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About trigonometric distributions for the description of failures of technical devices

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Abstract. *The purpose of this article is to offer and examine nonconventional trigonometric distributions in order to describe degradation failures of technical devices. Two methods for approximate description of reliability indices are proposed for an estimated value of mean time to failure. Firstly, as the parameter of a failure flow with operation time equal to mean time to failure tends to its stationary value equal to the opposite value of mean time to failure, it is offered to approximate the dependence of the parameter of a failure flow of the operation time by a piecewise linear function. Other reliability indices are defined using the Laplace transformation. For instance, the probability of reliable operation can be described by the cosine function, and the failure rate – by the tangent function. Secondly it is proposed to approximate the dependence of the density of failures distribution depending on the operation time by the sine function. Other reliability indices are defined using the Laplace transformation. For instance, the probability of reliable operation can be described by the function of squared cosine, and the failure rate – by the double tangent function. As a result of studies it has been concluded that since a failure rate of the offered distributions increases as the operation time increases, and the coefficient of variation is less than one, they can be used to describe degradation failures of technical devices. The obtained results have shown that reliability indices at these distributions are expressed by elementary functions and it can simplify the calculation of reliability indices of systems with different connections of their constituent elements.*

Keywords: *probability of reliable operation, failure rate, cosine distribution, the distribution of the cosine of the square.*

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Postulates

At present the reliability indices of technical devices (TD) are normally calculated presuming a constant rate of failure of their constituent elements. It corresponds with the case when the elements are affected only by sudden failures caused by external influences. Degradation failures of elements, related to internal processes of wear and ageing, are not taken into consideration. The latter does not correspond with reality. For instance, [1, 2] contain detailed descriptions of degradation processes causing wear and ageing of the elements of railway power supply, signalling and communication systems. Such processes lead to degradation failures of elements and are described in the reliability theory by a distribution class with an increasing failure rate function, i.e. IFR-distributions [3]. Let us call the elements with degradation failures the elements of an ageing type.

By means of acquisition and processing of information about failures recovered during the operation of the indicated system elements, only the estimations of constant parameter values of a failure flow щ [1] or mean times to failure as *Ф* = 1/щ [2] are obtained. However, it does not mean that the failure rate of the ageing type elements is a constant value.

According to the definition, the parameter of a failure flow is a ratio of the number of items failed in the time interval $n(dt)$ to the number of items under test in this interval dt provided that the failed items are replaced by the operable (new or repaired) ones, i.e. $\omega(t) = n(dt)/Ndt$, where N is the number of items under test which remains constant. From the reliability theory we know that the parameter of a failure flow at any distribution during operation tends to the stationary value equal to $\omega = 1/T$. It becomes apparent at the acquisition of statistics about failures of TD under actual operating conditions.

According to the definition, a failure rate is a ratio of the number of items failed in the time interval $n(dt)$ to the average number of the items N_{av}, operating without fail in this time interval dt, i.e. $\lambda(t) = n(dt)/N_{av}dt$. Due to failures of items, N_{av} decreases with each interval, however $\lambda(t)$ of the ageing type elements increases.

As the laws of distribution of mean time to failure of the ageing type elements are normally not known, the task of calculation of TD reliability indices has to be solved in the context of uncertainty.

The purpose of the paper is to offer and examine nonconventional trigonometric distributions in order to describe degradation failures of technical devices.

When only the value of mean time to failure can be estimated, for example, from the expression $T = 1/\omega$, then two methods can be offered for an approximate description of TD reliability indices.

