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Approach to ensuring reliability of complex systems based on parameter optimization of reliability diagrams

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Abstract. Aim. *Fulfillment of the requirements for the reliability indices of complex technical products and systems is one of the priority tasks to be solved along the stages of development and testing. It is advisable to define the parameter values of the elements of the complex system diagram at the design stage, optimally, in terms of the minimum of an efficiency/cost criterion, ensuring the fulfillment of the requirements for the system reliability.* ***Methods.*** *The main problem that impedes to solve the task of parameter optimization of a model of the reliability diagram is a significant instability of estimation of probability of reliable operation using a Monte-Carlo method (a significant dependence of the rate of estimation error of time). In such conditions an optimization search task could be solved on provision of a stepwise determination of the number of model experiments, which ensures the required accuracy of the estimation of probability of the system reliable operation, necessary for stable operation of the parameter optimization algorithm. The studies of characteristics of estimation of the system reliable operation allowed determining the interrelation of the estimation of reliable operation and the rate of estimation error, offering its approximation in form of a simple formula. The number of model experiments that ensure the required estimation accuracy, is defined using the developed formula determining the interrelation of the estimation of reliable operation and the rate of estimation error, and the known formulas determining the rate of error of the sum N of equally distributed independent random values. Use of the obtained formulas makes it possible to organize the work of the parameter optimization algorithm of the system reliability model by determining its parameters with the required accuracy using minimum computer resources in the context of instability of estimation of probability of the optimizable system reliable operation.* ***Results.*** *Efficiency of the offered approach to realize parameter optimization of a statistical model of the reliability diagram is shown on the sample of estimation of optimal parameters of the system reliability diagram variant, for which there is an analytical solution for the estimation of reliable operation probability. And the results of parameter optimization with the use of analytical value of the probability of reliable operation are the basis for estimation of the accuracy of the algorithm of parameter optimization of the system reliability model operating with the use of a Monte-Carlo method. It has been shown that the offered approaches ensure the convergence of the search algorithm and the required accuracy in estimation of the parameters of the system reliability diagram that optimally ensure the fulfillment of the requirements for the system reliability.* ***Conclusions.*** *The results described in the article confirm technical feasibility and economic viability of determination of optimal values of the system reliability parameters at the design stage. Obtained estimations are the basis for the system integration with required elements, or for the requirements to be set to their reliability, if the development of new elements is necessary. In case there are no elements with design characteristics of reliability, the required reliability of the system can be ensured by special technical redundancy measures and (or) by the creation of the system of technical maintenance and repair.*

Keywords: *reliability, reliability model, reliability diagram, reliable operation, optimization, optimization algorithm, convergence of a search algorithm.*

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Fulfilment of requirements to reliability indices of complex technical products and systems is one of the priority tasks to be solved along the stages of development and testing. If necessary it is possible to improve reliability either by improved reliability of system elements, or by taking technical measures on their redundancy and (or) on the creation of a maintenance system [1]. Technical measures lead to increase in system cost. The measures to be taken should be chosen with an efficiency/cost criterion. And efficiency of the measure taken is defined as an obtained growth of system reliability, and cost is defined as a respective growth of system cost.

Availability of such tool as a model of system reliability allows to formalize the development of recommendations to ensure the required reliability of the system under study. Reference [1] offers to solve the indicated task on the basis of development of search algorithms of optimal synthesis of reliability diagrams of sustems with redundancy. With no loss of entity the variant of uninterrupted power supply (UPS) configuration is further described. Its reliability diagram is shown in Figure 1 [1].

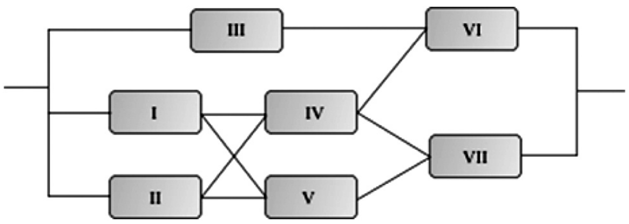

Fig. 1 – Variant of uninterrupted power supply (UPS) with redundancy

This article offers the approach to ensure reliability of complex systems based on parameter optimization of reliability diagrams.

