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# OPTIMIZATION OF PREVENTIVE REPLACEMENTS AND REPAIR UNDER CONDITIONS OF UNCERTAINTY

*The paper presents a methodological approach to definition and solution of tasks related to preventive replacement optimization and technical equipment repair under conditions of uncertainty of initial information.*

**Keywords:** *optimization, repair, replacement, information, uncertainty.*

## 1. Background

Nowadays, optimization calculations of parameters of preventive replacement and repair of technical equipment are carried out, as a rule, upon the assumption of strict verification and unambiguousness of initial information used and, consequently, strict unambiguousness of obtained solutions. The disadvantage of this approach is a known overestimation of optimization accuracy and impossibility to find out solutions economically close to a uniquely defined and formally optimal solution.

When solving practical tasks of optimization of preventive replacement and repair, more or less uncertainty of initial information is inevitable. It reveals itself in uncertain knowledge of numerical values of initial data or their probabilistic description. Therefore, it becomes evident that methods of optimization calculations in case of fully defined information are increasingly coming into conflict with reality. Real uncertainty of initial information requires a fundamentally new approach to task definition and solution. Optimization under uncertainty condition inevitably contains heuristic procedures that exclude complete formalization of this process. And it is a fundamental difficulty in formalization caused by incomplete knowledge that matters rather than the technical one.

Consideration of uncertainty factors while taking decisions on parameters of preventive replacement and repair has a number of benefits. First, it provides maximum approximation of formal methods of solution to real operational conditions. Second, it ensures compulsory multivariate calculations and a possibility of analyzing the consequences of decision making on their basis. Third, it gives a possibility of selecting the most flexible solutions among those that are practically identically cost-effective. Fourth, it gives a possibility of taking more justified decisions and of reducing the risk of over expenditure of funds stipulated by inaccurate knowledge.

## 2. Task definition

It is the definition of a task that requires special attention under uncertainty conditions. It involves the following: 1) description of technical and economic essence of a task, objectives and criteria of optimization; 2) mathematical formulization of a task, including the definition of a target function, constraints and composition of parameters whose uncertainty may affect the results of the solution.

Optimization consists in establishing such parameters of preventive replacement and repair that ensure maximum possible effect in certain conditions. The effect means a complete or partial achievement of certain objectives. Let us single out the most significant objectives and their corresponding criteria. It is evident that the system of preventive replacement and repair cannot be considered perfect if it is not enough cost-effective. Therefore, the main criterion of optimization is a minimum operating cost per unit for preventive replacement and repair and emergency recovery, with the damage of possible equipment failure taken into account. In some cases equipment failure may result in safety violation, e.g. violation of traffic safety of transport systems. Therefore, the second criterion of the definition of parameters for preventive replacement and repair should be the safety of equipment operation.

While optimizing preventive replacement and repair, it is impossible to consider cost efficiency and safety in isolation from reliability. It should be stressed that reliability gains sort of double importance for technical equipment. On the one hand, it affects significantly cost efficiency, and on the other hand, it predetermines safety to a great extent. In the context of comprehensive approach it is reasonable to consider that the main criteria are cost effectiveness and safety, and reliability is some means to achieve required values of these parameters. Then the optimization task may be formulated as follows: finding such values of preventive replacement and repair which allow to achieve minimum operating cost per unit and to secure acceptable safety related reliability of technical equipment.

In terms of mathematical formulization and common approach to its solution, the optimization task of preventive replacement and repair refers to research of operations [1]. It is reduced to finding such values of controlled parameters $U$ when under influence of uncontrolled $Z$ and fixed $W$ parameters the target function $C(U,Z,W)$ defining total operating costs per unit takes on minimum value. The periodicity $\tau$ and the depth $\alpha$ of preventive replacement and repair of equipment arise as the controlled parameter $U$. The fixed parameter $W$ is the cost of preventive replacement and repair $B$ and the cost of emergency recovery, with the damage of technical equipment failure $A$ taken into account. Probability of failure-free operation $P$ and probability $q$ of the event that equipment failure will be eliminated by minimum emergency repair appear as the uncontrolled parameter $Z$.

