

## About the optimization of overhead system maintenance

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**Abstract.** **Purpose** is to propose and study a mathematical model of optimization of maintenance of overhead devices, which considers the scope of recovery of service life. **Methods.** The analysis of this issue has proposed a strategy and a mathematical model of optimization of maintenance of overhead system, as a kind of a long length object that may undergo preventive replacements and overhauls with minimum emergency repair in case of failures of the overhead system. Besides, the paper describes several particular cases of the general model when performing only preventive replacements, or only preventive overhauls. To take into account the scope of service life recovery when performing a preventive overhaul, we use the parameter, which means the “age” of a long length object and which is defined as the difference between its pre-repair service life and inter-repair service life, related to the pre-repair service life. **Results.** At the given values of the number of preventive overhauls and scope of service life recovery, we obtained the expressions to define the optimal frequency of preventive overhauls and replacements of overhead system, as well as the optimal specific operating expenses. At the given values of the frequency of preventive replacements and scope of service life recovery, we obtained the expression to define the optimal number of preventive overhauls up to the replacement of overhead system. **Conclusion.** To take into account the scope of service life recovery after overhaul, it is advisable to use the parameter which is defined as the difference between pre-repair service life and inter-repair service life, related to the pre-repair service life of the overhead system. The proposed mathematical model of optimization of maintenance makes it possible to define the optimal frequency of preventive overhaul and replacements of overhead system, as well as the optimal number of overhaul for the period of the overhead system operating life under the given scope of recovery of service life.

**Keywords:** service life, scope of recovery, repair, replacement, mathematical model, optimization.

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### State of the art

According to [1], let us understand maintenance as a set of measures aimed to maintain and recover an operable condition of equipment, as well as to recover its service life.

Operation of the overhead system (OS) is accompanied by maintenance (M), current repairs (CR) and overhauls (O), as well as by reconstruction equivalent to preventive replacement [2,3]. Under the performance of maintenance by means of examinations, inspections, testing and measurements, only technical condition of OS is defined [3]. Besides, according to [4], when doing CR, only the recovery of operating capability takes place, but when doing overhauls, the recovery up to the certain level of the object’s service life is done. Full recovery of service life takes place only in case of replacement of OS equipment.

At present in the reliability theory [5,6] some methodological issues have been developed regarding optimization of preventive replacements (PRpl) with emergency replacements (ERpl), when initial reliability of devices is completely recovered, or PRpl with minimum emergency repairs (MER) in case of failures. The publications mentioned includes only two extreme cases of scope of service life recovery: no update when MER is done and full update when ERpl or PRpl is performed. But they are

the intermediate values of scope of recovery of the devices’ service life within these two extreme cases which are of practical interest.

**Purpose of this article** is to propose and study a mathematical model of optimization of overhead system maintenance, characterized by the extent of scope of service life recovery.

### Strategy and mathematical model of maintenance optimization

To consider the scope of service life recovery it is proposed to use the parameter  $a = T_{pr} - T_{ir}$  according to [7] which means “age” of the overhead system after the preventive overhaul.  $T_{pr}$  and  $T_{ir}$  here are pre-repair and inter-repair service life respectively [7]. In future, when developing mathematical models for maintenance optimization it is advisable to use a dimensionless parameter  $\alpha = a/T_{pr}$  to estimate the scope of service life recovery. If  $\alpha = 0$ , it means that replacement has been done. If overhaul is done, for example, in  $\phi$  time, then the OS “age” decreases from  $\tau$  to  $\alpha \cdot \tau$ .

From the perspective of reliability overhead system is considered to be an extended object with many different elements connected in series. In the process of troubleshoot-

ing only a separate damaged OS section is recovered, and practically, it does not affect the current reliability of OS as a whole. In this regard, let us consider the maintenance strategy under which failures are eliminated by minimum emergency repair, and after  $n$  of preventive overhauls the replacement of OS is done.

