullet\overline{6}}\bullet\overline{\overline{6}\bullet\overline{7}\bullet\overline{8}}\bullet\overline{\overline{5}\bullet\overline{7}\bullet\overline{9}}\bullet\overline{\overline{8}\bullet\overline{9}}\end{aligned}\right\}. \tag{10}$$

Based on (10), we deduce the following calculations formulas for TSE PC TPN (for UB and LB PC TPN):

$$\left.\begin{aligned}P^U_{s,t}&=\overline{\overline{p_5\cdot p_7\cdot p_9}\cdot\overline{p_5\cdot p_8}\cdot\overline{p_6\cdot p_7\cdot p_8}\cdot\overline{p_6\cdot p_9}}\\P^L_{s,t}&=\overline{q_5\cdot q_6}\cdot\overline{q_6\cdot q_7\cdot q_8}\cdot\overline{q_5\cdot q_7\cdot q_9}\cdot\overline{q_8\cdot q_9}\end{aligned}\right\}. \tag{11}$$

As in this case we have step-by-step «accumulation» of some numerical value as a result of multiplication (not additivity) then this *«step-by-step principle»* will be represented by arrows. Let us use our initial data (3) and calculate TSE PC TPN on EPA: UB $P^U_{s,t}$, and LB $P^L_{s,t}$ bear in mind (6), that $P_{s,t}$=0,766:

$$\left.\begin{aligned}P^U_{s,t}&=0,315\to0,589\to0,664\to0,84406=P^U_{s,t}\\P^L_{s,t}&=0,81\to0,7808\to769088\to0,75370624=P^L_{s,t}\end{aligned}\right\}. \tag{12}$$

Let us use the results and build the graph of dual image [17, 20] of the «behavior» of the TSE PC of our TPN (Fig. 1c, 2 and 3).

Analyzing the results we can see the paradox of classic EPA: a) UB PC TPN begins to accumulate from 0, and LB – from 1; b) TSE PC TPN are mutual intercrossing; c) after intercrossing TSE PC TPN diverge.

The axis of probabilities and its domains and intervals presented in Figure 4 should help us in the following questions.

Logically arguing, it is possible to understand that conjunctions and their additions (7) and (8) are nonequilibrium in different cases. For example, the arithmetical product of $\prod$ in initial state are set to entity (as the sum is set to «zero» in initial state):

$$\left.\begin{aligned}n&=\overline{1,N}\\n&:=0\end{aligned}\right\}\Rightarrow\prod:=1. \tag{13}$$

The conjunction in initial state also should be reduced to the form of «full possible event group»:

$$\left.\begin{aligned}n&=\overline{1,N}\\n&:=0\end{aligned}\right\}\Rightarrow\prod:=I. \tag{14}$$

Then cEEc in initial state should be reduced to the form of «impossible event»:

$$\left.\begin{aligned}n&=\overline{1,N}\\n&:=0\end{aligned}\right\}\Rightarrow\coprod=I-\prod=I-I=\otimes, \tag{15}$$

where ⊗ is the symbol of impossible event.

Let us set the complement of an arithmetic product to 1, as the numerical range varies from «zero» to «one»:

$$\left.\begin{aligned}n&=\overline{1,N}\\n&:=0\end{aligned}\right\}\Rightarrow\coprod=1-\prod=1-1=0. \tag{16}$$

We proceed from the fact that $I*I=I$, $I+I=I$, ⊗*⊗=⊗, ⊗+⊗=⊗, but I+⊗=⊗, I+⊗=I.

Let us clarify only two abbreviations in Figure 4: ZPE – zero-probability events; OPE – one-probability events [9–10]. Considering that in theory of combinational dependability all our calculations are based on numerical values in the interval from 0 to 1 (including these boundaries), let us formulate some statements and theorems.

