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ESTIMATION OF RELIABILITY FOR A MODEL OF ACCELERATED TESTING UNDER VARIABLE LOAD

The paper considers two interrelated problems: 1. Based on known characteristics of reliability under the conditions of constant load, how to find these characteristics for the case of arbitrary piecewise continuous functions of load. 2. The inverse problem consists in how to estimate the reliability parameters of a product under the conditions of constant loads, including those for small loads which correspond to normal modes, based on accelerated testing with variable (monotonically increasing) loads.

***Keywords**: reliability, variable mode, function of variable load, failure rate, resource function, accelerated testing.*

Estimation of reliability under the conditions of variable load

Estimation of reliability parameters of systems operating in variable modes under the impact of one or several variable factors (load, temperature etc) is one of the vital problems of the reliability theory. Also, in applied terms a very important problem is the reverse one, that is, based on accelerated testing with variable (for example, monotonically increasing) loads, to estimate the reliability parameters of a product under the conditions of constant loads, including those for small and medium loads corresponding to "normal modes" (which itself is one of the major tasks of accelerated testing).

Let $u(t)$ be load influencing a system at the instant of time $t \geq 0$ (note that, generally speaking, the load $u(t)$ can mean any variable factor influencing a system at the instant of time t and affecting its reliability), $F(t, u)$ function of failure-free operation time distribution, $f(t, u)$ respective density of distribution, $P(t, u) = 1 - F(t, u)$ function of reliability and $\lambda(t, u) = f(t, u)/P(t, u)$ function of system failure rate under the conditions of constant load $u(t) \equiv u$. Denote also as

$$\Lambda(t, u) = \int_0^t \lambda(z, u)\,dz$$

function of resource [1] (key function or risk function according to the terminology [2], [3]) under the conditions of constant load $u(t) \equiv u$.

Considered further is how, based on the abovementioned reliability parameters under the conditions of constant loads $u(t) \equiv u>0$, to find these parameters for the case of an arbitrary variable function of load $u(t)$ among sufficiently wide applications of the class of functions.

Suppose further that $u(t)$ is a piecewise continuous function, continuous from the right at $t \geq 0$ and having a limit from the left at each point $t0$, $u(t)>0$ with $t0$, function $\lambda(t,<)$ is continuous at $t \geq 0$, $u \geq 0$ and $\lambda(t,u)>0$ with $t>0$, $u>0$. Denote as $g(R, u)$ the function inverse of the function $\Lambda(t,u)$ for the first argument t with the fixed value of u.

Consider the process $[u(t), r(t), R(t)]$, where $u(t)$ is a specified function of load, $r(t)$ is a function of failure rate, and

$$R(t)=\int_0^t r(y)dy \tag{1}$$

is a resource function for the given function of load $u(t)$, $t \geq 0$. The value of $R(t)$ consistent with Equation (1) can be interpreted as the function of failure rate accumulated before the moment t. Another interpretation related to this and apparently suggested for the first time in [4] is that the quantity $R(t)$ characterizes the resource of a product used up before the moment of time t (if there has not been any failure before the moment t yet).

At the given current moment of time $t \geq 0$ an object is under the load $u(t)$. Taking into account that the function of resource accumulated by this moment is equal to $R(t)$, the process of $R(t)$ further accumulation at the moment of time t, according to its specified physical interpretation, has to start from the moment $\sigma_t = g[R(t), u(t)]$, for which the function of resource accumulated on the interval $(0, t)$ in constant load mode equal to $u(t)$ is identical to the value of $R(t)$. Thus, the value of the function of failure rate $r(t)$ at the given current moment of time t under the conditions of variable load shall be equal to

$$r(t)=\lambda[g(R_t,u_t), u_t] \tag{2}$$

(hereinafter we shall use the abridged notation $R_t = R(t)$, $u_t = u(t)$). From (2), considering that $r(t)=R'(t)$, we have a differential equation to define the function of resource $R_t = R(t)$ under the conditions of variable load function $u_t = u(t)$

$$R_t^{'} = \lambda[g(R_t,u_t), u_t] \tag{3}$$

with the initial condition $R(0)=0$. Whereupon the corresponding function of reliability under the conditions of variable load $u_t = u(t)$ is defined as $P(t) = exp[-R(t)]$.

