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Practical application of continuous distribution laws in the theory of reliability of technical systems

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Aim. *One of the stages of dependability analysis of technical systems is the a priori analysis that is usually performed at early design stages. This analysis a priori has known quantitative dependability characteristics of all used system elements. As unique, non-mass produced or new elements usually lack reliable a priori information on quantitative dependability characteristics, those are specified based on the characteristics of technical elements already in use. A priori information means information retrieved as the result of dependability calculation and simulation, various dependability tests, operation of facilities similar in design to the tested ones (prototypes). From system perspective, any research of technical object dependability must be planned and performed subject to the results of previous research, i.e. the a priori information. Thus, the a priori analysis is based on a priori (probabilistic) dependability characteristics that only approximately reflect the actual processes occurring in the technical system. Nevertheless, at the design stage, this analysis allows identifying system element connections that are poor from dependability point of view, taking appropriate measures to eliminate them, as well as rejecting unsatisfactory structural patterns of technical systems. That is why a priori dependability analysis (or calculation) is of significant importance in the practice of technical system design and is an integral part of engineering projects. This paper looks into primary [1] continuous distributions of random values (exponential, Weibull-Gnedenko, gamma, log normal and normal) used as theoretical distributions of dependability indicators. In order to obtain a priori information on the dependability of technical systems and elements under development, the authors present dependences that allow evaluating primary dependability indicators, as well as show approaches to their application in various conditions.* **Methods.** *Currently, in Russia there is no single system for collection and processing of information on the dependability of diverse technical systems [3] which is one of the reasons of low dependability. In the absence of such information, designing new systems with specified dependability indicators is associated with significant challenges. That is why the information presented in this article is based upon the collection and systematization of information published in Russian sources, analysis of the results of simulation and experimental studies of dependability of various technical systems and elements, as well as statistical materials collected in operation.* **Results.** *The article presents an analysis of practical application of principal continuous laws of random distribution in the theory of technical systems dependability that allows hypothesizing the possible shape of system elements failure models at early design stages for subsequent evaluation of their dependability indicators.* **Conclusions.** *The article may be useful to researchers at early stages of design of various technical systems as a priori information for construction of models and criteria used for dependability assurance and monitoring, as well as improvement of accuracy and reliability of derived estimates in the process of highly reliable equipment (systems) development.*

Keywords: *dependability, distribution, failure, operation time, density, mathematical expectation, dispersion.*

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Introduction

System failures can be described using models designed for application in various dependability-related tasks that treat differently the system of factors that are intrinsic to the nature of failure.

The random nature of failures over the course of technical systems and components operation allows describing those using probabilistic statistical methods. The most commonly used failure models are based on distributions of associated random values, i.e. times to failure of non-repairable items and times between failures of repairable items.

As the primary types of distributions of item times to failure we should emphasize the following ones [1]:

- exponential

- Weibull-Gnedenko
- gamma
- lognormal
- normal.

A review of the available literature sources on technology dependability resulted in the evaluation of practical application of those laws in the context of studying various technical objects. Based on the performed analysis, an appropriate a priori distribution of corresponding dependability criterion or indicator can be selected.

Exponential distribution

While being a special case of the Weibull-Gnedenko distribution (if α=1), the exponential distribution is of significant interest in itself as it adequately describes the distribution of element operation time within the period of normal operation. The practical popularity of the exponential law is explained by not only its various potential natural physical interpretations, but its exceptional simplicity and convenience of its simulation properties. Below are the formulas for identification of density and probability of no-failure over the time t as per this law:

$$f(t)=\lambda\cdot e^{-\lambda t};$$

$$P(t)=e^{-\lambda t},$$

where λ is the failure rate.

The expectation m and mean-square deviation m for the exponential distribution are expressed through its parameter:

$$m=1/\lambda,$$

$$\sigma=1/\lambda.$$

Mean time to first failure is equal to

$$T_{ave}=m=1/\lambda.$$

The exponential distribution is often used at the design stage when information on the dependability of the elements of the system in development is limited or absent. That is why it is often called the principal law of dependability [2]. The limiting factor of this law's application is the requirement of utmost simplicity of the failure and renewal streams (they must be ordinary, stationary and devoid of consequences) [3].