The change of the failure rate (FR) depending on the operation life under this strategy is shown in fig.1. After minimum emergency repairs the failure rate is not changed. After preventive overhauls (PO) with frequency  $x$  and scope of service life recovery  $\alpha$ , FR is reduced to  $\lambda(\alpha)$ , and after PRpl with frequency  $x_p$  it decreases to a zero level. At the time of PO and PRpl FR is  $\lambda(x+\alpha)$ . Here  $x$  and  $x_p$  are measured in units of service life.

The mathematical model of OS maintenance optimization under this strategy is defined from expression

$$y = (1 + n\gamma + \varepsilon \int_0^{x_p} \lambda(x) dx) / x_p \quad (1)$$

where  $y$  is the relative specific operating expenses;

$\gamma$  is the parameter of overhaul cost;

$\varepsilon$  is the parameter of cost of minimum emergency repair;

$\lambda$  is a failure rate;

The number of failures at  $0 - x_p$  interval is defined as follows:

$$\int_0^{x_p} \lambda(x) dx = \int_0^{\alpha} \lambda(x) dx + (n+1) \int_{\alpha}^{x+\alpha} \lambda(x) dx = n \ln P(\alpha) - (n+1) \ln P(x+\alpha), \quad (2)$$

Here  $P$  is the probability of reliable operation.

Substituting the values  $\int_0^{x_p} \lambda(x) dx$  from (2) to (1), and keeping in mind that  $x_p = \alpha + (n+1)x$ , we shall get the following mathematical model

$$y = \frac{1 + n\gamma + \varepsilon (n \ln P(\alpha) - (n+1) \ln P(x+\alpha))}{\alpha + (n+1)x}. \quad (3)$$

Let us consider two particular cases of the model (3):

with  $n=0$ , when  $\alpha=0$  (there are only replacements that completely recover the initial service life) we obtain the following mathematical model

$$y = (1 - \varepsilon \ln P(x)) / x,$$

that is known as the model of preventive replacements with minimum emergency repair in case of failure [5];

with  $n \rightarrow \infty$  (there are only overhauls that partly recover the initial service life) after we revealed the indeterminacy in (3) we shall obtain the following mathematical model

$$y = (\gamma - \varepsilon (\ln P(x+\alpha) - \ln P(\alpha))) / x,$$

that is known as the model of preventive overhauls with minimum emergency repair in case of failure [7].

Using expression (3) with given values of  $n$  and  $\alpha$ , the optimal frequency of preventive overhaul  $x_0$  and minimum specific operating expenses  $y_0$  could be defined from the condition  $\partial y / \partial x = 0$  as

$$\begin{aligned} & (\alpha + (n+1)x_0) \lambda(x_0 + \alpha) + (n+1) \ln P(x_0 + \alpha) - \\ & - n \ln P(\alpha) = \frac{(1 + n\gamma)}{\varepsilon}; \\ & y_0 = \varepsilon \lambda(x_0 + \alpha). \end{aligned}$$

Frequency of O can be defined from the expression

$$x = (x_p - \alpha) / (n+1). \quad (4)$$

$$\text{Then } x + \alpha = (x_p + n\alpha) / (n+1). \quad (5)$$

Substituting the obtained values of  $x$  and  $x + \alpha$  from (4) and (5) to expression (3), we shall transform it to the following form

$$y = \left( 1 + n\gamma + \varepsilon \left( n \ln P(\alpha) - (n+1) \ln P\left(\frac{x_p + n\alpha}{n+1}\right) \right) \right) / x_p \quad (6)$$

