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Research in behavior of the centre of failure free performance distribution density for redundant complex technical systems

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Abstract. Aim. *For complex highly-integrated technical systems that contain elements that vary in their physical nature and operating principles (combination of mechanical, electrical and programmable electronic components), complex dependability analysis appears to be challenging due to both qualitative and quantitative reasons (large number of elements and performed functions, poorly defined boundaries of interfunctional interaction, presence of hidden redundancy, static and dynamic reconfiguration, etc.). The high degree of integration of various subsystems erodes the boundaries of responsibility in the cause-and-effect link of failures. Thus, the definition of the strength and boundaries of interfunctional and cross-system interaction is of great value in the context of complex system analysis from the standpoint of locating bottlenecks, as well as reliable evaluation of the complex dependability level.* **Methods.** *In order to solve the tasks at hand, the authors propose a method that is based on the research of the behavior of the centroid of an area bounded above by the failure density function graph, below by the coordinate axis, from the right and left by the boundaries of the considered operation interval. Graphical analysis with construction of centroids is performed for each subsystem or structural unit of a complex technical system. After that, based on the partial centroids of the respective subsystems/units, the average centroid for the whole complex system is constructed. The authors suggest using the average centroid as a conditional universal measure of the average dependability level of highly-integrated technical systems that can be used in the development of specific design solutions. In this case, in particular, it is suggested to use the presented method for identification of the subsystem that, when redundant, ensures the highest all-around growth of dependability of the complex technical system as a whole. This condition is fulfilled by the subsystem/unit of which the partial centroid is situated at the longest distance from the average centroid. The assumptions presented in this article and the results obtained are tested by means of a short verification consisting in the calculation of the probability of no-failure of the system and subsystems, construction and analysis of respective graphs.* **Results.** *The method's implementation is presented using the example of a conventional mechatronic system. For the sake of briefness and focus the information is given in a simplified and abstract form. The application of the proposed method for analyzing complex technical systems dependability through the research of density function centroid introduced in this article was the target criterion of the method's development, i.e. identification of bottlenecks and areas with the highest potential for increasing the overall dependability. Further publications will be dedicated to proving the applicability of such entity as a centroid as a dependability evaluation criterion, as well as other applications of the presented method in complex technical systems dependability analysis.*

Keywords: *dependability, complex technical system, mechatronic system, failure density, probability of no-failure, centroid, Weibull-Gnedenko distribution law, redundancy.*

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Introduction

In the beginning, we should provide a description of the mathematical models used herein. Let us consider a conventional technical system of which the failure distribution $\lambda(t)$ is described with the Weibull-Gnedenko law:

$$\lambda(t)=\alpha\lambda_0 t^{\alpha-1},$$

$f(t)$ is the time to failure density function

$$f(t)=\lambda_0\alpha t^{\alpha-1}\exp\left(-\lambda_0 t^{\alpha}\right)$$

or

$$f(t)=\left(\frac{\alpha}{\beta}\right)\left(\frac{t}{\beta}\right)^{\alpha-1}\exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right),$$

where λ_0 is the initial failure density (if $t = 0$);
α is the parameter of the distribution shape;
β is the parameter of the breadth of distribution;

Table 1 Distribution law parameters of primary system units

Parameter	Electronic components	Software	Mechanical components
λ_0, h^{-1}	1×10^{-4}	0,005	1×10^{-7}
α	1	0,5	1,8
β	10000	40000	7742,6368
$\lambda(t)$	$\lambda_{el}(t) = 0{,}0001$	$\lambda_{SW}(t) = 0{,}0025\cdot t^{-0,5}$	$\lambda_{mech}(t) = 1{,}8\cdot10^{-7}t^{0,8}$
$f(t)$	$f_{el}(t)=10^{-4}e^{-10^{-4}t}$	$f_{SW}(t)=0.005*0.5t^{-0.5}e^{-0.005t^{0.5}}$	$f_{mech}(t)=10^{-7}*1{,}8t^{0{,}8}e^{-10^{-7}t^{1{,}8}}$

$$\beta = \lambda_0^{-1/\alpha}.$$

The first step is the definition of the time interval $[t_1;t_2]$ and bounding of the studied domain under the graph D_i (fig. 1).