According to [4 – 12], exponential distribution provides a good description of the dependability of technology operated after the end of the wear-in until significant degradation failures, i.e. within the period of normal operation when sudden failures take place. In [2, 7], it is said that the time to failure of technical systems with large numbers of serially connected elements can be described with this distribution if each of the elements individually does not significantly contribute to system failure. In case the failures of serially connected elements have an exponential distribution, then the system's own failures will be subject to that law and its failure rate will be equal to the sum of the elements' failure rates. Regard must be paid to the fact that systems that contain elements connected non-serially dependability-wise will not display exponential distribution despite the exponentiality of probabilities of failure free performance of its component elements [3].

As each element of a system in turn is itself a subsystem comprising several or commonly a larger number of elements, the total failure rate of the system's elements depends only on the number of faulty elements, while the time of repair of each faulty element has an exponential distribution. A failure of such subsystem is a failure of one of its elements that in maintenance is replaced with a new one. The net operating time of a subsystem defines that its failure flow is a sum of a large number of flows and, according to the Khinchin limiting theorem, it is asymptotically a Poisson stream. Therefore, we can conclude that the time interval between adjacent failures will have exponential distribution [13, 14].

The exponential law should be applied to those complex technical systems in which there are many different destructive processes that unfold simultaneously at different rates. However, as the difference in the rates at which the processes develop declines, the distribution approaches normal, if same-type destructive processes prevail, the distribution is exactly normal [11].

The authors of [2, 4, 6, 15] believe that in the context of solving problems related to complex system maintenance, if the renewal stream is simple, the exponential law should be applied when describing the renewal rate, labor intensity of current maintenance and failure recovery. In mass service, the intervals between repairs of equipment are also describable in terms of the exponential law [3].

As during normal operation sudden failures normally occur due to external effects, the replacement of an old element with a new one cannot influence the failure cause. Due to that, under exponential failure law there is no need for preventive measures, e.g. replacement of elements or their scheduled maintenance [16].

Some believe [2, 3] that if we consider the physical nature of sudden failures, the exponential law can be used to approximate the probabilities of no-failure of a large number of technical objects, primarily electronic equipment, electrical and electronic devices, hardware and software systems, etc.

However, despite the simplicity and universality the exponential law has a number of limitations. In particular, some papers [3, 13, 17] question the applicability of the exponential distribution law to sustained operation systems and over long intervals of time due to the following considerations:

– due to the fact that this distribution is characterized by "memorylessness", it has a significant disadvantage, i.e. contradiction with natural physical representations. This property means the absence of aging, i.e. a technical object does not age or, upon a certain time of operation will have a failure distribution identical to the one of a new object, which is inappropriate for the operation of many technical objects, especially over long periods of time [3,13, 17].

– in [3] it is claimed that the exponential distribution law is not applicable to complex technical systems, as due to non-simultaneity of the elements' operation and presence of failure aftereffects, the failure rate of a complex system cannot be permanent even if the failure rates of its elements are permanent. Therefore, this law cannot be used for dependability analysis of actual long-term operation technical systems, and the basic premises in the models are not adequate to the physical processes within the systems.

That points to the fact that you need to have sufficiently valid reasons to use exponential distributions, just like any other. Nevertheless, this distribution is common, which is due to the following:

– simplicity and dependence on only one parameter λ. This and the absence of aftereffect allow solving many tasks of the dependability theory and deliver solutions in an explicit analytical form;

– it has been proven that the time before failure of complex high-dependability repairable systems can be described with an exponential distribution under certain conditions (e.g. possibility to disregard the effect of materials "aging");

– under certain conditions the application of the exponential law in the cases when it is not appropriate allows achieving low dependability indicators, i.e. lower estimate, which is often acceptable.

Normal distribution law (Gaussian law)

The completeness of the theoretical research regarding the normal law, as well as comparatively simple mathematical properties make it the most attractive and easy to use. If the studied empirical data deviates from the normal law, there are the following ways it could be used:

- use it as the initial approximation. In many cases this assumption yields sufficiently accurate results;

- fit a transformation of the studied random value ξ, that would change the initial "non-normal" law into a normal one [14].

An important property of this law is its "self-reproducibility" that consists in the fact that the sum of any number of normally distributed random variables also follows the normal distribution law.

The distribution density of this law is defined by the formula:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-m)^2}{2\sigma^2}}.$$

Normal distribution dependability function is calculated using the following formula:

$$P(t) = \int_t^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx = 0,5 - F_0\left(\frac{t-m}{\sigma}\right),$$

where $F_0(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}} dx$ is the Laplace's function of which the values are tabulated.