The state of the system is defined by the state of its elements and by the logic of their interrelations in a diagram. The formula to define the state (*fault-free* – 1, *faulty* – 0) of the system in Figure 1 (states of the elements are specified by symbol ***S*** (state) with the respective indices) is as follows:

$$S_\Sigma = \{[(S_1 \vee S_2) \wedge S_4 \vee S_3] \wedge S_6\} \vee \vee[(S_1 \vee S_2) \wedge (S_4 \vee S_5) \wedge S_7]. \quad (1)$$

A logical formula to estimate the system state is defined by its structure, it does not depend on the type of law of probability of fault-free state of the elements, and formulation causes no difficulties. Analytic solution could be obtained for reliability diagrams with simple structure, therefore complex systems are analyzed based on statistical computation. A model of reliability of the system built on the basis of a logical formula under certain reliability indicators of system elements is functioning as follows. On the *i*-th modeling step, corresponding to the time interval t_i *N* of model experiments are done, and in each experiment the system state $S_{\Sigma ij}$ is defined in accordance with (1). Probability of reliable operation of the system $\hat{P}_i$ is assessed by averaging of the results of *N* of model experiments:

$$\hat{P}_i = \frac{1}{N}\sum_{j=1}^{N} S_{\Sigma_{i,j}}, \quad (2)$$

The obtained assessment of probability of system reliability corresponds to the results of analytic estimations $P(t)$ and contains the error of estimation $\Delta P(t)$:

$$\hat{P}(t) = P(t) + \Delta P(t). \quad (3)$$

Standard deviation (SD) of estimation error $\sigma_{\Delta P}(t)$ is defined by the number of model experiments *N* and SD of the event stream S_Σ:

$$\sigma_{\Delta P}(t) = \frac{\sigma_S(t)}{\sqrt{N}} \quad (4)$$

The results of statistical modeling by the estimation $\hat{P}_i$ (2) and SD $\sigma_S(t)$ of the event stream S_Σ under exponential law are shown in Figure 2. For modeling, the average time of reliable operation of elements is accepted as T_{fso} = 5000 h, *N* = 10000.

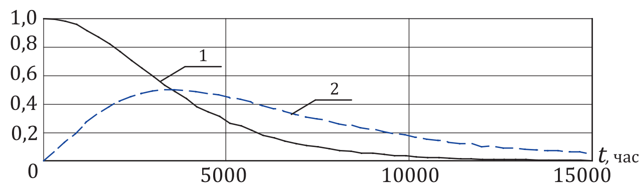

Fig. 2. Dependence of estimation of probability of reliable operation $\hat{P}(t)$ (1) and SD of the event stream $\sigma_S(t)$ (2) on time

The character of the processes $\hat{P}(t)$ and $\sigma_S(t)$, predefinition of their values in the row of points on the time axis point at a possible functional interdependency of $\sigma_S(t)$ and $\hat{P}(t)$, which can be defined by a grapho-analytical method. Dependency graph $\sigma_S = f(\hat{P})$ made in accordance with Figure 2 is shown in Figure 3. Viable regression function $\sigma_S = f(\hat{P})$ for the dependency in Figure 3 is a circular curve equation $\sigma_S^2 + (\hat{P}-0.5)^2 = 0.5^2$. In reference to a desired variable σ_S this equation takes the form:

$$\sigma_s = \sqrt{0,25 - (\hat{P} - 0,5)^2} \quad (5)$$

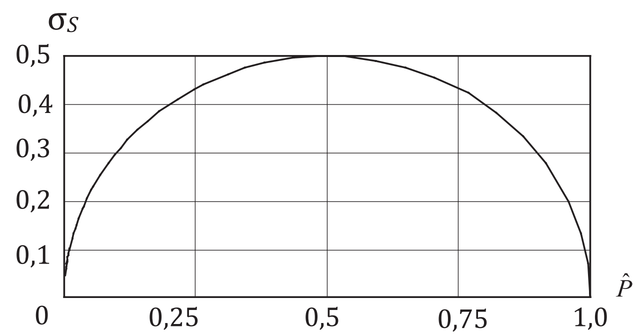

Fig.3 Dependency of SD of the system state stream σ_S on estimation of probability of reliable operation $\hat{P}$

Based on (5), with consideration of (4), the formula for a stepwise determination of the number of model experiments necessary to ensure a required value of estimation error SD $\sigma_{\Delta P req}$ at the *i*-th point of modeling is as follows:

$$N_{req i} = \frac{\hat{P}_i}{\sigma_{\Delta Preq}} * (1 - \hat{P}_i) \quad (6)$$

Modeling in accordance with (6) makes it possible to estimate the system reliability with the defined accuracy at all points of the modeling time interval.

