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Identification of dependability indicators of manufactured samples of radioelectronic systems

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Abstract. *The article deals with the identification of dependability of manufactured samples of radioelectronic systems. This task belongs to the class of a posteriori analysis. In order to identify the dependability characteristics of equipment, upon production of a pilot batch one performs a posteriori analysis whose first stage is the statistical test (ST). There are a lot of methods for such tests that primarily depend on identifying the time of test completion (r – to failure of r systems, T – upon reaching operation time T, n – to failure of all systems, as well as mixed ones) and the ability to replace failed systems with healthy ones. Such tests are necessary because at the design stage a designer does not possess complete a priori information that would allow identifying the dependability indicators in advance and with a sufficient accuracy. An important source of dependability information is a system for collection of data on product operational performance. There are two primary types of dependability tests. One of them is the determinative test intended for evaluation of dependability indicators. It is typical for mass-produced products. Another type of test is the control test designed to verify the compliance of a system's dependability indicators with the specifications. This paper is dedicated to the first type of tests. It shows the procedure for statistical tests of radioelectronic systems using various procedures. Evaluation of the mean time to failure $\hat{t}^*$ is usually performed by means of the method of maximum likelihood. The essence of the method is that in the process of statistical data processing the likelihood function is found, while the required parameter ($\hat{t}^*$ is the evaluation of parameter t^*) equals to the argument value under which the likelihood function is maximal. The evaluation of the mean time to failure $\hat{t}^*$ is a point estimate of the initial parameter t^*, which in turn is a random value and within a specific test can take any positive value from 0 to ∞. Therefore, in addition to the point estimation an interval estimation of the measured parameter is usually performed. That means that estimation $\hat{t}^*$ identifies the confidence interval ($\hat{t}^*_L, \hat{t}^*_U$) in which the value of the measured parameter t^* with a specified probability is found. Here $\hat{t}^*_L, \hat{t}^*_U$ are respectively the lower and upper limits of a confidence interval. The article considers two procedures of testing pilot batches of radioelectronic systems, and for each of them the following dependability indicators are defined: evaluation of mean time to failure; confidence interval of mean time to failure. It is shown that for the purpose of identifying the mean time to failure, test procedure [n, V, r] is more efficient than procedure [n, B, r].*

Keywords: *radioelectronic system, time to failure, test duration.*

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Introduction

At the current stage of the society's development when the concept of information technology has become ingrained in many people's minds everyone now depends on the security of personal information. If valuable information is communicated with a delay or is inaccurate, a person, company or nation as a whole may face serious consequences. For that reason the requirements for the dependability and availability of radioelectronic systems of information communication and processing are becoming more demanding.

The dependability characteristics of radioelectronic systems (RES) are identified in two stages: a priori analysis that consists in approximate calculation of system dependability based on known quantitative (probabilistic) characteristics of its elements' dependability, as well as a posteriori analysis upon production of a pilot batch of equipment [1-5]. A posteriori analysis provides more accurate results for a specific manufactured batch [6], therefore this stage is of importance as regards the manufacturing process.

Problem definition and solution

In order to identify the dependability characteristics of equipment, upon production of a pilot batch one performs a posteriori analysis whose first stage is the statistical test (ST). There are a lot of methods for such tests that primarily depend on identifying the time of test completion (r – to failure of r systems, T – upon reaching operation time T, n – to failure of all systems, as well as

Figure 1. Instants of failure and failure cycle

mixed ones) and the ability to replace failed systems with healthy ones. [7].

Upon completion of statistical data processing, the calculated characteristics are validated against the specifications and requirements of regulatory documents by public authorities as required.

Identification of dependability characteristics based on testing of pilot batches of RES according to [n, B, r] procedure

1. Problem specification

It is assumed that tests are performed on a pilot batch of 100 (n = 100) RESs without replacement of failed systems to 20 (r = 20) failures. The resulting sample must correspond to the theoretical failure flow model, i.e. the probability density function (PDF) of the failure cycle must correspond to the model as follows

$$w(y_i) = \lambda(n-i+1)e^{-\lambda(n-i+1)y_i} \tag{1}$$

where λ=1/t^* is the failure rate of one system, t^* *is* the mean time to failure of one system, *(n – i + 1)* is the number of systems involved in the tests inclusive of the failed ones.

Such sample can be acquired out of the value *x* evenly distributed over the interval (0; 1) according to formula

$$y_i = -\frac{1}{\lambda(n-i+1)}\ln(x).$$

Let the mean time to failure of one system be 1000 hours.