The mean time before failure is equal to $T_{ave}=m$, and the relation between the time to first failure $\overline{T_1}$ and value T_{ave} is expressed with the formula:

$$\overline{T_1} = T_{ave} + \frac{\sigma\sqrt{2/\pi}}{\left[1+F\left(\frac{T_{ave}}{\sigma\sqrt{2}}\right)\right]} e^{-\frac{T_{ave}^2}{2\sigma^2}}.$$

The failure rate for the normal distribution is the increasing function that is defined by formula:

$$\lambda(t) = \frac{\sqrt{\frac{2}{\pi}} e^{-\frac{(t-T_{ave})^2}{2\sigma^2}}}{\sigma\left[1-F\left(\frac{t-T_{ave}}{\sigma\sqrt{2}}\right)\right]},$$

where $F(\ )$ is the function integral of the form

$$F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx.$$

The normal law is used to describe the dependability of technical facilities over the period of aging [2, 4, 6, 18]. In a number of sources [6 – 9, 19 – 22] it is stated that it is used in the cases when the failures are gradual and are caused by directional physicochemical changes due to deterioration (aging), and the coefficient of variation $\upsilon \leq 0,3 \div 0,4$ [7, 23]. Under stable conditions and modes of operation during this period the Gaussian distribution matches well with the mean and gamma-percentile life distribution [7], as well as machine's original life [4].

It is important to note that normal distribution of time before failure comes from the uniformity of technical objects quality, permanent average rate of deterioration and realization of deterioration as they long move and intertwine until failures start occurring [16].

The particular feature of the normal law application is as follows: if σ values are low compared to mean time before failure m, the density is fairly close to zero within a significant time interval, which lets us conclude that within this interval the probability of failure is very low. That reflects the fact that granted the deterioration level is high and accumulated deterioration is low the probability of failure is low. That is exactly the reason for the forced replacements (repairs) at low levels of deterioration that enable a low probability of failure between repairs [16]. In turn, for the exponential distribution that reaches maximum density if $t = 0$, most failures occur at the beginning of operation.

The statistical analysis, e.g. in [2, 11] of the test and operation results of mechanical units and metal structures subject to intense deterioration, aging and fatigue, shows that strength and load distribution are described with the normal law with associated probability densities. In some units a combination of exponential and normal distributions was observed. Such composed distribution is possible if units and parts of a device are simultaneously subject to sudden and deterioration failures. In hydraulic carrying systems and geared pumps the normal law describes the time between failures [11]. It must be noted that such random values as measurement and manufacturing errors, etc. also follow the Gaussian law. [16].

Logarithmically normal distribution

A random value ξ is lognormally distributed when its logarithm is distributed normally. A lognormal random value is affected by a large number of mutually independent factors, while the effect of each individual factor is "uniformly insignificant" and equally possible in sign. Unlike in the case of normal distribution, the sequential nature of the effect of random factors means that the random gain caused by the action of each further factor is proportional to the already achieved studied value.

The distribution density is defined by the formula

$$f(t) = \frac{1}{st\sqrt{2\pi}} e^{-\frac{(\ln t - \mu)^2}{2s^2}},$$

where μ and s are the parameters evaluated by the results of n tests to failure;

$$\mu = \frac{1}{n}\sum_{i=1}^{n} \ln t_i;$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\ln x_i - \mu\right)^2}.$$

For the lognormal law, the dependability function is as follows:

$$P(t) = \frac{1}{\sqrt{2\pi}} \int_{\frac{\ln\left(t/\mu\right)}{s}}^{\infty} e^{-t^2/2}\, dt.$$

The expectation of the time before failure and mean-square deviation are determined from the formula:

$$T_{ave} = m = e^{\left(\mu + s^2/2\right)};$$

$$\sigma = \sqrt{e^{2\mu + s^2}\left(e^{s^2} - 1\right)}.$$

In the case of lognormal distribution the failure rate will be equal to [23]

$$\lambda(t) = \frac{0,4343 e^{-\frac{(\lg t - \mu)^2}{2s^2}}}{st\sqrt{2\pi}F\left(\frac{\mu - \lg t}{s}\right)}.$$

A lognormal distribution is a distribution of positive variables, hence it is somewhat more accurate than the normal distribution [2]. It describes the behavior of the time between failures of objects that «strengthen» over time. The «strengthening» causes a slow decrease of deterioration rate. That is why before using the lognormal distribution it is necessary, on the basis of the physical nature of the deterioration process and, if possible, analysis of deterioration realizations behavior, to establish whether the studied technical objects have a tendency for «strengthening» [16].

