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Regarding the planning of testing scope for new equipment samples

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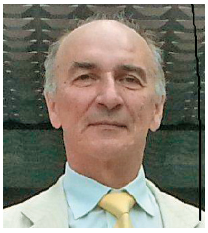
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Abstract. Purpose. *This article describes the issues of planning testing scope for high-reliable objects. The development and manufacture of new samples of equipment is accompanied by a task to define their reliability characteristics. It is based on the fact that there are requirements related to the necessity to specify the above mentioned characteristics in certificates and technical descriptions of the products supplied to the market. The most objective way to define reliability characteristics of the products is a field test. But under the manufacture of complex expensive objects there is no opportunity to introduce a batch with lots of finished products for testing. Thus there is a task to define the duration of field testing and scope of products to be tested, provided there are requirements for the accuracy of estimations related to the objects' reliability characteristics obtained as the result of testing. Planning of the scope is based on the requirements of a manufacturer related to a necessity to confirm the value of lower bound of reliable operation probability with a predefined confidence level. Two tasks are solved in this work. The first task is to define the scope of testing of a batch with finished products N_0 for a time moment t_0, for which a customer's requirement would be fulfilled related to the achievement of the lower bound of probability of reliable operation, specified with a confidence probability 1 – . This task is solved using a non-parametric approach. The second task is to define a required scope of test N_{t1} of the equipment of this type for the time moment different from the moment of first studies $t_1 \neq t_0$. Here one solves the question: how are N_{t0} and N_{t1} correlated? The scope of tests N_{t1} is defined based on the determination of confidence levels providing with the same accuracy of indices as in point t_0. This task is solved with a semiparametric approach. When solving the second task, the parameterization of mean time to failure distribution is used. Three types of distribution are studied: exponential law, Weibull distribution and distribution with linear function of a failure rate. The considered types of distribution laws help to study the behavior of the objects with a decreasing, constant and increasing function of failure rate.* **Methods.** *The formulas for calculation of test scope for different durations of a test-run are derived. Dependence of scope on the duration of a test-run and on a real level of probability of reliable operation is studied as well. Scope planning and respective studies are carried out for different behavior models of a failure rate of the product.* **Conclusion.** *Obtained results give the basis for a well-reasoned approach to the planning of scope of tests of high-reliable objects. The study results showed that the longer a test-run is, the fewer objects are required to be introduced for a test. Dependence is non-linear; it is specified by the parameterization of the failure rate function. Analogous dependence was also obtained for the probability of reliable operation: the higher the PRO is, the fewer objects are required to be tested.*

Keywords: *planning of scope of tests, duration of a test-run, probability of reliable operation, failure rate, lower bound of probability of reliable operation, confidence probability level.*

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Introduction

The development and manufacture of new samples of equipment is accompanied by a task to define their reliability characteristics. It is based on the fact that there are requirements related to the necessity to specify the above mentioned characteristics in the certificates and technical descriptions of the products supplied to the market. The most objective way to define reliability characteristics of the products is a field test. But it should be noted that under the manufacture of complex expensive objects there is no opportunity to introduce a batch with lots of finished products for testing. Thus there is a task to define the duration of field testing and scope of products to be tested, provided there are requirements to the accuracy of estimations related to the objects' reliability characteristics obtained as the result of testing.

Before proceeding with a task definition it is necessary to consider the behavior of indices to be used for the definition. Let us take probability of reliable operation (PRO) – $P(t)$ as an index to be defined. Accuracy of non-parametric estimation of this index is specified by a dispersion calculated by formula:

$$D\left(\hat{P}(T)\right) = \frac{P(t)\left(1 - P(t)\right)}{N},$$

where N is scope of tests during which a PRO is estimated. Therefore, we will get a dispersion value depending on the test scope and a PRO value. The higher the PRO value is, the lower the dispersion is. The dispersion gets its maximum value at the PRO level equal to 0.5. Then, as the level is less than 0.5, the dispersion is getting lower again.

The dispersion of the estimation of PRO index is related to another characteristic of accuracy – a lower confidence estimation of PRO calculated with a specified confidence probability 1–α. As there is a task to estimate the objects' reliability characteristics, it is implied that a researcher does not have any a priori information about these indices. A production manufacturer expects that the equipment supplied to the market must be high-reliable. He assigns a task for a researcher about the timing and scope of tests which may ensure a certain level of the equipment reliability. A task can be defined based on the following considerations. A manufacturer formulates some requirements to the lower confidence estimation of PRO ($\underline{P}_0$), which should be provided with a specified confidence probability 1–α, i.e.

$$\Pr(P(t_0) \geq \underline{P}_0) = 1 - \alpha.$$

This value will be a lower critical value. *Initial value of the lower confidence level of PRO is estimated by the methods of calculation of a structural reliability* [1, 2]. We will define a required number of the objects N_0 to be tested based on this value of the lower confidence estimation of PRO. A required scope of tests that was derived when solving this task, N_0 will ensure the achievement of the specified PRO levels – $[\underline{P}_0, 1]$ with confidence probability 1 – α.