After the limits of the operation period and thus the area D_i have been defined, the subsequent analysis consists in the identification of the area S_i

$$S_i = \iint_{D_i} dfdt = \int_{t_1}^{t_2} f_i(t)dt$$

and calculation of the coordinates $(\overline{t_i};\overline{f_i})$ of the centroid of the respective domain D_i

$$\overline{f_i} = \frac{1}{S_i}\iint_{D_i} f\ dfdt,$$

$$\overline{t_i} = \frac{1}{S_i}\iint_{D_i} t\ dfdt.$$

A number of important observations must be made:

The area under curve *f(t)* within the interval $[t_1;\ t_2]$ is the realization probability of a random value of which the distribution is described with the corresponding function *f(t)*. Therefore, from the dependability theory point of view, area S_i of domain D_i is the probability of system failure within the specified time of operation and possesses associated properties, in particular,

$$\lim_{t\to\infty} S_i = 1;$$

Let us call the X-axis coordinate $\overline{t}$ of the centroid of the area under graph *f(t)* calculated for the time interval $[0;\ t_i]$, the "**relative mean time to failure**" of which the limit tends to the true value of the mean time to failure if $t_i\to\infty$:

$$\lim_{t_i\to\infty}\left(\frac{1}{S}\int_0^{t_i} tf(t)dt\right) = \frac{1}{1}\cdot\int_0^{\infty} tf(t)dt = T_1,$$

where T_1 is the mean time to failure [1].

A numerical evaluation of the mean time to failure can be performed by limiting the T_1 range of calculation to t_i, then the mean time to failure will be defined with a certain error even subject to integral expansion by parts:

$$T_1 = \int_0^{t_i} tf(t)dt = -tP(t)\Big|_0^{t_i} + \int_0^{t_i} P(t)dt.$$

Thus, within the interval [0; t$_i$], the centroid X-axis coordinate is the ratio between mean time to failure and probability of such failure.

The centroid Y-axis coordinate $\overline{f}$ characterizes the «**relative failure density of a facility near the mean time to failure**".

Part 1. Input data for calculation

For complex technical systems that contain elements that vary in their physical nature and operating principles (e.g. combination of mechanical, electronic and software components), complex dependability analysis appears to