As stated above, one of the main tasks to be solved during the development of complex/complicated technical systems is the task to fulfil reliability requirements which is formulated as follows:

$$P(t_{req}) \geq P_{req}. \tag{7}$$

It means that for the defined time interval the system shall assure reliability not worse than the required reliability. In case the requirement is not fulfilled, it is necessary to change the system to improve reliability of the elements in service (average time of their reliable operation).

Determination of minimum required values of average time of reliable operation of the elements assuring the implementation of (7) can be formalized and realized based on parametric identification of systems [2].

For parametric identification of a reliability model, quality can be indicated by the functional which is a measure of concordance of reliability probability estimation obtained on the model with this parameter vector with the required probability value:

$$Q(w) = [P_{\text{req}} - \hat{P}(w, t_{\text{req}})]^2, \tag{8}$$

where w is a parameter vector of a reliability model, whose components are average time intervals of reliable operation of the reliability diagram elements.

Minimization of functional (8) by vector w under the conditions of its high dimensions can be organized using the first-order gradient-based algorithms. In this case stepwise minimization $Q(w)$ is done. Every step of minimization is made after its direction is defined which is set by derivative $Q(w)$ for the parameter vector w:

$$w_i = w_{i-1} + \Delta_i \cdot \mu_i, \tag{9}$$

where $i \in [1 \ldots n]$;

μ_i is a unit vector in the direction of derivative $\frac{dQ(w)}{dw}$, defined at the i-th step;

Δ_i is a value of the minimization step; it is chosen equal to the step of parameter modification at the calculation of derivative $\frac{dQ(w)}{dw}$; a value of the last step Δ_n is calculated by a straight-line interpolation of the interval from the correlation between $\hat{P}_{n-1}(t_{req.})$, $P_{req.}$ and $\hat{P}_n(t_{req.})$.

Components of the vector μ determinate sensitivity (8) to the changes of values of parameter vector elements and are calculated by the formula:

$$\mu_{ki} = \frac{\hat{P}^{*}_{i-1,w_k}\left(t_{req.}\right) - \hat{P}_{i-1}\left(t_{req.}\right)}{C_{ik}}, \tag{10}$$

where $\hat{P}^{*}_{i-1,w_k}(t_{req.})$ is the estimation of probability of fault-free state of the system at the moment t_{req} at the increase of the element w_k by the modification step value Δ;

$\hat{P}_{i-1}(t_{req.})$ is the estimation of probability of a fault-free state of the system at the moment $t_{req.}$ at the previous step of optimization;

c_{ik} is the estimation coifficient of the cost of modification of the k-th element at the i-th step (increasing with the increment of average time of reliable operation).

Convergence of a search algorithm that means the penetration of parameter vector estimation at the last step of the algorithm to the permissible area of their optimal values can be provided under respective accuracy of determination of the vector μ_i (10), which is defined by the estimation accuracy of probability of a fault-free state of the system.

Requirements for the accuracy of probability estimation can be determinated by a statistical model on the basis of analytical solution available for the reliability diagram shown in Figure 1 [3]:

$$\begin{aligned} P(t) = {} & P_3(t)\cdot P_6(t) + \left(P_1(t) + P_2(t) - P_1(t)\cdot P_2(t)\right)\cdot \\ & \cdot\{P_4(t)\cdot[(1-P_3(t))\cdot P_6(t) + (1-P_6(t))\cdot P_7(t)] + \\ & +(1-P_3(t)\cdot P_6(t))\cdot(1-P_4(t))\cdot P_5(t)\cdot P_7(t)\} \end{aligned} \tag{11}$$