Constraints in the form of equality and inequality may be imposed upon the parameters of the target function. Probability of failure-free operation defining the state of technical equipment depends on the parameters $\tau$ and $\alpha$ and is described by the distribution function $F$. According to safety conditions, the probability of failure-free operation should be not lower than a certain acceptable level $P_\partial$. As preventive replacement and repair aim at failure prevention of technical equipment, their periodicity should be lower than the time to failure $T$. Then in general, the optimization task of equipment preventive replacement and repair may be formalized in the following way:

$$C\,(\tau,\alpha) = min\ C(\tau,\alpha;A,B;P,q) \text{ with } P = F(\tau,\alpha),\ \alpha\geq 0,\ 0<\tau<T,\ P\geq P_\partial \qquad (1)$$

Let us introduce the criterion $C$ according to which optimal parameters of preventive replacement and repair subject to minimum operating cost per unit are defined, and the criterion $P$ according to which parameters of preventive replacement and repair subject to acceptable safety related probability of failure-free operation is defined. While executing preventing replacement and repair in practice, there is a range of constraints on materials, number of personnel, duration, weather conditions, etc. Times of preventive replacement and repair of different equipment should be coordinated between each other. In the framework of criteria $C$ and $P$ it is difficult, sometimes impossible, to take into account the effect of a set of constraints and factors, a range of which can be given in a qualitative form only and does not affect search for optimal preventive replacement and repair parameters. In this regard it seems reasonable not to formalize such constraints but to take them into account for final decision making.

If for parameters of the target function $Z$, $W$ and the distribution function $F$ there are unambiguous statistical characteristics, then the optimization task in question (1) would be of definite probability. If for $Z$ and $W$ there are data on their possible range and $F$ may be given by series of possible distribution functions, then task (1) should be solved in the form of probabilistic indefinite statement. Thus, the mathematical principles of task solution of preventive replacement and repair optimization significantly depend on a degree of certainty of initial information about distribution function of failure-free operation and target function parameters. In this regard a formal method of optimization should be preceded by the analysis of initial data whose uncertainty affects results of the task solution.

## 3. Analysis of uncertainty factors

The task of analysis is to classify and to give qualitative description of uncertainty factors in order to find methodological and practical difficulties in optimizing and to set out the ways of their solving as well as further to establish a degree of influence of these factors on the accuracy of optimization of equipment preventive replacement and repair. Optimization accuracy may be estimated by target function deviations from optimal value by the influence of the parameter of interest to us as the coefficient $K = C / C_o$, where $C$ is the target function value with parameter deviation; $C_o$ is the target function value with the optimal parameter.

With the help of classification [2], the initial information in optimization tasks of preventive replacement and repair can be divided into four categories: 1) deterministic; 2) probabilistically definite, when functions and parameters of the random variables distribution are known; 3) probabilistically indefinite, when function of the random variables distribution are unknown and 4) properly indefinite.

In practice, as a rule, no one has to use "completely" uncertain information as it is possible to get the required minimum of approximate information by one way or another, including expert analysis, for any parameter. Deterministic information is referred to cost of preventive replacement and repair $B$, whose average value is uniquely defined by regulations. Information of emergency recovery cost as part of the parameter $A$ can be referred to conditionally deterministic information as it cannot be defined uniquely due to some dependence on a number of random factors such as suddenness of equipment failure, qualification of maintenance personnel, etc. Damage information due to equipment failure in case of sudden and sometimes insufficiently defined character can be referred to probabilistically definite or probabilistically indefinite information.

In practice, a great challenge occurs in accurate estimation of repair depth $\alpha$ and its contribution to equipment reliability variation as quality of preventive repair depends on a number of random factors which are difficult to take into account, such as equipment state, quality of spare parts and repair procedures, qualification of service personnel, etc. Subject to statistics, information on the parameter $\alpha$ can be categorized as probabilistically definite or probabilistically indefinite information.