If during the tests it turned out that the object's reliability is higher than it was expected by the customer – $\underline{P}_0^* \geq \underline{P}_0$, it means that based on the predefined scope N_0, the reliability index to be estimated is obtained with a higher accuracy – $\underline{P}_0^*$. In this case to achieve the result with a predefined accuracy $\underline{P}_0$, fewer tests N_0^* are required.

Task definition

Therefore, having familiarized with this reasoning one can set the first task of the study which is to define the scope of tests of the batch with finished products N_0 so that for any value $\underline{P}_0^* \geq \underline{P}_0$, predefined with a confidence probability 1–α, a correlation for the required scope of tests $N_0^* \leq N_0$ shall be fulfilled.

When solving the task let us assume that a failure rate function is defined by one of the formulas [1]:

$$\lambda(t) = \lambda; \qquad (1)$$

$$\lambda(t) = \lambda_1 + \lambda_2 t; \qquad (2)$$

$$\lambda(t) = \lambda_1 t^{\lambda 2}. \qquad (3)$$

Expression (1) (a rate is constant) is common with an exponential distribution of mean time to failure, formula (2) is basic for a distribution with linear failure rate and function (3) is basic for Weibull distribution.

To simplify calculations let us transform the model under consideration to the following form:

$$\lambda(t) = \lambda g(t), \qquad (4)$$

where g(t)=1 corresponds to the exponential distribution,

g(t)=a+bt corresponds to the distribution with linear function of the failure rate, (5)

g(t)=t^a corresponds to the Weibull distribution. (6)

Function of the failure rate g(t) must satisfy two basic requirements:

$$g(t) \geq 0,$$

$$G(t) = \int_0^t g(\tau)d\tau \to \infty \text{ with } t \to \infty.$$

Besides we shall assume that coefficients in (5), (6) a, b are known, the one which is unknown and estimated by the sampling is λ.

Planning of scope of tests in non-parametric statement

Let us proceed with a task solving now. We shall solve the task in non-parametric statement. We know [5] that for any t the number of products that have not failed up to the moment t is distributed by a binomial law

$$N\hat{P}(t) = \mu_N(t) \sim Bin\big(N, P(t)(1-P(t))\big)$$

$$\Pr\big(\hat{P}(t) \ge \underline{P}\big) = \sum_{i=k}^{N} C_N^i P^i(t)\big(1-P(t)\big)^{N-i} = \\ = I_{P(t)}(k, N-k+1),$$

where $I_x(a,b)$ is an incomplete beta-function, $k = \lceil N\underline{P} \rceil$ is an expression $N\underline{P}$ rounded to the larger one. It is not possible to find an accurate analytical solution of equation

$$I_{P(t)}(k, N-k+1) = 1-\alpha$$

because it contains two unknown variables N and $P(t)$. Let us study approximate ways of the task solving.

If to use a central limit theorem, we shall obtain a normal law for the PRO estimation at the limit:

$$\hat{P}(t) \sim Norm\left(P(t); \frac{P(t)(1-P(t))}{N}\right). \tag{7}$$

This makes it possible to form a one-sided confidence set:

$$\Pr\big(\hat{P}(t) \ge \underline{P}\big) = 1 - \Phi\left(\frac{\underline{P} - P(t)}{\sqrt{P(t)(1-P(t))}}\sqrt{N}\right) = \alpha,$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left(-u^2/2\right)du$ is the function of distribution of the standard normal law – $Norm(0;1)$.

Therefore, estimation of the required scope of tests that would guarantee the fulfilment of the predefined requirements shall be defined by formula

$$N_{t_0} = \frac{P_0 \cdot (1-P_0) \cdot u_{1-\alpha}^2}{(\underline{P}_0 - P_0)^2}, \tag{8}$$

where $u_{1-\alpha}$ is a quantile of standard normal distribution $Norm(0;1)$ of level $1-\alpha$.