Note that the hereinafter used universal interpretation of $R(t)$ as a function of accumulation of used-up resource is surely not evident in all cases, all the more so in relation to objects of different physical nature. Yet, statistical inferences derived owing to this interpretation have quite a natural qualitative character and can serve at least a good preliminary model to solve the problems specified above.

Let us consider special cases and consequences of Equations (2), (3). In the most simple special case when the function of load $u(t)$ is piecewise continuous, these equations provide a solution identical to the solution known earlier [4], [5] for piecewise constant modes.

Example 1. (Exponential model) Let us consider a special case when the function of failure rate $\lambda(\mathrm{t,u})$ in each constant mode (i.e. under constant load $u(t) \equiv u$) depends on the value of load but does not depend on the time t, that is

$$\lambda(\mathrm{t}, u) = \lambda(u) \tag{4}$$

for any $u \geq 0$, $t \geq 0$. In other words, in each constant mode under constant load $u(t) \equiv u$ the time of failure-free operation has an exponential distribution with $\lambda = \lambda(u)$ parameter. In this case, Equations (2), (3) lead to

$$r(t) = \lambda(u_t) \tag{5}$$

i.e. in this case the function of failure rate under the conditions of variable load is calculated by means of a simple substitution of the load function $u_t = u(t)$ in the function of failure rate for the case of constant loads (4) for u.

In the general case, the similar quantity $\tilde{r}(t) = \lambda(t, u_t)$, which is calculated via substitution of u_t in the function $\lambda(\mathrm{t}, u)$ for the second argument u, shall be named as a simplified estimation for the function of failure rate $r(t)$ under the conditions of variable load $u_t = u(t)$. In the exponential model this simplified estimation provides an accurate solution (5). In the general case such estimation is inaccurate as it does not take into account the resource already used up by the moment of time t. However, for some natural conditions of monotony, namely if the function of load $u(t)$ monotonically increases with t, and the function of failure rate $\lambda(t, u)$ monotonically increases with t and with u, the simplified estimation $\tilde{r}(t) = \lambda(t, u_t)$ provides an upper estimation for the function of failure rate $r(t)$ under the conditions of variable load $u(t)$. Which is analogous to the results known earlier (for constant modes) [2], [6] − [12], for distributions with an increasing function of failure rate.

Example 2. (Weibull model) Let $\lambda(t, u) = \mathrm{c}ut$, $\Lambda(t, u) = \mathrm{c}ut^2/2$, where $\mathrm{c} > 0$ is a constant, i.e. under the conditions of constant loads the time of system failure-free operation has a Weibull-Gnedenko distribution with the form parameter $\alpha = 2$, and with the function of failure rate proportional to the load imposed on a system. Let $u(t) = vt$, where the function of load monotonically increases with the permanent velocity $v > 0$. In this case $\mathrm{g}(R_t, u_t) = \sqrt{2R_t/(cu_t)}$, and Equation (3)

$$R_t' = \sqrt{2cu_tR_t} = \sqrt{2cvtR_t}$$

looks like

$$R(t) = (2/9)\ \mathrm{c}vt^3,\ r(t) = (2/3)\mathrm{c}vt^2.$$

That is, under the conditions of a variable load monotonically increasing with the constant velocity $u(t) = vt$, the time of failure-free operation has a Weibull-Gnedenko distribution with the form parameter $\alpha = 3$. The simplified estimation of failure rate under the conditions of variable load in this case looks like $\tilde{r}(t) = \lambda(t, u_t) = \mathrm{c}u_t t = cvt^2$.

Equations (2), (3) presented above, therefore, allow us to find a distribution of the time of failure-free operation and reliability parameters of a product under the conditions of the arbitrary piecewise continuous variable function of load $u(t)$, if the parameters under the conditions of constant loads are known. Consider further the inverse task, that is, using these equations, how to estimate reliability parameters under the conditions of constant loads on the basis of accelerated testing under the conditions of variable (monotonically increasing) load.