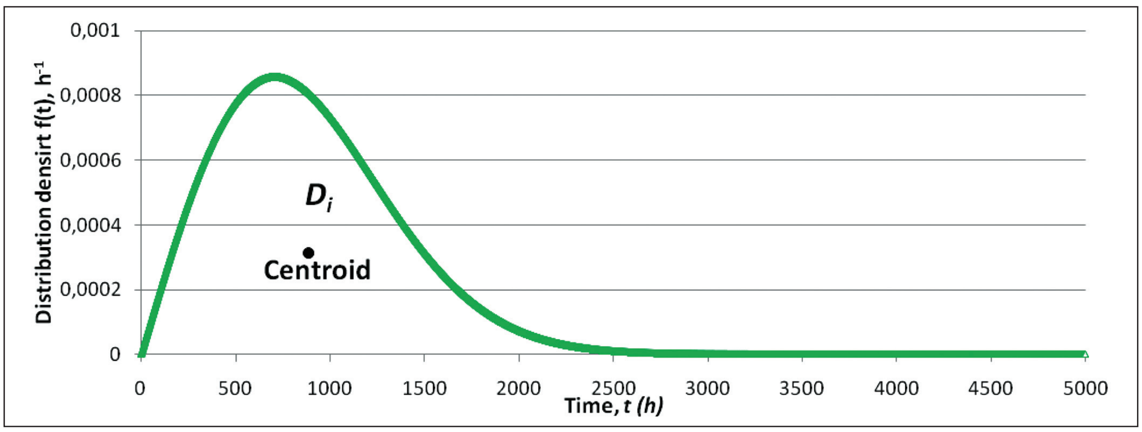

Figure 1 – Failure density of a technical system and location of centroid D_i

Table 2 – Centroid coordinates of a mechatronic system units

#	Parameter	Operation time range, h				
		0 – 2500	0 – 5000	0 – 7500	0 – 10 000	0 – ∞
Electr.	S_i	0.2212	0.3935	0.5276	0.6321	1
	$\bar{f}$, h^{-1}	4.447×10^{-5}	4.016×10^{-5}	3.6811×10^{-5}	3.4198×10^{-5}	6.412×10^{-6}
	$\bar{t}$, h	1198	2292	3285	4180	10000
SW	S_i	0.2212	0.2978	0.3514	0.3935	1
	$\bar{f}$, h^{-1}	1.0637×10^{-4}	7.5797×10^{-5}	6.2311×10^{-5}	5.4285×10^{-5}	4.5914×10^{-5}
	$\bar{t}$, h	781.7	1521	2234.4	2925	80000
Mechan.	S_i	0.1225	0.3656	0.611	0.795	1
	$\bar{f}$, h^{-1}	2.9855×10^{-5}	4.2475×10^{-5}	4.5175×10^{-5}	4.3353×10^{-5}	3.7695×10^{-7}
	$\bar{t}$, h	1584.55	3054.64	4324.27	5328.75	6885.42

Table 3 – Coordinates of the average centroid of a mechatronic system

Parameter	Operation time range, h				
	0 – 2500	0 – 5000	0 – 7500	0 – 10 000	0 – ∞
$\bar{f}$, h^{-1}	6.023×10^{-5}	5.281×10^{-5}	4.8099×10^{-5}	4.39453×10^{-5}	1.75677×10^{-5}
$\bar{t}$, h	1188	2289.21	3281.22	4144.5	32295.14

be challenging due to both qualitative and quantitative reasons (large number of performed functions, poorly defined boundaries of interfunctional interaction, presence of hidden redundancy, static and dynamic reconfiguration, etc.) [2, 3]. This property manifests itself in the forced transition from design to function analysis when each individual function is submitted to analysis, while system dependability as a whole is characterized with the vector of dependability indicators of all the functions [4, p. 91]. In practice, a combination of structural and functional study of system dependability is used, as well as analysis of various special situations caused by functional failures and/or external events, combination of deductive and inductive methods of analysis (e.g. a combination of failure mode, effects and criticality analysis (FMECA) and failure/fault tree analysis (FTA).

Let us consider the method that involves the identification of failure density centroids for a conventional mechatronic system that in a single device contains an electronics unit, software (SW) and mechanical components.

It is suggested to study the centroid behavior successively within the operation time intervals [0; 2500], [0; 5000], [0; 7500], [0;10000] hours and individually within the interval [0; +∞]. The research will assume that the system operates

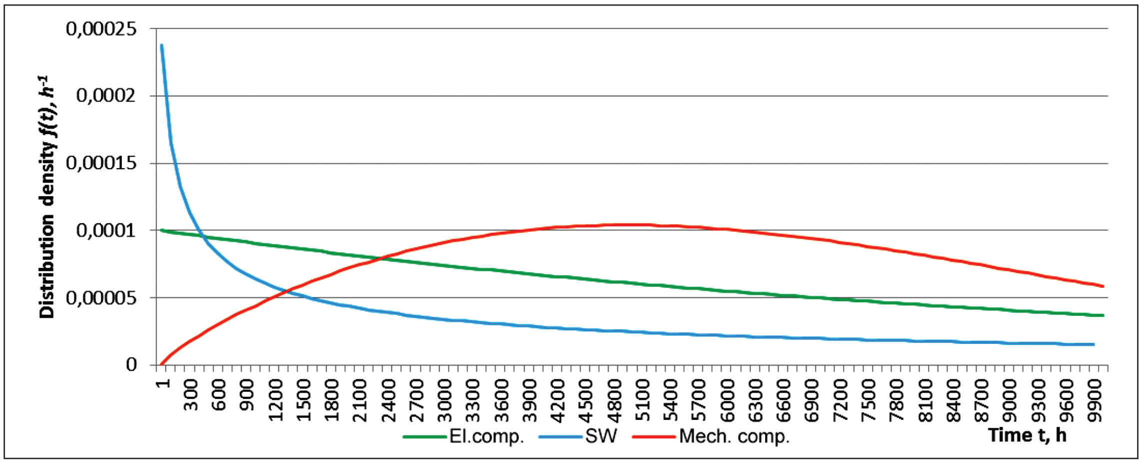

Figure 2 – Failure density functions of a mechatronic system's units

Table 4 – Location of partial centroids of a mechatronic system's units with rate 1 hot redundancy