This distribution also describes the following: renewal processes; longevity of products operating during the aging period when the deterioration increment is proportional to instantaneous deterioration [7, 14, 19]; operation times in the situation of rapid «burnout» of undependable elements; failures occurring as the result of material fatigue, in particular, description of operation time of ball bearings [3, 7].

In general, lognormal distribution adequately describes the times to failure of complex technical systems (tractors, automobiles, special heavy-duty vehicles, etc.), as well as electronic equipment [2].

Gamma distribution

Gamma distribution has a two-parameter distribution with the shape parameter ($\alpha > -1$) and scale parameter ($\beta > 0$):

$$f(t) = \frac{t^{\alpha}}{\beta^{\alpha} G(\alpha)} e^{-\frac{t}{\beta}}.$$

The probability of no-failure is defined using the formula:

$$P(t) = \int_{t}^{\infty} \frac{x^{\alpha-1}}{\beta^{\alpha} G(\alpha)} e^{\frac{x}{\beta}} dx = 1 - I\left(\alpha, \frac{t}{\beta}\right),$$

where $G(\alpha) = \int_{0}^{\infty} x^{\alpha-1} e^{-x} dx$ is the gamma function;

$I(\alpha, t) = \frac{1}{G(\alpha)}\int_{0}^{t} x^{\alpha-1} e^{-x} dx$ is the incomplete gamma function.

The expectation (mean time between failures) and mean-square deviation for the gamma distribution are equal to:

$$T_{ave} = m = \alpha\beta;$$

$$\sigma = \sqrt{\alpha}\beta.$$

The failure rate formula is as follows:

$$\lambda(t) = \frac{t^{\alpha} e^{-t/\beta}}{\int_t^{\infty} x^{\alpha-1} e^{-x/\beta} dx}.$$

Gamma distribution serves to describe deterioration failures; failures due to damage accumulation; description of operation time of complex technical systems with redundant elements; renewal time distribution [2, 7, 10, 16]. It can also be used for longevity (lifetime) analysis of certain technical objects [17].

Gamma distribution has a number of useful properties:

1. If $\alpha<1$ the failure rate monotonically decreases which corresponds to a rapid "burnout" of undependable elements.

2. If $\alpha>1$ the failure rate increases, gradual deterioration and aging of elements takes place.

3. If $\alpha=1$ gamma distribution matches the exponential one and can be used to describe the probability of failures in normal operation of a technical system [18].

Given the above, we can conclude that gamma distribution may be used at all stages of the lifecycle: wearing-in ($\alpha<1$), normal operation ($\alpha=1$) and aging ($\alpha>1$) [20].

4. If $\alpha>10$ gamma distribution practically matches the normal one and therefore can be used to describe the probability of failures of aging units, mechanisms and other elements [16, 18]. Also, if $P(t)\to\infty$ the gamma distribution approaches the normal distribution law. For this reason it is often used for approximation of those unimodal, but nonsymmetrical distributions that are poorly approximated with normal distribution [9, 12].

5. If α is a positive integer, then in [2, 7] the gamma distribution is also called Erlang distribution.

6. If $\lambda=1/2$ and α is divisible by 1/2, then the gamma distribution matches the ch-square distribution [2, 7].

On the assumption of [13] in respect to the tasks solved in terms of the Laplace transformation the gamma distribution can be conveniently used to approximate natural distributions.

In [11, 12], the following definition is given: gamma distribution is the characteristic of the time of failure occurrence in complex electromechanical systems in cases when sudden failures of elements take place at the initial stage of operation or system debugging, i.e. it is a convenient characteristic of the time of failure occurrence in the equipment during the wear-in period.

In complex technical systems that consist of elements of which the probability of no-failure has an exponential distribution, the probability of no-failure of the system as a whole will have a gamma distribution [11].

The distribution of the failure occurrence time in complex technical systems with redundancy (granted that failure flows of the primary and all backup systems are simple) can also be described with a gamma distribution [12]. Similarly, in cases of cold or combined redundancy the probability of no-failure of the system follows the generalized gamma distribution [3]. That said, it has been established [24] that in redundant systems (both repairable and non-repairable) there are always hidden failures, while the efficiency of their detection is quite limited. Those factors have a significant effect on system dependability and require more detailed model design (e.g. using Markov or semi-Markov process).