In practice, special challenge is faced when choosing the distribution function $F$ due to poor statistics on equipment failure. It is possible to define a distribution function by the available method of mathematical statistics [3] when the number of failures exceeds fifty. In this case information on $F$ will be probabilistically definite, or otherwise, it will be probabilistically indefinite as it is possible to get several possible distribution functions. Information on the parameter $q$ subject to the volume of statistics on equipment failures can be probabilistically definite or probabilistically indefinite.

Uncertainty of initial information leads to methodological and practical difficulties in solving tasks of preventive replacement and repair optimization. In addition, dimension of the task to be solved increases significantly as a large number of possible combinations of information on distribution function of failure-free operation probability and target function parameters arises. For example, if three types of the function $F$ are given and each of the parameters $A$, $\alpha$, $q$ is given by three values, then in this case, there are 81 combinations of the information used. Each combination corresponds to an optimal value of preventive replacement and repair parameter when solving a task.

Thus, uncertainty of initial information leads to ambiguity of optimization task solution. Calculation can determine an area only within which every periodicity of preventive replacement and repair will be optimal with one or another combination of initial information. Academician L.A. Melentyev called this area as “an ambiguity area of optimal solutions”[2]. Practical consequence of ambiguity of the initial information is that ambiguity in results of optimization task solution leads to ambiguity when selecting

