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RELIABILITY OF CONTAMINATION DISPERSION AWAY FROM A SINGLE POLLUTION SOURCE

Dispersion of contaminants moving away from a pollution source. A method is offered to predict atmospheric contamination with various production and maintenance wastes. There are a number of mathematical models describing contamination dispersion. The selection of a model depends on a set of factors. A model is defined by the purpose of calculation and, in its turn, defines a solution approach. This paper considers only two models. The basic guidelines of the paper are general.

Keywords: *pollution source, model, concentration, atmosphere, random choice, regeneration, pollution density*

1. Problem statement

Let us simulate the process of air pollution from a single source. In parallel, let us specify conditions when the reliability of dispersion is provided.

Contamination is evaluated by the concentration of contaminants as they move away from a source. Contaminants come from the atmosphere as well as from a source. Denote the percent of such contaminants as k. The peculiarity of k is that it can be either positive or negative. The atmosphere shall not be regarded in more detail.

Assume that two identical volumes v_1 and v_2 are located at a fixed distance from each other. Name v_1 as input and v_2 as output measurement of air pollution concentration. The input measurement v_1 is defined by physical processes that take place in a pollution source. The output measurement is defined by dispersion and atmospheric influence.

Let us set a fixed distance l between v_1 and v_2. For example, $l = 100$ m. Denote n_0 as concentration of contaminants in v_1 (input measurement) and n_1 as concentration of contaminants in v_2 (output measurement). Due to the simulation approach, each individual measurement is random.

The process of extracting contaminants from a source in a certain period of time $t = t_0$ minutes after beginning becomes statistically stationary, and the mean concentration and dispersion of contaminants $\overline{n}_0$, $\overline{\sigma}_0{}^2$ can be considered as constant.

Assume t_1 as a time period, this elapsed allowing us to regard the process in the volume v_2 as statistically stationary as well. For the fixed distance l we shall match such random time t_1 that with t_1 exceeded we have an inequation between concentrations

$$n(t_0) > n(t_1) \quad (1)$$

The fulfillment (1) indicates that dispersion really takes place at one hundred meter distance.

Note that the direction of a relation between v_1 and v_2 doesn't matter. Mismatch with the vector of air movement only means that relation is made in the direction of projection velocity vector.

Let the whole sector in the area nearest to the source be split into fixed length sections.

The choice of time t_1 is related to contradictory requirements. On the one hand, the increase of t_1 gives a higher guarantee for the fulfillment (1), on the other hand, the redundancy of t_1 rules out the time minimally requisite to correlate with the distance of 100 m. The following way for choosing t_1 is offered.

1. The minimum value of t_0 is defined. To that end the values of $rnd(t_0)$ are checked for a number of possible t_0. As a result, we have the minimum values as $t_0 = 0.05$, i.e. extraction of contaminants in the volume v_1 rapidly brings to a statistically stationary state.

2. Starting from t_0, we increase t_1 with increment by 0.05 at each step. The procedure finishes at that step where $rnd(t_1)$ operation results become feasibly different as to order of figures, i.e. statistical stationarity breaks. In this case we have $t_1 = 0.35$.

3. Assume the maximum random quantity of the last step finally as t_1.

Set maximum intervals t_m^k between neighboring measurements. Then $t_0 + t_m^1$ is a time interval wherein measurement following t_0 is located. In our case we have $t_m^1 = 0.3$.

Let us choose a random time of $t_1 \in (t_0 + t_m^1)$ measurement following t_0 using the function of $rnd(t_0 + t_m^1) \to t_1$. We have $t_1 = 0.247$.

Name t_0 point and posterior time points defined as to the same method as regeneration points Tp0, Tp1, Tp2 ... For the second regeneration point we have

$$rnd(t_1 + t_m^2) \to t_2 \text{ etc.}$$

(It is not worth setting $t_m^k \to \infty$; it practically doesn't make sense).

Therefore, we have a continuity of measurement points wherein dispersion is reliable if $n(t_0) > n(t_1) > n(t_2) > ...$ Here $n(t_k)$ is the concentration of contaminants at the moment of measurement at Tp k point.