Analitical reliability model (11) used as the object of a search algorithm excludes stochastic character of the constituents used to calculate derivative (10), but it provides unconditional convergence of a search algorithm. The obtained values of the reliability model parameter vector bringing the minimum of (8) are the basis for the estimation of convergence of the search algorithm of optimization, built above the statistical reliability model. Besides, operating with an analytical reliability model helps to estimate an acceptable level of determination errors μ_{ki} (10) providing with the required parameters of convergence of the statistical algorithm. In this regard (10) is extended with a summand simulating an error of calculation of the vector of the derivative μ:

$$\mu_{ki\text{ш}} = \mu_{ki}\cdot(1 + \sigma_{\text{ш}}\cdot\xi_{nki}), \tag{12}$$

where ξ_{nki} is a mumber from the sample of normal random numbers with a zero mathematical expectation and unit variance;

$\sigma_{\text{ш}}$ is a specified rate (SD) of the introduced relative error of the calculation of the derivative vector components.

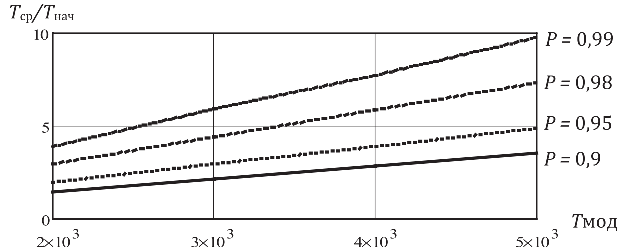

Fig. 4 Dependency of the required level of the system reliability on the specified duration and probability of reliable operation

Table 1. Results of calculation of optimal parameters of reliability of the elements ensuring reliability of the system and initial data for the determination of allowable errors in calculation of derivatives.

No. of element	$T = 2000$			$T = 3000$			$T = 4000$			$T = 5000$		
	$P = 0{,}9$	$P = 0{,}95$	$P = 0{,}99$	$P = 0{,}9$	$P = 0{,}95$	$P = 0{,}99$	$P = 0{,}9$	$P = 0{,}95$	$P = 0{,}99$	$P = 0{,}9$	$P = 0{,}95$	$P = 0{,}99$
1	1,29	1,7	3,18	1,86	2,51	4,82	2,44	3,32	6,33	3,03	4,14	7,97
2	1,29	1,7	3,18	1,86	2,51	4,82	2,44	3,32	6,33	3,03	4,14	7,97
3	1,42	1,98	3,84	2,2	3,03	5,88	3	4,09	7,74	3,81	5,15	9,77
4	1,39	1,89	3,59	2,08	2,84	5,46	2,77	3,79	7,18	3,47	4,73	9,05
5	1,25	1,62	3	1,74	2,33	4,52	2,24	3,06	5,92	2,75	3,79	7,45
6	1,76	2,57	5,47	2,75	3,93	8,35	3,73	5,27	10,95	4,72	6,63	13,82
7	1,61	2,32	4,95	2,4	3,43	7,51	3,16	4,53	9,81	3,9	5,63	12,35
T_{av}	1,43	1,97	3,89	2,13	2,94	5,91	2,83	3,91	7,75	3,53	4,89	9,77
Maximum deviation of parameter from an optimal value, %												
$\Sigma_n = 30$ %	4,6	3,3	4,3	4,1	4,9	1,1	4,6	4,9	4,3	5,0	4,2	3,4
$\Sigma_n = 50$ %	10,1	10,4	9,3	12,5	13,9	6,6	3,9	8,2	7,7	12	10,5	5,4
Minimum values of components of the vector of derivatives μ_k (*10^{-3})												
μ_k min	3,8	1,86	0,31	3,35	1,52	0,2	3,84	1,27	0,15	2,63	0,97	0,12

The value σ_n leading, for instance, to not more than 5 % level of error of determination of the reliability model parameters in relation to their optimal values defines the requirements for the accuracy of estimations made to check the probability of fault-free state of the system.

Figure 4 and Table 1 show the dependency of the required level of the system reliability (relative to the value $T_{init} = 5000$ h) on the specified duration and probability of reliable operation, obtained with an analytical model of optimization.