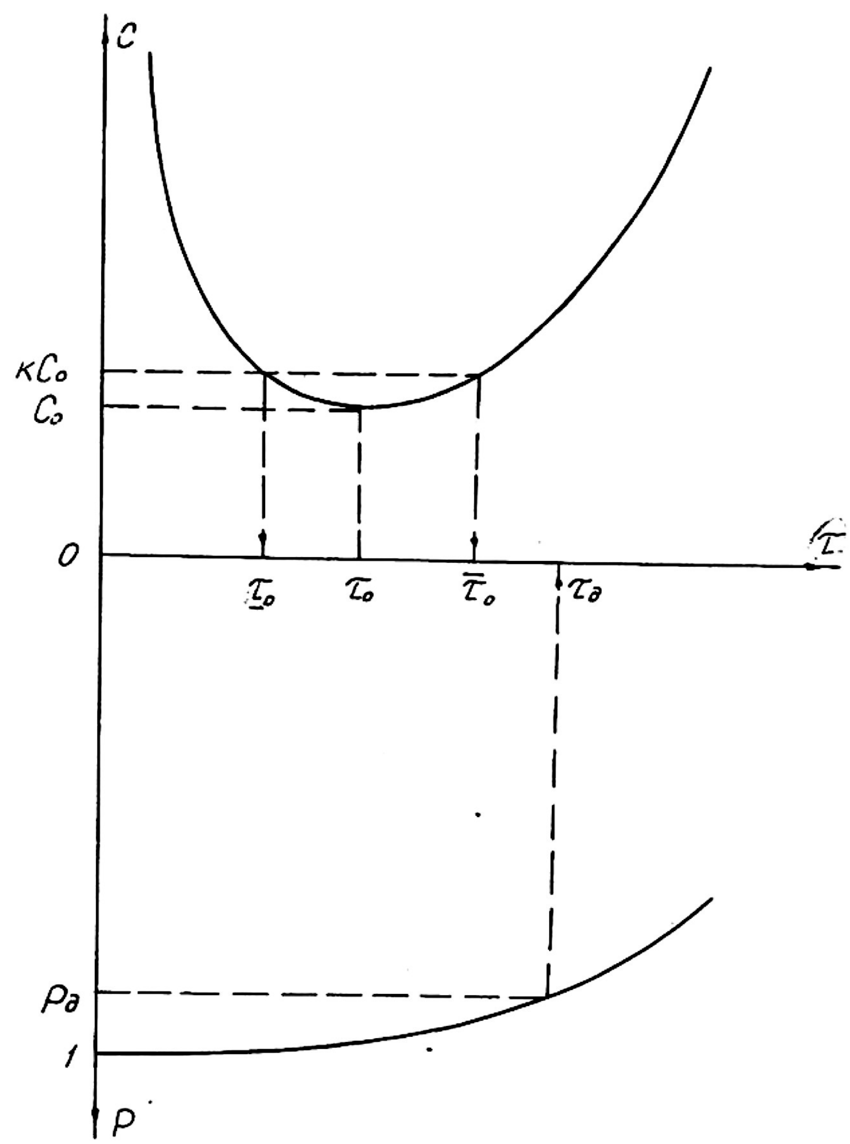


Fig. 1. The approach to solving of task 13 for case 1

preventive replacement and repair parameters. It is clear that under these conditions a final decision should be taken by humans on a heuristic basis and such "subjective" choice is inevitable in case of ambiguity of initial information.

Difficulties and negative consequences related to ambiguity of initial information may be overcome by two approaches: 1) reduction of information ambiguity; 2) development of relevant methods of optimization and decision making under ambiguity conditions. Works following the first approach are extremely effort-consuming and expensive. Moreover, no efforts in this case will make it possible to nullify ambiguity of initial information. Therefore, it is of great interest to carry out researches following the second approach with the aim of creating methods which allow us to take reasonable decisions on practically optimal preventive replacement and repair parameters of technical equipment with ambiguity of initial information.

## 4. Methodological principles of task solution

Methods of solving optimization tasks for preventive replacement and repair significantly depend on certainty of the initial information used. Let us provide classification of optimization tasks in accordance with a degree of certainty of the information on a distribution function of failure-free operation probability and target function parameters. Let us consider three degrees of information certainty about the distribution function: 1) $F$ and its parameters are known; 2) variance coefficient $V$ is only known; 3) time to failure $T$ is only known. The first degree corresponds to probabilistically definite information, the second and the third ones correspond to probabilistically indefinite information. Let us consider three degrees of information certainty of the parameters $A, \alpha, q$: 1) deterministic; 2) probabilistically definite; 3) probabilistically indefinite.

The types of possible optimization tasks are represented in the form of matrix (Table 1). Type of task is characterized by a double index: the first index means a row number, and the second one means a column number. The row numbers correspond to a degree of certainty of information about the distribution function, the column numbers corresponds to the target function parameters. All the tasks, except for 11,12, are probabilistically indefinite. By now the reliability theory has only developed task solution methods for 11 [4,5] and 31 [6]. Since in practice for preventive replacement and repair optimization, the required initial information is, as a rule, probabilistically definite or probabilistically indefinite, the development of methods for task solution of 12,13,22,23,32 and 33 type is of great interest.

**Table 1**

| **Degree of information certainty about the function $F$** | **Degree of information certainty about the parameters $A, a, q$** | | |
|---|---|---|---|
| | **1. Deterministic** | **2. Probabilistically definite** | **3. Probabilistically indefinite** |
| $F$ is known | 11 | 12 | 13 |
| $V$ is known | 21 | 22 | 23 |
| $T$ is known | 31 | 32 | 33 |

One of the possible methodological approaches to solving the above mentioned tasks is as follows. Solution of probabilistically indefinite tasks is reduced to their probabilistically definite equivalent. To this end, in case of conditionally deterministic and probabilistically definite information, the target function $A, \alpha, q$ are given by mathematical expectation, and in case of probabilistically indefinite information, the function is given by a range of values. In the latter case when the required initial data are unavailable, the target function parameters are estimated by expertise. The unknown distribution functions $F$ are chosen heuristically. In addition, it is necessary to give several possible distribution functions for more reliable solutions to be obtained.