Now it is easy to find a correlation between distance and time. For the start period $(0, t_0]$, time spent as per unit of distance length is

$$\gamma = \frac{t_0}{l} \quad (2)$$

For t_1 point, an extra distance from a reference point is $l_1 = \frac{t_1 - t_0}{\gamma}$, and we have the whole distance $l + l_1$ etc.

Let us consider the time spent as per unit of distance as $\gamma = const$, although we can record the changes of γ at regeneration points.

For each k -th regeneration, we can find the probability of its occurrence

$$q_k = \int_{t_{k-1}}^{t_k} p_k(t)b(t)dt \tag{3}$$

where $p_k(t)$ is event probability, t point belongs to the interval $(t_{k-1}, t_k]$ (corresponds to the k -th regeneration), $b(t)$ is distribution density of contamination concentration in the volume v_2.

Therefore, distance as such is excluded from consideration.

The fulfillment (1) will be defined by the distribution of a random quantity t on the time axis. Then in the near vicinity of a source, the scheme in the example looks like (Fig.1)

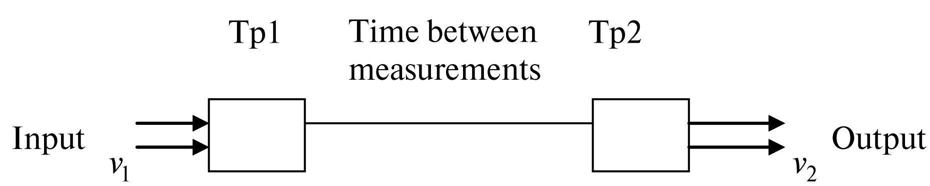

Fig. 1

Tp1 andTp2 are the first two points of regeneration. Let p_{n_1} be the probability of contamination concentration n_1 at the moment of measurements at Tp1, and p_{n_2} the probability of contamination concentration n_2 at Tp2 at the same moment of measurements as at Tp1. However, at Tp2 the concentration of contaminants is made up from n_2 and possible k changes for the time t. The values of k can be negative. Then the atmospheric influence facilitates dispersion. Since we should assume the worst scenario, the negative values of k are not taken into account.

Without taking into account the negative values of k, let us introduce the atmospheric concentration as

$$m_1 = 1 - n_1, \; m_2 = 1 - n_2 - k . \tag{4}$$

Both distributions involved in (3) are specified pursuant to the analysis of physical processes in a source and atmosphere. The applied problem of dispersion starts to be solved through selecting $p_k(t)$ probability and $b(t)$ dispersion.

2. Reliability of contamination dispersion in the nearest vicinity of a source

The conditions of reliability mean the purpose of $p_k(t)$, $b(t)$ satisfying (1), (2), (3), (4). According to (1), n_2 concentration shall drop as t grows.

Let us consider m_1 changes as a Poisson random process. For Tp k point, we have

$$p_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

As far as $b(t)$ is concerned, for the nearest vicinity one is advised [1] to use the Gaussian distribution law (Fig. 2).