Model 1 with acceleration factor

Let, under the conditions of constant loads, i.e. with $u(t) \equiv u$, the function of resource $\Lambda(t,u)$ and the function of failure rate $\lambda(t,u)$ look like

$$\Lambda(t,u) = \mathrm{k}(u)\Lambda(t) \ , \ \lambda(\mathrm{t},u) = \mathrm{k}(u)\lambda(t)$$

where $\Lambda(t) = \Lambda(t,u_0)$ is the function of resource, and $\lambda(t) = \lambda(t,u_0)$ is the function of failure rate under the conditions of some (constant) basic load u_0. In this model the form of distribution of the failure-free operation time for different values of constant load $u(t) \equiv u$ looks the same and is defined by the function $\Lambda(t)$, and the impact of load is taken into account through "acceleration factor" $\mathrm{k}(u)$. In this case the main task is to define the relation between the acceleration factor $\mathrm{k}(u)$ and the value of load $u \geq 0$ following the results of accelerated testing.

The function g(R , u) in this case is defined by the expression g(R , u) = $\Lambda^{-1}\left[R/\mathrm{k}(u)\right]$, where $\Lambda^{-1}(z)$ is the function inverse of the function $\Lambda(t)$. Equation (4) for this model appears to be

$$R_t' = \mathrm{k}(u_t)\ \lambda\left[\Lambda^{-1}\left(\frac{R_t}{\mathrm{k}(u_t)}\right)\right] \tag{6}$$

where $u_t = u(t)$ is the function of load. Consider the case wherein

$$\Lambda(t) = \beta t^{\alpha},\ \lambda(t) = \alpha\beta t^{\alpha-1},$$

i.e. the time of failure-free operation under the conditions of constant loads has the Weibull-Gnedenko suggesting further that the form parameter is $\alpha \geq 1$, that is, the function of failure rate $\lambda(t)$ increases monotonically which is a natural physical assumption for most of technical systems. Equation (6) in this case writes as follows

$$R_t' = \alpha\beta^{1/\alpha}\,\mathrm{k}^{1/\alpha}(u_t) R_t^{\frac{\alpha-1}{\alpha}} \tag{7}$$

whence

$$R(t) = \beta\left[\int_0^t \mathrm{k}^{1/\alpha}(u_z)dz\right]^{\alpha} \tag{8}$$

Then the function of reliability under the conditions of variable load $u_t = u(t)$ is defined as P(t)= $exp[-R(t)]$.

These formulas allow us to find the distribution of the time of a product's failure-free operation under the conditions of variable load if the dependence of the acceleration factor $\mathrm{k}(u)$ is known in relation to load. Consider the inverse task when the function $\mathrm{k}(u)$ is unknown, and one has to estimate it upon results of accelerated testing with a variable, monotonically increasing load.

Generally speaking, the form and appearance of the distribution of the time of failure-free operation under the conditions of variable load can be significantly different from those of this distribution under the conditions of constant load, and can substantially depend on the velocity of load increase, and in the general case on the type of the load $u(t)$. Therefore, a class of distributions that serves a basis for estimating parameters under the conditions of variable load, shall be broader compared to the used class of distributions for the case of constant load (in this case compared to the class of Weibull-Gnedenko distributions).

Obviously, the acceleration factor $\mathrm{k}(u)$ shall increase monotonically as the load u increases. Hence, we shall further use the class M of all functions of the acceleration factor $\mathrm{k}(u)$ satisfying this natural physical limitation. For precision, denote as M the class of all the functions $\mathrm{k}(u) \geq 0$ continuous at $u \geq 0$ with the continuous derivative $k'(u) > 0$ for all $u \geq 0$, for $u > 0$. Also, introduce the class U of all the continuous functions of load $u(t) \geq 0$, $t \geq 0$ with the continuous derivative $u'(t)$ for all $t \geq 0$ and such that $u(0) = 0$, $u'(t) > 0$ for $t > 0$.

For this fixed function of load $u(t) \in U$, introduce further the L class of all the solutions $R_t = R(t)$ for Equation (7) generated by the above M class of functions $\mathrm{k}(u)$. In other words, L is a class of all functions of resource belonging to type (8) for different possible functions of the acceleration factor $\mathrm{k}(u)$ of the M class.

Note that in some situations it is evident beforehand that in absence of load the condition $\mathrm{k}(0) = 0$ shall be satisfied (for example, in absence of load, a product is switched off and cannot failure in this state). In order to take into account these situations, introduce the subclass $M_0 \subset M$ of all M functions $\mathrm{k}(u)$ meeting the supplementary condition $\mathrm{k}(0) = 0$.