In order to solve the obtained equivalent probabilistically definite problem, it is possible to use the known methods which allow finding optimal solutions corresponding to mathematical expectation of target function of the following type

$C(\tau/\alpha) = \min M[C(\tau; A, B, \alpha; P, q)]$, where $M$ is expectation sign.

As a result of this problem solution, it is necessary to define not only formally optimal parameters of preventive replacement and repair for every preset distribution function $F$, but also an area of all possible conditionally optimal values based on a range of variations of the target function $A$, $\alpha$, $q$ or the given optimization precision factor $K$. Under uncertainty of initial information, formal methods of solving defined tasks are a necessary, although auxiliary tool that allows us to automate an extremely effort-consuming but necessary process of research for conditionally optimal values of preventive replacement and repair

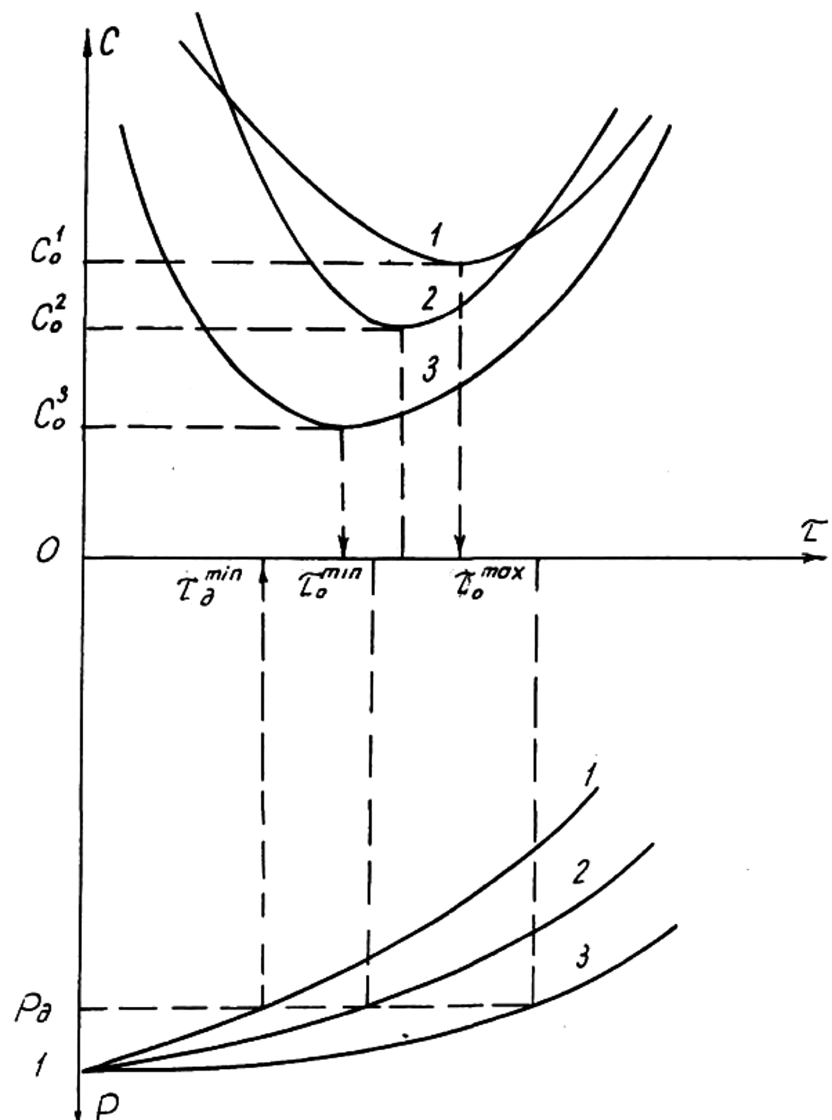


Fig. 2. The approach to solving of tasks 21, 22, 31 and 32 for case 3

parameters when using computers. Final decision making is carried out by an expert taking into account operation experience and involving additional unformalized criteria.

Task solution of preventive replacement and repair optimization using the proposed methodological approach consists of the following stages.