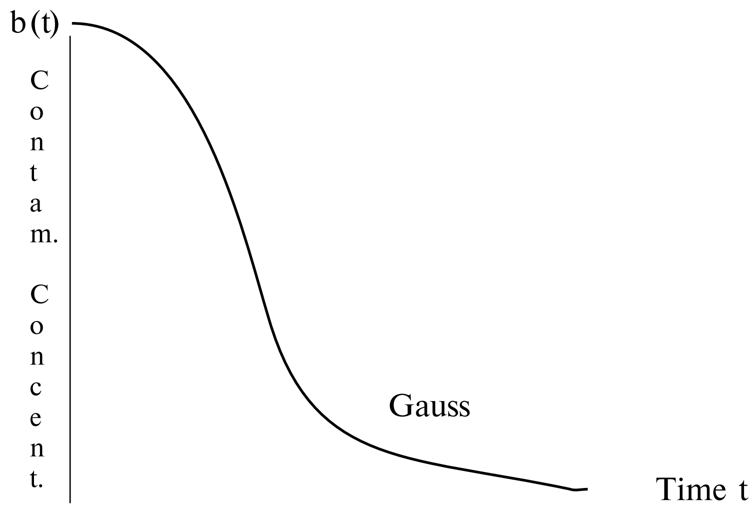

Fig. 2. Diagram of *tg*α distribution

The integral (3) in elementary functions is not calculated. Poisson-Gauss distribution parameters: λ, a, σ. If $a = 0, \sigma = 1$, then we can have the opportunity for each $t \geq 0$ to use $\Phi'(t)$ tables [2]. Let $t_2 - t_1 = \Delta t$ small. Then, using the intermediate value theorem, we can derive

$$\frac{\Delta\Phi(t)}{\Delta t} = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} = b(t)$$

The definition of $b(t)$ restricts to this. With $a \neq 0, \sigma \neq 1$,

$$t_2 = \frac{t-a}{\sigma}, \quad t_1 = \frac{t - \Delta t - a}{\sigma}$$

Let us use $\Phi'(t)$ tables to verify the conclusion made earlier: the minimum time $t_0 = 0.05$. As to the table in Fig. 5, we have $\Phi'(t_0) = 0.3989$.

Then the dispersion parameter defined as to basic value, for t_0, is

$$\sigma = \frac{\sqrt{0.3989}}{0.05} = 12.6.$$

However, the time of measurement can vary from 0 to 0.9. Thus, the other boundary for σ is

$$\sigma = \frac{\sqrt{0.2661}}{0.05} = 10.32$$

Lets us finally calculate

$$\sigma = rnd(12.6 - 10.32) + 10.32 = 11.65$$

The high value of σ indicates rapid dispersion in the nearest vicinity of a source.
For comparison let us calculate the boundaries of σ for $t_1 = 0.247$. We have

$$\sigma = rnd(1.99 - 1.037) + 1.037 = 1.37.$$

In fact, the decrease of σ as to table (see Fig. 5) can be radical as well as gradual as time changes at the 10-minute interval of observations (see Fig. 3).

Let α be the angle of change of contamination concentration for the observation time defined as linear approximation. Fig. 3 shows the diagram of $tg\alpha$ calculation as to table [2], wherein minutes from 0 to 9 are split into 10 divisions with the step of 0.1.

We shall find $tg\alpha$ with $a = 0, \sigma = 1$. The values of $b(t)$ are found using tables $b(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$. The table in Fig.5 gives $\Delta t = 0.1$. The maximum tabular initial time is equal to 9.

In the Table in Fig. 5, the basic values of $b(t)$ are taken from a standard table, and the final value is derived as linear approximation according to the diagram in Fig.3. The basic result is $tg\alpha$ quantities.

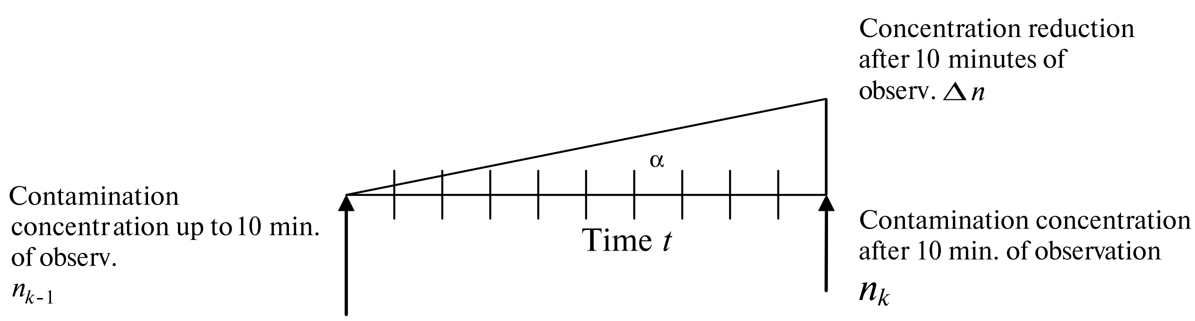

Fig. 3. Diagram of $tg\alpha$ calculation for the 10-minute observation

Fig. 3 specifies the type of the first line of the table in Fig. 5, independent of the type of distribution $b(t)$. Suppose that the initial concentration in v_2 is equal to $n_0 = 0$, without possible intakes from the air (4) taken into account, and after 10 minutes of observation the concentration n_1 becomes equal to that near to the source of pollution.