#	Parameter	Operation time range, h				
		0 – 2500	0 – 5000	0 – 7500	0 – 10 000	0 – ∞
Electr.	S_i	0.04893	0.1548	0.2784	0.39958	1
	$\bar{f}, h^{-1}$	1.23×10^{-5}	1.849×10^{-5}	2.1255×10^{-5}	2.2162×10^{-5}	1.6666×10^{-5}
	$\bar{t}, h$	1613.74	3119	4512.56	5793.2	15000
SW	S_i	0.04893	0.08869	0.12351	0.15482	1
	$\bar{f}, h^{-1}$	9.8646×10^{-6}	9.0145×10^{-6}	8.439×10^{-6}	7.9991×10^{-6}	2.94458×10^{-6}
	$\bar{t}, h$	1187.8509	2321.9787	3422.724	4495.5467	140000
Mech.	S_i	0.015	0.1337	0.37338	0.63207	1
	$\bar{f}, h^{-1}$	6.023×10^{-6}	2.4287×10^{-5}	3.9767×10^{-5}	4.4709×10^{-5}	6.0746×10^{-11}
	$\bar{t}, h$	1937.86	3768.7818	5406.2758	6762.2095	9086.04

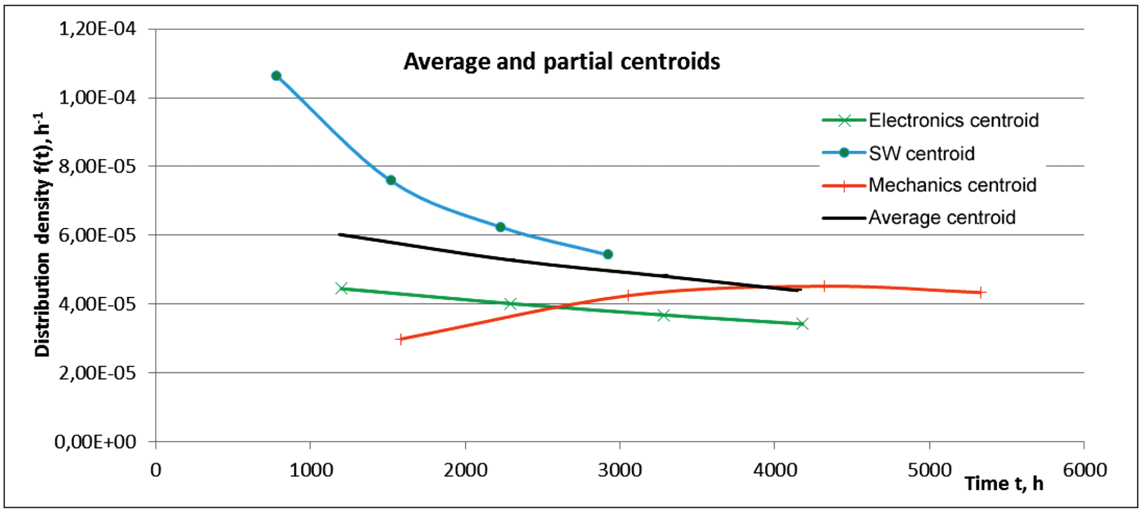

Figure 3 – Location of the average and partial centroids for a mechatronic system

without recovery. The calculation results are presented in table and graphs (fig. 3 – fig. 6).

Now, we suggest considering the system comprehensively by defining the «average centroid» using the formula:

$$\bar{f}_0 = \frac{\bar{f}_{el} + \bar{f}_{SW} + \bar{f}_{mech}}{3},$$

$$\bar{t}_0 = \frac{\bar{t}_{el} + \bar{t}_{SW} + \bar{t}_{mech}}{3}.$$

Part 2. System redundancy

Let us analyze the behavior of the average centroid subject to changing reference parameters. In order to achieve equivalent changes for each of the three subsystems, it is suggested to modify the dependability parameters in such a way as if the system had a rate 1 hot redundancy (dual redundancy). The time to failure density function of a redundant system appears as follows:

$f(t) = 2\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)$ for exponential law,

$f(t) = 2\alpha\lambda_0 t^{\alpha-1} e^{-\lambda t^{\alpha}}\left(1 - e^{-\lambda t^{\alpha}}\right)$ for the Weibull-Gnedenko law

or

$$f(t) = \left(\frac{\alpha}{\beta}\right)\left[\frac{t}{\beta}\right]^{\alpha-1} e^{-\left[\frac{t}{\beta}\right]^{\alpha}}\left(1 - e^{-\left[\frac{t}{\beta}\right]\alpha}\right).$$

The area under the redundant system graph must be equal to the square of the corresponding area of a non-redundant system.