For this fixed function of load $u(t) \in U$, introduce further the subclass $L_0 \subset L$ of all solutions of Equation (7) generated by the M_0 class, in other words, L_0 is a class of all functions of resource belonging to type (8) for different possible functions of the acceleration factor $\mathrm{k}(u)$ of the M class and such that $\mathrm{k}(0) = 0$.

The L class of functions of resource $R(t)$, therefore, assigns different possible distributions of the time of a product's failure-free operation for the given function of load $u(t) \in U$. Correspondingly, the L_0 class assigns the same distributions under the conditions of the load function $u(t) \in U$, with the supplementary condition $\mathrm{k}(0) = 0$.

For approximation of the function of failure rate $r(t)$ and the function of resource $R(t)$, for distribution of mean time to failure under the conditions of variable load $u(t) \in U$, we shall further use a parametric class of the above functions looking like

$$r(t,\theta) = \sum_{l=m\,l}^{n} \theta t^{l}, \quad R(t,\theta) = \sum_{l=m}^{n} \frac{\theta_l}{l+1} t^{l+1}, \theta \in \Theta \tag{9}$$

where $m \leq n$, $\theta = (\theta_m, \theta_{m+1}, \ldots, \theta_n)$ is the vector of parameters taking on values out of the set

$$\Theta = \{\theta : \theta_l \geq 0,\ l = m,...,n\}$$

The appropriate parametric class of distribution functions for approximation of the distribution of failure-free operation time under the conditions of variable load $u(t) \in U$ looks like

$$F(t,\theta) = 1 - exp[-R(t,\theta)], \theta \in \Theta \quad (10)$$

Note that for any distribution of this class, the function of failure rate $r(t,\theta)$ increases monotonically at t, this being a natural physical limitation.

Also, taking into account the above condition of monotony of the function $k(u)$, the condition shall be satisfied

$$R(t,\theta) \in L \text{ for any } \theta \in \Theta \quad (11)$$

In other words, for the solution of the inverse task to be the function of the acceleration factor $k(u)$ increasing monotonically at u, it is required that the condition (11) should be satisfied. If the supplementary limitation $k(0) = 0$, is used in relation to the acceleration factor, then correspondingly a stricter condition should be satisfied

$$R(t,\theta) \in L \text{ for any } \theta \in \Theta \quad (12)$$

where L ,L_0 are the above classes of resource functions for the time of failure-free operation under the conditions of variable functions of load $u(t) \in U$, for which, in case of solving the inverse problem, a required physical limitation of monotony is automatically satisfied for the function of the acceleration factor $k(u)$.

Theorem 1. The condition (11) is satisfied if and only if one of the following two conditions is fulfilled

$$\alpha - 1 = m < n \quad (13)$$

$$\alpha - 1 < m \leq n \quad (14)$$

Proof. Let the function of load $u(t) \in U$ and the vector of parameters $\theta \in \Theta$. Then, for the given function of resource $R(t,\theta)$, in compliance with (7), for any $t \geq 0$, the equation is satisfied

$$k(u_t) = \left(\frac{1}{\beta\alpha^{\alpha}}\right)\frac{r^{\alpha}(t,\theta)}{R^{\alpha-1}(t,\theta)} \quad (15)$$

whence, taking into account that the function $u_t = u(t)$ increases strictly monotonically at $t \geq 0$,

$$k(u) = \left(\frac{1}{\beta\alpha^{\alpha}}\right)\frac{r^{\alpha}(t_u,\theta)}{R^{\alpha-1}(t_u,\theta)} \quad (16)$$

where $t_u = t(u)$ is the function inverse of the function of load $u(t) \in U$. Under these conditions the inverse function $t(u)$ exists and is continuous for all $u \geq 0$, with the derivative $t'(u) > 0$, $u > 0$ continuous at $u > 0$. Hence, the defined in (16) function of the acceleration actor (solution of the inverse problem) is $\mathrm{k}(u) \in M$, i.e. it satisfies the main condition of monotony specified above if and only if the equation is fulfilled

$$\frac{d}{dt}\left[\frac{r^{\alpha}(t,\theta)}{R^{\alpha-1}(t,\theta)}\right] > 0 \tag{17}$$