1. Initial data on failures, cost of preventive replacement and repair and emergency recovery as well as equipment failure damage shall be collected, and based on these data, the required initial information on the distribution function $F$ and the target function $A$, $B$ , $\alpha$, $q$ for task solution shall be defined.

2. According to a degree of initial information certainty in Table 1, the type of optimization task to be solved shall be chosen.

3. A set of the distribution function $F$ shall be chosen, a range of values for the target function $A$, $\alpha$, $q$ shall be estimated and the optimization precision factor $K$ shall be given for probabilistically indefinite tasks.

4. According to the criterion $C$:

- In case of probabilistically definite tasks, the optimal periodicity of preventive replacement and repair and its relevant minimum of the target function shall be calculated;

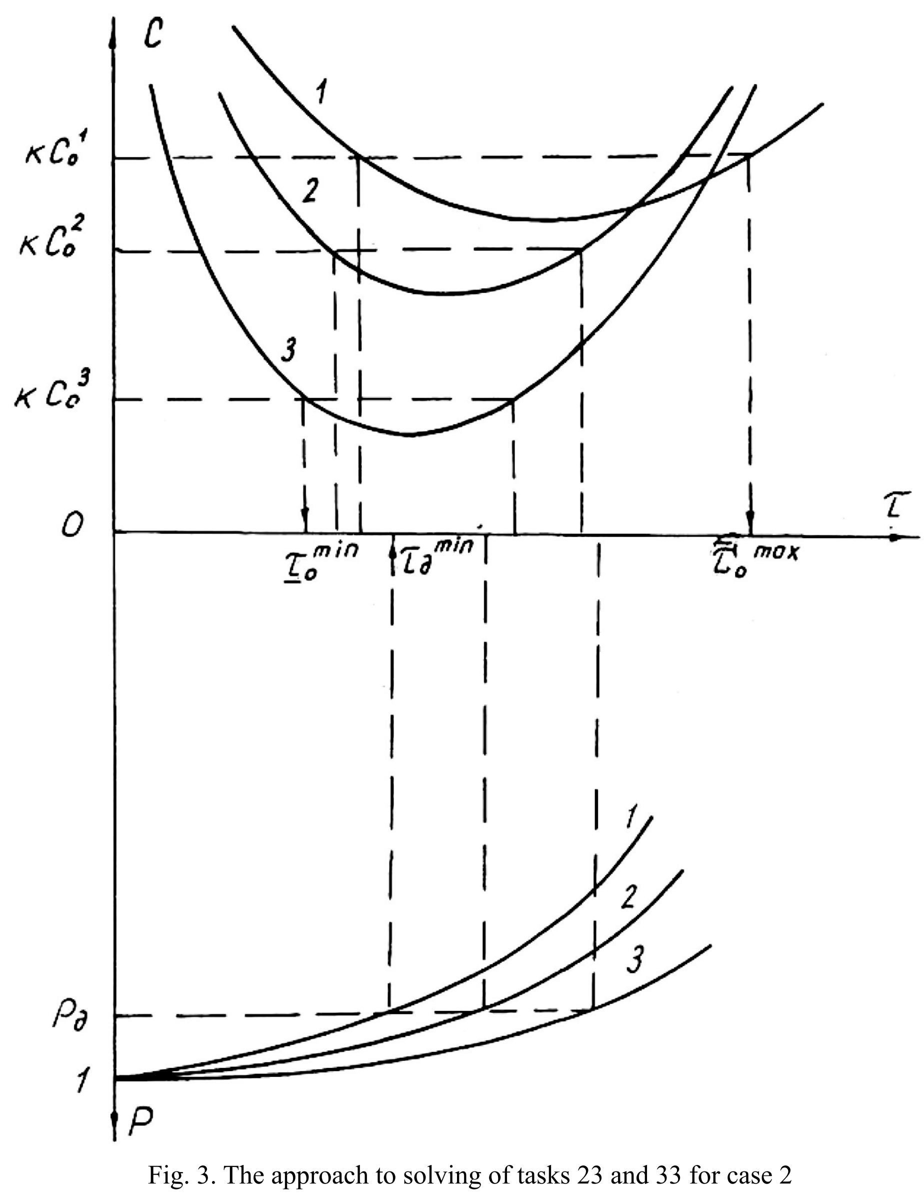


Fig. 3. The approach to solving of tasks 23 and 33 for case 2

- In case of probabilistically indefinite tasks, for a set of the distribution function $F$ we shall calculate a set of optimal periodicities of preventive replacement and repair and a set of minimal mathematical expectations of the target function related to them. Then a range of lower and upper values of the periodicity shall be defined with the given factor $K$, and an area of conditionally optimal values of preventive replacement and repair shall be set.

5. According to the criterion P:

- In case of probabilistically definite tasks, the acceptable safety related periodicity value of preventive replacement and repair shall be calculated;

- In case of probabilistically indefinite tasks, for a set of the distribution function $F$ we shall define a set of the acceptable safety related periodicity values of preventive replacement and repair. The minimum value of this set shall be chosen.