Part 3. Research of centroid behavior

Let us make and verify the assumption that at each specific operation time interval the subsystem (or unit)

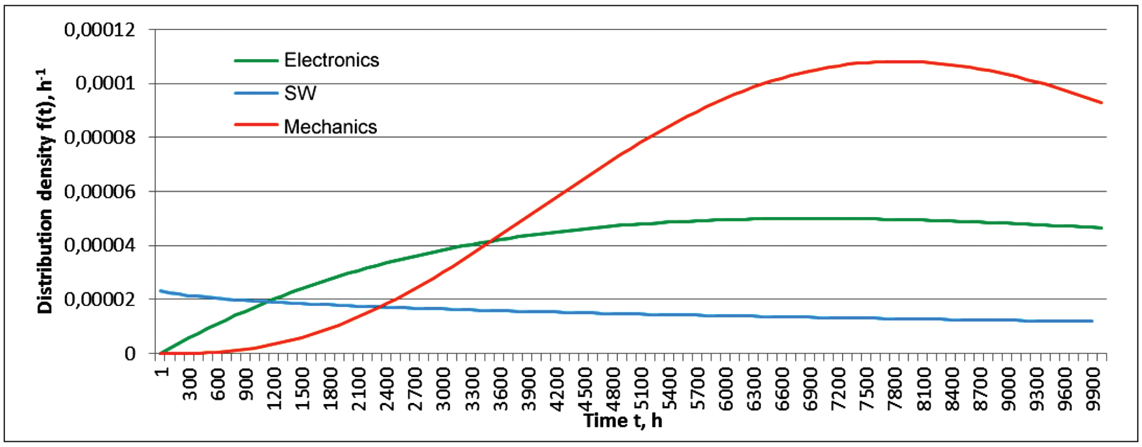

Figure 4 – Failure density functions of a mechatronic system's units for the case of rate 1 hot redundancy

Table 5 – Distance between the partial and average centroids BEFORE redundancy

Subsystem		Operation time range, h			
		0 – 2500	0 – 5000	0 – 7500	0 – 10 000
Electronics	$\rho_{\bar{f}}$	1.57617×10^{-5}	1.26507×10^{-5}	1.1288×10^{-5}	9.74733×10^{-6}
	$\rho_{\bar{t}}$	9.916666667	2.786666667	3.776666667	35.41666667
SW	$\rho_{\bar{f}}$	4.61383×10^{-5}	2.29863×10^{-5}	1.4212×10^{-5}	1.03397×10^{-5}
	$\rho_{\bar{t}}$	406.3833333	768.2133333	1046.823333	1219.583333
Mechanics	$\rho_{\bar{f}}$	3.03767×10^{-5}	1.03357×10^{-5}	2.924×10^{-6}	5.92333×10^{-7}
	$\rho_{\bar{t}}$	396.4666667	765.4266667	1043.046667	1184.166667

Table 6 – Coordinates of the average centroid of a mechatronic system with rate 1 hot redundancy

Redundant subsystem		Operation time range, h			
		0 – 2500	0 – 5000	0 – 7500	0 – 10 000
Electronics	$\bar{f}, h^{-1}$	4.95083×10^{-5}	4.55873×10^{-5}	4.29137×10^{-5}	3.99333×10^{-5}
	$\bar{t}, h$	1326.663333	2564.88	3690.41	4682.316667
SW	$\bar{f}, h^{-1}$	2.80632×10^{-5}	3.05498×10^{-5}	3.01417×10^{-5}	2.85167×10^{-5}
	$\bar{t}, h$	1323.466967	2556.206233	3677.331333	4668.0989
Mechanics	$\bar{f}, h^{-1}$	5.22877×10^{-5}	4.6748×10^{-5}	4.62963×10^{-5}	4.43973×10^{-5}
	$\bar{t}, h$	1305.853333	2527.2606	3641.891933	4622.403167
Full redundancy	$\bar{f}, h^{-1}$	5.22877×10^{-5}	4.6748×10^{-5}	4.62963×10^{-5}	4.43973×10^{-5}
	$\bar{t}, h$	1579.816967	3069.920167	4447.1866	5683.652067

of which the partial centroid is the most remote from the average centroid has the highest influence of the location of the average centroid and thus the highest potential on local changes of the whole system's dependability level