for all $t > 0$. The equation is equivalent to the inequality

$$\alpha r'(t,\theta) R(t,\theta) > (\alpha - 1) r^2(t,\theta)$$

or, considering (9), to the inequality

$$\sum_{i=m}^{n} i\theta_i t^{i-1} \sum_{j=m}^{n} \left(\frac{\theta_j}{j+1}\right) t^{j+1} > (\alpha - 1) \sum_{i=m}^{n} \theta_i t^i \sum_{j=m}^{n} \theta_j t^j$$

whence this inequation is equivalent to the following one

$$\sum_{l=2m}^{2n} b_l t^l > 0, \quad t > 0$$

where

$$b_l = \frac{1}{2} \sum_{(i,j)\in A_l} c_{ij} \theta_i \theta_j,$$

where the coefficients are

$$c_{ij} = \alpha\left(\frac{i}{j+1} + \frac{j}{i+1}\right) - 2(\alpha - 1)$$

and the summation is done through a set of indices

$$A_l = \{(i,j):\ i + j = l;\ \ m \leq i \leq n, m \leq j \leq n\}$$

The minimum function

$$\mathrm{f}(x,y) = \frac{x}{y+1} + \frac{y}{x+1}$$

at the set

$$A_l' = \{(x,y):\ x + y = l,\ \ x \geq 0,\ y \geq 0\}$$

is reached at the symmetrical point $x = y = l/2$. Hence,

$$c_{ij} \geq \left(\frac{2}{l+2}\right)(l+2-2\alpha)$$

for all $(i,j) \in A_l$, where $2m \leq l \leq 2n$, hence, Inequality (17) is fulfilled with any $t > 0$, if the condition (13) or the condition (14) is satisfied.

Therefore, satisfaction of (13) or (14) is sufficient for (11). We can easily see further that satisfaction of (13) or (14) is also necessary for (11). Indeed, if $m < \alpha - 1$, then, according to (16), $\mathrm{k}(u) \to \infty$ for $u \to 0$. If $\alpha - 1 = m = n$, then the function $\mathrm{k}(u)$ is an identical constant. Thus, satisfaction of one of the conditions (13) or (14) is also necessary for (11). The theorem is proven.

Theorem 2. The condition (12) is satisfied if and only if the condition is fulfilled

$$\alpha - 1 < m \leq n$$

Proof. Let the function of load be $u(t) \in U$ and the vector of parameters be $\theta \in \Theta$. Then, following (15), (16), the Equation is satisfied

$$\mathrm{k}(0) = \begin{cases} \dfrac{\theta_m}{\beta\alpha}, & \mathit{if} \quad m = \alpha - 1 \\ 0, & \mathit{if} \quad m > \alpha - 1 \end{cases}$$

then, considering that $L_0 \in L$, the proof ensues from the previous theorem.

Theorems 1, 2 set conditions, for which the use of the parametric class of distributions defined above in (9), (10) to estimate the function of resource (t) and the function of distribution $F(t) = 1 - exp[-R(t)]$ under the conditions of variable load $u(t) \in U$ is correct in that the resulting estimation of the acceleration factor $\mathrm{k}(u)$ based on this estimation satisfies the necessary physical limitation of monotony $\mathrm{k}(u) \in M$, and, if need, the supplementary limitation $\mathrm{k}(0) = 0$

Let further $\widehat{\theta} = (\widehat{\theta}_m, ..., \widehat{\theta}_n)$ be the vector of estimations of parameters as to distribution of the time of a product's failure-free operation under the conditions of accelerated testing with variable load $u(t) \in U$. Where the parameters $m \leq n$ are chosen on the basis of the previous theorems 1, 2. Then, in compliance with (16), the estimation of the function of the acceleration factor (solution of the inverse problem) is found by using the formula

$$\widehat{\mathrm{k}}(u) = \left(\frac{1}{\beta\alpha^{\alpha}}\right)\frac{r^{\alpha}(t_u, \widehat{\theta})}{R^{\alpha-1}(t_u, \widehat{\theta})} = \left(\frac{1}{\beta\alpha^{\alpha}}\right)\sum_{l=m}^{n}\widehat{\theta}_l t^l(u) \Big/ \sum_{l=m}^{n}\frac{\widehat{\theta}_l}{l+1}t^{l+1}(u) \tag{18}$$

where $t_u = t(u)$ is the function inverse of the function of load $u(t)$.