6. The obtained values according to the criterion $C$ and $P$ shall be compared, and an area of the appropriate values of preventive replacement and repair periodicity shall be defined.

7. Analysis of the obtained area of the appropriate values shall be performed, and final decision on practically optimal preventive replacement and repair values shall be made.

Let us illustrate the solution of the optimization task at stage 4, 5 and 6.

Task 11 and 12. According to the criterion $C$, we find the optimal value of preventive replacement (preventive repair) periodicity $\tau_o$ and the minimum value of the target function $C_o$, and according the criterion $P$, we find the acceptable safety related periodicity $\tau_\partial$ of preventive replacement (preventive repair). Then the appropriate periodicity values $\tau_{\text{ц}}$ are in the range: with $\tau_o < \tau_\partial$, $\tau_o \le \tau_{\text{ц}} \le \tau_\partial$, and with $\tau_o > \tau_\partial$, $\tau_{\text{ц}} \le \tau_\partial$.

Task 13. According to the criterion $C$, $\tau_o$ and $C_o$ , shall be defined as well as the lower $\underline{\tau}_o$ and upper $\overline{\tau}_o$ values of preventive replacement (preventive repair) with the given factor $K$. According to the criterion $P$, $\tau_\partial$ shall be defined. Then we obtain as follows: 1) with $\overline{\tau}_o < \tau_\partial$, $\underline{\tau}_o \le \tau_{\text{ц}} \le \overline{\tau}_o$; 2) with $\underline{\tau}_o < \tau_\partial < \overline{\tau}_o$, $\underline{\tau}_o \le \tau_{\text{ц}} \le \tau_\partial$; 3) with $\underline{\tau}_o > \tau_\partial$, $\tau_{\text{ц}} \le \tau_\partial$.

Figure 1 presents the solving approach to task 13 for case 1 which shows that the appropriate values of periodicity with the given optimization precision $K$ are in the range $\underline{\tau}_o \ldots \overline{\tau}_o$ defined according to the criterion $C$. Thus, in this case the main criterion for the periodicity choice of preventive replacement (preventive repair) is minimum operating cost per unit.