The distance between the points is found according to the known formula:

$$\rho_{\bar{t}} = \left|\bar{t}_i - \bar{t}_0\right|,$$

$$\rho_{\bar{f}} = \left|\bar{f}_i - \bar{f}_0\right|.$$

According to the results given in table 5, the partial centroid most remote from the average one both on the

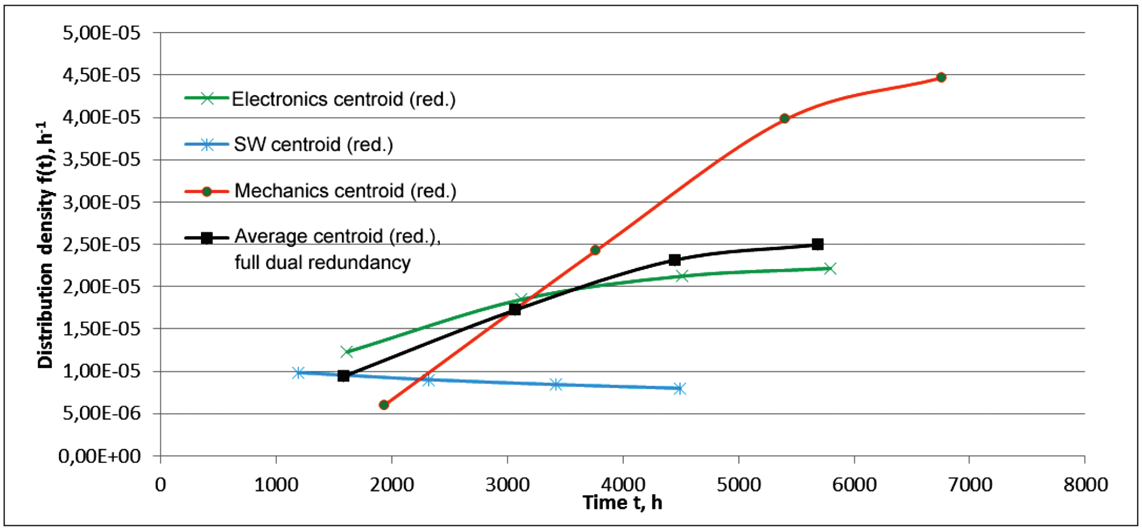

Figure 5 – Location of the average and partial centroids for a mechatronic system with rate 1 hot redundancy

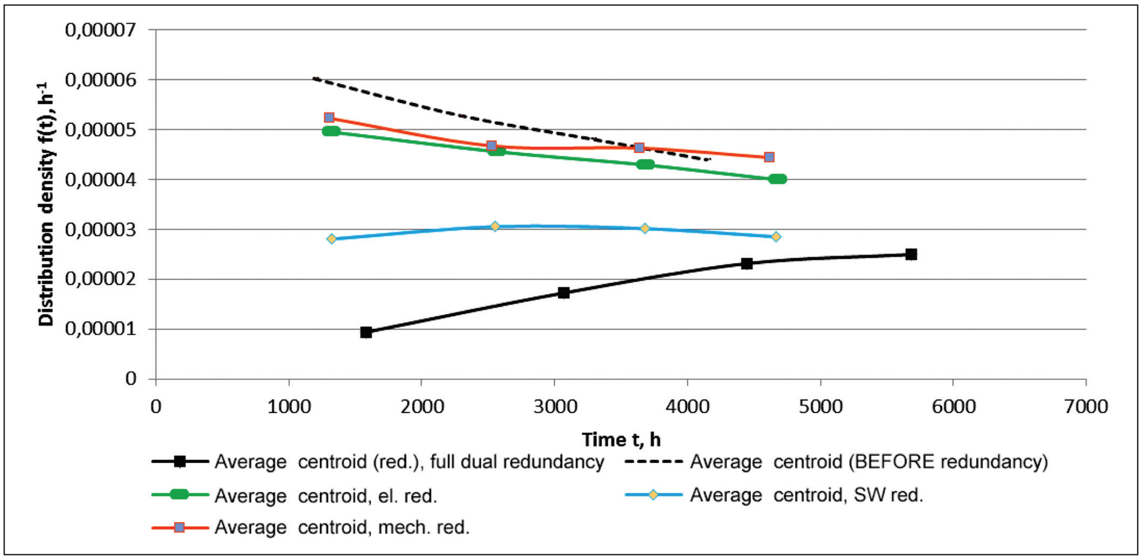

Figure 6 – Coordinates of the average centroid of a mechatronic system with redundancy of one unit

X-axis and Y-axis belongs to the software unit. Let us evaluate the change of location of the average centroid of a mechatronic system depending on the redundancy of either individual or all units.