As the result, we get the failure pattern shown in Figure 1.

2. Estimation of mean time to failure of a pilot batch

Estimation of mean time to failure $\hat{t}^{\bullet}$ is usually performed by means of the method of maximum likelihood. The essence of the method is that in the process of statistical data processing the likelihood function is found, while the required parameter ($\hat{t}^{\bullet}$ is the evaluation of parameter t^*) equals to the argument value under which the likelihood function is maximal (Fig. 2).

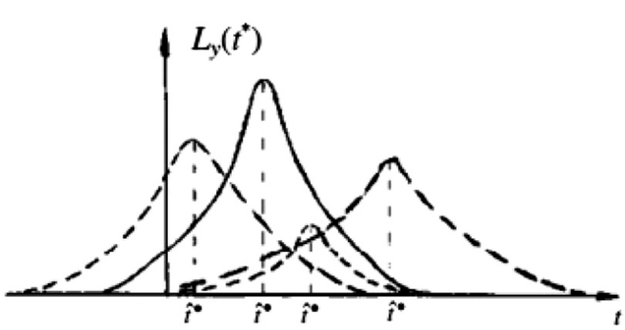

Figure 2. Possible formula for the likelihood function

The likelihood function equals to the joint probability density of intervals y_i subject to their independence

$$L_y(t^*) = \prod_{i=1}^{r} w(y_i) \tag{2}$$

Then, the estimation of mean time to failure will equal to the extremum of the likelihood function

$$\hat{t}^{\bullet} = \hat{t}^{\bullet}_{МП} = \arg\max L_y(t^*).$$

In order to find the extremum of the likelihood function, the following equation must be solved

$$\frac{\partial L_y(t^*)}{\partial t^*} = 0.$$

As any monotone function of likelihood function is also a likelihood function, then in order to simplify the solution we can use the equation

$$\frac{\partial \ln L_y(t^*)}{\partial t^*} = 0.$$

Given (1) and (2) we deduce:

$$L_y(t^*) = \prod_{i=1}^{r} w(y_i) = \prod_{i=1}^{r} \lambda(n-i+1)e^{-\lambda(n-i+1)y_i} = \\ = e^{-\sum_{i=1}^{r}\lambda(n-i+1)y_i}\prod_{i=1}^{r}\frac{1}{t^*}(n-i+1),$$

$$\ln L_i(t^{\bullet}) = \sum_{i=1}^{r}\left[\ln(n-i+1) - \ln t^{\bullet} - \lambda(n-i+1)y_i\right]. \tag{3}$$

If we replace λ =1/$t^*$and differentiate:

$$\frac{\partial \ln L_y(t^*)}{\partial t^*} = \sum_{i=1}^{r}\left[-\frac{1}{t^*} + \frac{n-i+1}{t^*}y_i\right].$$

As a result, we obtain an evaluation of mean time to failure that in our case equals to:

$$\hat{t}^{\bullet} = \hat{t}^{\bullet}_{МП} = \tfrac{1}{r}\left[\sum_{i=1}^{r}(n-i+1)y_i\right] = 952{,}39 \text{ (hours)} \tag{4}$$

3. Total operation time of all systems to the r^{th} failure

Expression (4) shall be rearranged to time points:

$$\hat{t}^{\bullet} = \tfrac{1}{r}\left[\sum_{i=1}^{r}(n-i+1)(t_i - t_{i-1})\right] = \\ = \tfrac{1}{r}\sum_{i=1}^{r}(n-i+1)t_i - \tfrac{1}{r}\sum_{i=1}^{r}(n-i+1)t_{i-1} = \\ = \tfrac{1}{r}\left[\sum_{i=1}^{r}t_i + (n-r)t_r\right], \tag{5}$$

where $(n–r)t_r$=(100–20)·203,43=16274,4 (hours) is the total operation time of non-failed systems;

$\sum_{i=1}^{r}t_i = 2366{,}6$ (hours) is the total time of no-failure of all failed systems;

$\sum_{i=1}^{r}t_i + (n-r)t_r = 18641$ (hours) is the total operation time of all systems to the r^{th} failure.

4. Confidence interval of mean time to failure

The estimation of mean time to failure $\hat{t}^{\bullet}$ is a point estimate of the initial parameter t^*, which in turn is a random value and within a specific test can take any positive value from 0 to ∞. Therefore, in addition to the point estimation, an interval estimation of the measured parameter is usually performed. That means that estimation $\hat{t}^{\bullet}$ identifies the confidence interval $(\hat{t}_L^{\bullet}, \hat{t}_U^{\bullet})$ in which the true value of the measured parameter t^* with a specified probability is found

$$P\left\{\hat{t}_L^{\bullet} < t^* < \hat{t}_U^{\bullet}\right\} = \gamma, \tag{6}$$

where γ is the confident probability (or confidence coefficient), $\hat{t}_L^{\bullet}, \hat{t}_U^{\bullet}$ are respectively the lower and upper limits of the confidence interval.