Cosine distribution

Firstly, as the parameter of a failure flow with $t = T$ tends to its stationary value equal to $1/T$, it is proposed to approximate the dependence $\omega(t)$ by a piecewise linear function of the following type [4]:

$$\text{with } t < T\ \omega(t) = t/T^2;\ \text{with } t \geq T\ \omega(t) = 1/T. \tag{1}$$

Other indices are defined using the Laplace transformation. The distribution density $f(t)$ shall be found from the equation connecting it in an operator form with the parameter of a failure flow $f(s) = \omega(s)/(1+\omega(s))$ as

$$f(t) = (1/T)\sin(t/T). \tag{2}$$

Then, the probability of reliable operation $P(t)$ and the failure rate $\lambda(t)$ shall be defined from the equations:

$$P(t) = 1 - \int_0^t f(t)dt = \cos(t/T), \tag{3}$$

$$\lambda(t) = f(t)/P(t) = (1/T)\mathrm{tg}(t/T). \tag{4}$$

The argument t/T in the formulas used to define reliability indices is measured in radians. Let us call the obtained distribution the cosine distribution whose range of definition lies within the interval $0 < t/T < \pi/2$.

Improper integral of distribution density within the range of distribution definition as per [5] shall be equal to one. Let us check

$$\int_0^{\pi T/2} (1/T)\sin(t/T)dt = 1.$$

Coefficient of variation of distribution is defined from the equation

$$V = \mu_2^{0,5}/\mu_1, \tag{5}$$

where μ_1 is the first initial moment;
μ_2 is the second central moment;

$$\mu_1 = \int_0^{\pi T/2} tf(t)dt = \int_0^{\pi T/2} (t/T)\sin(t/T)dt = T,$$

$$\mu_2 = \int_0^{\pi T/2} (t-\mu_1)^2 f(t)dt =$$

$$= \int_0^{\pi T/2} (t-T)^2 (t/T)\sin(t/T)dt = (\pi - 3)T^2.$$

Putting the values μ_1 and μ_2 into equation (5), we shall get $V = 0,376$.

As a failure rate of this distribution in accordance with (4) is a monotonously increasing function of time, and the value of the coefficient of variation is less than one, it refers to the class of IFR-distributions and can be used to describe degradation failures of technical devices. Article [4] contains the definitions of asymmetry and kurtosis coefficients and notes that the cosine distribution can be represented in the Pearson's range by a point with the coordinates $p^2 = 0,18$ и $\beta = 2,23$. It is shown that according to [3] a cosine function is also the distribution with an increasing mean failure rate, the distribution of a "new is better than the used" type and the distribution of a "new is averagely better than the used" type.

Equations (1), (2), (3) and (4) were used to define the dependences of reliability indices of the relative time of operation t/T of the distribution offered. The calculation results are brought together in Table 1.

Table 1

t/T	0	0,2	0,4	0,6	0,8	1,0	1,2	1,4	$\pi/2$
$T f(t)$	0	0,20	0,39	0,56	0,72	0,84	0,93	0,98	1,0
$T \lambda(t)$	0	0,20	0,42	0,68	1,03	1,56	2,57	5,80	∞
$T \omega(t)$	0	0,20	0,40	0,60	0,80	1,0	1,0	1,0	1,0
$P(t)$	1	0,98	0,92	0,83	0,70	0,54	0,36	0,17	0

Distribution of the cosine of the square

Secondly, it is proposed to approximate the dependence of density of TD failures distribution $f(t)$ depending on the operation time t by the sine function of the following type

$$f(t) = (1/T)\sin(2t/T) \tag{6}$$

with a range of definition $0 < t < \pi T/2$.

Improper integral of distribution density within the range of distribution definition as per [5] shall be equal to one. Let us check

$$\int_0^{\pi T/2} (1/T)\sin(2t/T)dt = 1.$$

Considering that $\sin(2t/T) = 2\sin(t/T)\cos(t/T)$, let us introduce the equation (6) in the following form

$$f(t) = (2/T)\sin(t/T)\cos(t/T). \tag{6a}$$

The probability of reliable operation $P(t)$ with consideration of (6) is defined based on the equation

$$P(t) = 1 - \int_0^t f(t)dt = (1 + \cos(2t/T))/2. \tag{7}$$

Considering that $\cos(2t/T) = \cos^2(t/T) - \sin^2(t/T)$, let us introduce the equation (7) in the following form

$$P(t) = \cos^2(t/T). \tag{7a}$$

Let us call the obtained distribution the distribution of the cosine of the square.

