

Method for calculating gamma-percentile time to failure and failure rate of resistive position sensors of control systems

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Abstract. Aim. Traditionally, the dependability indicators of resistive position sensors based on wire-wound potentiometers used in various control systems are confirmed by means of appropriate dependability tests or tests of comparable products. For cases of non-availability of comparable products test data or significant changes in the product's design and materials, a method for short-term dependability testing and dependability indicators forecasting is required. Calculations of dependability indicators are to be based on statistical information on the variations of properties and parameters over the course of dependability testing along with research findings regarding the physical patterns, descriptions of process kinetics that cause such variations. **Methods.** The analysis of physical processes that cause catastrophic changes in resistive position sensors has shown that under electrical loads thermal and electrical fields form that cause electrokinetic, thermoelectric, thermo-diffusion effects. In all cases the rates of physical and chemical processes are functions of material temperature, have temperature dependence and are described with the Arrhenius equation. The conducted research allowed establishing that variations of the position sensors' impedance are largely defined by the processes occurring in the resistive element. The temporal dependence of impedance can be described with a logarithmic, exponential or polynomial dependence. **Results.** Mathematical models that describe physical and chemical processes occurring in resistive position sensors in operation allowed developing a scientifically grounded calculation and experimental method for short-term reliability testing. The method includes the description of thermal and electrical modes, durability testing conditions and timing. It is shown that the results of such tests are used in subsequent statistical processing for the purpose of forecasting dependability values. Gamma-percentile time to failure and failure rate are evaluated by means of forecasting the degradation of the acceptance criterion values. The dependence of acceptance criteria values acquired in the course of the tests is approximated by a straight line, exponential curve or a polynomial equation. The form of the approximating line for forecasting the value of gamma-percentile time to failure and failure rate is defined analytically based on the adopted model that describes the physical and chemical processes occurring in potentiometers in operation. The value of acceptability criteria of gamma-percentile time to failure required by the performance specifications and technical regulations is identified through extrapolation of the approximating line as a continuation of the chosen approximating curve (straight line). **Conclusions.** The provided test data for short and long-term reliability corresponds to the calculated values of dependability indicators, which confirms the applicability of the developed calculation method. The application of the proposed method allows reducing the scope and duration of costly dependability tests.

Keywords: resistive potentiometers, short-term reliability testing, forecasting, gamma-percentile time to failure.

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Introduction

Resistive (potentiometric) position sensors based on wire-wound potentiometers find wide application in automated guidance, supervision and control systems, and they are primary sources of information for such systems. Currently, despite the availability of digital potentiometers, the interest for wire-wound potentiometers is still high. Among the reasons of their continued popularity are strong accuracy

characteristics and high dependability. The durability of some items may reach 5 mil operating cycles. According to regulatory documents, the dependability indicators of potentiometers are the gamma-percentile time to failure (T_γ) and failure rate. In practice, the following methods are used for confirming potentiometers compliance with the specified dependability criteria: experimental based on short and long-term reliability tests, computational and experimental, computational based on the results of comparable

products testing [1]. Normally, short-term reliability tests are conducted at the stage of research and development activities. Based on the obtained test data or comparable product tests, the required potentiometer dependability criteria are confirmed. However, using the test results of comparable products is often complicated due to non-availability of such data or due to significant modifications to product design and used materials.

Obviously, the methods for confirmation of potentiometer compliance with the specified dependability requirements are to combine statistical information on the variations of properties and parameters over the course of dependability testing and research findings regarding the physical patterns, descriptions of process kinetics that cause such variations.

Analysis of physical and chemical processes occurring in products

The correctness of calculation results is defined by the compliance of the adopted model with the actual patterns of physical and chemical processes occurring in the products. Under electrical loads, thermal and electrical fields form within potentiometers that cause electrokinetic, thermoelectric, thermo-diffusion and other effects. The physical and chemical processes cause changes in the electrical parameters of potentiometers.

Under electrical loads, the materials that compose potentiometer elements develop additional fields and physical and chemical processes, primarily:

- thermal field distortion (uneven heating) and associated thermomechanical stress,
- electrical field distortion and field gradient that causes local overheating and ruptures,
- electrolysis, ionization and other localized processes.