The calculation results given in table 6 and fig. 6 show that the most significant position change of the average centroid in a mechatronic system is achieved through redundancy of the subsystem of which the partial centroid was most remote from the average one, i.e. the software unit. That is how the biggest growth of dependability of a mechatronic system in general is achieved.

The study of centroid behavior allows evaluating the contribution of varied system elements into the overall dependability through superposition and, most importantly, identifying the subsystem that most contributes to the overall dependability level of the product.

Let us express the result in terms of the probability of no failure (PNF) parameter *P(t)*:

$$P(t) = \exp\left(-\lambda_0 t^{\alpha}\right) = \exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right),$$

$$f(t) = -\frac{d}{dt}P(t) = \frac{d}{dt}Q(t).$$

A mechatronic system is an integrated complex of electromechanical, electrohydraulic, electronic elements and computer devices that continuously and dynamically exchange energy and information united by a common system of automated control that includes elements of artificial intelligence [5]. As it is labor intensive and often incorrect to reflect the interconnections between electronic, software and mechanical elements of a complex system at the system level (diversity of performed functions, intrinsic time of subsystems' operation, hidden redundancy, static and dynamic reconfiguration, etc.), in our reasoning in order to express the resulting probability of no-failure of a mechatronic system as a whole as the limits of the confidence interval we will be using a “corridor” of the TBF functions of serial and parallel connection of electronic, software and mechanical units, as well as their arithmetical mean value.

Conclusion

According to the graph given in fig. 7, by the end of the operation time range the TBF function of the software unit is the highest out of all the systems, therefore it is not obvious that the redundancy of the software unit has the highest

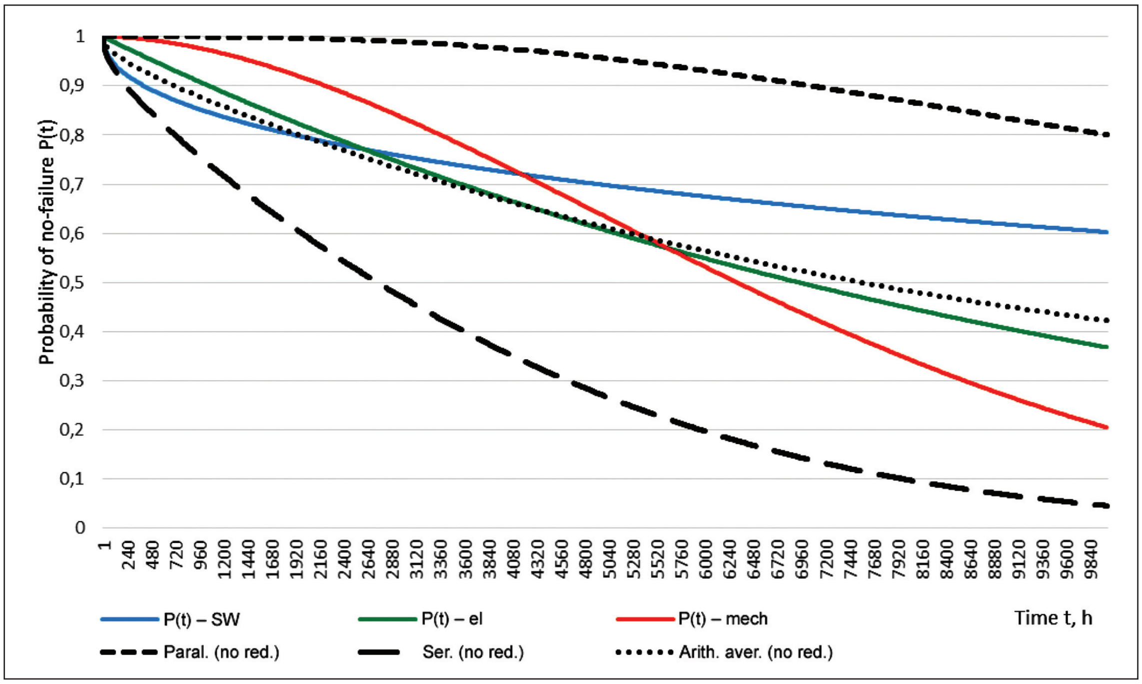

Figure 7 – Probability of no-failure of mechatronic system units without redundancy