A failure rate $\lambda(t)$ with consideration of (6a) and (7a) is defined as

$$\lambda(t) = f(t)/P(t) = (2t/T)\,\mathrm{tg}(t/T). \tag{8}$$

The parameter of a failure flow $\omega(t)$ is defined through the Laplace transformation of the following type $\omega(s) = f(s)/(1 - f(s))$ with consideration of (6) as follows

$$\omega(s) = \left(\sqrt{2}/T\right)\sin\left(\sqrt{2}\cdot t/T\right). \qquad (9)$$

The coefficient of variation of distribution is defined using the equation (5).

$$\mu_1 = \int_0^{\pi T/2} tf(t)dt = \int_0^{\pi T/2} (t/T)\sin(2t/T)dt = \pi T/4,$$

$$\mu_2 = \int_0^{\pi T/2} (t-\mu_1)^2 f(t)dt =$$

$$\int_0^{\pi T/2} \left(t - \frac{\pi T}{4}\right)^2 \sin(2t/T)dt = \left(\frac{\pi^2}{16} - 0,5\right)T^2.$$

Putting the values μ_1 and μ_2 into equation (5), we shall get $V = 0,435$.

Equations (6), (7a), (8) and (9) were used to define the dependences of reliability indices of the relative time of operation t/T of the distribution offered. The calculation results are brought together in Table 2 and are represented in Figures 1 and 2.

Table 2

t/T	0	0,2	0,4	0,6	$\pi/4$	1,0	1,2	1,4	$\pi/2$
$T f(t)$	0	0,39	0,72	0,93	1,0	0,91	0,68	0,33	0
$T \lambda(t)$	0	0,40	0,84	1,36	2,0	3,12	5,14	11,6	∞
$T \omega(t)$	0	0,39	0,76	1,06	1,26	1,40	1,39	1,30	1,10
$P(t)$	1,0	0,96	0,85	0,68	0,50	0,29	0,13	0,03	0

Formula (8) and Figure 1 show that a failure rate is monotonously increasing with the time of operation and, considering that the value of the variation coefficient is less than one, the offered distribution refers to the class of IFR-distributions and can be used to describe degradation failures of technical devices' elements. Figure 1 also shows that the parameter of a failure flow tends to the value equal to $1/T$. It confirms a famous provision of the reliability theory that at any distribution, the parameter of a failure flow in the course of operation tends to the steady value which is the opposite to the mean time to failure value.

As it is seen from Figure 2, the probability of TD reliable operation decreases in the course of operation, and with the value $t = \pi T/2$ it verges towards the zero value. And a curve $P(t)$ is at first up-convex, and then down-convex.

For comparison Figures 3 – 5 show the curves of distribution density, failure rate and the parameter of a failure flow of the cosine distribution (marked with blue 1) and the distribution of the cosine of the square (marked with red 2), built with the use of data from Tables 1 and 2. As