Example 3. Let $\alpha = 2$, $m = 1$, $n = 4$ and the function of load look like $u(t) = vt$, i.e. the load increases linearly with the constant velocity $v > 0$. In this case the inverse function $t(u) = u / v$ and the estimation (18) of the relation between an acceleration factor and load looks like in Fig. 1.

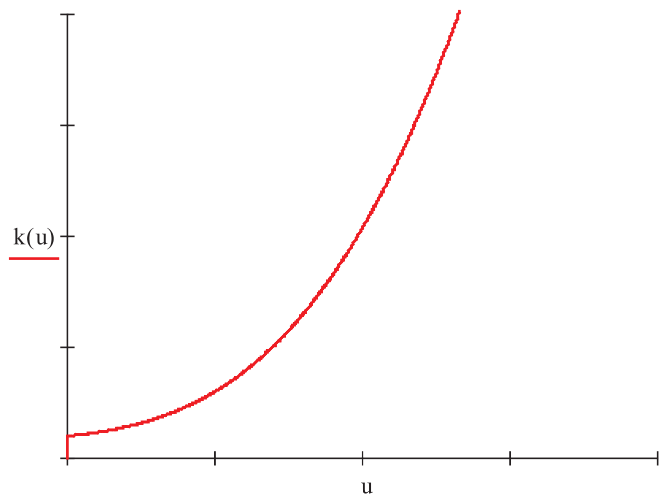

Fig.1. Relation between acceleration factor and load

3. Model 2 with acceleration factor

Consider the case when under the conditions of constant loads, the function of resource $\Lambda(t,u)$ and the function of failure rate $\lambda(t,u)$ look like

$$\Lambda(t,u)=\Lambda(\mathrm{k}_u t),\ \lambda(t,u)=\mathrm{k}_u\lambda(\mathrm{k}_u t),$$

where $\Lambda(t)=\Lambda(t,u_0)$ resource function, and $\lambda(t)=\lambda(t,u_0)=\Lambda'(t)$ is function of failure rate under the conditions of some basic (constant) load u_0. The function $\mathrm{k}_u=\mathrm{k}(u)$ in this model as in the previous model 1 means "acceleration factor" when moving from one (constant) load u_0 to another (constant) load u. This said, the model actually assumes a linear deterministic relation $\xi_u\equiv\xi_0/\mathrm{k}_u$ between random times to failure ξ_u, ξ_0 under the conditions of the above constant loads u, u_0. Simplification to this model as to the previous model 1 is that the form of distribution of failure-free operation ξ_u for different values of (constant) load u looks identical and is defined by the function $\Lambda(t)$. the impact of load in this case is taken into account through the acceleration factor (on time axis) $\mathrm{k}(u)$, which in this model accordingly takes on some other physical meaning. (In the previous model the quantity $\mathrm{k}(u)$ has the meaning of the acceleration factor on the axis of values of the function of failure rate or the function of resource.)

In this case the function g(R , u) is defined by the expression

$$\mathrm{g}(R,u)=\Lambda^{-1}(R)/\mathrm{k}(u)$$

where $\Lambda^{-1}(R)$ is the function inverse of the function $\Lambda(t)$. Equation (3) for this model looks like

$$R_t'=k(u_t)\lambda[\Lambda^{-1}(R_t)], \qquad (19)$$

where $u_t=u(t)$ is the function of load. From (19),

$$\int_0^R\frac{dz}{\lambda[\Lambda^{-1}(z)]}=\int_0^t k(u_z)dz$$

Whence, after the substitution of variables $z = \Lambda\ (t)$,

$$R(t) = \Lambda\left(\int_0^t k(u_z)dz\right)$$

Following this, the function of reliability under the conditions of variable load is defined as $\mathrm{P}(t) = exp\left[-R(t)\right]$. This expression for this model was derived by Cox and Oakes in [3], directly based on the above linear relation between mean times to failure (scale correlation) $\xi_u \equiv \xi_0 / \mathrm{k}_u$. In this particular case the above equations (2), (3) provide an answer matching the solution [3] derived using some other considerations.