Changes in the potentiometer electrical impedance in time are due to processes of oxidation, diffusion, as well as solid body reactions. In all cases the rates of physical and chemical processes in potentiometer materials are functions of material temperature, have a temperature dependence and are described with the Arrhenius equation [2].

$$\tau^{-1} = v_0 \cdot \exp\left(\frac{-Q}{kT}\right),$$

where v_0 is the frequency multiplier, Q is the energy of activation, τ^{-1} is the relaxation rate, k is the Boltzmann constant.

In operation, materials used in potentiometers develop various physical and chemical processes that may cause significant deviations of the potentiometer's electrical impedance from the reference value. As the specific electrical impedance of the frame and coatings is significantly higher than the specific electrical impedance of the resistive element's material, it should be expected that if the electrical impedance of the frame and coatings is subject to significant variations, the overall impedance of the

potentiometer accurate to measurement error value will remain constant. Thus, the variation of a potentiometer's overall impedance is defined by the processes occurring in the resistive element.

The passage of electrical current causes a potentiometer to generate heat. The amount of heat generated in unit time is defined by Joule-Lenz's law. Based on the volume of the potentiometer in the steady state, a certain temperature distribution is established that is defined by the resistor design and the current that passes through it. Any process that causes changes in the structure and composition is defined by the displacement of atoms and ions, i.e. the diffusion. The rate of diffusion is defined by the diffusion coefficient that depends on the temperature and energy of activation.

In the case of chemical gas corrosion there is a number of dependencies that describe the variation of the thickness of the oxide film as the function of time. The overall impedance of a potentiometer is a function of the thickness of the oxide film. The growth of the films can be described with:

a) a parabolic equation of the n -th degree [3]:

$$h^n = kt + \gamma,$$

b) a logarithmic equation [3]:

$$h = k \ln(t) + \gamma,$$

where k is a coefficient that depends on the temperature, n, γ are constants.

The dependence of the acceptability criteria, e.g. variation of impedance, on the time is represented by the following functional relations:

$$1) Z = a \lg(t),$$

$$2) Z = at + b,$$

$$3) Z = t^a.$$

The dependence of impedance variation in time on the temperature, load and its starting rate is represented as [4]:

$$Z = Z_0 \cdot e^{\frac{-B}{T}} \cdot P^\beta (1 + \alpha \cdot f) \cdot f(\tau),$$

where Z is the acceptance criterion, relative variation of overall impedance, %, Z_0 is the coefficient with the dimension of the acceptance criterion, B is the energy coefficient that characterizes the potentiometer's activation energy, T is the temperature, ^0K , P is the electric load, W , f is the starting rate, $1/h$, α is the cycling coefficient, $f(\tau)$ is the time parameter.

By means of transformations and introduction of additional designations, this mathematical model is written in the form of linear polynomial with coded explanatory variables [3]:

$$y(x) = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3.$$

Method for dependability testing of potentiometers

The application of mathematical models that describe physical and chemical processes occurring in resistive

position sensors in operation allows developing a scientifically grounded calculation and experimental method for short-term reliability testing (stage 1). The results of such tests are used in further statistical processing for the purpose of forecasting the values of dependability criteria (stage 2).

Short-term reliability tests of potentiometers (1000 hours) are carried out in two cycles. The first cycle includes 3 stages:

1. Hot soaking under $(85+3)^\circ\text{C}$, direct current and appropriate power of 1 W for 400 h. The accuracy of voltage control is $\pm 5\%$.
2. Humidity soaking under increased air humidity according to method 207-2 of GOST 20.57.406 without electric load for 96 h.
3. Wearing tests are conducted with rotation of potentiometer axis within not less than 90 % of the operating angle and rotation speed of 100 revolutions per minute with the number of axis rotations of 41600 for low-resistance potentiometers and 83300 rotations for high-resistance potentiometers.

The second test cycle includes the following stages:

1. Hot soaking (modes as in the first cycle) for 400 h;
2. Soaking in normal environmental conditions under 1 W direct current for 96 hours. The accuracy of voltage control is $\pm 5\%$.
3. The wearing test is similar to the one in the first cycle.

Potentiometers are deemed successfully tested for short-term reliability if the relative variation of the overall impedance is not more than $\pm 2\%$ and there is no mechanical damage.