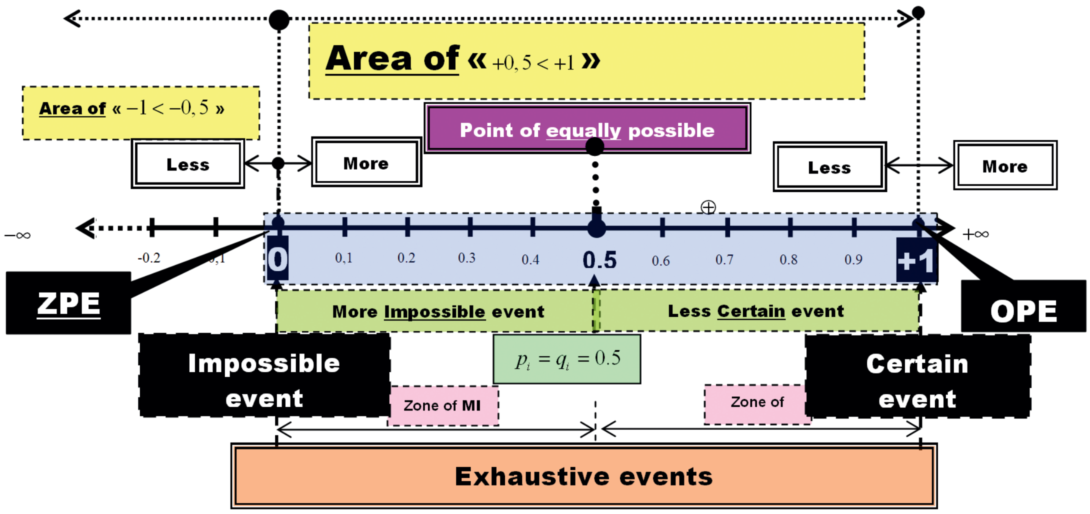

Figure 4. Event probability axis

Statement 1 (Fig. 4). With the increasing number of cofactors in the form of probabilistic numerical values the value of their product falls dramatically and converges to ZPE domain.

Statement 2 (Fig. 3). With the increasing number of logical cofactors, the veracity of their logical chain decreases.

In [16] it is written: «**When you remove the brackets remember the rule:** $p\cdot p=p$». Let us name this rule by the author's name of this article: «Bogatyrev's rule».

Theorem 1. The connectedness event of TPN described by cEEc is redundant; every of TPN is cEEc where WC of RG edges are its cofactors, i.e.:

$$\left.\begin{matrix} f_1=\{a,b\}\\ f_2=\{a,c\}\end{matrix}\right\}\Rightarrow\left[\left(\overline{\overline{a\bullet b}\bullet\overline{a\bullet c}}\right)>\left(\overline{\overline{a\bullet b}*\overline{a\bullet c}}=a\bullet\overline{\bar b\bullet\bar c}\right)\right].\quad(17)$$

The proof:

$$\left(\left(\overline{\overline{a\cdot b}\cdot\overline{a\cdot c}}\right)\Leftrightarrow\left(a\cdot\overline{\bar b\cdot\bar c}\right)\right)\Rightarrow$$

$$\Rightarrow\left(\left(I-(I-a\cdot b)\cdot(I-a\cdot c)\right)\Leftrightarrow\left(a\cdot(I-(I-b)\cdot(I-c))\right)\right)\Rightarrow$$

$$\Rightarrow\begin{pmatrix} I-\left(I-a\cdot b-a\cdot c+a^2\cdot b\cdot c\right)\Big)\Leftrightarrow\\ \Big(a\cdot\left(I-\left(I-b-c+b\cdot c\right)\right)\end{pmatrix}\Rightarrow$$

$$\Rightarrow\begin{pmatrix} \left(I-I+a\cdot b+a\cdot c-a^2\cdot b\cdot c\right)\Leftrightarrow\\ \left(a\cdot\left(I-I+b+c-b\cdot c\right)\right)\end{pmatrix}\Rightarrow$$

$$\Rightarrow\left(\left(a\cdot b+a\cdot c-a^2\cdot b\cdot c\right)\Leftrightarrow\left(a\cdot b+a\cdot c-a\cdot b\cdot c\right)\right)\Rightarrow$$

$$\left(\underline{\underline{(-a)\Leftrightarrow(-I)}}\right).$$

Turning to the stochastic side of «endspiel» and remembering that $(0<p_{(a,b,c)}<1)$ it is easy to see that the comparison result is performed as follows $((-p_a)>-1)$, q.e.d. ($\Leftrightarrow$ is the symbol «compare»).