4. General model with acceleration factor

Let

$$\Lambda(t,u) = \Lambda(t,\ \mathrm{k}_u)\ ,\ \lambda(\mathrm{t},u) = \ \lambda(t,\ \mathrm{k}_u) \tag{20}$$

where $\lambda(\mathrm{t},u) = \ \Lambda_t^{'}\ (\mathrm{t},u)$. Distribution of the time of failure-free operation in this model depends on the load through the parameter $\mathrm{k}_u = \mathrm{k}(u)$, which as in the previous models has the meaning of "acceleration factor" depending on the value of load. (The previous models 1 and 2 are obviously specific case of this model.) In this case the inverse problem specified above is restricted to the definition of dependency $\mathrm{k}(u)$ upon results of accelerated testing under the conditions of variable load $u\,(t)$.

Equation (2) for this model can be written this way

$$r(t) = \lambda\,[\sigma_{\mathrm{t}},\ \mathrm{k}(u_t)] \tag{21}$$

where at each current moment of time $t \geq 0$ the value $\sigma_{\mathrm{t}} = \ \mathrm{g}(\,R_t, u_t\,)$ is calculated by the equation

$$\Lambda\left[\,\sigma_{\mathrm{t}},\ \mathrm{k}(u_t)\right] = R(t) \tag{22}$$

Let $u_t = u(t),\ t \geq 0$ be the specified function of load and $N = \{u : u = u(t),\ 0 \leq t \leq T\}$ is a set of all values of the function $u(t)$ on the time interval $0 \leq t \leq T$, where T is the moment of completion of the testing under the conditions of variable load $u(t)$. Let the function of load $u(t)$ have a continuous derivative and increase strictly monotonically on the interval $0 \leq t \leq T$, $u(0) = 0$, with the set $N = [0\ , d\,]$, where $d = u(T)$. And let $R(t)$ be the statistical estimation of the function of resource upon results of resting under the conditions of load $u(t)$, and $r(t) = R'(t)$ be the corresponding estimation of the function of failure rate. Using Equations (21), (22), after substitution of the variables $u = u(t)$, the solution of the inverse problem – the function of the acceleration factor $\mathrm{k}_u = \mathrm{k}(u)$ – is then defined using the system of equations

$$\lambda(\sigma,\mathrm{k}) = r(t_u) \tag{23}$$

$$\Lambda(\sigma,\mathrm{k}) = R(t_u) \tag{24}$$

in relation to the pair (σ, k), for each $u \in N$. After that the function of reliability under the conditions of constant loads is defined as

$$\mathrm{P}(t,u) = exp\left[\Lambda - (t, \mathrm{k}_u)\right],\ u \in N$$

The system of Equations (23), (24), therefore, allows us to find a solution of the inverse problem for the general model (22) with acceleration factor.

Example 4. For model 2 considered above (with acceleration factor), the function of resource is $\Lambda(t,\ \mathrm{k}_u) = \Lambda(\mathrm{k}_u t)$, and the function of failure rate is $\lambda(t, \mathrm{k}_u) = \mathrm{k}_u \lambda(\mathrm{k}_u t)$. The system of Equations (23), (24) in this case looks like

$$\mathrm{k}\lambda(\mathrm{k}\sigma) = r(t_u),\ \Lambda(\mathrm{k}\sigma) = R(t_u)$$

whence the expression for acceleration factor is

$$\mathrm{k}(u) = \frac{r(t_u)}{\lambda\left\{\Lambda^{-1}\left[R(t_u)\right]\right\}}$$

this providing a solution of the inverse problem for this model.

Therefore, Equations (2), (3), (23), (24) derived above allow us to estimate the time distribution of failure-free operation and reliability parameters under the conditions of the arbitrary piecewise continuous function of load based on known parameters under the conditions of constant loads. Also, these equations allow us to solve the inverse problem as well, i.e. to estimate reliability parameters under the conditions of constant loads upon results of accelerated testing under the conditions of a variable, monotonically increasing load, and this being one of the key tasks of accelerated testing. Note also that in terms of applications it can be very interesting to further generalize the presented results to more general models, including nonparametric ones, as well as to construct guaranteed (confident) evaluations for reliability parameters based on the results of accelerated testing.

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