Even in such (worst) case $tg\alpha = 0.1 \quad \alpha \leq 75`$. We can neglect the changes of concentration and suppose that $n_1 \approx 0$. In other words, the 10-minute interval will be considered as always sufficient for dispersion of any contamination in v_2. It is such interval that is foreseen in the tables [2].

Fig. 4 assumes that the entire part of the atmosphere considered starting from a pollution source is divided into linear sections. The calculation of random time t^k gives respectively 15, 24.885, 25.848, 28.506, 30.311 (minutes), with the time of observation and some reserve taken into account.

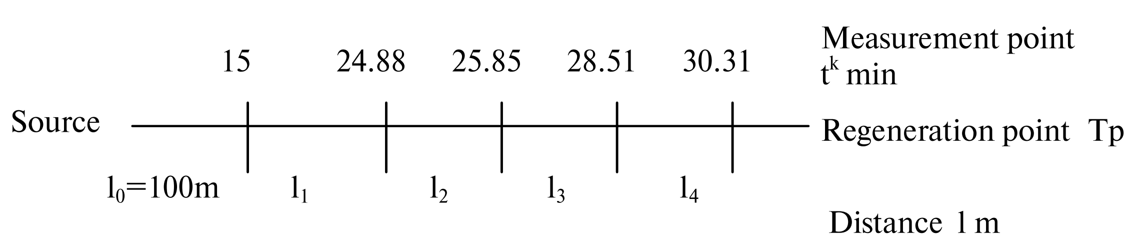

Fig. 4. Results for 5 Tp

Small $tg\alpha$ quantities for all n values prove that the concentration primarily changes at the first 15-minute interval.

The table in Fig.5 is made up on the basis of the standard table [2]. But the standard table already has an approximately linear character as regards lines as well as columns. This is what justifies the scheme in Fig.5 and the calculation of final values and $tg\alpha$ in the table in Fig. 5.

Fig. 5. Table of results in Gauss model

Time *n*	Time of change e.g. 1,01	Change time interval with a given time of change	Final value	tgα	Basic value
0	0	0-10	0,3973	tg 0,0112	0,3989
1	0,1	0-10	0,3918	tg 0,052	0,3970
2	0,2	0-10	0,3825	tg 0,085	0,3910
3	0,3	0-10	0,3697	tg 0,0117	0,3814
4	0,4	0-10	0,3538	tg 0,0145	0,3683
5	0,5	0-10	0,3352	tg 0,0169	0,3521
6	0,6	0-10	0,3144	tg 0,0188	0,3332
7	0,7	0-10	0,2920	tg 0.0203	0,3123
8	0,8	0-10	0,2685	tg 0,0212	0,2897
9	0,9	0-10	0,2444	tg 0,0217	0,2661
10	1	0-10	0,2203	tg 0,0217	0,2420
11	1,1	0-10	0,1965	tg 0,0214	0,2179
12	1,2	0-10	0,1736	tg 0,0206	0,1942
13	1,3	0-10	0,1518	tg 0,0196	0,1714
14	1,4	0-10	0,1315	tg 0,0182	0,1497
15	1,5	0-10	0,1127	tg 0,0168	0,1295
16	1,6	0-10	0,0957	tg 0,0152	0,1109
17	1,7	0-10	0,0804	tg 0,0136	0,0940
18	1,8	0-10	0,0669	tg 0,0121	0,0790
19	1,9	0-10	0,0551	tg 0,0105	0,0656
20	2	0-10	0,0449	tg 0,00819	0,0540
21	2,1	0-10	0,0363	tg 0,00693	0,0440
22	2,2	0-10	0,0290	tg 0,00585	0,0355
23	2,3	0-10	0,0229	tg 0,00486	0,0283
24	2,4	0-10	0,0180	tg 0,00396	0,0224
25	2,5	0-10	0,0139	tg 0,00274	0,0175
26	2,6	0-10	0,0107	tg 0,00261	0,0136
27	2,7	0-10	0,0081	tg 0,00207	0,0104
28	2,8	0-10	0,0061	tg 0,00162	0,0079
29	2,9	0-10	0,0046	tg 0,00126	0,0060
30	3	0-10	0,0034	tg 0,0060	0,0044
31	3,1	0-10	0,0025	tg 0,009	0,0033
32	3,2	0-10	0,0018	tg 0,0072	0,0024
33	3,3	0-10	0,0013	tg 0,0054	0,0017
34	3,4	0-10	0,0009	tg 0,0036	0,0012
35	3,5	0-10	0,0006	tg 0,000	0,0009
36	3,6	0-10	0,0004	tg 0,0027	0,0006
37	3,7	0-10	0,0003	tg 0,0018	0,0004
38	3,8	0-10	0,0002	tg 0,0019	0,0003
39	3,9	0-10	0,0001	tg 0,0009	0,0002