Corollary 1.1. As in EPA only LAM is used then according to (17) in EPA (the first in (8)) only the redundant TPN CoC is always described that leads to the crossing of UB from the *bottom* of the exact value of PC TPN thereby this UB is the false estimate.

Corollary 1.2. To eliminate this lack, it is necessary to use LSM rules instead of LAM when describing UB PC TPN in EPA [17, 20]. Then the redundant description of TPN CoC is impossible and the false UB PC TPN on EPA becomes true, i.e. in lower LB PC TPN. The LSM rules for SCh in FC (the first in (8) and [17, 20]) are the following:

$$\left.\begin{matrix}\bar a*\overline{a\bullet b}=\bar a\\ \overline{a\bullet b}*\overline{a\bullet c}=\overline{a\bullet\overline{\bar b\bullet\bar c}}\\ \overline{a\bullet\overline{\bar b\bullet\bar c}}*\overline{b\bullet d}=\bar a\bullet\overline{b\bullet d}+a\bullet\bar b\bullet\bar c\\ \bar a*\bar b=\bar a\bullet\bar b\end{matrix}\right\}.\quad(18)$$

Theorem 2. The connectedness of TPN described by conjunction of cEEc where WC of RG edges is its cofactors is redundant, i.e.:

$$\left.\begin{matrix} r_1=\{a,b\}\\ r_2=\{a,c\}\end{matrix}\right\}\Rightarrow\left[\left(\overline{\overline{\bar a\bullet\bar b}\bullet\overline{\bar a\bullet\bar c}}\right)>\left(\overline{\bar a\bullet\bar b}*\overline{\bar a\bullet\bar c}=\overline{\bar a\bullet\overline{b\bullet c}}\right)\right].\quad(19)$$

The proof: Let us also transfer the comparison to «endspiel»:

$$\left(\left(\overline{\overline{\bar a\bullet\bar b}\bullet\overline{\bar a\bullet\bar c}}\right)\Leftrightarrow\left(\overline{\bar a\bullet\overline{b\bullet c}}\right)\right)\Rightarrow\left(\left(\left(I-\bar a\bullet\bar b\right)\bullet\left(I-\bar a\bullet\bar c\right)\right)\Leftrightarrow\right.$$

$$\Leftrightarrow\left[\left(I-\bar a\bullet(I-b\bullet c)\right)\right)=\left(I-\bar a\bullet\left(I-\left(I-\bar b\right)\bullet\left(I-\bar c\right)\right)\right)=$$

$$=\left(I-\bar a\bullet\left(I-I+\bar b+\bar c-\bar b\bullet\bar c\right)\right)=\left(I-\bar a\bullet\left(\bar b+\bar c-\bar b\bullet\bar c\right)\right)=$$

$$=\left(I-a\bullet b-a\bullet c+\bar a\bullet\bar b\bullet\bar c\right)\Big]\Rightarrow$$

$$\Rightarrow\begin{bmatrix}\left(I-\bar a\bullet\bar b-\bar a\bullet\bar c+\bar a^2\bullet\bar b\bullet\bar c\right)\Leftrightarrow\\ \left(I-\bar a\bullet\bar b-\bar a\bullet\bar c+\bar a\bullet\bar b\bullet\bar c\right)\end{bmatrix}\Rightarrow\left(\bar a\Leftrightarrow\otimes\right).$$

Turning to the stochastic side of «endspiel» and remembering that ($0<p_{(a,b,c)}<1$), the inequality $q_a>0$ is always true. Therefore, the initial (19) is true, q.e.d.

Corollary 2.1. As in EPA only LAM is used then according to (19) in EPA (the second in (8)) only the redundant TPN CoC is always described that leads to the crossing of the LB from *above* of the exact value of PC TPN thereby this LB is the false estimate (note: we shouldn't forget that here we use GDI [17, 20]).