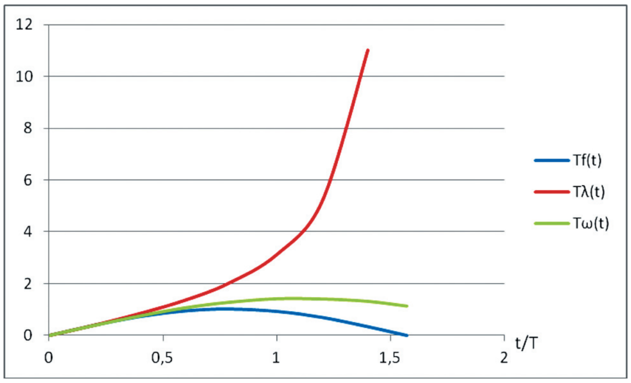

Fig. 1

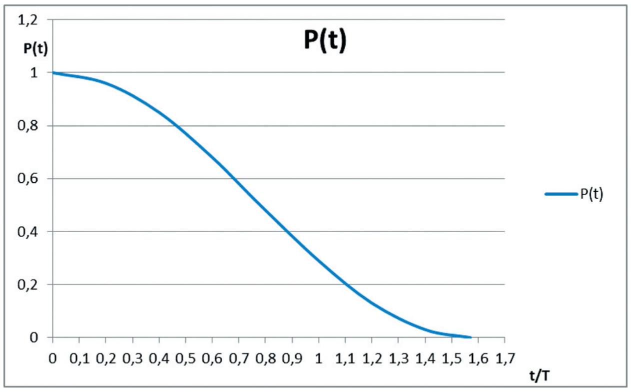

Fig. 2

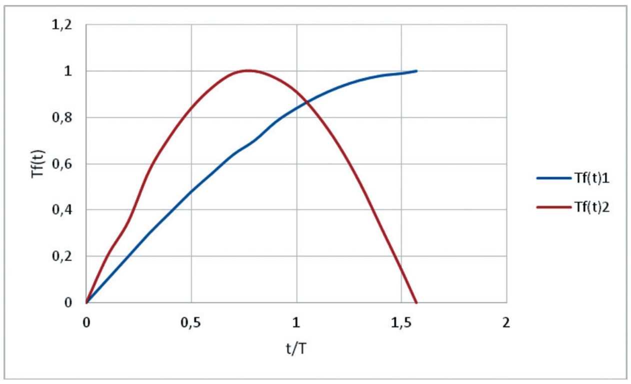

Fig. 3

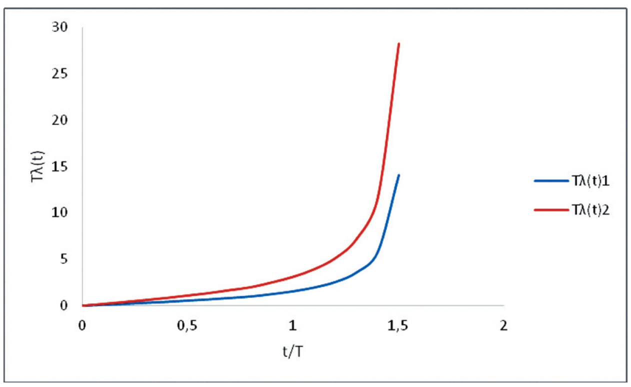

Fig. 4

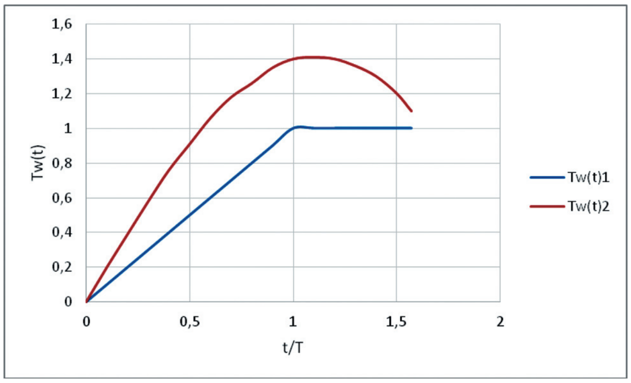

Рис. 5

we can see from formulas (4), (8) and Figure 4, a failure rate at the distribution of the cosine of the square increases in the course of operation is twice as fast as in case of the cosine distribution.

Conclusion

In the case when only mean time to failure is defined, and we know that TD elements are exposed to wear and ageing, it is reasonable to use the offered cosine distribution and distribution of cosine of the square to describe TD degradation failures in the context of uncertainty. The obtained results show that reliability indices at these distributions are expressed by elementary functions and it can simplify the calculations of reliability indices of systems with different connections of their constituent elements.

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