Weibull-Gnedenko distribution

The Weibull-Gnedenko distribution is a two-parameter distribution with the shape parameter α and scale parameter β that is characterized by the probability density function:

$$f(t) = \frac{\alpha t^{\alpha-1} e^{-\left(\frac{t}{\beta}\right)^{\alpha}}}{\beta^{\alpha}}.$$

The connections between dependability indicators appear as follows:

$$P(t) = e^{-\left(\frac{t}{\beta}\right)^{\alpha}},$$

$$T_{ave} = \beta G\left(1+\frac{1}{\alpha}\right),$$

$$\sigma = \beta\sqrt{G\left(1+\frac{2}{\alpha}\right) - G^2\left(1+\frac{1}{\alpha}\right)},$$

$$\lambda(t) = \frac{\alpha}{\beta^{\alpha}} t^{\alpha-1}.$$

This law has a wide range of use as it bridges over the fields of application of a number of other distributions, but is described with more complex formulas [4]. It can be used to describe:

- lifetimes of ball bearings, threads, splined shafts and other parts with simultaneous deterioration of several working faces [4];
- times to failure with simultaneous occurrence of sudden and deterioration failures [4];
- probability of no-failure of mechanical elements during aging or deterioration [3, 11, 12];
- lifetime of components of metal structures, supporting systems, support and rotation systems, hydraulic and electric drive systems in connection with fatigue and sudden failure (coefficient of variation of 0.35 – 0.70) [22].
- failure distribution during wear-in [2, 3, 21];
- operation times of special-purpose complex technical systems (mobile installations) in operation [2, 18];
- operation times of parts and components of automobiles, handling and other machinery subject to fatigue failures, ball bearings times to failure [2, 3];

- distribution of mean and gamma-percentile life subject to fatigue failure in stable conditions and modes of operation [7].

In a number of cases [3, 8, 10] the Weibull-Gnedenko distribution is universal due to the following properties;

– if α=1 it transforms into exponential distribution;

– if α<1 failure density and rate functions decrease;

– if α>1 failure density and rate functions increase;

– if α=2 function $\lambda(t)$ is linear, the distribution transforms into the Rayleigh distribution with density $f(t) = 2te^{-\lambda t^2}$;

– if α=3,3 the distribution is close to normal.

Due to its universality the Weibull-Gnedenko distribution is recommended for priority application when processing experimental data on the dependability of technical facilities in situations when the type of the distribution function is not initially known [15]. Additionally, all natural distributions are approximated much better with this distribution rather than the exponential distribution [9].

Like the gamma distribution, the Weibull-Gnedenko distribution well suits the approximation of natural distributions at various lifecycle stages: wear-in (α<1), normal operation (α=1) and aging (α>1) [2, 14, 20].

Also, a complex technical system that is considered as a single structural entity comprising a large number of elements in each of which the time before failure is subject to gamma distribution, but the parameters of such distributions slightly vary from element to element, will have a distribution close to the Weibull-Gnedenko distribution [16, 20]. It should be noted that many technical objects contain large numbers of identical or similar in design elements that operate in similar conditions (e.g. an internal combustion engine has a number of cylinders, electronic equipment has a large number of capacitors, resistors, etc.). If the repeating elements of a technical system define the time before failure of the system, that produces a structure that would have the Weibull-Gnedenko distribution [16].

From the point of view of the physical nature of failures, the Weibull-Gnedenko distribution adequately describes the time before failure of many electronic equipment elements in case the failure of such elements is considered [16] as a deviation of a parameter beyond the specified limits.

Conclusion

In conclusion, it should be noted that beside the above mentioned types of distributions, solving some tasks involves special types (several dozen in total), as well as discontinuous distributions that are not covered in this article. Distributions have various transitions and connections. Despite the existing goodness measures of the chosen theoretical and empiric distributions, all of them provide the answer to the following question: whether or not there are good grounds for discarding a hypothesis for the chosen distribution? The authors note that any data can be made to fit the multi-parametric law even if it does not correspond to real physical phenomena [7]. Thus, while choosing the type of distribution and its parameters one must first take into consideration the physical nature of the occurring processes and events.

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