Corollary 2.2. To eliminate this lack, it is necessary to use LSM rules instead of LAM when describing LB PC TPN in EPA [17, 20]. Then the redundant description of CoC TPN is impossible and the *false* LB PC TPN on EPA becomes true and transforms into upper UB PC TPN. The LSM rules for SCu in FC (the second in (8) and [17, 20]) are the following:

$$\left.\begin{array}{c} a*\overline{\bar{a}\bullet\bar{b}}=a \\ \overline{\bar{a}\bullet\bar{b}}*\overline{\bar{a}\bullet\bar{c}}=\overline{\bar{a}\bullet\overline{b\bullet c}} \\ \overline{\bar{a}\bullet\overline{b\bullet c}}*\overline{\bar{b}\bullet\bar{d}}=a\bullet\overline{\bar{b}\bullet\bar{d}}+\bar{a}\bullet b\bullet c \\ a*b=a\bullet b \end{array}\right\}. \qquad (20)$$

Graphic interpretation of the theorems 1 and 2 is presented in Figure 5.

Thus, the LSM rules and OsLF will be as follows:

Lower bound (based on SCh):

$$\left.\begin{array}{c} \bar{a}*\overline{a\bullet b}=\bar{a} \\ \overline{a\bullet b}*\overline{a\bullet c}=\overline{a\bullet\overline{\bar{b}\bullet\bar{c}}} \\ \overline{a\bullet\overline{\bar{b}\bullet\bar{c}}}*\overline{b\bullet d}=\bar{a}\bullet\overline{b\bullet d}+a\bullet\bar{b}\bullet\bar{c} \\ \bar{a}*\bar{b}=\bar{a}\bullet\bar{b} \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left(E^{L}_{s,t}=\coprod_{e=1}^{m_F}*\overline{f}_e\Big|(e=0)\Rightarrow\coprod_{e=0}^{m_F}*I\right)\le E_{s,t}, \qquad (21)$$

where $|-$ «under conditions», and

Upper bound (based on SCu):

$$\left.\begin{array}{c} a*\overline{\bar{a}\bullet\bar{b}}=a \\ \overline{\bar{a}\bullet\bar{b}}*\overline{\bar{a}\bullet\bar{c}}=\overline{\bar{a}\bullet\overline{b\bullet c}} \\ \overline{\bar{a}\bullet\overline{b\bullet c}}*\overline{\bar{b}\bullet\bar{d}}=a\bullet\overline{\bar{b}\bullet\bar{d}}+\bar{a}\bullet b\bullet c \\ a*b=a\bullet b \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left(E^{U}_{s,t}=\prod_{e=1}^{m_R}*\overline{r}_e\Big|(e=0)\Rightarrow\prod_{e=0}^{m_R}*I\right)\ge E_{s,t}. \qquad (22)$$

This completes the description of the main results.

4. Example

Let us take as an example «the bridge» (Fig. 1c), whose SCh and SCu sets are represented accordingly in (1) and (2) and initial data on AF edges are in (3). Let us calculate LB PC TPN «behavior» based on OsLF formal rules (19):

$$E^{L}_{s,t}=\overline{I*\overline{\overline{5\bullet7\bullet9}}}=\underline{\underline{5\bullet7\bullet9}}_1\rightarrow\overline{\overline{\overline{5\bullet7\bullet9}*\overline{5\bullet8}}}=$$
$$=\overline{\overline{5\bullet\overline{\overline{7\bullet9}\bullet\bar{8}}}}=\underline{\underline{5\bullet\overline{\overline{7\bullet9}\bullet\bar{8}}}}_2\rightarrow\overline{\overline{5\bullet\overline{\overline{7\bullet9}\bullet\bar{8}}}*\overline{6\bullet7\bullet8}}=\ldots$$
$$\ldots=\underline{\underline{\left(\overline{\bar{\bar{5}}\bullet\overline{6\bullet7\bullet8}+5\bullet\overline{7\bullet9}\bullet\bar{\bar{8}}}\right)}}_3\rightarrow$$
$$\rightarrow\overline{\left(\overline{\overline{\bar{5}\bullet\overline{6\bullet7\bullet8}+5\bullet\overline{7\bullet9}\bullet\bar{8}}}\right)*\overline{6\bullet9}}=\ldots$$
$$\ldots=\left(\underline{\underline{\overline{\bar{5}\bullet6\bullet\overline{\overline{\overline{7\bullet8}\bullet\bar{9}}}+5\bullet\overline{\overline{\bar{7}\bullet\bar{6}}\bullet9}\bullet\bar{8}}}}_4\right). \qquad (23)$$