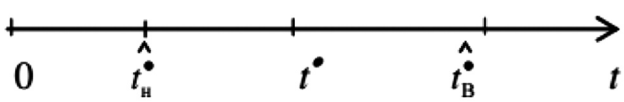

Figure 3. Confidence interval

In order to identify the confidence interval, we need to know the distribution function of estimation probability. For that purpose, we must transform formula (6) in such a way as to use normalized quantities

$\begin{matrix}\hat{t}_L^{\bullet} = \hat{t}^{\bullet}(1-\varepsilon_1) \\ \hat{t}_U^{\bullet} = \hat{t}^{\bullet}(1+\varepsilon_2)\end{matrix}$, where $\hat{t}_U^{\bullet} - \hat{t}_L^{\bullet} = \hat{t}^{\bullet}(\varepsilon_1 + \varepsilon_2)$ is the length of the interval.

Then, (6) rewrites as

$$P\left\{\hat{t}^{\bullet}(1-\varepsilon_1) < t^* < \hat{t}^{\bullet}(1+\varepsilon_2)\right\} = \gamma \tag{7}$$

Given that

$$\begin{matrix} t^* > \hat{t}^{\bullet}(1-\varepsilon_1) \\ t^* < \hat{t}^{\bullet}(1+\varepsilon_2) \end{matrix} \text{ or } \begin{matrix} \hat{t}^{\bullet} < \dfrac{t^*}{1-\varepsilon_1} \\ \hat{t}^{\bullet} > \dfrac{t^*}{1+\varepsilon_2} \end{matrix},$$

formula (7) is as follows

$$P\left\{\frac{t^*}{1+\varepsilon_2} < \hat{t}^{\bullet} < \frac{t^*}{1-\varepsilon_1}\right\} = \gamma,$$

$$\text{or } P\left\{\frac{1}{1+\varepsilon_2} < \frac{\hat{t}^{\bullet}}{t^*} < \frac{1}{1-\varepsilon_1}\right\} = \gamma. \tag{8}$$

Thus, we need to find the PDF of the value $\frac{\hat{t}^{\bullet}}{t^*}$.

Out of (5) we can deduce that the total time to failure equals to

$$t_\Sigma = \hat{t}^{\bullet} r = \sum_{i=1}^{r} (n-i+1) y_i = 18844,43 \text{ (hours)}. \tag{9}$$

The probability density function of intervals y_i is known (1). In this law, the variable must be replaced in order to deduce the standard probability density with the variance equal to 1.

Let us denote by

$$z_i = \frac{n-i+1}{t^*} \cdot 2y_i; \frac{\partial y_i}{\partial z_i} = \frac{t^*}{2(n-i+1)}. \tag{10}$$

Then $w(z_i) = \frac{1}{2} e^{\frac{z_i}{2}}$ is the exponential density with unit variance.

It is known that in this case $\sqrt{z_i}$ has a Gaussian distribution, while $\sum_{i=1}^{r} z_i$ is distributed over $\chi^2(2r)$ with $2r = 40$ degrees of freedom, which is commonly used in statistics for processing of experimental data.

Given (9) and (10)

$$\sum_{i=1}^{r} z_i = \frac{2t_\Sigma}{t^*}.$$

Let us introduce the variable

$$\tau = \sum_{i=1}^{r} z_i = \frac{2t_\Sigma}{t^*} = \frac{2\hat{t}^{\bullet} r}{t^*},$$

τ is distributed over $\chi^2(2r)$ with $2r$ degrees of freedom; r is the number of failures.

The distributions $\chi^2(2r)$ are tabulated. For a large number of degrees of freedom this distribution tends to normal.

Let $\frac{1}{1+\varepsilon_2} = \alpha_2$ & $\frac{1}{1-\varepsilon_1} = \alpha_1$, then formula (8) for the confidence interval works out to

$$P\left\{2r\alpha_2 < \tau < 2r\alpha_1\right\} = \gamma. \tag{11}$$

Fig. 4 shows the PDF χ^2, the crosshatched area under the curve is the confident probability γ (the whole area under the PDF, as we know, equals to one). As shown in Fig. 4, the confidence interval can be plotted on the axis τ differently, i.e. the solution is ambiguous.