Using expression (6) with given values of  $n$  and  $\alpha$ , the optimal frequency of preventive replacements  $x_{p0}$  and minimum specific operating expenses could be defined from the condition  $\partial y / \partial x_p = 0$  as

$$\begin{aligned} & x_{p0} \lambda\left(\frac{x_{p0} + n\alpha}{n+1}\right) + (n+1) \ln P\left(\frac{x_{p0} + n\alpha}{n+1}\right) - \\ & - n \ln P(\alpha) = \frac{(1 + n\gamma)}{\varepsilon}; \\ & y_0 = \varepsilon \lambda\left(\frac{x_{p0} + n\alpha}{n+1}\right). \end{aligned}$$

Using the expression (6) with given values of  $x_p$  and  $\alpha$ , the optimal number of overhauls  $n_0$  could be defined from the condition  $\partial y / \partial n = 0$  as

$$\frac{x_p - \alpha}{n_0 + 1} \lambda\left(\frac{x_p + n_0 \alpha}{n_0 + 1}\right) + \ln P\left(\frac{x_p + n_0 \alpha}{n_0 + 1}\right) - \ln P(\alpha) = \gamma / \varepsilon.$$

## Conclusion

To take into account the scope of service life recovery after overhaul, it is advisable to use the parameter which is defined as the difference between pre-repair service life and inter-repair service life, related to the pre-repair service life of the overhead system.

The proposed mathematical model of optimization of maintenance makes it possible to define optimal frequency of preventive overhaul and replacements of overhead system, as well as optimal number of overhaul for the period of the

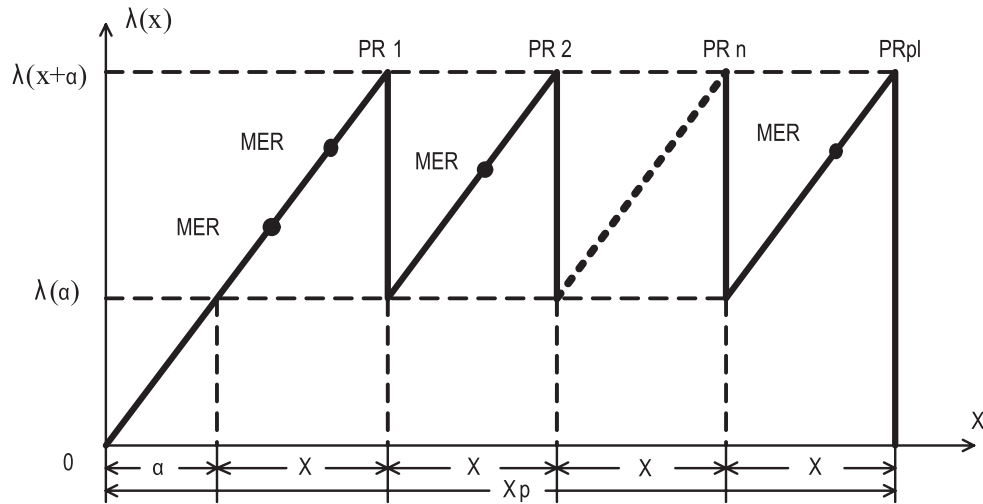


Fig. 1. Change of failure rate due to preventive OR and replacement with minimum emergency repairs

overhead system operating life under the given scope of recovery of service life.

## References

1. GOST 32192 – 2013. Dependability in railway technics. General concepts. Terms and definitions.
2. STO RZD 1.12.001 – 2007. The devices for electrification and power supply. Maintenance and repair. Basic requirements.
3. Rules of construction and technical operation of the overhead system of electrified railways (ИЭ – 868). – M.: Transizdat, 2002. – 184 p.
4. GOST 18322 Equipment maintenance and repair system. Terms and definitions.
5. Barlow, R., Proshan, F.: Mathematical theory of reliability. – M.: Soviet radio, 1961. – 488 p.
6. Beichelt F, Franken P. Reliability and maintenance: Math-

ematical method. M: Radio I Svyaz Press; 1988. – 392 p.

7. Volodarsky V.A. About optimization of preventive replacements and repairs of technical devices // Dependability. – 2011.- No. 2. – P. 49-59.

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