The results at the corresponding step after LSM operands is marked by double underline. If we exclude from (21) all intermediate logical operations then we obtain the following:

$$E^{L}_{s,t}=5\bullet7\bullet9_1\rightarrow5\bullet\overline{\overline{7\bullet9}\bullet\bar{8}}_2\rightarrow$$
$$\rightarrow\left(\overline{\bar{5}\bullet\overline{6\bullet7\bullet8}+5\bullet\overline{7\bullet9}\bullet\bar{8}}\right)_3\rightarrow$$
$$\rightarrow\left(\overline{\bar{5}\bullet6\bullet\overline{\overline{\overline{7\bullet8}\bullet\bar{9}}}+5\bullet\overline{\overline{\bar{7}\bullet\bar{6}}\bullet9}\bullet\bar{8}}\right)_4. \qquad (24)$$

Let us use our initial data (3) and show the dynamic of «normal» behavior of the LB PC TPN obtained according to the stated in the article OsLF rules (19), which allowed estimates to stop their mutual crossing and transform from LB to UB PC TPN which was the distinctive feature of «classic» EPA. The increasing LB PC TPN are the following:

$$P^{L}_{s,t}=0,315_1\rightarrow0,463_2\rightarrow0,631_3\rightarrow\underline{\underline{0,766}}_4=P_{s,t}. \qquad (25)$$

It could be seen that at the last 4th step LB PC TPN is equal to the exact value of PC TPN, estimated in (6).

Let us describe the dynamic of UB PC TPN «normal» behavior using OsLF method, according to the rules (22):