Task 21, 22, 31 и 32. According to the criterion $C$, for a set of the distribution function $\{F^1, F^2, \ldots, F^n\}$ we shall define a set of the optimal periodicity values $\{\tau_o^1, \tau_o^2, \ldots, \tau_o^n\}$ and its corresponding set of minimum values of the target function $\{C_o^1, C_o^2, \ldots, C_o^n\}$. The area of the conditionally optimal periodicity values is defined by the value range $\tau_o^{min} \ldots \tau_o^{max}$. According to the criterion $P$, a set of the acceptable safety-related values of the periodicity $\{\tau_\partial^1, \tau_\partial^2, \ldots, \tau_\partial^n\}$ shall be defined and the minimum $\tau_\partial^{min}$ shall be chosen. Then we obtain: 1) with $\tau_o^{max} < \tau_\partial^{min}$, $\tau_o^{max} \le \tau_{\text{ц}} \le \tau_\partial^{max}$; 2) with $\tau_o^{min} < \tau_\partial^{min} < \tau_o^{max}$, $\tau_o^{min} \le \tau_{\text{ц}} \le \tau_\partial^{min}$; 3) with $\tau_o^{min} > \tau_\partial^{min}$, $\tau_{\text{ц}} \le \tau_\partial^{min}$. Figure 2 presents the solving approach to task 21, 22, 31 and 32 for case 3 with three distribution functions $F$. The figure shows that the appropriate values of periodicity are in the value range $0 \ldots \tau_\partial^{min}$ defined by the criterion $P$. Thus, in this case the main criterion for periodicity choice is equipment safety.

Task 23 and 33. According to the criterion $C$, for the set $\{F^1, F^2, \ldots F^n\}$, the set $\{\tau_o^1, \tau_o^2, \ldots, \tau_o^n\}$ and its corresponding set $\{C_o^1, C_o^2, \ldots, C_o^n\}$ as well as the set of upper and lower periodicity values $\{\underline{\tau}_o^1, \overline{\tau}_o^1; \underline{\tau}_o^2, \overline{\tau}_o^2; \ldots; \underline{\tau}_o^n, \overline{\tau}_o^n\}$ with the given $K$ shall be defined. The area of the conditionally optimal values of periodicity is defined by the range of values $\underline{\tau}_o^{min} \ldots \overline{\tau}_o^{max}$. According to the criterion $P$, the set $\{\tau_\partial^1, \tau_\partial^2, \ldots, \tau_\partial^n\}$ shall be defined, and $\tau_\partial^{min}$ shall be chosen out of it. Then we obtain: with 1) $\overline{\tau}_o^{max} < \tau_\partial^{min}$, $\underline{\tau}_o^{min} \le \tau_{\text{ц}} \le \overline{\tau}_o^{max}$; 2) with

$\underline{\tau}_o^{\min} < \tau_\partial^{\min} < \overline{\tau}_o^{\max}$, $\underline{\tau}_o^{\min} \le \tau_ц \le \tau_\partial^{\min}$; 3) with $\underline{\tau}_o^{\min} > \tau_\partial^{\min}$, $\tau_ц \le \tau_\partial^{\min}$. Figure 3 presents the solving approach to task 23 and 33 for case 2 with three distribution functions. The Figure shows that the appropriate periodicity values are in the value range $\underline{\tau}_o^{\min}$ $\tau_\partial^{min}$ defined according to the criteria *C* and *P*. Thus, in this case it is possible to provide a cost efficient value of total operating costs for the given optimization precision as well as a failure-free operation probability value stipulated by safety requirements.

## Conclusion

In the framework of an integrated approach, it is reasonable to consider minimum operating costs per unit and an acceptable level of equipment failure-free operation probability in terms of safety requirements as the main criteria of preventive replacement and repair optimization.

The solution of preventive replacement and repair optimization becomes more complicated due to uncertainty of initial information which reveals itself in errors of numerical values of target function parameters and in inadequate description of the distribution function of reliability values. Uncertainty of initial information increases the dimension of a task and leads to ambiguity of its solution results and choice of periodicity for equipment preventive replacement and repair.

It is reasonable to reduce the solution of probabilistically indefinite tasks of preventive replacement and repair optimization to their probabilistically definite equivalent by giving a value range of the target function parameters and choosing the most probable distribution function of reliability parameters.

The aim of formalized solution of probabilistically indefinite tasks is to define areas of conditionally optimal periodicity values of preventive replacement and repair whose depth depends on a degree of certainty of initial information. Final choice of decisions should be made by experts using additional unformalized criteria.

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