Gamma-percentile time to failure if $\gamma = 95\%$ and failure rate are evaluated by means of forecasting the degradation of the acceptance criterion values (ACP) obtained as the result of short-term reliability tests in the following order.

1. In each i -th ($i = 1, 2, \dots, N$) time cross-section using the measured ACP values (relative variation of impedance) Y_{ij} ($j = 1, 2, \dots, n$) the ACP value in the i -th time cross-sections is identified subject to dispersion Y_i :

$$Y_i = m_i \pm \frac{KS_i}{\sqrt{n}},$$

where m_i is the average ACP value, S_i is the mean square value of ACP in the i -th time cross-section, K is the quantile of Student's t -distribution, of which the values are chosen subject to confidence probability $P = 0,95$.

2. The calculated values of Y_i are used as experimental points that are later approximated by one of the following equations:

- a) straight line,
- b) exponential curve,
- c) polynomial equation.

The value of gamma-percentile time to failure is identified by means of extrapolation of approximating lines as a

continuation of the chosen curve (straight line) constructed according to the least squares methods.

The values of Y_i can be with one sign (plus or minus) or different signs (plus and minus). In the first case the approximation does not take the sign into consideration, while in the second case the approximation is based on absolute values of ACP deviation. The most satisfactory approximating line is chosen based on the minimal discrepancy between the experimental and calculated values of ACP deviation using the least squares method.

3. Calculation of gamma-percentile operation time for the case of straight line approximation:

$$Y = b_0 + b_1 x.$$

where x is the products testing time, b_0, b_1 are coefficients calculated according to formulas:

$$b_0 = \frac{\sum_{i=1}^N y_i \sum_{i=1}^N x_i^2 - \sum_{i=1}^N y_i x_i \sum_{i=1}^N x_i}{N \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i \right)^2},$$

$$b_1 = \frac{N \sum_{i=1}^N y_i x_i - \sum_{i=1}^N y_i \sum_{i=1}^N x_i}{N \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i \right)^2}.$$

The value Y for the specified value of gamma percentile time to failure x is identified using the formula:

$$Y = b_0 + b_1 x.$$

In each time cross-section, the value of discrepancy is identified between the experimental values Y_e and the values Y_p identified using the calculated straight line. Accumulated discrepancy is calculated:

$$\sum_{i=1}^N S_{SL}^2 = \sum_{i=1}^N (Y_p - Y_e)^2.$$

4. Calculation of gamma-percentile operation time for the case of exponential approximation:

$$Y = 1 - e^{-kx},$$

where k is the coefficient that defines the rate of exponential curve growth.

For the last two time cross-sections, the values of coefficients k_1, k_2 are identified:

$$-k_1 = \frac{\ln(1 - Y_{1NORM})}{x_1}, -k_2 = \frac{\ln(1 - Y_{2NORM})}{x_2},$$

where x_1, x_2 are the last two time cross-sections, Y_{1NORM}, Y_{2NORM} are normalized values of ACP deviation in the second to last (Y_1) and last time cross-sections (Y_2).

The values Y_1, Y_2 are normalized to the maximum allowable ACP deviation (ΔY_{alw}) in accordance with the

values given in the performance specification or technical regulations:

$$Y_{1NORM} = \frac{Y_1}{\Delta Y_{ALW}}, Y_{2NORM} = \frac{Y_2}{\Delta Y_{ALW}}.$$

As the calculated value of k , the average of the following coefficients is adopted: $k = \frac{k_1 + k_2}{2}$.

The value Y for the specified value of gamma percentile time to failure x is identified using the formula:

$$Y = (1 - e^{-kx}) + \Delta Y_{BGN},$$

where ΔY_{BGN} is the ACP value in the first cross-section.

Accumulated discrepancy is calculated: $\sum_{i=1}^N S_{exp}^2 = \sum_{i=1}^N (Y_p - Y_e)^2$.

5. Calculation of gamma-percentile operation time for the case of polynomial approximation:

$$Y = b_0 + b_1x + b_2x^2 + b_3x^3,$$

where b_0, b_1, b_2, b_3 are polynomial coefficients.