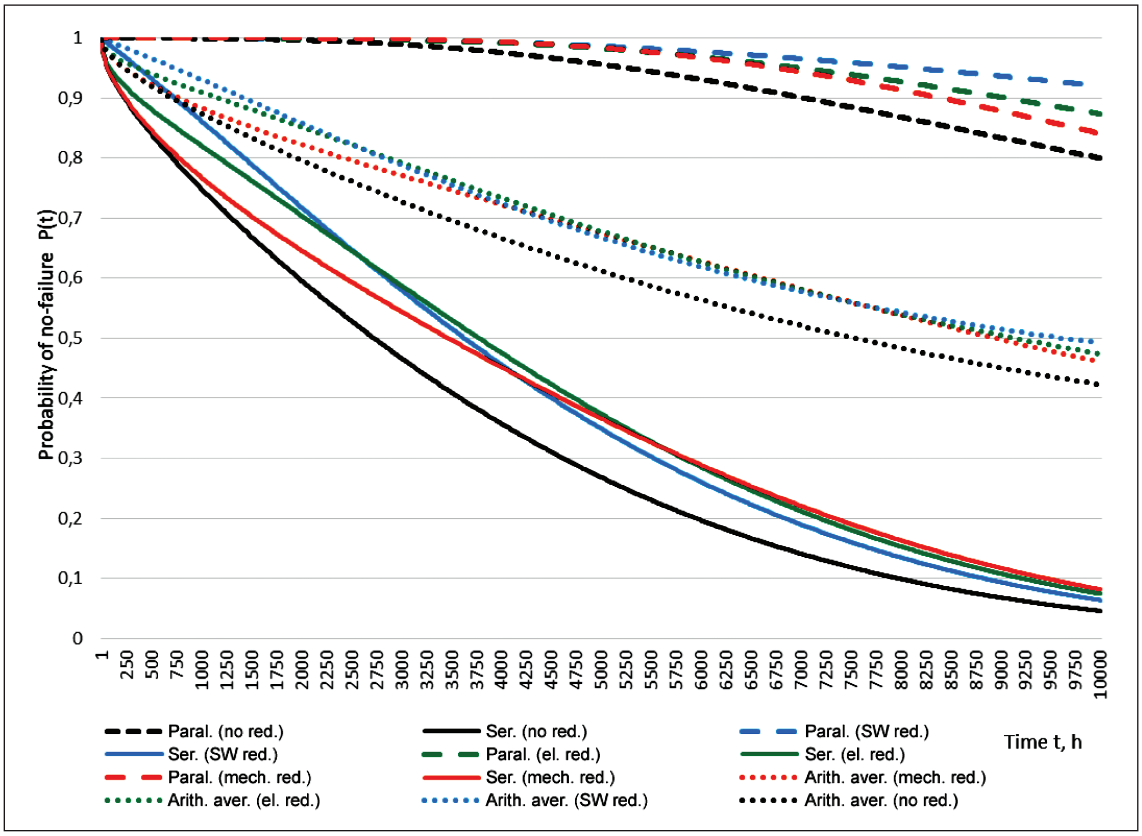

Figure 8 – Probability of no-failure of mechatronic system with one redundant unit

impact on the overall level of dependability.

The calculation results of the probability "corridor" for cases of individual units' redundancy are presented in fig. 8. It can be seen that the largest upward shift of the "corridor" is achieved by software unit redundancy. The highest total difference between the arithmetic averages of the partial "corridor" and the mechatronic system without redundancy also belongs to the software unit.

The research of the behavior of failure density centroids for components of complex technical systems allows evaluating the degree of mutual influence of subsystems and identifying their contribution into the overall level of dependability of a complex technical system.

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