Variable P_0 is unspecified in expression (8). Let us estimate it based on the following considerations. It was noted earlier that PRO distribution can be approximate by the normal law (7). Thus, for an approximate estimation for P_0 we can offer a point in the middle of interval $[\underline{P}_0, 1]$. Therefore

$$P_0 = (1 + \underline{P}_0)/2.$$

And finally we can write down

$$N_{t_0} = \frac{(1-\underline{P}_0)(1+\underline{P}_0)u_{1-\alpha}^2}{(1-\underline{P}_0)^2} = \frac{(1+\underline{P}_0)u_{1-\alpha}^2}{1-\underline{P}_0}.$$

Due to the fact that under the planning of testing scope a PRO value in point t_0 is unknown, let us study the dependences of the required scope of products to be tested on value P_0. When performing the calculations the following values of model parameters were taken: $\underline{P}_0$=0.93; α_0 = 0,1; t_0=360; t=540; k = 0,004. The calculations were performed for a linear model of the failure rate (2). The graph of change of the required scope of observations depending on the PRO estimation is shown in Fig. 1. Based on the results shown in the graph we can make the following conclusion: the higher the product's reliability is, the fewer products are required to be introduced for testing to confirm the PRO value. And the dependence is explicitly non-linear.

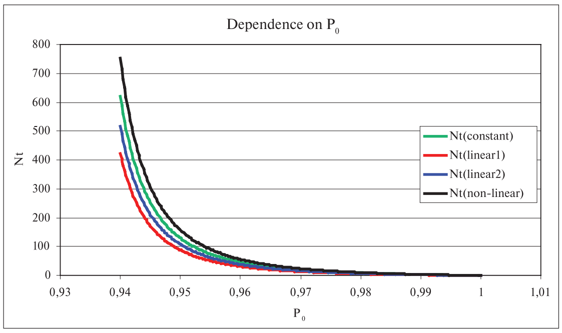

Fig. 1. Dependences of testing scope on P_0.

Semiparametric method of planning of testing scope in point $t_1 \neq t_0$

Let us solve the other task now. We shall define the required scope of tests of the equipment of a given type for another moment of time $t_1 \neq t_0$. Let us denote the required scope of tests as N_{t_1}. And besides let us answer the question: how are N_{t_0} and N_{t_1} correlated? Scope of tests N_{t_1} shall be defined based on the specified confidence bounds that ensure the same accuracy of indices as in point t_0.

Estimation of the number of tests at an arbitrary moment of time t N_t is defined by the formula similar to (8)

$$N_t = \frac{P(t) \cdot (1-P(t)) \cdot u_{1-\alpha}^2}{\big(\underline{P}(t) - P(t)\big)^2}. \tag{9}$$

Let us notice that in (9) values of the variable $\underline{P}(t)$ and $P(t)$ are unknown. Let us define them. We shall use the opportunity that $\underline{P}(t)$ should belong to the same curve of the lower bound of PRO, estimated as per model (4):

$$\underline{P}_0 = \exp\left(-\bar{\lambda} \cdot G(t_0)\right) \text{ and } \underline{P}(t) = \exp\left(-\bar{\lambda} \cdot G(t)\right).$$

Having taken the logarithm of two equations, getting rid of $\underline{\lambda}$ we obtain the correlation for lower confidence bounds of PRO:

$$\frac{\ln \underline{P}_0}{\ln \underline{P}(t)} = \frac{G(t_0)}{G(t)} \text{ or } \underline{P}(t) = \underline{P}_0^{\,G(t)/G(t_0)}.$$

We shall have the same correlation for PRO estimates

$$P(t)=P_0^{G(t)/G(t_0)}. \qquad (10)$$

If to substitute (10) in (9) and divide by (8) we will get:

$$\frac{N_t}{N_{t_0}}=\frac{P_0^{G(t)/G(t_0)}\cdot\left(1-P_0^{G(t)/G(t_0)}\right)}{P_0\cdot\left(1-P_0\right)}\cdot\left(\frac{\underline{P}_0-P_0}{\underline{P}_0^{G(t)/G(t_0)}-P_0^{G(t)/G(t_0)}}\right)^2,$$

Then we obtain the estimation for the required scope of tests at an arbitrary moment of time t:

$$N_t=\frac{P_0^{G(t)/G(t_0)}\cdot\left(1-P_0^{G(t)/G(t_0)}\right)\cdot u_{1-\alpha}^2}{\left(\underline{P}_0^{G(t)/G(t_0)}-P_0^{G(t)/G(t_0)}\right)^2}. \qquad (11)$$

If $\lambda G(t)$ is short, then from (11) we will have

$$N_t=\frac{\lambda\cdot u_{1-\alpha}^2}{\left(\lambda-\bar{\lambda}\right)^2}\cdot\frac{1}{G(t)}+o\left(\lambda G(t)\right).$$

As $\lambda=-\frac{\ln P_0}{G(t_0)}$ and $\bar{\lambda}=-\frac{\ln \underline{P}_0}{G(t_0)}$, then we will asymptotically get the result

$$N_t=\left[\left(\frac{\ln P_0}{\ln P_0-\ln \underline{P}_0}\right)^2\frac{u_{1-\varepsilon}^2}{1-P_0}\right]\cdot\frac{G(t_0)}{G(t)}.$$