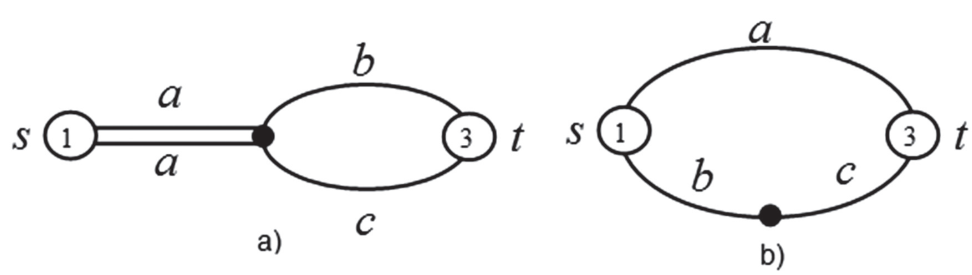

Figure 5. Graphic interpretation

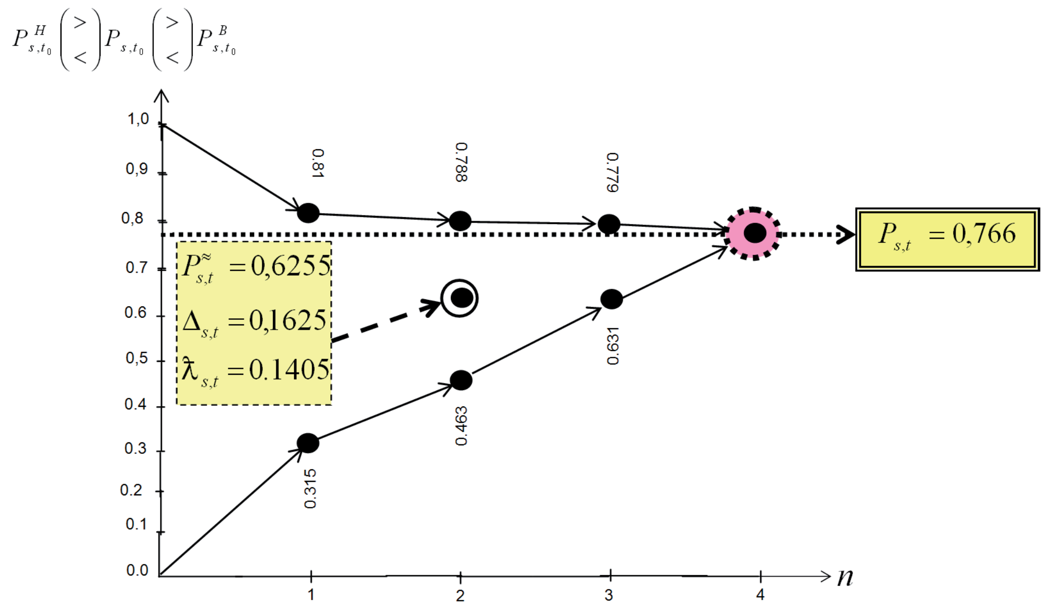

Figure 6. Graph of the dual image of the behavior of the PC TSE for the "bridge" under the adopted input data

$$\forall l_{(5\le \nu \le 9)} \in L\left[\, p_{(5\le \nu \le 9)} = 0,(5 \le \nu \le 9)\right]$$

$$E^{U}_{s,t} = I \bullet \overline{\bar{5} \bullet \bar{6}} = \underline{\underline{\overline{\bar{5} \bullet \bar{6}}}}_1 \to \left(\overline{\bar{5} \bullet \bar{6}} * \overline{\bar{6} \bullet \bar{7} \bullet \bar{8}} = \underline{\underline{\overline{\bar{6} \bullet \overline{5 \bullet \overline{\bar{7} \bullet \bar{8}}}}}}_2 \right) \to$$

$$\to \left(\overline{\bar{6} \bullet \overline{5 \bullet \overline{\bar{7} \bullet \bar{8}}}} * \overline{\bar{5} \bullet \bar{7} \bullet \bar{9}} \right) = \left(\underline{\underline{6 \bullet \overline{\bar{5} \bullet \bar{7} \bullet \bar{9}}}} + \ldots + \underline{\underline{\bar{6} \bullet 5 \bullet \overline{\bar{7} \bullet \bar{8}}}} \right)_3 \to$$

$$\to \begin{pmatrix} \left(6 \bullet \overline{\bar{5} \bullet \bar{7} \bullet \bar{9}} + +\bar{6} \bullet 5 \bullet \overline{\bar{7} \bullet \bar{8}}\right) * \overline{\bar{8} \bullet \bar{9}} = \\ = \underline{\underline{6 \bullet \overline{\bar{9} \bullet \overline{\bar{5} \bullet \bar{7}} \bullet 8} + \bar{6} \bullet 5 \bullet \bar{8} \bullet \overline{\bar{7} \bullet 9}}} \end{pmatrix}_4 . \qquad (26)$$

Using our initial data (3) we can obtain numerical values of falling UB PC TPN equal to the PC TPN exact value:

$$P^{U}_{s,t} = 0{,}8 \to 0{,}788 \to 0{,}779 \to \underline{\underline{0{,}766}} = P_{s,t}. \qquad (27)$$

As a result, UB PC TPN became also equal to the exact PC TPN value calculated in (6).

Based on the results (25) and (27) it is possible to construct the graph of the dual image [17, 20] of LB PC TPN increasing and UB PC TPN falling dynamic for the exact PC TPN value. This graph is presented in Figure 6. Here it is also shown that FIP estimation procedure was realized at the 2[d] step, the estimative parameters of the calculations were obtained under which the operator should decide whether he continuous calculations or not.

5. Conclusion

In summary, we think that we (co-authors) completely solve the task. Based on the results we can see that deduced EPA doesn't belong to *NP*-class. These new assessments based on human intelligence belong to *IN*-class. This approach is very popular among experts involved in structural dependability analysis of complex systems.

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