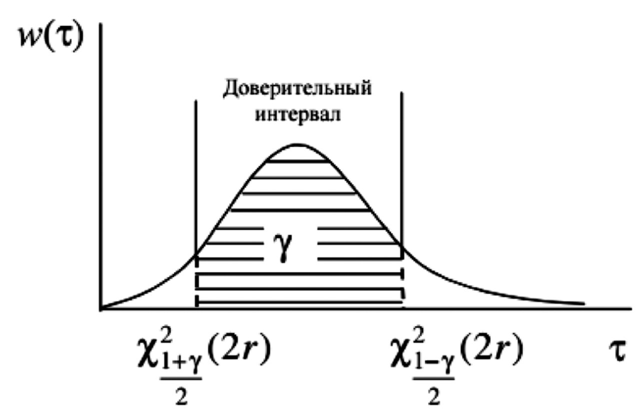

Figure 4. Probability density function χ^2

That is usually done to make the interval limits cut on the right and left identical areas under the curve equal to $\frac{1-\gamma}{2}$.

Then, the lower limit of the confidence interval $\chi^2_{\frac{1-\gamma}{2}}(2r)$ is $\left(\frac{1-\gamma}{2}\right)\%^{th}$ distribution point $\chi^2(2r)$, while the upper limit of the confidence interval $\chi^2_{\frac{1-\gamma}{2}}(2r)$ is $\left(\frac{1-\gamma}{2}\right)\%^{th}$ distribution point $\chi^2(2r)$, of which the values are identified in accordance with the $\chi^2(2r)$ inverse distribution tables.

Further, based on (11),

$$P\left\{\chi^2_{\frac{1+\gamma}{2}}(2r) < \tau < \chi^2_{\frac{1-\gamma}{2}}(2r)\right\} = \gamma,$$

or

$$P\left\{\chi^2_{\frac{1+\gamma}{2}}(2r) < \frac{2\hat{t}^{\bullet}r}{t^*} < \chi^2_{\frac{1-\gamma}{2}}(2r)\right\} = \gamma, \qquad (12)$$

or

$$P\left\{\frac{2\hat{t}^{\bullet}r}{\chi^2_{\frac{1-\gamma}{2}}(2r)} < t^* < \frac{2\hat{t}^{\bullet}r}{\chi^2_{\frac{1+\gamma}{2}}(2r)}\right\} = \gamma.$$

From (12) it is seen that the lower limit of the confidence interval is equal to

$$t^* > \frac{2\hat{t}^{\bullet}r}{\chi^2_{\frac{1-\gamma}{2}}(2r)} = \hat{t}^{\bullet}_L,$$

while the upper limit is respectively equal to

$$t^* > \frac{2\hat{t}^{\bullet}r}{\chi^2_{\frac{1-\gamma}{2}}(2r)} = \hat{t}^{\bullet}_U.$$

Thus it can be established that for our tested radiotechnical system under a confident probability of 80% the true value of t^* lies in the range from $\hat{t}^{\bullet}_L = \frac{2\hat{t}^{\bullet}r}{\chi^2_{\frac{1-\gamma}{2}}(2r)} = 735{,}3$ (*hours*) to $\hat{t}^{\bullet}_U = \frac{2\hat{t}^{\bullet}r}{\chi^2_{\frac{1-\gamma}{2}}(2r)} = 735{,}3$ (*hours*).

5. Test duration

The duration of test corresponds with the moment of the r^{th} failure when the test stops. For the [n, B, r] procedure this value is random and it is important to evaluate it both for the contractor and the customer.

The PDF of this value is hard to find, as $T = t_r = \sum_{i=1}^{r} y_i$, while the values y_i are heterogeneous (depend on i (1)). So, let us just identify the average value (expectation) and variance.

Average test duration

$$m(T) = m(t_r) = \sum_{i=1}^{r} m\{y_i\} = \hat{t}^{\bullet}\sum_{i=1}^{r}\frac{1}{n-i+1}. \qquad (13)$$

Let us write (13) as a series $m(t_r) = \hat{t}^{\bullet}\sum_{k=n-r+1}^{n}\frac{1}{k}$, where $k=n-i+1$.

Let us denote $\phi(m) = \sum_{k=1}^{m}\frac{1}{k}$,

then $m(t_r) = \hat{t}^{\bullet}\left[\phi(m) - \phi(n-r)\right]$

It is known that if $m>>1$, the function $\phi(m) \approx \ln m$.