Table 1 also represents the results of estimation of the effect of noise contamination of the derivative vector in accordance with (12) on the results of parameter estimation by a search algorithm.

Analysis of the results listed in Table 1 shows that the acceptable relative level of error of estimation of the derivative components (10) made with a statistical reliability model leading to not more than 5 % of deviations in the determination of the reliability model parameters is the level $\sigma_n = 30$ %.

Absolute level of estimation error is defined from the formula

$$\sigma_{\Delta P} = \mu_{kmin} * \sigma_n. \quad (13)$$

Keeping in mind that SD of the constituents of (10) $\hat{P}^*_{i-1,w_k}(t_{req.})$ and $\hat{P}_{i-1}(t_{req.})$ are practically equal, the acceptable SD value of error μ_k shall exceed acceptable values $\sigma_{\Delta P}(t)$ approximately by 1,4.

Figure 5 shows the graph $\sigma_{\Delta Pad}(P)$ obtained with an analytical model based on the allowable value σ_n. with consideration of the μ_k behavior depending on the estimation of probability of reliable operation of the system, and its approximation built on the basis of (5) on the probability interval $P \geq 0.75$and the polynomial aP^2+bP+c (a= -1.25; b=1; c= 0.17) on the interval $P < 0.75$, providing the required accuracy within the probability interval $P \leq 0.99$.

The formula for approximation of dependency $\sigma_{\Delta Pad}(P)$ on the probability interval $P \geq 0.75$ is as follows:

$$\sigma_{\Delta Pad}(P) = \sigma_S \cdot (1-P) \cdot 10^{-2}. \quad (14)$$

Based on (14) and (4), the formula for a stepped determination of number of the experiments N_{reqi} required to ensure the convergence of a search algorithm operating with a statistical model on the interval $P \geq 0.75$ is as follows:

$$N_{reqi} = 10^4/(1 - \hat{P}_i) \quad (15)$$

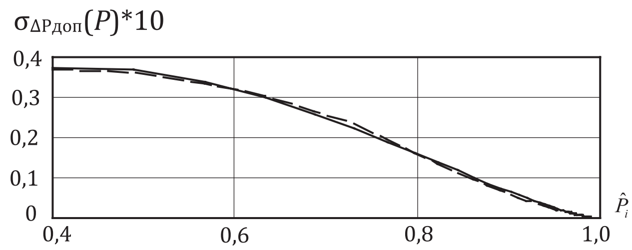

Fig. 5 Dependency of the value of an allowable error in the estimation of reliability probability (full line) on the estimation of probability of reliable operation of the system and its approximation (dashed line)

The executed estimation of optimal parameters of the system, whose reliability model corresponds to Figure 1, using a statistical model corresponding to formula (1) showed that at the determination of N_{req} in accordance with (15) there is a convergence of a search algorithm and estimation of the system parameters with the accuracy not lower than ±5 %. In the experiment carried out, an average error of parameter determination in relation to the results listed in Table 1 is 3,03 %. The obtained formulas are determined by the characteristics of the state stream and they do not depend on complexity of the system reliabilty diagram.

The values listed in the table show the minimum reauired value of average time of reliable operation of the diagram elements (Figure 1) to ensure the selected reliability requirements (in relation to initial values T_{in} = 5000 h). The results of optimization algorithm generally comply with the results expected based on the location of elements in the reliability diagram. Maximum deviations of optimal values of average time of reliable operation of the elements in relation to their average values (T_{av}) are 21 – 41 % depending on time requirements and probability of the system reliable operation. It shows economic and technical viability of determination of optimal (minimum required) values of the system reliability parameters at the design stage. In case there are no elements with design characteristics of reliability, the required reliability of the system can be ensured either by special technical redundancy measures and (or) in case of operational and technical feasibility by the creation of the system of technical maintenance and repair.

Application of the offered approach makes it possible at the design stage to define the parameter values of the elements in the scope of reliability diagrams of complex systems served to ensure optimal fulfillment of requirements for the system reliability. The obtained formulas to define the convergence parameters are the basis for computational stability and efficiency of an optimization algorithm operating with a statistical model of the reliability diagram.

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