For the purpose of polynomial coefficient calculation, the matrix of time cross-sections (X) and the ACP values matrix (Y) are constructed:

$$\begin{bmatrix} \sum_{i=1}^N Y_i \\ \sum_{i=1}^N Y_i x_i \\ \sum_{i=1}^N Y_i x_i^2 \\ \sum_{i=1}^N Y_i x_i^3 \end{bmatrix} = \begin{bmatrix} N & \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 & \sum_{i=1}^N x_i^3 \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 & \sum_{i=1}^N x_i^3 & \sum_{i=1}^N x_i^4 \\ \sum_{i=1}^N x_i^2 & \sum_{i=1}^N x_i^3 & \sum_{i=1}^N x_i^4 & \sum_{i=1}^N x_i^5 \\ \sum_{i=1}^N x_i^3 & \sum_{i=1}^N x_i^4 & \sum_{i=1}^N x_i^5 & \sum_{i=1}^N x_i^6 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

The value Y for the specified value of gamma percentile operation time x is identified using the formula:

$$Y = b_0 + b_1x + b_2x^2 + b_3x^3.$$

Accumulated discrepancy is calculated: $\sum_{i=1}^N S_{POL}^2 = \sum_{i=1}^N (Y_i - Y)^2$.

6. Values $\sum_{i=1}^N S_{SL}^2, \sum_{i=1}^N S_{EXP}^2, \sum_{i=1}^N S_{POL}^2$ are compared and the

lowest one is identified. The value obtained by means of the approximation formula with the lowest accumulated discrepancy is taken as the calculated value Y for the specified value of gamma-percentile time to failure x (in hours).

7. The calculated value Y is compared with the maximum allowable ACP deviation (ΔY_{ALW}). If the condition $Y < \Delta Y_{ALW}$ is fulfilled, the potentiometers comply with the specified requirement for gamma-percentile time to failure.

8. Calculation results verification in the case of a straight line or polynomial equation approximation of experimental points through calculation of homogeneity of variance using Cochran's Q test, as well as model validity check using Fisher's ratio test.

9. Product failure rate over the gamma percentile time to failure is identified using the formula:

$$\lambda = \frac{1 - P}{T_\gamma}.$$

Conclusion

In order to validate the above method, short-term reliability tests were performed on 24 wire-wound potentiometers. According to the results of short-term reliability tests not a single catastrophic or parametric failure was identified, while the electrical parameters of the resistors were within the specified requirements.

As per the proposed method, the gamma percentile evaluation of time to failure was performed that equaled to $T_\gamma = 10000$ hours.

As the result of experimental data processing, exponential approximation was chosen for forecasting potentiometer ACP within the time of period of 10000 hours. The ACP value (Y) within the operation time $x = 10000$ hours was calculated using the formula:

$$Y = 1 - e^{-kx} = 1,166 \, \%.$$

The comparison of the calculated value $Y = 1,166 \, \%$ with the maximum allowable ACP deviation $\Delta Y_{ALW} = \pm 2\%$ shows that the condition $Y < \Delta Y_{ALW}$ is fulfilled, therefore the products comply with the specified requirements in terms of gamma-percentile time to failure if $\gamma = 95 \, \%$. The graph of ACP variation over the operation time of 10000 hours is given in Figure 1.

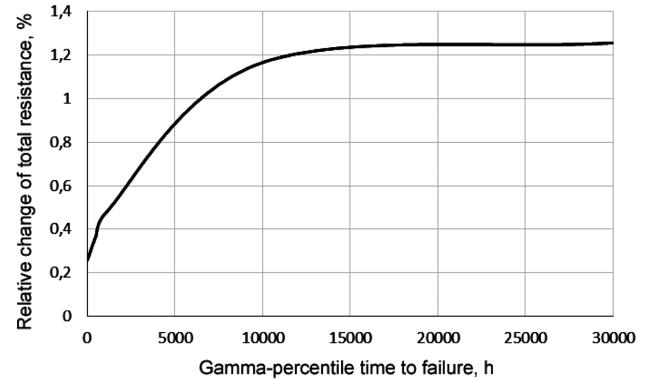


Figure 1. Graph of relative variation of impedance over the operation time of 10000 hours

In order to confirm the calculations of gamma-percentile time to failure, 10000-hour long-term reliability tests were carried out on the same wire-wound potentiometers. Failures were not recorded, the ACP values of the potentiometers did not exceed allowable figures.

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