The first line of the table in Fig. 5 can be used to demonstrate the smallness of $tg\alpha$ in case of non-Gaussian distributions as well.

Take Poisson-Pareto model of concentration changes. Fig. 6 shows the function of $b(t)$ for this case.

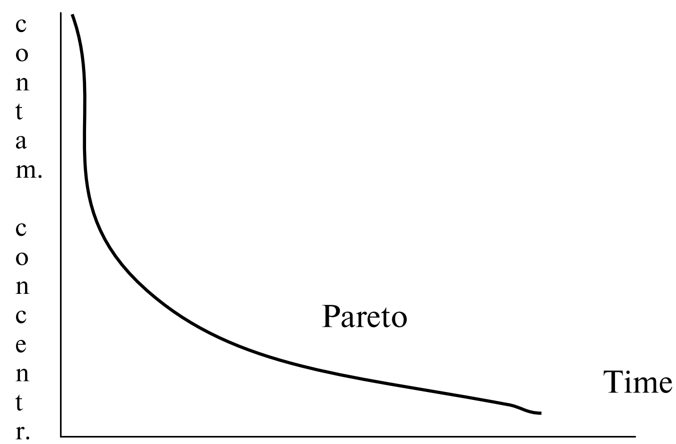

Fig. 6. Representation of distribution density

Fig. 7 presents the first line of Pareto distribution density.

Fig. 7. Example of *tgα* smallness

Time *n*	Time of change e.g. 1,01	Change time interval with a given time of change	Final value	tgα	Basic value
0	0.0	10	0.000	0.05	0.5
1	0.1	10	0.00174	0.0498	0.0498

Conclusion

The reliability of dispersion of contaminants means the satisfaction of four conditions listed in the paper. The paper presents the simulation of contamination spread process involving the reliability check.

Using the example of two points one of which corresponds to the location of a pollution source and the other one is spaced at l distance away from it, we show that with that arrangement of the points, we can correlate a distance l with time t_1, measurement of contamination concentration with whose expiration the contamination process at a far point stabilizes.

Since atmospheric flows have not been defined beforehand, measurements are expected to be reproduced ten times with random outcomes. To use the worst scenario, we choose the biggest outcome.

The paper presents a procedure of choice optimization, as the worst scenario can give an unnecessary overshoot.

For each measurement point the method takes into account the possibility of extra inflows of contaminants from the air, apart from the observed source.

The suggested method is accepted if one has to combine sections with different directions and different lengths.

The basic used model defining t_1 is a Gaussian distribution advised for the nearest vicinity of a source. The paper presents a table of Laplace transform of functions changed for the sake of the problem in question $\Phi'(t)$.

All the operations as to selection of t_1 are easily feasible as they are made within the linear approximation allowable for table $\Phi'(t)$.

The further development of the method for predicting reliability for farther distances and other extensions require the application of various models presented in the literature.

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