This formula could be reduced as follows:

$$\frac{N_{t_1}}{N_{t_0}}=\frac{G(t_0)}{G(t_1)} \text{ or } N_t\cdot G(t)=const.$$

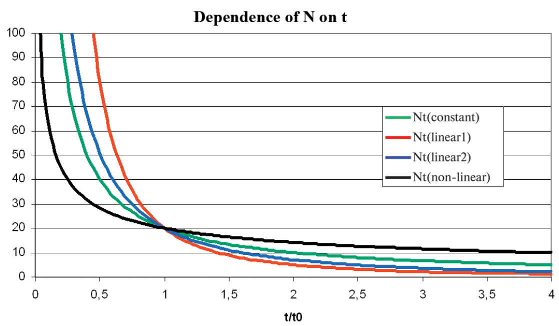

Fig. 2. Dependence of testing scope of the duration of a test-run

We shall study the obtained results. Let us calculate the required scope of tests depending on the duration of a test-run.

Fig. 2 shows the change of testing scope depending on the duration of a test-run in a relative time scale t/t_0. Input parameters of the model along the calculations were taken on the following level: P_0=0.999; $\underline{P}_0$=0.97; α_0 = 0,1; t_0=360.

Green graph corresponds to the case when $g(t)$=1 or $\lambda(t)$=λ (rate is constant). Red graph corresponds to the case when $g(t)=t$ or $\lambda(t)=\lambda t$ (rate is growing linearly). Blue graph corresponds to the case when $g(t)=1+kt$ or $\lambda(t)=\lambda(1+kt)$ (rate is growing linearly from point (0, λ)), with k = 0,004. As a slope ratio k increases, the dependence of scope of observations converges fast to the graph for a linear failure rate. Black graph corresponds to the case when $g(t)=1/\sqrt{t}$ (rate decreases as per the law $\lambda(t)=\lambda/\sqrt{t}$). The results shown in figure 2 can be illustrated by the calculations. Let us consider the exponential law of mean time to failure distribution ($G(t)=t$). For this model, if with t_0=20 hrs we should perform N_{t_0}=100 tests, then for t_1=200 hrs we will get the testing scope $N_{t_1}=N_{t_0}\frac{t_0}{t_1}=100\cdot\frac{20}{200}=1010$. For a linearly increasing failure rate $\lambda(t)=\lambda t$, respectively $G(t)=t^2$ we will get the result: if for t_0=20 hrs we should perform N_{t_0}=100 tests, then for t_1=200 hrs the number of tests shall be

$$N_{t_1}=N_{t_0}\frac{t_0^2}{t_1^2}=100\cdot\frac{20^2}{200^2}=1.$$

The studies performed for a parametric model of the linear function of a failure rate showed that the increase of probability P_0 in point t_0, under the rest constant input parameters of the model leads to a significant reduction of testing scope N_t (see Fig. 3). Along the calculations input parameters took the following values: $\underline{P}_0$=0.99; α_0 = 0,1; t_0=360; k=0,004. As the result of the performed calculations the result obtained earlier was confirmed: the higher the product's reliability is, the fewer objects are required to be introduced for the tests N_t.

Conclusion

We obtained the results allowing for a well-reasoned approach to the planning of scope of tests of high-reliable objects. The information provided by a manufacturer in relation to the necessity to confirm a lower bound of probability of reliable operation with a predefined confidence probability is used as initial information. The formulas derived in the article made it possible to study the dependence of testing scope on the duration of a test-run and on the probability of reliable operation of the product. The studies showed that the longer a test-run is the fewer products are required to be introduced for testing. And the dependence is non-linear, associated with the parametrization of the failure rate function. Similar dependence was got for the probability of reliable operation as well: the higher the product's PRO is, the fewer objects are required to be tested.

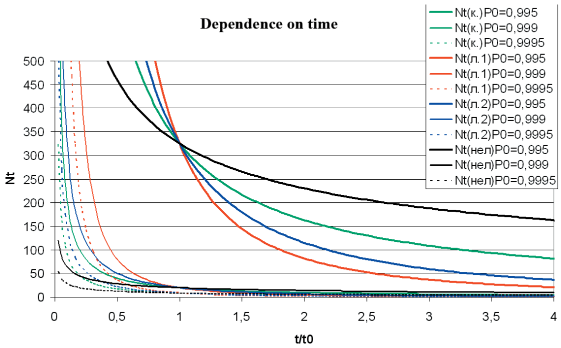

Fig. 3. Dependence of testing scope on time at different P_0

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