Then if $r>>1$, the average test duration

$$m(t_r) \approx \hat{t}^{\bullet}\ln\frac{n}{n-r}.$$

If $n = r$, then $m(t_r) = \hat{t}^{\bullet}\phi(n) = \hat{t}^{\bullet}\ln n = 4385{,}93$ (hours).

The variance of test duration equals to

$$D(t_r) = \hat{t}^{\bullet 2}\sum_{i=1}^{r}\frac{1}{(n-i+1)^2}. \qquad (14)$$

Let us denote $\phi(m) = \sum_{k=1}^{m}\frac{1}{k^2}$, then

$$D(t_r) = \hat{t}^{\bullet 2}\left[\phi(n) - \phi(n-r)\right].$$

If $n \to r$, then $D(t_r) \to \hat{t}^{\bullet 2}\cdot\frac{\pi^2}{6}$.

I.e. the variance of test duration decreases as r grows, but tends not to zero, but a constant number, therefore this procedure is not very efficient.

Identification of dependability indicators based on the results of test of a specific pilot batch of RESs according to the [n, V, r] procedure

1. Problem specification

It is assumed that the conditions of this problem are comparable with those of the above one. As a result of the test, a sample of instants of failure was obtained.

The failure model for one system is

$$w(t) = \lambda e^{-\lambda t}, t > 0, \lambda > 0,$$

or

$$w(t) = \frac{1}{t^*}e^{-\frac{t}{t^*}}, t > 0,$$

where t^* is the average time to failure.

The sample of intervals $y_i = t_i - t_{i-1}$ is homogenous and is governed by probability density

$$w(y_i) = n\lambda e^{-n\lambda y_i},$$

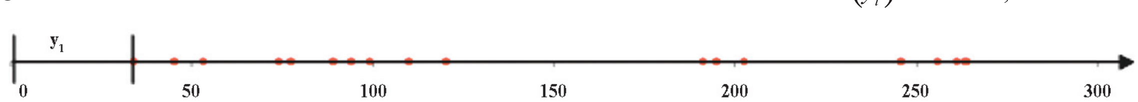

Figure 5. Instants of failure and failure cycle

where $n\lambda$ is the collective failure rate of the systems involved in the test.

2. Identification of mean time to failure

The estimation of the mean time to failure $\hat{t}^{\bullet}$ can also be performed by means of the maximum likelihood method.

For the purpose of the current task, the likelihood function represents the PDF of the intervals y under the given value of the parameter t^*

$$L_y\left(t^*\right)=\prod_{k=1}^{r} w\left(y_k\right)=\left(\frac{n}{t^*}\right)^r \cdot e^{-\sum_{k=1}^{r}\frac{-n}{t^*}y_k}.$$

The maximum likelihood estimation of $\hat{t}^{\bullet}$ is defined as the parameter that corresponds to the maximum of the likelihood function

$$\frac{\partial \ln L_y\left(t^*\right)}{\partial t^*}=\frac{-r}{t^*}-\frac{n}{t^{*r}}\cdot\sum_{k=1}^{r} y_k=0.$$

Then the estimation is

$$\hat{t}^{\bullet}=\frac{n}{r}\sum_{k=1}^{r} y_k=\frac{n\cdot t_r}{r}=1319,85 \text{ (hours)},$$

where $n\cdot t_r=t_\Sigma=26397$ (hours) is the total time to failure shared by both test plans. It implies that $\hat{t}^{\bullet}=\frac{t_\Sigma}{r}$, and the quality of estimation is identical to that of the procedure [n, B, r] under identical t_Σ and r.

3. Average duration of test

$$m\left\{t_r\right\}=\sum_{k=1}^{r} m\left\{y_k\right\}=\frac{r}{n}t^*.$$

If $n = r$, then $m\left\{t_r\right\}=t^*=1319,85$ (hours), which is less than under the procedure [n, B, r].

Variance of the duration of test

$$D\left\{t_r\right\}=\sum_{k=1}^{r} D\left\{y_k\right\}=\frac{r}{n^2}\left(t^*\right)^2.$$

If $n = r$, then $D\left\{t_r\right\}=\frac{\left(t^*\right)^2}{n}$ and tends to zero if n increases. Therefore, this test procedure is more efficient compared to [n, B, r].

Conclusions

The article examined two procedures for testing pilot batches of radioelectronic systems and for each of them the following dependability indicators were identified:

- estimation of mean time to failure;
- confidence interval of mean time to failure;
- it is shown that for the purpose of identifying the mean time to failure test procedure [n, V, r] is more efficient than procedure [n, B, r].

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